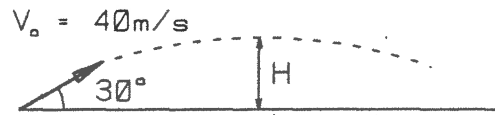


1. A projectile fired at  $30^\circ$  from the horizontal with an initial velocity of 40 meters per second will reach a maximum height  $H$  above the horizontal of:



- a.  81.5 m                      d.  24.8 m  
 b.  20.4 m                      e.  141 m  
 c.  6.2 m

$$y = -\frac{gt^2}{2} + V_0 \sin \theta \cdot t + y_0$$

$$= -\frac{9.81 \frac{\text{m}}{\text{s}^2} (2.04\text{s})^2}{2} + 40 \sin 30 (2.04)$$

$$y = 20.4 \text{ m}$$

$$V_f = -gt + V_0 \sin \theta$$

$$0 = -9.81 \frac{\text{m}}{\text{s}^2} t + 40 \sin 30$$

$$t = 2.04 \text{ s}$$

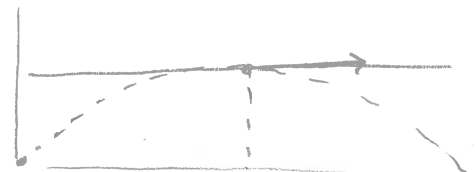
2. At the highest point on its trajectory the radius of curvature of the path of the projectile in problem 1 (above) would be:

- a.  zero.  
 b.  infinity.  
 c.  equal to the maximum elevation  $H$  (answer to problem 1).  
 d.  122 m.  
 e.  163 m.

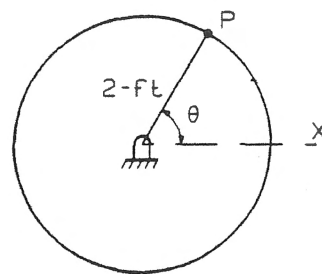
$$a_n = \frac{v^2}{r}$$

$$r = \frac{v^2}{a_n}$$

$$r = \frac{(V_0 \cos \theta)^2}{g} = \frac{(40 \frac{\text{m}}{\text{s}} \cos 30)^2}{9.81 \frac{\text{m}}{\text{s}^2}}$$



3. The gear starts from rest and the angular position of line OP is given by  $\theta = 2t^3 - 7t^2$  where  $\theta$  is in radians and  $t$  in seconds. The magnitude of the total acceleration of point P when  $t = 2$  seconds is:



- a.  38  $\text{fps}^2$                       d.  20  $\text{fps}^2$   
 b.  10  $\text{fps}^2$                       e.  19  $\text{fps}^2$   
 c.  32  $\text{fps}^2$

$$\theta = 2t^3 - 7t^2$$

$$\omega = 6t^2 - 14t$$

$$\alpha = 12t - 14 = 12(2s) - 14 = 14 \frac{\text{ft}}{\text{sec}^2}$$

4. The acceleration of a particle moving along a straight line is directly proportional to its displacement:  $a = 2s$  where  $a$  is in meters per second squared and  $s$  is in meters. If the particle has a velocity of  $+2$  m/s as it passes through the origin, its velocity at  $s = 4$  m will be:

- a.  18 m/s                      d.  4.5 m/s  
 b.  4.0 m/s                      e.  6 m/s  
 c.  3.5 m/s

$$a = 2s$$

$$v_0 = 2 \frac{\text{m}}{\text{s}}$$

$$s_0 = 0 \text{ m}$$

$$v^2 + v_0^2 + 2a(s - s_0), \quad a = 2 \cdot s$$

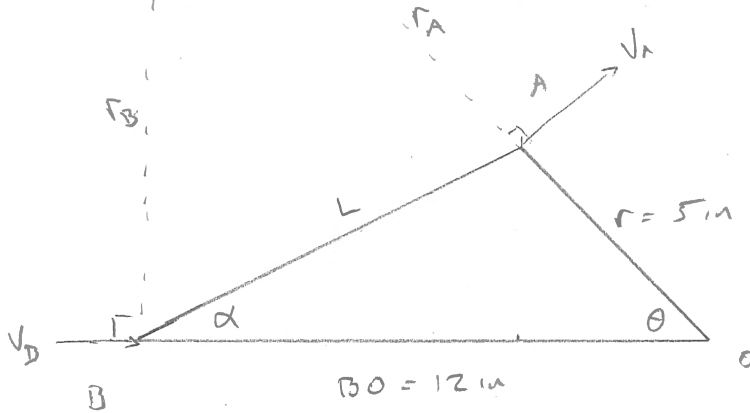
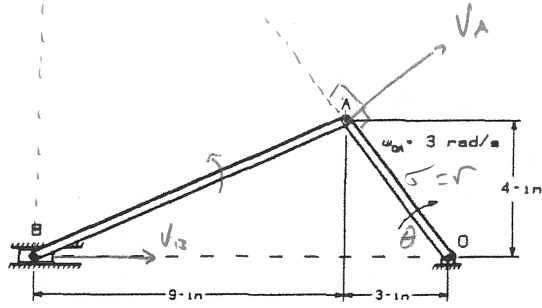
$$v^2 + v_0^2 + 2 \cdot 2(s - s_0)$$

$$v^2 + \left(2 \frac{\text{m}}{\text{s}}\right)^2 + 4(4\text{m} - 0)$$

$$v = 4.47 \frac{\text{m}}{\text{s}}$$

5. Member OA has a constant angular velocity of 3 radians per seconds clockwise. For the position shown B is moving to the right with a velocity of:

- a.  12 ips.
- b.  9 ips.
- c.  16 ips.
- d.  15 ips.
- e.  zero (it is instantaneously at rest).



$$OB = 12 \text{ in}$$

$$\theta = \tan^{-1} \frac{4}{3} = 53.13^\circ$$

$$V_A = r_A \omega = (5 \text{ in}) 3 \frac{\text{rad}}{\text{sec}} = 15 \frac{\text{in}}{\text{sec}}$$

$$V_B = r_B \omega$$

$$\omega = \frac{V_A}{r_A} = \frac{V_B}{r_B}$$

$$\cos \theta = \frac{12}{r_A + r}$$

$$r_A = \frac{12}{\cos \theta} - r = 15 \text{ in}$$

$$\tan \theta = \frac{r_B}{12}$$

$$r_B = 12 \tan \theta = 16 \text{ in}$$

$$V_A = V_B \cdot \frac{OB - r \cos \theta}{OB \sin \theta}$$

$$15 \frac{\text{in}}{\text{sec}} = V_B \cdot \frac{12 \text{ in} - (5 \text{ in}) \cos 53.13}{(12 \text{ in}) \sin 53.13}$$

$$V_B = 16 \frac{\text{in}}{\text{sec}}$$

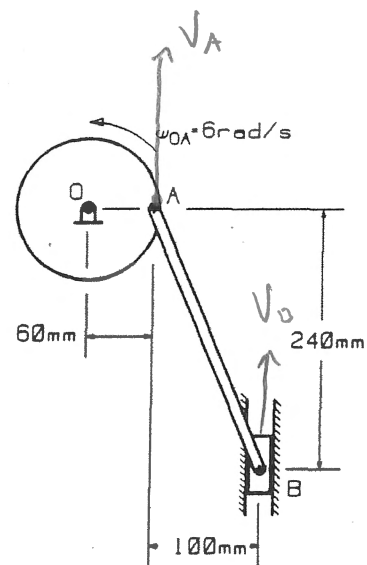
$$\frac{V_A}{r_A} = \frac{V_B}{r_B}$$

$$\frac{15 \frac{\text{in}}{\text{sec}}}{15 \text{ in}} = \frac{V_B}{16 \text{ in}}$$

$$V_B = 16 \text{ in}$$

6. Wheel OA is rotating counterclockwise with a constant angular velocity of 6 radians per second. At the instant shown the angular velocity of member AB is zero, and the angular acceleration of AB is:

- zero.
- $9 \text{ rad/s}^2$  counterclockwise.
- $9 \text{ rad/s}^2$  clockwise.
- $6 \text{ rad/s}^2$  counterclockwise.
- $8.3 \text{ rad/s}^2$  clockwise.



VELOCITIES ARE PARALLEL

$$\therefore \omega_{AB} = 0$$

NO INSTANTANEOUS CENTER



7. The 10 pound ball is supported by a cord and is swinging in the vertical plane. At the instant shown the velocity of the ball is 3 fps, and the tension in the cord is:

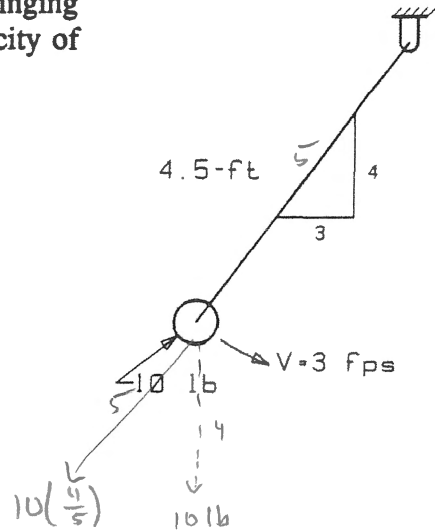
- a.  28.0 lb.  
 b.  8.0 lb.  
 c.  6.0 lb.  
 d.  10.0 lb.  
 e.  8.6 lb.

$$T - 10\left(\frac{4}{5}\right) = ma$$

$$T - 10\left(\frac{4}{5}\right) = M \frac{V^2}{r}$$

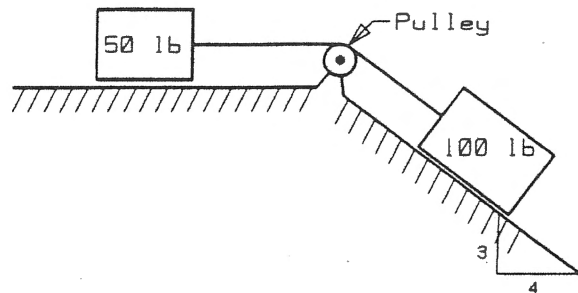
$$T = 10 \text{ lb} \left( \frac{3^2}{4.5} \right) + 10\left(\frac{4}{5}\right)$$

$$T = 10 \text{ lb}$$



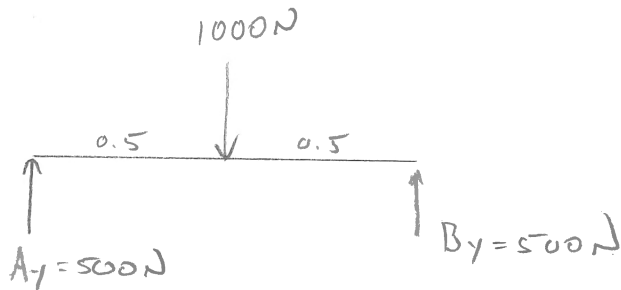
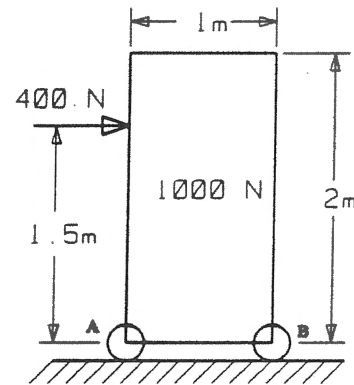
8. The two bodies shown move on frictionless planes and are connected by a flexible cord. The tensile load in the cord is:

- a.  20 lb.  
 b.  50 lb.  
 c.  60 lb.  
 d.  80 lb.  
 e.  110 lb.



