NAME: ______________________________

# Exam 02: Chapters 16–19

## Instructions

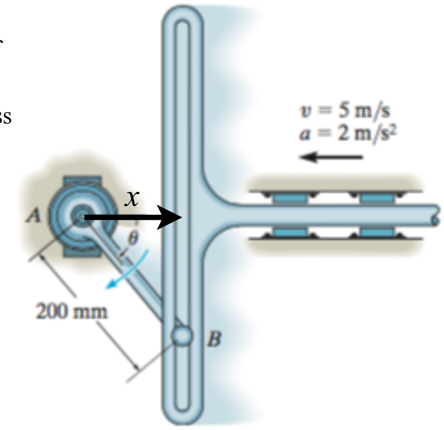
- ***Solve six of the following problems to the best of your ability.*** You have two hours in which to complete this exam.
- ***Choose one problem from each chapter, then select two additional problems to complete.*** Clearly and unambiguously note which six problems you have solved and wish to have scored. If you work on more than six problems, I will not choose for you which of them to grade
- ***You may use your calculator and your textbook.*** If you solve a system of equations using your calculator, note this on your paper. If you need to use WolframAlpha to solve a trig equation, you will be given access to a mobile device.
- ***Read and follow the directions carefully.*** Pay attention to the hints!! They are there for a reason!!
- ***Solve using the method required by the problem statement.*** If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- ***Show all your work.*** Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.

## Scoring

| Problem | Grade This? | Points | Problem | Grade This? | Points |
|---|---|---|---|---|---|
| 01: Chapter 16 | | /25 | 05: Chapter 18 | | /25 |
| 02: Chapter 16 | | /25 | 06: Chapter 18 | | /25 |
| 03: Chapter 17 | | /25 | 07: Chapter 19 | | /25 |
| 04: Chapter 17 | | /25 | 08: Chapter 19 | | /25 |

# Problem 01 Chapter 16

When $\theta = 60°$, the slotted guide rod is moving to the left with a velocity $v = -5$m/s and an acceleration $a = -2$m/s$^2$ and. Determine the angular velocity ω and angular acceleration α of the link AB at this instant.




Hint: Start with sign convention! Let to the left be negative $x$. Using A as the origin, express the horizontal (you do not need the vertical!) position of B as a function of $\theta$. Then start taking time derivatives!

$$x = r\cos\theta$$
$$\dot{x} = -r\sin\theta\,\dot{\theta}$$
$$\ddot{x} = -r[\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}]$$

$$\dot{x} = -r\sin\theta\,\dot{\theta} = v = -5\text{m/s}$$
$$(0.2\text{m})\sin 60°\,\omega = 5\text{m/s}$$
$$\dot{\theta} = \omega = 28.9\text{rad/s} \quad (+ \text{ means } \curvearrowright)$$

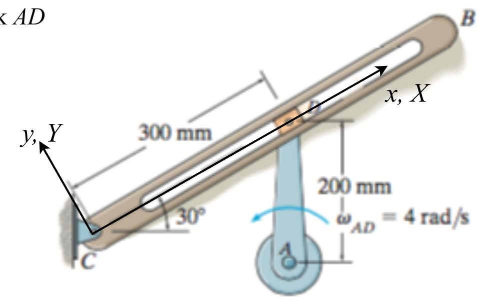
$$\ddot{x} = -r[\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}] = a = -2\text{m/s}^2$$
$$(0.2\text{m})[(28.9\text{rad/s})^2\cos 60° + \ddot{\theta}\,\sin 60°] = 2\text{m/s}^2$$
$$\ddot{\theta} = \alpha = -470\text{rad/s}^2 \quad (- \text{ means } \curvearrowleft)$$

# Problem 02 Chapter 16

Block $D$ of the mechanism is confined to move within the slot of guide $CB$ while the link $AD$ rotates with constant $\omega_{AD} = 4$ rad/s. Determine the angular velocity $\omega_C$ and angular acceleration $\alpha_C$ of CB at the instant shown.

Hint: Practically identical to 16.139!