9. The homogeneous 1000 newton crate moves on small frictionless rollers of negligible mass. The combined normal reaction on the front rollers at B is:

- a.    400 N  
 b.    500 N  
 c.    700 N  
 d.    ~~1100 N~~  
 e.    1000 N (i.e. it is tipping).

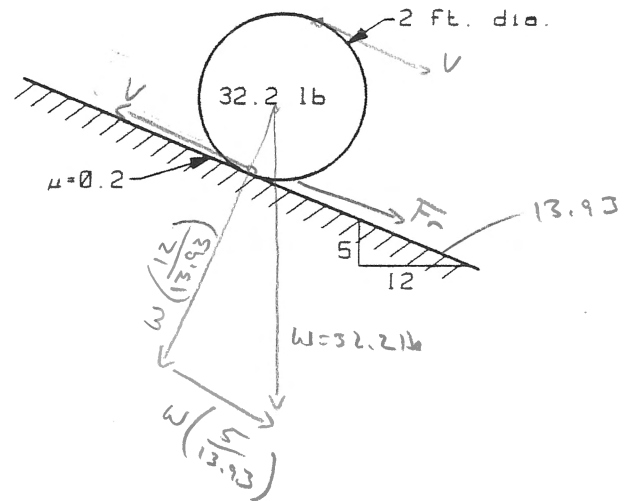


$$\hookrightarrow \sum M_A = 0 = B_y(1\text{m}) - 1000(0.5) - 400\text{N}(1.5)$$

$$B_y = 1100\text{N}$$

10. The 32.2 pound homogeneous cylinder is released from rest on the inclined plane. The angular acceleration of the cylinder after it is released will be:

- a.  13.4 rad/s<sup>2</sup>
- b.  12.4 rad/s<sup>2</sup>
- c.  3.2 rad/s<sup>2</sup>
- d.  5.9 rad/s<sup>2</sup>
- e.  8.3 rad/s<sup>2</sup>



$$F_r = M a_t$$

$$a_t = r \alpha$$

$$N = 32.2 \left( \frac{12}{13.93} \right) = 27.74 \text{ lb}$$

$$F_r = 0.2 N = 5.55 \text{ lb}$$

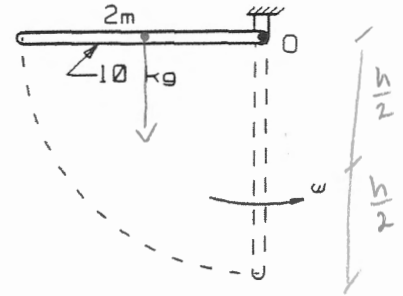
$$\sum F_x = -F_r + W \left( \frac{5}{13.93} \right) = M a_t$$

$$-5.55 + 32.2 \left( \frac{5}{13.93} \right) = M r \alpha$$

$$6.01 = 32.2(1) \alpha$$

$$\alpha = 0.19 \frac{\text{rad}}{\text{sec}^2}$$

11. A slender rod 2 meters long and having a mass of 10 kilograms is released from rest in the horizontal position. It swings counterclockwise in the vertical plane while pivoted about point 'O'. Its angular velocity as it reaches the vertical position is:



- a.  1.2 rad/s                      d.  9.8 rad/s  
 b.  7.7 rad/s                      e.  3.8 rad/s  
 c.  19.6 rad/s

$$mg h_c = \frac{1}{2} I \omega^2$$

$$mg \left( \frac{h}{2} \right) = \frac{1}{2} \left( \frac{1}{3} m r^2 \right) \omega^2$$

$$g \frac{h}{2} = \frac{1}{2} \cdot \frac{1}{3} r^2 \omega^2$$

$$9.8 \left( \frac{2}{3} \right) (1m) = \frac{1}{6} (2m)^2 \omega^2$$

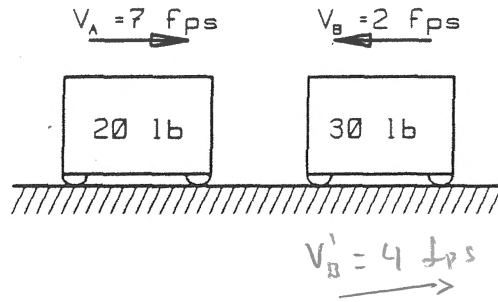
$$\omega = 3.84 \frac{\text{rad}}{\text{s}}$$

12. Two identical cylinders R and S are released simultaneously from rest at the top of two inclined planes having the same length and slope. Cylinder R rolls without slipping while cylinder S moves down a perfectly smooth plane. The two cylinders reach the bottom of their respective planes:

- a.  at the same instant.  
 b.  with the same angular velocity.  
 c.  with the same linear velocity of the mass centers.  
 d.  with the same kinetic energy.  
 e.  with none of the above.

Same Kinetic Energy

13. Carts A and B have weights and initial velocities as shown. The velocity of cart B immediately after impact is observed to be 4 ft/sec to the right. The velocity of cart A immediately after impact is:



- a.    1 fps<sup>2</sup>                      d.    5.6 fps<sup>2</sup>  
 b.    ~~X~~ 2 fps<sup>2</sup>                      e.    10 fps<sup>2</sup>  
 c.    4 fps<sup>2</sup>

$$M_1 V_1 + M_2 V_2 = M_1 V_1' + M_2 V_2'$$

$$(20)(7) + (30)(-2) = 20 V_1' + (30)(4)$$

$$V_1' = -2 \text{ fps}$$

14. The carts in problem No. 13 rebound with a coefficient of restitution of:

- a.    ~~X~~ 0.67                      d.    1.2  
 b.    0.22                      e.    2.0  
 c.    0.40

$$e = \frac{V_2' - V_1'}{V_1 - V_2} = \frac{4 - (-2)}{7 - (-2)} = 0.667$$

**DYNAMICS-1**

How many degrees of freedom does a coin rolling on the ground have?

- (A) one            (B) two            → (C) three            (D) five

2 TRANSLATIONAL

1 ROTATIONAL



**DYNAMICS-2**

What is the definition of instantaneous velocity?

- (A)  $v = dx dt$             (B)  $v = \int x dt$   
→ (C)  $v = \frac{dx}{dt}$             (D)  $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta t}{\Delta x}$

**DYNAMICS-3**

A car travels 100 km to city A in 2 h, then travels 200 km to city B in 3 h. What is the average speed of the car for the trip?

- (A) 45 km/h      (B) 58 km/h      → (C) 60 km/h      (D) 66 km/h

$$V_{AVE} = \frac{100 \text{ km} + 200 \text{ km}}{2 \text{ h} + 3 \text{ h}} = 60 \frac{\text{km}}{\text{h}}$$

**DYNAMICS-4**

The position of a particle moving along the  $x$ -axis is given by  $x(t) = t^2 - t + 8$ , where  $x$  is in units of meters, and  $t$  is in seconds. Find the velocity of the particle when  $t = 5$  s.

- (A) 9.0 m/s      (B) 10 m/s      (C) 11 m/s      (D) 12 m/s

$$V = \frac{d}{dt} \cdot x(t) = 2t - 1 \Big|_{t=5} = 9 \frac{\text{m}}{\text{s}}$$

**DYNAMICS-5**

If a particle's position is given by the expression  $x(t) = 3.4t^3 - 5.4t$  m, what is most nearly the acceleration of the particle at  $t = 5$  s?

- (A) 1.0 m/s<sup>2</sup>      (B) 3.4 m/s<sup>2</sup>      (C) 18 m/s<sup>2</sup>      → (D) 100 m/s<sup>2</sup>

$$V = \frac{d}{dt} x(t) = 10.2t^2 - 5.4$$

$$a = \frac{d}{dt} V = 20.4t \Big|_{t=5s} = 102 \frac{m}{s^2}$$

**DYNAMICS-6**

A car starts from rest and moves with a constant acceleration of 6 m/s<sup>2</sup>. What is the speed of the car after 4 s?

- (A) 18 m/s      (B) 24 m/s      (C) 35 m/s      (D) 55 m/s



**DYNAMICS-7**

A car starts from rest and has a constant acceleration of  $3 \text{ m/s}^2$ . What is the average velocity during the first 10 s of motion?

- (A) 12 m/s      (B) 13 m/s      (C) 14 m/s      → (D) 15 m/s

$$V_f = V_o + at = 0 + 3 \frac{\text{m}}{\text{s}^2} (10\text{s}) = 30 \frac{\text{m}}{\text{s}}$$

$$V_{\text{AVE}} = \frac{\Delta X}{\Delta t}$$

$$S_f = S_o + V_o t + \frac{1}{2} at^2 = 0 + 0 + \frac{1}{2} (3 \frac{\text{m}}{\text{s}^2}) (10)^2 = 150 \text{ m}$$

$$V_{\text{AVE}} = \frac{150 \text{ m}}{10 \text{ s}} = 15 \frac{\text{m}}{\text{s}}$$

**DYNAMICS-8**

A truck increases its speed uniformly from 13 km/h to 50 km/h in 25 s. What is most nearly the acceleration of the truck?

- (A) 0.22  $\text{m/s}^2$       → (B) 0.41  $\text{m/s}^2$       (C) 0.62  $\text{m/s}^2$       (D) 0.92  $\text{m/s}^2$

$$V_f = V_o + at$$

$$\left(\frac{1000\text{m}}{\text{km}}\right) \left(\frac{\text{h}}{3600\text{s}}\right) 50 \frac{\text{km}}{\text{h}} = 13 \frac{\text{km}}{\text{h}} \left(\frac{\text{h}}{3600\text{s}}\right) \left(\frac{1000\text{m}}{\text{km}}\right) + a(25\text{s})$$

$$a = 0.41 \frac{\text{m}}{\text{s}^2}$$

**DYNAMICS-9**

A bicycle moves with a constant deceleration of  $-2 \text{ m/s}^2$ . If the initial velocity of the bike is  $4.0 \text{ m/s}$ , how far does it travel in  $3 \text{ s}$ ?

- (A)  $2.0 \text{ m}$       (B)  $2.5 \text{ m}$        $\rightarrow$  (C)  $3.0 \text{ m}$       (D)  $4.0 \text{ m}$

$$S_f = S_o + V_o t + \frac{1}{2} a t^2$$

$$\Delta S = (4 \frac{\text{m}}{\text{s}})(3 \text{ s}) + \frac{1}{2} (-2 \frac{\text{m}}{\text{s}^2})(3 \text{ s})^2$$

$$\Delta S = 3 \text{ m}$$

$$V_o = 4 \frac{\text{m}}{\text{s}}$$

$$a = -2 \frac{\text{m}}{\text{s}^2}$$

$$t = 3 \text{ s}$$

**DYNAMICS-10**

A ball is dropped from a height of  $60 \text{ m}$  above ground. How long does it take to hit the ground?

- (A)  $1.3 \text{ s}$       (B)  $2.1 \text{ s}$        $\rightarrow$  (C)  $3.5 \text{ s}$       (D)  $5.5 \text{ s}$

$$y = -\frac{gt^2}{2} + V_o(\sin\theta)t + y_o$$

$$60 \text{ m} = \frac{9.81 t^2}{2}$$

$$t = 3.5 \text{ s}$$

## DYNAMICS-11

A man driving a car at 65 km/h suddenly sees an object in the road 20 m ahead. Assuming an instantaneous reaction on the driver's part, what constant deceleration is required to stop the car in this distance?

- (A) 7.1 m/s<sup>2</sup>      (B) 7.5 m/s<sup>2</sup>      (C) 8.0 m/s<sup>2</sup>      → (D) 8.1 m/s<sup>2</sup>

$$V_f^2 = V_o^2 + 2a(\Delta s)$$

$$0 = (18.06 \frac{m}{s})^2 + 2a(20m)$$

$$a = -8.15 \frac{m}{s^2}$$

$$V_f = 0$$

$$V_o = 65 \frac{km}{h}$$

$$S_f = 20m$$

$$V_o = 65 \frac{km}{h} \left( \frac{1000m}{km} \right) \left( \frac{h}{3600s} \right)$$

$$V_o = 18.06 \frac{m}{s}$$

## DYNAMICS-12

A ball is thrown vertically upward with an initial speed of 24 m/s. Most nearly how long will it take for the ball to return to the thrower?

- (A) 2.3 s      (B) 2.6 s      (C) 4.1 s      → (D) 4.9 s

$$V_f^2 = V_o^2 + 2a \Delta s$$

$$0 = (24 \frac{m}{s})^2 - 2(9.81 \frac{m}{s^2})(S_f - 0)$$

$$S_f = 29.36m$$

$$y = \frac{gt^2}{2}$$

$$29.36m = \frac{9.81 t^2}{2}$$

$$t = \sqrt{2(29.36)} = 4.84s$$

$$t (up + down) = 2t$$

$$V_o = 24 \frac{m}{s}$$

**DYNAMICS-13**

A projectile is launched upward from level ground at an angle of  $60^\circ$  from the horizontal. It has an initial velocity of 45 m/s. How long will it take before the projectile hits the ground?

$\theta = 60^\circ$   
 $V_0 = 45 \frac{m}{s}$

- (A) 4.1 s      (B) 5.8 s       $\rightarrow$  (C) 7.9 s      (D) 9.5 s

$$V_y = -gt + V_0 \sin \theta =$$

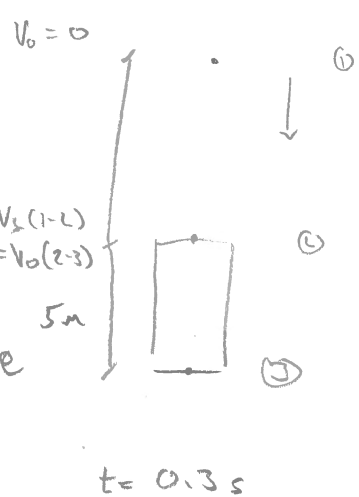
$$0 = -9.81 \frac{m}{s^2} t + 45 \frac{m}{s} \cdot \sin 60$$

$$t = 3.97 (2) = 7.95 s$$

**DYNAMICS-14**

A man standing at a 5 m tall window watches a falling ball pass by the window in 0.3 s. From approximately how high above the top of the window was the ball released from a stationary position?

- (A) 8.2 m      (B) 9.6 m      (C) 12 m      (D) 21 m



$$V = \frac{\Delta x}{\Delta t} = \frac{5m}{0.3s} = 16.67 \frac{m}{s}$$

1-2  $V_f^2 = V_0^2 + 2a \Delta s$   
 $V_f^2 = 7 + (2)(4)a$

2-3  $V_f^2 = V_0^2 + 2a \Delta s$

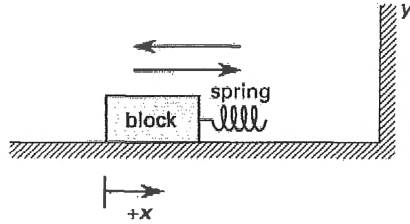
$$V_f^2 = 0 + (2)(10)a$$

$$a = \frac{V_f}{20}$$

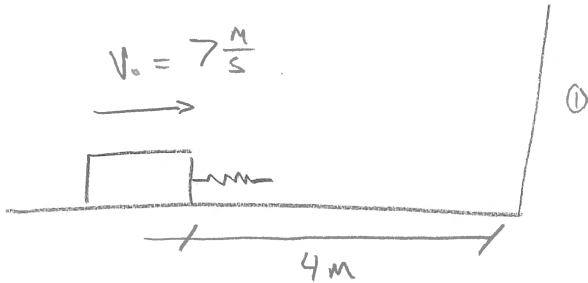
1-2  $V_f^2 = 7 + 8 \left( \frac{V_f^2}{20} \right)$

**DYNAMICS-15**

A block with a spring attached to one end slides along a rough surface with an initial velocity of 7 m/s. After it slides 4 m, it impacts a wall for 0.1 s, and then slides 10 m in the opposite direction before coming to a stop. If the block's deceleration is assumed constant and the contraction of the spring is negligible, what is the average acceleration of the block during impact with the wall?

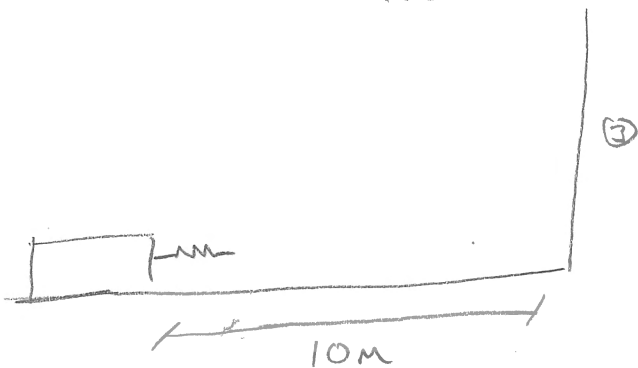
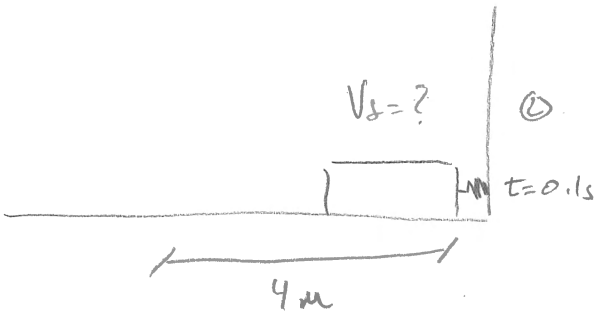


- (A)  $-120 \text{ m/s}^2$     (B)  $-100 \text{ m/s}^2$     (C)  $-99 \text{ m/s}^2$     (D)  $-49 \text{ m/s}^2$



1-2

$$V_f^2 = V_0^2 + 2a(\Delta s)$$



**DYNAMICS-17**

A train with a top speed of 75 km/h cannot accelerate or decelerate faster than  $1.2 \text{ m/s}^2$ . What is the minimum distance between two train stops in order for the train to be able to reach its top speed?

- (A) 300 m      (B) 350 m      → (C) 360 m      (D) 365 m

$$V_s = 75 \frac{\text{km}}{\text{h}} = 20.83 \frac{\text{m}}{\text{s}}$$

$$a = 1.2 \frac{\text{m}}{\text{s}^2}$$

TOP SPEED AT  
MID POINT



$$V_0 = 0$$

$$V_s = 20.83 \frac{\text{m}}{\text{s}}$$

1-2

$$V_f^2 = V_0^2 + 2a\Delta s$$

$$\left(20.83 \frac{\text{m}}{\text{s}}\right)^2 = 0 + 2\left(1.2 \frac{\text{m}}{\text{s}^2}\right)\Delta s$$

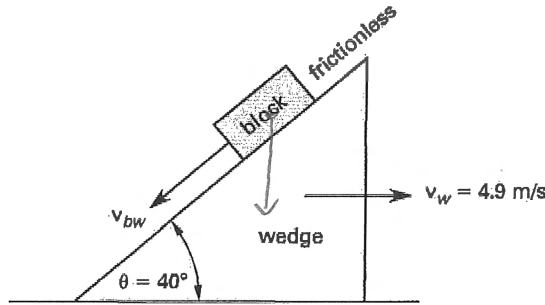
$$\Delta s = 180 \text{ m}$$

1-3 TOTAL DISTANCE

$$2 \cdot \Delta s = 361.69 \text{ m}$$

**DYNAMICS-18**

A block with a mass of 150 kg slides down a frictionless wedge with a slope of  $40^\circ$ . The wedge is moving horizontally in the opposite direction at a constant velocity of 4.9 m/s. What is most nearly the absolute speed of the block 2 s after it is released from rest?