Set up solution similar to example problems:




| Motion of Moving Reference Frame $x,y$ | Motion of D w/resp to Moving Ref Frame $x,y$ | Motion of D w/resp to Fixed Frame $X,Y$ |
|---|---|---|
| $v=0$ | $\vec{r}_{D/C} = (0.300\text{m})\,\hat{\mathbf{i}}$ | $\vec{r}_{AD} = l_{AD}(\sin 30°\,\hat{\mathbf{i}} + \cos 30°\,\hat{\mathbf{j}})$ |
| $a=0$ | $\vec{v}_{D/C} = (v_{DC})\,\hat{\mathbf{i}}$ | $\vec{v}_D = \vec{\omega}_{AD} \times \vec{r}_{AD} = [\omega_{AD}\hat{\mathbf{k}}] \times [l_{AD}(\sin 30°\,\hat{\mathbf{i}} + \cos 30°\,\hat{\mathbf{j}})]$ |
| $\Omega=\omega_C$ | $\vec{a}_{D/C} = (a_{DC})\,\hat{\mathbf{i}}$ | $\vec{a}_D = \vec{\alpha}_{AD} \times \vec{r}_{AD} - \omega_{AD}^2 \vec{r}_{AD} = 0 - [\omega_{AD} l_{AD}(\sin 30°\,\hat{\mathbf{i}} + \cos 30°\,\hat{\mathbf{j}})]$ |
| $\dot{\Omega}=\alpha_C$ | $\vec{v}_D = \vec{v}_C + \vec{\Omega} \times \vec{r}_{D/C} + \vec{v}_{D/C}$<br>$\vec{a}_D = \vec{a}_C + \dot{\vec{\Omega}} \times \vec{r}_{D/C} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{D/C}) + 2\vec{\Omega} \times \vec{v}_{D/C} + \vec{a}_{D/C}$ | |

$$\vec{v}_D = (4\text{rad/s})\,\hat{\mathbf{k}} \times [(0.200\text{m})(\sin 30°\,\hat{\mathbf{i}} + \cos 30°\,\hat{\mathbf{j}})]$$
$$\vec{v}_D = (-0.693\,\hat{\mathbf{i}} + 0.400\,\hat{\mathbf{j}})\,\text{m/s}$$
$$\vec{a}_D = -[(4\text{rad/s})^2(0.200\text{m})(\sin 30°\,\hat{\mathbf{i}} + \cos 30°\,\hat{\mathbf{j}})]$$
$$\vec{a}_D = -(1.60\,\hat{\mathbf{i}} + 2.77\,\hat{\mathbf{j}})\,\text{m/s}^2$$

$$\vec{v}_D = 0 + \omega_C\hat{\mathbf{k}} \times (0.300\text{m})\,\hat{\mathbf{i}} + v_{DC}\hat{\mathbf{i}}$$
$$\vec{v}_D = v_{DC}\hat{\mathbf{i}} - (0.300\text{m})\,\omega_C\hat{\mathbf{j}}$$
$$\vec{a}_D = 0 + \alpha_C\hat{\mathbf{k}} \times (0.300\text{m})\,\hat{\mathbf{i}} + (\omega_C\hat{\mathbf{k}}) \times (\vec{\Omega} \times \vec{r}_{D/C}) + 2(\omega_C\hat{\mathbf{k}}) \times (v_{DC}\hat{\mathbf{i}}) + (a_{DC}\hat{\mathbf{i}})$$

$$\vec{v}_D = v_{DC}\hat{\mathbf{i}} + (0.300\text{m})\,\omega_C\hat{\mathbf{j}} = (-0.693)\,\hat{\mathbf{i}} + (0.400)\,\hat{\mathbf{j}}$$
$v_{DC} = (-0.693)\,\text{m/s}$

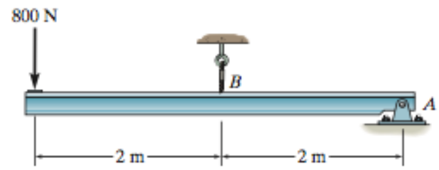
$\omega_C = 1.33\text{rad/s}$

$$\vec{a}_D = [a_{DC} - (1.33)^2(0.3)]\,\hat{\mathbf{i}} + [(0.300\text{m})\,\alpha_C - 2(1.33)(0.693)]\,\hat{\mathbf{j}} = (-1.60)\,\hat{\mathbf{i}} - (2.77)\,\hat{\mathbf{j}}$$
$a_{DC} = 2.13\text{m/s}^2$