- (A) 8.9 m/s    (B) 9.4 m/s    (C) 9.5 m/s    (D) 9.8 m/s

$$V_B = V_w + W_{B/w}$$

$$F = ma$$

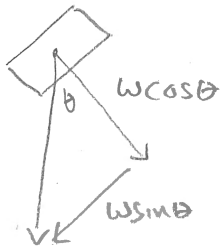
$$V_s = V_o + at$$

$$M = 150 \text{ kg}$$

$$t = 2 \text{ s}$$

$$\theta = 40^\circ$$

①



$$W = 150 \text{ kg} (9.81 \frac{\text{m}}{\text{s}^2})$$

$$= 1471.5 \text{ N}$$

$$F = ma$$

$$W \sin \theta = ma$$

$$a = \frac{mg \sin \theta}{m}$$

$$a = g \sin \theta$$

ALT

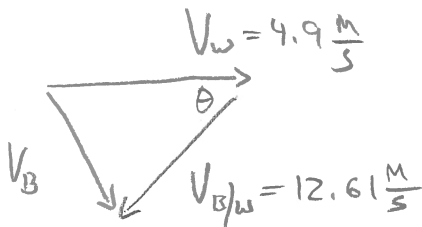
$$V(z) = \int a(z) = \int_0^z g \sin 40^\circ dt$$

$$V(z) = 12.61 \frac{\text{m}}{\text{s}}$$

$$V_{B/w} = V_o + at = 0 + g t \sin \theta$$

$$V_{B/w} = (9.81 \frac{\text{m}}{\text{s}^2}) (\sin 40^\circ) (2 \text{ s}) = 12.61 \frac{\text{m}}{\text{s}}$$

②



LAW OF COSINES

$$V_B^2 = V_w^2 + V_{B/w}^2 - 2 V_w \cdot V_{B/w} \cos \theta$$

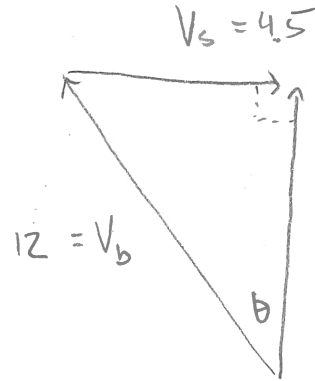
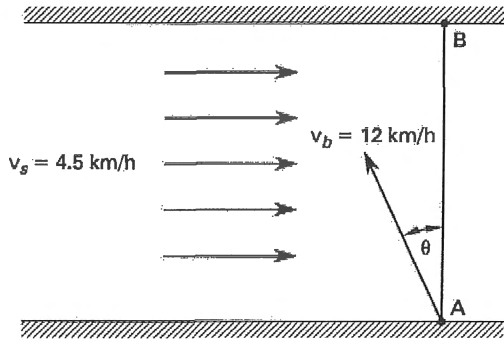
RELATIVE VELOCITY

$$V_B = V_w + V_{B/w}$$

$$V_B = 9.4 \frac{\text{m}}{\text{s}}$$

**DYNAMICS-19**

A stream flows at  $v_s = 4.5$  km/h. At what angle,  $\theta$ , upstream should a boat traveling at  $v_b = 12$  km/h be launched in order to reach the shore directly opposite the launch point?



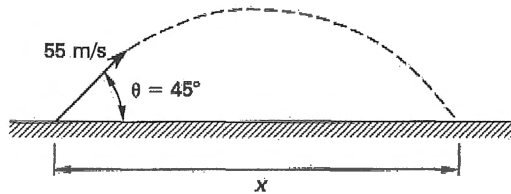
- (A) 22°      (B) 24°      (C) 26°      (D) 28°

$$\sin \theta = \frac{4.5}{12}$$

$$\theta = \sin^{-1} \frac{4.5}{12} = 22.02^\circ$$

**DYNAMICS-20**

An object is launched at  $45^\circ$  to the horizontal on level ground as shown. What is the range of the projectile if its initial velocity is 55 m/s? Neglect air resistance.



- (A) 309 m      (B) 617 m      (C) 624 m      (D) 680 m

$$V_y = V_0 - gt$$

$$0 = 55 \frac{m}{s} \cdot \sin 45 - 9.81 \frac{m}{s^2} \cdot t$$

$$t = 4s$$

$$2t = 8s$$

$$S_y = S_0 + V_0 t + \frac{1}{2} g t^2$$

$$S_y = 0 + 55(8) \cos 45 = 311 m$$

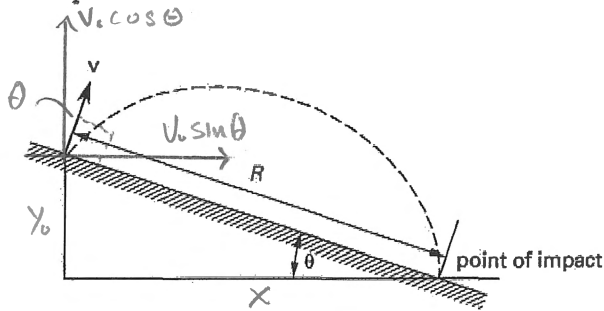
ALT

$$R = \frac{V^2 \sin(2\theta)}{g} = 308.36 m$$



## DYNAMICS-21

A projectile is fired with a velocity,  $v$ , perpendicular to a surface that is inclined at an angle,  $\theta$ , with the horizontal. Determine the expression for the distance  $R$  to the point of impact.



$$(A) R = \frac{2v^2 \sin \theta}{g \cos^2 \theta}$$

$$(B) R = \frac{2v^2 \sin \theta}{g \cos \theta}$$

$$(C) R = \frac{2v \cos \theta}{g \sin \theta}$$

$$(D) R = \frac{2v \sin \theta}{g \cos \theta}$$

$$X = X_0 + V_0 t + \frac{1}{2} a t^2$$

$$Y = Y_0 + V_0 t + \frac{1}{2} a t^2$$

$$X = 0 + V_0 t + 0$$

$$Y = R \sin \theta + V_0 t \cos \theta + \frac{1}{2} a t^2$$

$$R \cos \theta = V_0 t \sin \theta$$

$$0 = R \sin \theta + V_0 t \cos \theta + \frac{1}{2} a t^2$$

$$t = \frac{R \cos \theta}{V_0 \sin \theta}$$

$$0 = R \sin \theta + V_0 \left( \frac{R \cos \theta}{V_0 \sin \theta} \right) \cos \theta + \frac{1}{2} g \left( \frac{R \cos \theta}{V_0 \sin \theta} \right)^2$$

$$\left[ 0 = R \sin \theta + \frac{R \cos^2 \theta}{\sin \theta} + \frac{g R^2 \cos^2 \theta}{2 V_0^2 \sin^2 \theta} \right] \left( \frac{\sin \theta}{R} \right)$$

$$0 = \sin^2 \theta + \cos^2 \theta - \frac{g R \cos^2 \theta}{2 V_0^2 \sin \theta}$$

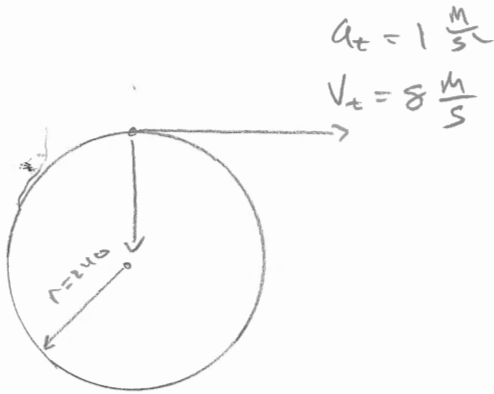
$$0 = 1 - \frac{g R \cos^2 \theta}{2 V_0^2 \sin \theta}$$

$$R = \frac{2 V_0^2 \sin \theta}{g \cos^2 \theta}$$

**DYNAMICS-22**

A cyclist on a circular track of radius  $r = 240$  m is traveling at 8 m/s. His speed in the tangential direction (i.e., the direction of his travel) increases at the rate of  $1 \text{ m/s}^2$ . What is most nearly the cyclist's total acceleration?

- (A)  $-0.9 \text{ m/s}^2$     (B)  $0.7 \text{ m/s}^2$     (C)  $0.9 \text{ m/s}^2$      $\rightarrow$  (D)  $1.0 \text{ m/s}^2$

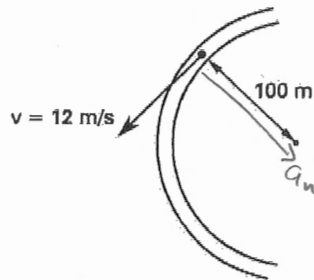


$$a_n = \frac{v_t^2}{r} = \frac{(8 \frac{\text{m}}{\text{s}})^2}{240 \text{ m}} = 0.27 \frac{\text{m}}{\text{s}^2}$$

$$\begin{aligned} a_{\text{TOT}} &= \sqrt{a_n^2 + a_t^2} \\ &= \sqrt{0.27^2 + 1^2} \\ &= 1.03 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

**DYNAMICS-23**

A motorcycle moves at a constant speed of  $v = 12 \text{ m/s}$  around a curved road of radius  $r = 100$  m. What is most nearly the magnitude and general direction of the motorcycle's acceleration?

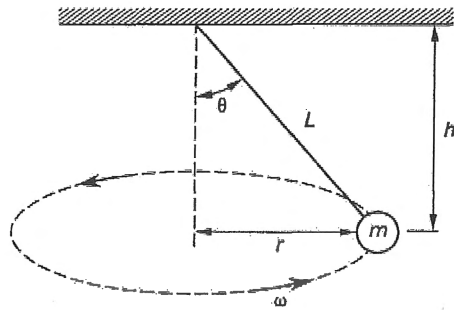


$$a_n = \frac{v^2}{r} = \frac{(12 \frac{\text{m}}{\text{s}})^2}{100 \text{ m}} = 1.44 \frac{\text{m}}{\text{s}^2}$$

- (A)  $1.1 \text{ m/s}^2$  away from the center of curvature  
 (B)  $1.1 \text{ m/s}^2$  toward the center of curvature  
 (C)  $1.4 \text{ m/s}^2$  away from the center of curvature  
 $\rightarrow$  (D)  $1.4 \text{ m/s}^2$  toward the center of curvature

**DYNAMICS-24**

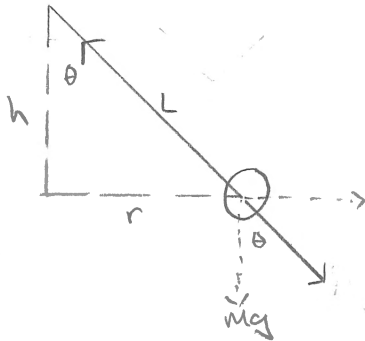
A pendulum of mass  $m$  and length  $L$  rotates about the vertical axis. If the angular velocity is  $\omega$ , determine the expression for the height  $h$ .



(A)  $h = \frac{g \cos \theta}{\omega^2} \rightarrow$  (B)  $h = \frac{g}{\omega^2}$

(C)  $h = \frac{g}{\omega^2 \cos \theta}$

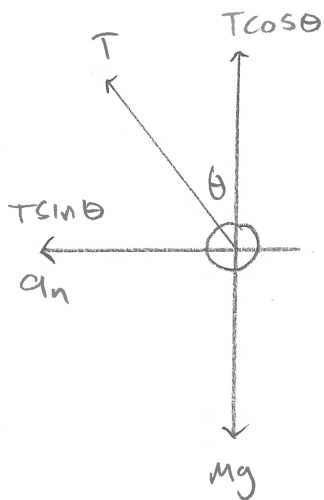
(D)  $h = \frac{gL \cos \theta}{\omega^2}$



$\cos \theta = \frac{h}{L}$

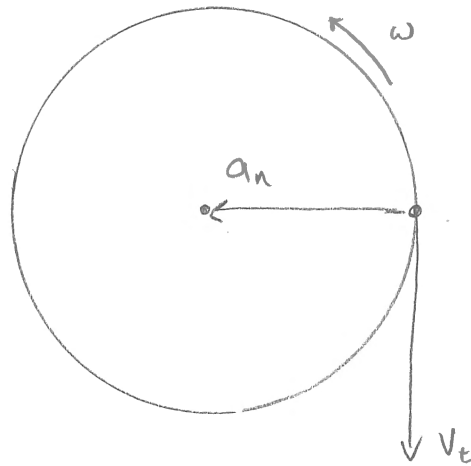
$\tan \theta = \frac{r}{h}$

③  $r = h \tan \theta$



$\Sigma F_x = -T \sin \theta = -M a_n$   
 $-T \sin \theta = -M r \omega^2$   
 ①  $T = \frac{M r \omega^2}{\sin \theta}$

$\Sigma F_y = T \cos \theta = mg$   
 ②  $T \cos \theta = mg$



① ②  
 $\frac{M r \omega^2 \cos \theta}{\sin \theta} = mg$

① ② ③

$M \omega^2 (h \tan \theta) \frac{\cos \theta}{\sin \theta} = mg$

$\omega^2 h \frac{\sin \theta \cdot \cos \theta}{\cos \theta \sin \theta} = g$

$\omega^2 h = g$

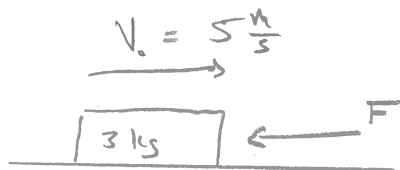
$h = \frac{g}{\omega^2}$

## DYNAMICS-25

A 3 kg block is moving at a speed of 5 m/s. The force required to bring the block to a stop in  $8 \times 10^{-4}$  seconds is most nearly

$$F = ma$$

- (A) 10 kN      (B) 13 kN      (C) 15 kN      → (D) 19 kN



$$V_f = 0 \text{ @ } t = 8(10)^{-4} \text{ s}$$

$$F = ma = 3 \text{ kg} \left( -625 \frac{m}{s^2} \right)$$

$$F = -18750 \text{ N} = -18.7 \text{ kN}$$

$$V_f = V_0 + at$$

$$0 = 5 \frac{m}{s} + 8(10)^{-4} a$$

$$a = -625 \frac{m}{s^2}$$

## DYNAMICS-26

A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is the greatest acceleration that the car can reach without breaking the rope?

- (A)  $1.2 \text{ m/s}^2$       → (B)  $2.5 \text{ m/s}^2$       (C)  $3.8 \text{ m/s}^2$       (D)  $4.5 \text{ m/s}^2$



$$F = ma$$

$$2000 \text{ N} = 800 \text{ kg} \cdot a$$

$$a = 2.5 \frac{m}{s^2}$$

**DYNAMICS-27**

A force of 15 N acts on a 16 kg body for 2 s. If the body is initially at rest, how far is it displaced by the force?

- (A) 1.1 m      (B) 1.5 m      → (C) 1.9 m      (D) 2.1 m

$$F = m \frac{\Delta v}{\Delta t} = m \left( \frac{V_f - V_0}{t} \right)$$

$$15 \text{ N} = (16 \text{ kg}) \frac{(V_f - 0)}{2 \text{ s}}$$

$$V_f = 1.88 \frac{\text{m}}{\text{s}}$$

$$\textcircled{D} \quad F = ma$$

$$a = \frac{F}{m} = \frac{15 \text{ N}}{16 \text{ kg}} = 0.94 \frac{\text{m}}{\text{s}^2}$$

$$\textcircled{B} \quad S_f = S_0 + V_0 t + \frac{1}{2} a t^2$$

$$\Delta s = \frac{1}{2} a t^2$$

$$= \frac{1}{2} (0.94 \frac{\text{m}}{\text{s}^2}) (2)^2 = 1.88 \text{ m}$$

**DYNAMICS-28**

A car of mass  $m = 150 \text{ kg}$  accelerates in 10 s from rest at a constant rate to a speed of  $v = 6 \text{ m/s}$ . What is the resultant force on the car due to this acceleration?

- (A) 75 N      → (B) 90 N      (C) 95 N      (D) 98 N

$$V_0 = 0$$

$$V_f = 6 \frac{\text{m}}{\text{s}}$$

$$t = 10 \text{ s}$$

$$V_f = V_0 + at$$

$$6 \frac{\text{m}}{\text{s}} = 0 + a (10 \text{ s})$$

$$a = 0.6 \frac{\text{m}}{\text{s}^2}$$

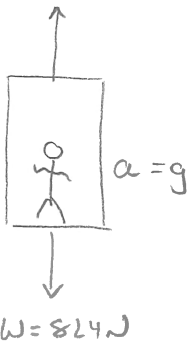
$$F = ma = (150 \text{ kg}) (0.6 \frac{\text{m}}{\text{s}^2}) = 90 \text{ N}$$

**DYNAMICS-29**

A man weighs himself twice in an elevator. When the elevator is at rest, he weighs 824 N; when the elevator starts moving upward, he weighs 932 N. Most nearly how fast is the elevator accelerating, assuming constant acceleration?

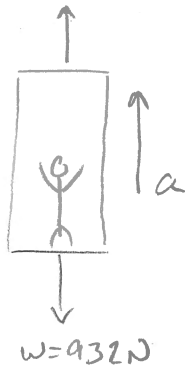
- (A) 0.64 m/s<sup>2</sup>    (B) 1.1 m/s<sup>2</sup>    → (C) 1.3 m/s<sup>2</sup>    (D) 9.8 m/s<sup>2</sup>

AT REST



$M = 84 \text{ kg}$

MOVE UP



$M = 95.01 \text{ kg}$

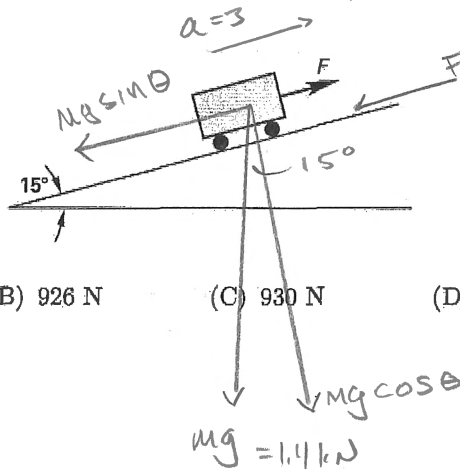
$F = ma$

$932 \text{ N} - 824 \text{ N} = 84 \text{ kg} \cdot a$

$a = 1.29 \frac{\text{m}}{\text{s}^2}$

**DYNAMICS-30**

A truck weighing 1.4 kN moves up a slope of 15°. What is the force generated by the engine if the truck is accelerating at a rate of 3 m/s<sup>2</sup>? Assume the coefficient of friction is  $\mu = 0.1$ .