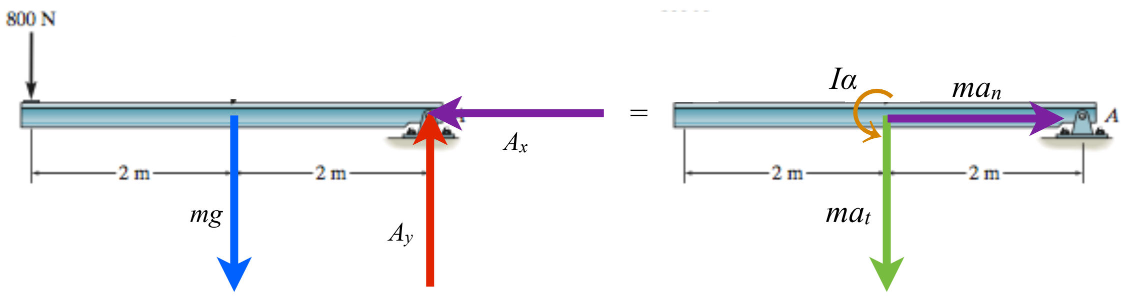
$\alpha_C = -3.08\text{rad/s}^2$

## Problem 03 Chapter 17

If the cord at *B* suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin *A*, and the angular acceleration $\alpha$ of the 120-kg beam. Treat the beam as a uniform slender rod.




Hint: *Do not over-think*; straightforward $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ and $\Sigma \boldsymbol{M} = I\boldsymbol{\alpha}$. Note that because of rotation with respect to A, you have a normal (centripetal) and tangential acceleration components (but then apply your initial condition of released from rest...).




$$\sum F_x = A_x = ma_n = m(\omega^2)\left(\frac{l}{2}\right) = 0$$

$$\sum F_y = 800\text{N} + mg - A_y = ma_t = m(\alpha)\left(\frac{l}{2}\right)$$

$$\sum M_G = (800\text{N} + A_y)\left(\frac{l}{2}\right) = I\alpha = \left(\frac{ml^2}{12}\right)\alpha$$

$$800\text{N} - A_y + mg = \left(\frac{ml}{2}\right)\alpha$$

$$800\text{N} + A_y = \left(\frac{ml}{6}\right)\alpha$$

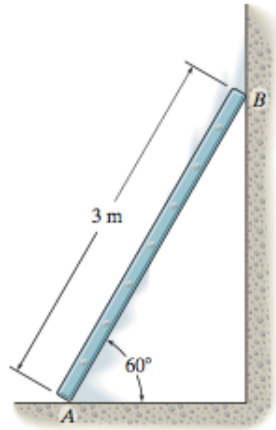
$$1600\text{N} + (120\text{kg})(9.8\text{m/s}^2) = \left[\frac{4(120\text{kg})(4\text{m})}{6}\right]\alpha$$

$$\alpha = 8.68\text{rad/s}^2$$

$$A_y = \left[\frac{(120\text{kg})(4\text{m})}{6}\right](8.68\text{rad/s}^2) - 800\text{N} = -106\text{N}$$

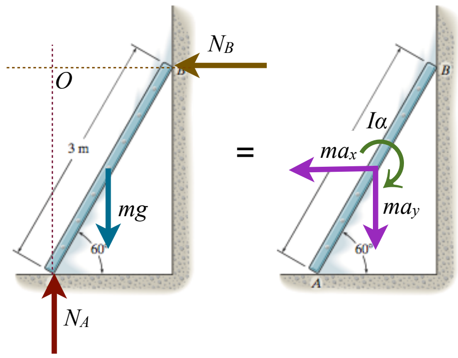
# Problem 04

Chapter 17




The slender rod has mass $m = 12$ kg. At the instant shown, $\theta = 60°$, and the rod's angular velocity is $\omega = 2$ rad/s. Determine the angular acceleration $\alpha$ of the rod, and the reaction forces at $A$ and $B$.