$Fr = \mu N = (0.1) mg \cos \theta$   
 $= 0.1(1.4) \cos 15 = 135.23 \text{ N}$

$1400 \text{ N} = m \cdot (9.81)$

$m = 142.71 \text{ kg}$

- (A) 876 N    → (B) 926 N    (C) 930 N    (D) 958 N

$\sum F_x = F - Fr - mg \sin \theta = ma$

$F = ma + Fr + mg \sin \theta = (142.71)(3 \frac{\text{m}}{\text{s}^2}) + 135.23 \text{ N} + 1400 \sin 15$

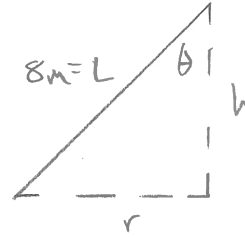
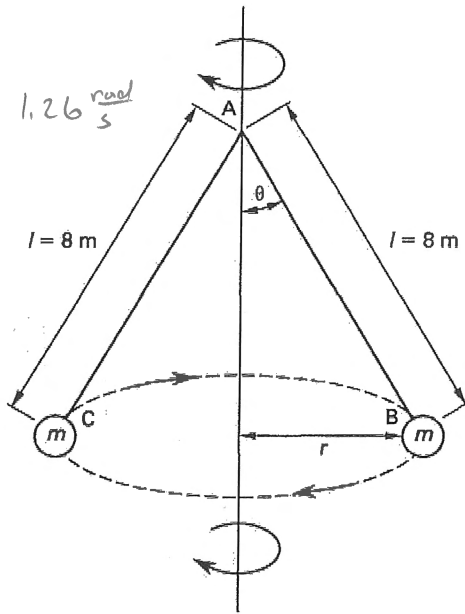
$F = 925 \text{ N}$

**DYNAMICS-32**

A simplified model of a carousel is illustrated. The 8 m long arms AB and AC attach the seats B and C, each with a mass of 200 kg, to a vertical rotating shaft. What is the maximum angle of tilt,  $\theta$ , for the seats, if the carousel operates at 12 rpm?

$$12 \frac{\text{rev}}{\text{min}} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 1.26 \frac{\text{rad}}{\text{s}}$$

$$\omega = 1.26 \frac{\text{rad}}{\text{s}}$$

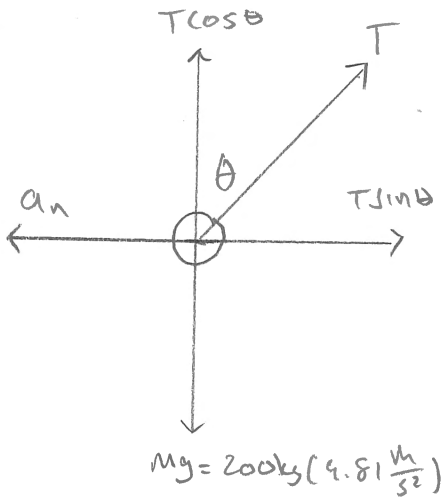


$$\sin \theta = \frac{r}{8\text{m}}$$

$$r = 8\text{m} \sin \theta$$

- (A) 39°      (B) 40°      (C) 45°      (D) 51°

$$F = ma = m r \omega^2 = m L \sin \theta \omega^2$$

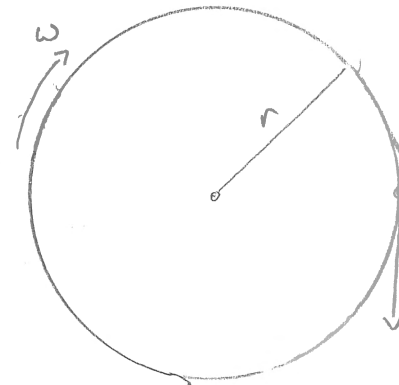


$$Mg = 200\text{kg} \left( 9.81 \frac{\text{m}}{\text{s}^2} \right)$$

$$W = 1962 \text{ N}$$

$$\sum F_x = T \cos \theta = W$$

$$T = \frac{1962 \text{ N}}{\cos \theta}$$



$$\sum F_y = T \sin \theta = M a_n = M r \omega^2$$

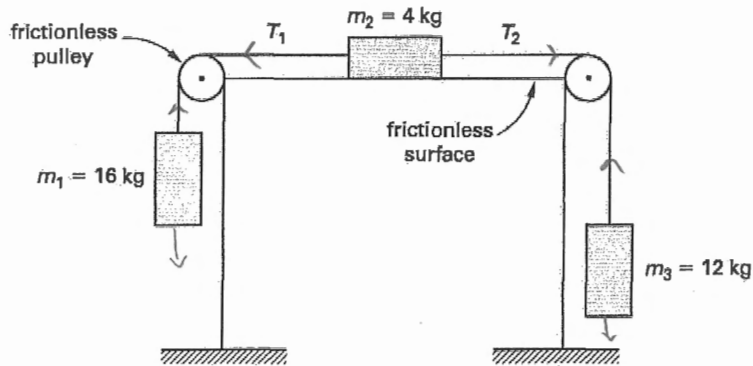
$$T \sin \theta = M r \omega^2$$

$$\frac{1962 \cdot \sin \theta}{\cos \theta} = 200\text{kg} (8 \sin \theta) \left( 1.26 \frac{\text{rad}}{\text{s}} \right)^2$$

$$\cos \theta =$$

## DYNAMICS-34

Three masses are attached by a weightless cord as shown. If mass  $m_2$  is exactly halfway between the other masses and is located at the center of the flat surface when the masses are released, what is most nearly its initial acceleration? Assume there is no friction in the system and that the pulleys have no mass.



- (A)  $1.0 \text{ m/s}^2$     (B)  $1.2 \text{ m/s}^2$     (C)  $9.8 \text{ m/s}^2$     (D)  $12 \text{ m/s}^2$

$$T_1 - T_2 = (m_1 + m_2 + m_3)a$$

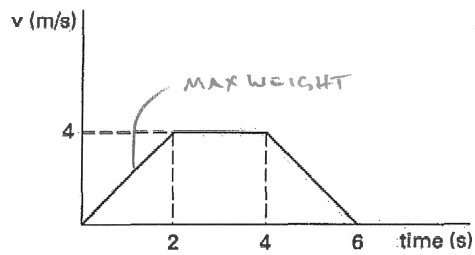
$$m_1 g - m_3 g = (m_1 + m_2 + m_3)a$$

$$a = \frac{g(m_1 - m_3)}{m_1 + m_2 + m_3} = 1.23 \frac{\text{m}}{\text{s}^2}$$



**DYNAMICS-35**

The maximum capacity (occupant load) of an elevator is 1000 N. The elevator starts from rest, and its velocity varies with time as shown in the graph. What is most nearly the maximum additional tension in the elevator cable due to the occupants at full capacity? Neglect the mass of the elevator.

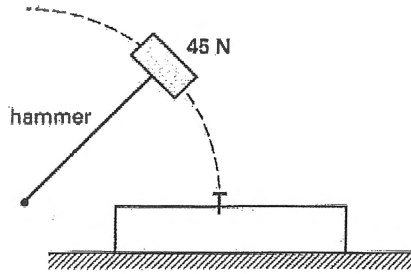


- (A) 960 N      (B) 1000 N      → (C) 1200 N      (D) 1400 N

$$\begin{aligned}
 F &= F_0 + Ma = F_0 + \frac{F}{g} \cdot \frac{\Delta v}{\Delta t} \\
 &= 1000 \text{ N} + \frac{1000 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \cdot \frac{4 \frac{\text{m}}{\text{s}}}{2 \text{ s}} = 1200 \text{ N}
 \end{aligned}$$

## DYNAMICS-37

A lead hammer weighs 45 N. In one swing of the hammer, a nail is driven 1.5 cm into a wood block. The velocity of the hammer's head at impact is 4.5 m/s. What is most nearly the average resistance of the wood block?



$$W = 45 \text{ N}$$

$$S = 1.5 \text{ cm} = 0.015 \text{ m}$$

$$V = 4.5 \frac{\text{m}}{\text{s}}$$

$$M = \frac{F}{a} = \frac{45 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = 4.59 \text{ kg}$$

- (A) 3090 N → (B) 3100 N (C) 3920 N (D) 4090 N

$$T_1 + V_1 = T_2 + V_2 + U$$

ENERGY IS NOT CONSERVED  
DUE TO THE RESISTANCE OF  
THE BLOCK

$$V_1 = U$$

$$\frac{1}{2} M V^2 = F X$$

$$\frac{1}{2} (4.59 \text{ kg}) (4.5 \frac{\text{m}}{\text{s}})^2 = F (0.015 \text{ m})$$

$$F = 3098 \text{ N}$$

## DYNAMICS-39

A 580 N man is standing on the top of a building 40 m above the ground. What is his potential energy relative to the ground?

- (A) 10 kJ      (B) 12 kJ      (C) 20 kJ      → (D) 23 kJ

$$T = mgh = 580 \text{ N} \cdot 40 \text{ m} = 23200 \text{ N}\cdot\text{m}$$

$$23200 \text{ J}$$

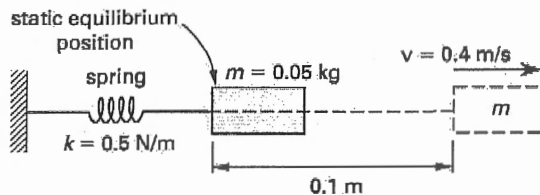
$$J = \text{N}\cdot\text{m}$$

$$W = \frac{J}{s}$$

$$N = k_s \frac{\text{m}}{\text{s}^2}$$

## DYNAMICS-41

A 0.05 kg mass attached to a spring (spring constant,  $k = 0.5 \text{ N/m}$ ) is accelerated to a velocity of 0.4 m/s. What is the total energy for the body in the following diagram? Neglect the spring mass.



$$k = 0.5 \frac{\text{N}}{\text{m}}$$

$$m = 0.05 \text{ kg}$$

$$v_f = 0.4 \frac{\text{m}}{\text{s}}$$

- (A) 0.0025 J      (B) 0.0040 J      (C) 0.0065 J      (D) 0.0092 J

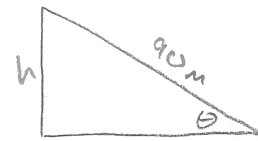
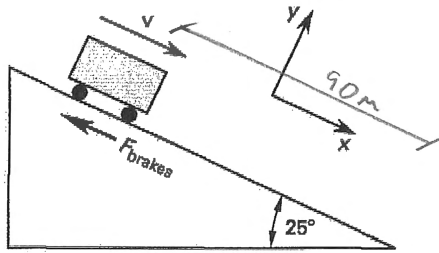
POWER/WORK AND ENERGY

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k s^2 = \frac{1}{2} (0.05 \text{ kg}) (0.4 \frac{\text{m}}{\text{s}})^2 + \frac{1}{2} (0.5 \frac{\text{N}}{\text{m}}) (0.1 \text{ m})^2$$

$$E = 0.0065 \text{ J}$$

**DYNAMICS-42**

A 1000 kg car is traveling down a  $25^\circ$  slope. At the instant that the speed is 13 m/s, the driver applies the brakes. What constant force parallel to the road must be generated by the brakes if the car is to stop in 90 m?



$$\sin \theta = \frac{h}{90}$$

$$h = 90 \sin 25^\circ$$

$$h = 38.04 \text{ m}$$

- (A) 1290 N      (B) 2900 N      → (C) 5080 N      (D) 8630 N

$$V_0 = 13 \frac{\text{m}}{\text{s}}$$

$$m = 1000 \text{ kg}$$

$$T_1 + V_1 = T_2 + V_2 + U$$

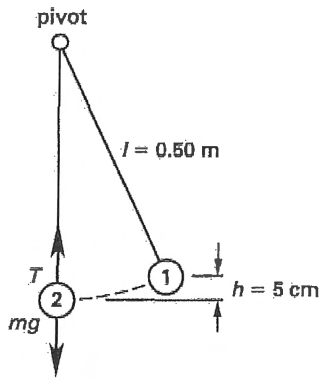
$$mgh + \frac{1}{2} m v^2 = Fx$$

$$(38.04)(1000 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) + \frac{1}{2}(1000 \text{ kg})(13 \frac{\text{m}}{\text{s}})^2 = F(90 \text{ m})$$

$$F = 5085 \text{ N}$$

**DYNAMICS-44**

A simple pendulum consists of a 100 g mass attached to a weightless cord. If the mass is moved laterally such that  $h = 5$  cm and then released, what is the maximum tension in the cord,  $T$ ?



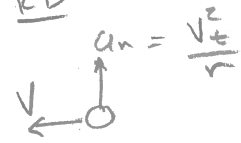
- (A) 1.08 N      (B) 1.12 N      (C) 1.18 N      (D) 1.25 N

ENERGY  
 $T = \text{KINETIC}$   
 $V = \text{POTENTIAL}$

FTBD



KD



$$m = 100g \left( \frac{kg}{1000g} \right)$$

$$h = 0.05 \text{ m}$$

$$T_1 + V_1 = T_2 + V_2$$

$$mgh = \frac{1}{2} m v^2$$

$$\sqrt{2gh} = v = \sqrt{2(9.81 \frac{m}{s^2})(0.05m)}$$

$$v = 0.99 \frac{m}{s}$$

$$F = ma_n = m \frac{v^2}{r} = T - W$$

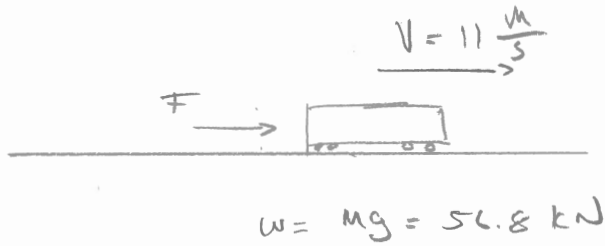
$$T = mg \frac{v^2}{r} + mg$$

$$T = 20.23 \text{ N}$$

## DYNAMICS-45

A stationary passenger car of a train is set into motion by the impact of a moving locomotive. What is the impulse delivered to the car if it has a velocity of 11 m/s immediately after the collision? The weight of the car is 56.8 kN.

- (A) 45.5 kN·s    (B) 57.5 kN·s    (C) 63.7 kN·s    (D) 64.1 kN·s



$$\text{IMPULSE} = F \cdot t$$

$$F = m \frac{dv}{dt}$$

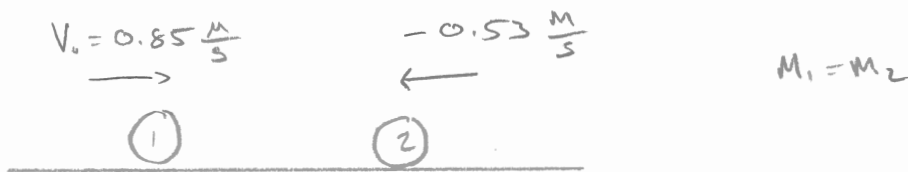
$$F dt = m dv = \frac{56.8 \text{ kN}}{9.81 \frac{\text{m}}{\text{s}^2}} \cdot 11 \frac{\text{m}}{\text{s}} = 63.69 \text{ kN} \cdot \text{s}$$

## DYNAMICS-47

Two identical balls hit head-on in a perfectly elastic collision. Given that the initial velocity of one ball is 0.85 m/s and the initial velocity of the other is -0.53 m/s, what is the relative velocity of each ball after the collision?

- (A) 0.85 m/s and -0.53 m/s  
 (B) 1.2 m/s and -0.72 m/s  
 (C) 1.2 m/s and -5.1 m/s  
 (D) 1.8 m/s and -0.98 m/s

ELASTIC COLLISION → PERFECTLY PLASTIC  $e=1$



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

$$v_1 + v_2 = v_1' + v_2'$$

$$0.85 - 0.53 = v_1' + v_2'$$

$$(v_1 - v_2)e = v_2' - v_1'$$

$$v_1' + v_2' = 0.32$$

$$-v_1' + v_2' = 1.38$$

$$v_1' + v_2' = 0.32$$

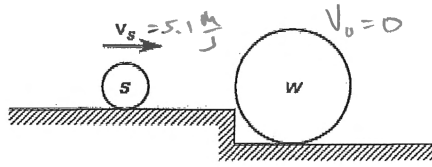
$$-v_1' + v_2' = 1.38$$

$$v_1' = -0.53 \frac{m}{s}$$

$$v_2' = 0.85 \frac{m}{s}$$

## DYNAMICS-48

A steel ball weighing 490 N strikes a stationary wooden ball weighing 490 N. If the steel ball has a velocity of 5.1 m/s at impact, what is its velocity immediately after impact? Assume the collision is central and perfectly elastic.



$$M_s = M_w$$

- (A) -5 m/s      (B) -2 m/s      (C) 0 m/s      (D) 5 m/s

$$M_1 v_1 + M_2 v_2 = M_1 v_1' + M_2 v_2'$$

$$v_1 = v_2' + v_2'$$

$$5.1 \frac{m}{s} = v_1' + v_2'$$

$$v_2' = 5.1 - v_1'$$

$$\left[ \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 = \frac{1}{2} M_1 v_1'^2 + \frac{1}{2} M_2 v_2'^2 \right] \frac{2}{M}$$

$$v_1^2 = v_1'^2 + v_2'^2$$

$$5.1^2 = v_1'^2 + v_2'^2$$

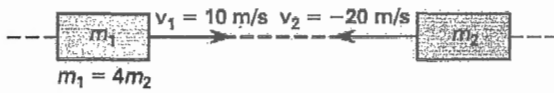
$$5.1^2 = v_1'^2 + (5.1 - v_1')^2$$

$$v_1' = 0$$



## DYNAMICS-49

Two masses collide in a perfectly inelastic collision. Given the data in the illustration, find the velocity and direction of motion of the resulting combined mass.



PERFECTLY INELASTIC

$$e = 0$$

MASSES STICK TOGETHER

- (A) The mass is stationary.  
 (B) 4 m/s to the right  
 (C) 5 m/s to the left  
 (D) 10 m/s to the right

$$M_1 = 4M_2$$

$$M_1 v_1 + M_2 v_2 = (M_1 v_1' + M_2 v_2')$$

$$M_1 v_1 + M_2 v_2 = (M_1 + M_2) v'$$

$$4M v_1 + M v_2 = (4M + M) v'$$

$$M(4v_1 + v_2) = 5M v'$$

$$4v_1 + v_2 = 5v'$$

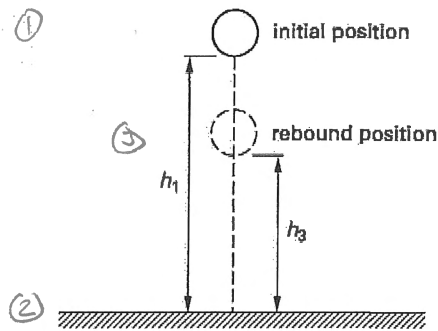
$$4\left(10 \frac{M}{s}\right) + \left(-20 \frac{M}{s}\right) = 5v'$$

$$v' = 4 \frac{M}{s}$$



## DYNAMICS-50

A ball is dropped onto a solid floor from an initial height,  $h_0$ . If the coefficient of restitution,  $e$ , is 0.90, how high will the ball rebound?



- (A)  $0.45h_1$     (B)  $0.81h_1$     (C)  $0.85h_1$     (D)  $0.90h_1$

1-2

$$mgh_1 = \frac{1}{2} m v_2^2$$

$$v_2^2 = 2gh_1$$

$$v_2 = \sqrt{2gh_1}$$

2-3

$$\frac{1}{2} m v_2'^2 = mgh_3$$

$$v_2'^2 = 2gh_3$$

$$v_2' = \sqrt{2gh_3}$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{\sqrt{2gh_3} - 0}{0 - \sqrt{2gh_1}} = 0.9$$

$$0.9 = \frac{\sqrt{2gh_3}}{\sqrt{2gh_1}}$$

$$0.9^2 = \left( \frac{\sqrt{2gh_3}}{\sqrt{2gh_1}} \right)^2$$

$$0.81 = \frac{2gh_3}{2gh_1}$$

$$h_3 = 0.81 h_1$$

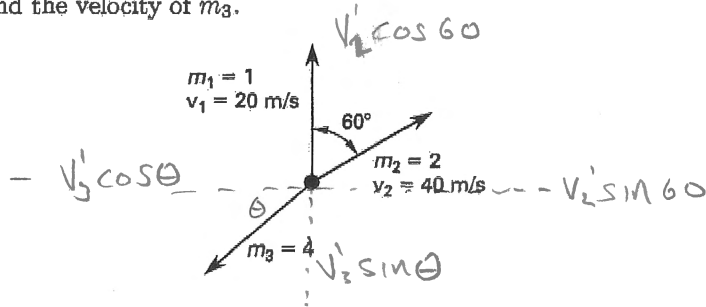
NOTE:  $e$  CANNOT BE USED TO DETERMINE HEIGHT, SUCH AS:

$$h_3 = e h_1 = 0.9 h_1$$

$e$  IS PROPORTIONAL TO VELOCITY ONLY (KINETIC ENERGY)

## DYNAMICS-51

A mass suspended in space explodes into three pieces whose masses, initial velocities, and directions are given in the illustration. All motion is within a single plane. Find the velocity of  $m_3$ .