Hint: Free body equation!!!! Force summation is totally straightforward. Extend the lines of action of the normal forces $N_A$ and $N_B$, and where they intersect is your bet bet for summing torques. To have enough equations to make a determinate system, also use relative accelerations:
$\boldsymbol{a}_G = \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/B} - \omega^2 \boldsymbol{r}_{G/B}$ and $\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} - \omega^2 \boldsymbol{r}_{A/B}$




$$\sum F_x = N_B = ma_x$$

$$\sum F_y = mg - N_A = ma_y$$

$$(\curvearrowright +)\ \sum M_O = (mg)\left(\frac{l}{2}\right)\cos 60° = I\alpha + (ma_y)\left(\frac{l}{2}\right)\cos 60° + (ma_x)\left(\frac{l}{2}\right)\sin 60°$$

$$(mg)\left(\frac{l}{2}\right)\cos 60° = \left(\frac{ml^2}{12}\right)\alpha + (ma_y)\left(\frac{l}{2}\right)\cos 60° + (ma_x)\left(\frac{l}{2}\right)\sin 60°$$

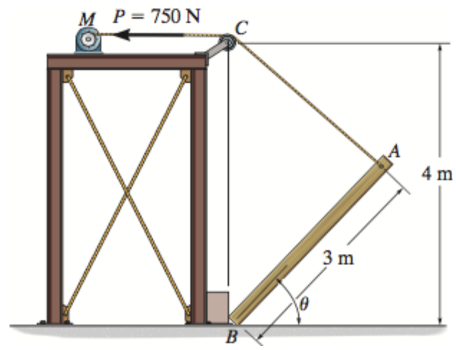
$$g\cos 60° = \left(\frac{l}{6}\right)\alpha + (a_y)\cos 60° + (a_x)\sin 60°$$

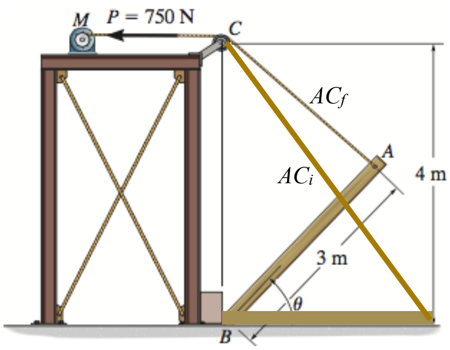
$$\vec{a}_G = \vec{a}_B + \vec{\alpha} \times \vec{r}_{G/B} - \omega^2 \vec{r}_{G/B}$$

# Problem 05 Chapter 18

Motor $M$ exerts a constant force $P = 750$ N on the rope. The post has $m = 100$-kg and is at rest when $\theta = 0°$. Use the work-energy theorem to determine the angular velocity $\omega$ of the post at the instant $\theta = 60°$.

Hint: Neglect the mass of the pulley and its size, and consider the post as a slender rod. Also, that trick with the rope length! Work done on the post by the motor = force × distance, but you have to calculate the distance as the change in length of the rope: $AC_i - AC_f$.




$AC_i = 5\text{m}$

$AC_f = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 30°}\ \text{m} = 2.05\text{m}$

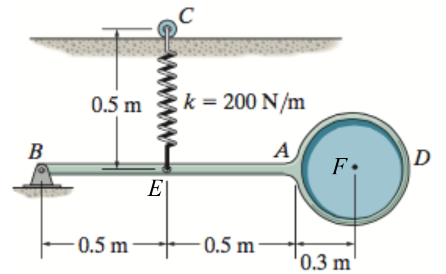
$$T_1 + \sum_{1\to 2} U = T_2$$

$$0 + P(AC_i - AC_f) - (mg)\left(\frac{l}{2}\right)\sin\theta = \frac{1}{2} I_B \omega^2 = \frac{1}{2}\left[\frac{1}{3} m l^2\right]\omega^2$$

$$(750\text{N})(5-2.05)\text{m} - (100\text{kg})(9.81\text{m/s}^2)\left(\frac{3\text{m}}{2}\right)\sin 60° = \frac{1}{2}\left(\frac{1}{3}\right)(100\text{kg})(3\text{m})^2\omega^2$$

$\omega = 2.50\text{rad/s}$

## Problem 06 Chapter 18




The pendulum consists of a slender rod ($m_{AB}$ = 6kg) fixed to a thin disk ($m_D$ = 15kg). The spring has an unstretched length $l_o$ = 0.2 m, and the pendulum is released from rest. Use conservation of energy to determine the angular velocity $\omega$ of the pendulum when it and rotates clockwise 90° from its initial position shown. The roller at C allows the spring to always remain vertical.

Hint: Just be very careful with your moments of inertia.

Let the datum for gravitational energy be at the initial position of the pendulum; when $\theta$ = 90°, gravitational potential energy will be negative!

$$T_1+V_1=T_2+V_2$$

$$0+\frac{1}{2}k(l_1-l_o)^2=\frac{1}{2}I_B\omega^2+\frac{1}{2}k(l_2-l_o)^2-(m_{AB}g)h_E-(m_Dg)h_F$$

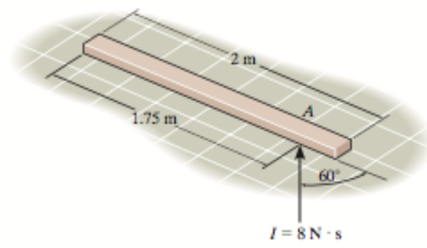
$$I_B=\frac{1}{3}m_{AB}l^2+\frac{1}{2}m_Dr^2+m_Dd_F^2=\frac{1}{3}(6\text{kg})(1\text{m})^2+\frac{1}{2}(15\text{kg})(0.3\text{m})^2+(15\text{kg})(1.3\text{m})^2=28.0\text{kg}\cdot\text{m}^2$$

$$\frac{1}{2}(200\text{N/m})(0.5\text{m}-0.2\text{m})^2=\frac{1}{2}(28.0\text{kg}\cdot\text{m}^2)\omega^2+\frac{1}{2}(200\text{N/m})(1.0\text{m}-0.2\text{m})^2-[(6\text{kg})(0.5\text{m})+(15\text{kg})(1.3\text{m})](9.81\text{m/s}^2)$$

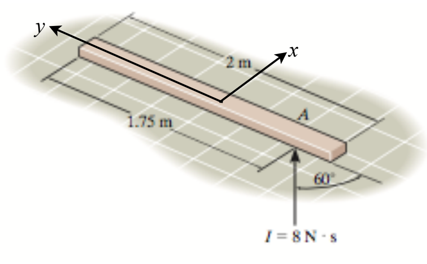
$$\omega=3.44\text{rad/s}$$

# Problem 07 Chapter 19

The slender rod ($m$ = 4 kg) is initially at rest on a smooth floor. It is kicked so as to receive a horizontal impulse $I$ = 8 N·s at point A as shown. Use the impulse-momentum theorem to determine the linear velocity and angular velocity of the mass center of the rod.




Hint: Linear velocity vector! $x$- and $y$-components!




$$mv_{xi} + I_x = mv_x$$

$$0 + (8\text{N}\cdot\text{s})\,\sin 60^\circ = (4\text{kg})\,v_x$$

$$v_x = 1.73\text{m/s}$$

$$mv_{yi} + I_x = mv_y$$

$$0 + (8\text{N}\cdot\text{s})\,\cos 60^\circ = (4\text{kg})\,v_y$$

$$v_y = 1.0\text{m/s}$$

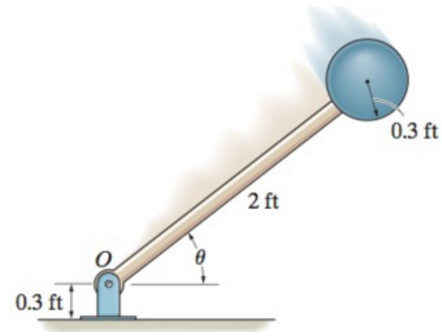
$$H_{oi} + \sum M_o\, t = H_{of} = I_o\omega = \frac{ml^2}{12}\omega$$

$$0 + (8\text{N}\cdot\text{s})\,\sin 60^\circ\,(0.75\text{m}) = \frac{(4\text{kg})\,(2\text{m})^2}{12}\omega$$

$$\omega = 3.90\text{rad/s}$$

# Problem 08

## Chapter 19




The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_i = 0°$, determine the angle $\theta_f$ of rebound after the ball strikes the wall and the pendulum swings back up to the point of momentary rest. The coefficient of restitution $e = 0.8$.

Hint: Suspiciously similar to Problem 19.47...