- (A) 20 m/s → (B) 23 m/s (C) 35 m/s (D) 40 m/s

$$\sum p_x = 0 = 2(40)\sin 60 - 4(v_3)\cos \theta$$

$$v_3 \cos \theta = 17.3 \quad \cos \theta = \frac{17.3}{v_3}$$

$$\sum p_y = 0 = (1)(20) + (2)(40)\cos 60 - (4)v_3 \sin \theta$$

$$v_3 \sin \theta = 15 \quad \sin \theta = \frac{15}{v_3}$$

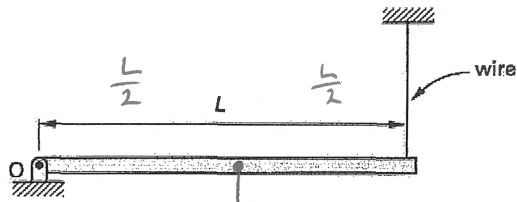
$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{15}{17.3} \cdot \frac{v_3}{v_3}$$

$$\theta = \tan^{-1} \frac{15}{17.3} = 40.43^\circ$$

$$v_3 = \frac{17.3}{\cos 40.43} = 22.9 \frac{\text{m}}{\text{s}}$$

## DYNAMICS-53

A uniform beam of weight  $W$  is supported by a pin joint and a wire. What will be the angular acceleration,  $\alpha$ , at the instant that the wire is cut?



(A)  $\frac{g}{L}$

(B)  $\frac{3g}{2L}$

(C)  $\frac{2g}{L}$

(D)  $\frac{Wg}{L}$

$$a_n = \frac{v^2}{r} = r\omega^2 = r\alpha$$

$$F = \frac{L}{2}$$

$$F = ma_n = Mr\alpha$$

$$Mg = Mr\alpha$$

$$\alpha = \frac{g}{r} = \frac{g}{\frac{L}{2}}$$

$$\alpha = \frac{2g}{L}$$

MOMENT

$$\sum M_o = I\alpha$$

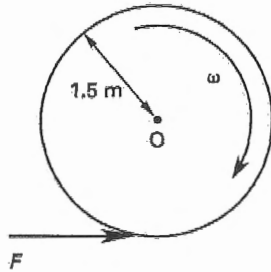
$$I = \frac{ML^2}{3}$$

$$Mg \cdot \frac{L}{2} = \frac{ML^2}{3} \alpha$$

$$\alpha = \frac{3g}{2L}$$

## DYNAMICS-54

A thin circular disk of mass 25 kg and radius 1.5 m is spinning about its axis with an angular velocity of  $\omega = 1800$  rpm. It takes 2.5 min to stop the motion by applying a constant force,  $F$ , to the edge of the disk. The force required is most nearly



- (A) 7.2 N      (B) 16 N      → (C) 24 N      (D) 32 N

$$\omega = 1800 \frac{\text{REV}}{\text{MIN}} \cdot \frac{2\pi}{\text{REV}} \cdot \frac{\text{MIN}}{60\text{s}} = 188.5 \frac{\text{rad}}{\text{s}}$$

$$M = 25 \text{ kg}$$

$$t = 2.5 \text{ MIN} \left( \frac{60\text{s}}{\text{MIN}} \right) = 150\text{s}$$

$$F = M a_n = M r \omega^2 = (25 \text{ kg})(1.5 \text{ m}) \left( 188.5 \frac{\text{rad}}{\text{s}} \right)^2 = 1332 \text{ kN}$$

NOPE

DIDNT ACCOUNT FOR TIME

$$\omega_f = \omega_0 + \alpha t$$

RIGID BODY MOMENT

$$0 = 188.5 + \alpha(150)$$

$$M = I \alpha \quad I = \frac{M r^2}{2}$$

$$\alpha = -1.25 \frac{\text{rad}}{\text{s}^2}$$

$$F \cdot r = \frac{M r^2}{2} \alpha$$

$$F = 23.44 \text{ N}$$

**DYNAMICS-55**

A mass,  $m$ , of 0.025 kg is hanging from a spring whose spring constant,  $k$ , is 0.44 N/m. If the mass is pulled down and released, what is the period of oscillation?

- (A) 0.50 s      (B) 1.2 s       $\rightarrow$  (C) 1.5 s      (D) 2.1 s

$$mg = k \delta$$

$$0.025 \text{ kg} (9.81 \frac{\text{m}}{\text{s}^2}) = 0.44 \frac{\text{N}}{\text{m}} \delta$$

$$\delta = 0.56 \text{ m}$$

$$\tau = \frac{2\pi}{\sqrt{\frac{g}{\delta}}} = \frac{2\pi}{\sqrt{\frac{9.81 \frac{\text{m}}{\text{s}^2}}{0.56 \text{ m}}}} = 1.5 \text{ s}$$

$$\tau = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{0.44}{0.025}}} = 1.5 \text{ s}$$

**DYNAMICS-56**

A body hangs from an ideal spring. What is the frequency of oscillation of the body if its mass,  $m$ , is 0.015 kg, and  $k$  is 0.5 N/m?

- (A) 0.51 Hz      (B) 0.66 Hz      (C) 0.78 Hz       $\rightarrow$  (D) 0.92 Hz

$$\tau = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{0.5 \frac{\text{N}}{\text{m}}}{0.015 \text{ kg}}} = 1.09 \text{ s}$$

$$f = \frac{1}{\tau} = 0.92 \text{ Hz}$$

**DYNAMICS-57**

What is the natural frequency,  $\omega$ , of an oscillating body whose period of oscillation is 1.8 s?

- (A) 1.8 rad/s      (B) 2.7 rad/s       $\rightarrow$  (C) 3.5 rad/s      (D) 4.2 rad/s

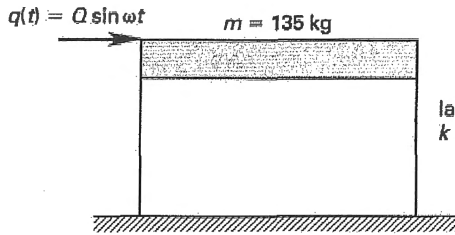
$$\tau = \frac{2\pi}{\omega}$$

$$1.8 \text{ s} = \frac{2\pi}{\omega}$$

$$\omega = 3.49 \frac{\text{rad}}{\text{s}}$$

**DYNAMICS-58**

A one-story frame is subjected to a sinusoidal forcing function  $q(t) = Q \sin \omega t$  at the transom. What is most nearly the frequency of  $q(t)$ , in hertz, if the frame is in resonance with the force?



$$K = 5 \frac{\text{kN}}{\text{cm}} \left( \frac{1000 \text{ N}}{\text{kN}} \right) \left( \frac{100 \text{ cm}}{\text{m}} \right)$$

$$K = 500000 \frac{\text{N}}{\text{m}}$$

$$M = 135 \text{ kg}$$

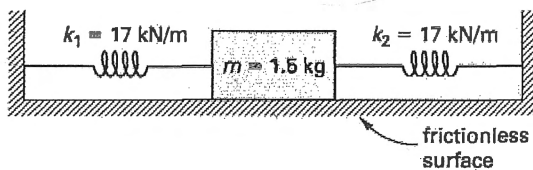
- (A) 2.6 Hz      (B) 2.9 Hz      (C) 3.6 Hz      → (D) 9.7 Hz

$$\zeta = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{57105^5}{135}}} = 0.15$$

$$f = \frac{1}{\zeta} = \frac{1}{0.1} = 9.69 \text{ Hz}$$

**DYNAMICS-59**

In the mass-spring system shown, the mass,  $m$ , is displaced 0.09 m to the right of the equilibrium position and then released. Find the maximum velocity of  $m$ .



- (A) 0.3 m/s      (B) 5 m/s      (C) 8 m/s      → (D) 14 m/s

$$T_1 + V_1 = T_2 + V_2$$

$$2 \cdot \frac{1}{2} (17 \frac{\text{kN}}{\text{m}}) \left( \frac{1000 \text{ N}}{\text{kN}} \right) (0.09 \text{ m}) = \frac{1}{2} (1.5 \text{ kg}) v^2$$

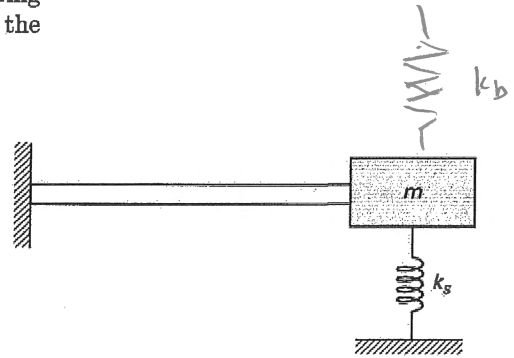
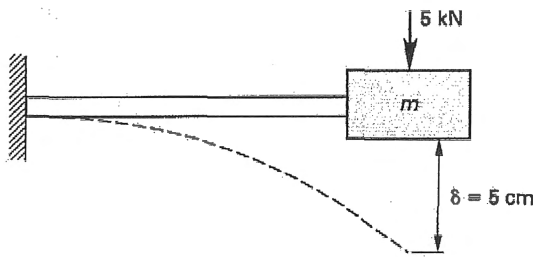
$$\frac{1}{2} k s^2 + \frac{1}{2} k s^2 = \frac{1}{2} m v^2$$

$$v = 13.5 \frac{\text{m}}{\text{s}}$$

POTENTIAL ENERGY FOR 2 SPRINGS

## DYNAMICS-60

A cantilever beam with an end mass,  $m = 7000$  kg, deflects 5 cm when a force of 5 kN is applied at the end. The beam is subsequently mounted on a spring of stiffness,  $k_s = 1.5$  kN/cm. What is most nearly the natural frequency of the mass-beam-spring system?



- (A) 1.5 rad/s      (B) 3.1 rad/s      (C) 6.0 rad/s      (D) 6.3 rad/s

$$m = 7000 \text{ kg}$$

$$\delta = 5 \text{ cm} \left( \frac{m}{1000 \text{ cm}} \right) = 0.05 m$$

$$F = 5000 \text{ N}$$

$$k = 1.5 \frac{\text{kN}}{\text{cm}} \left( \frac{1000 \text{ cm}}{m} \right) \left( \frac{1000 \text{ N}}{\text{kN}} \right) = 150(10)^3 \text{ N}$$



FE Review-Dynamics

1. A particle's curvilinear motion is represented by the equation  $s(t) = 20t + 4t^2 - 3t^3$ . What is most nearly the particle's initial velocity?

- (A) 20 m/s
- (B) 25 m/s
- (C) 30 m/s
- (D) 32 m/s

$$s(t) = 20t + 4t^2 - 3t^3$$

$$v(t) = \frac{ds(t)}{dt} = 20 + 8t - 9t^2$$

$$v(0) = 20 \frac{m}{s}$$

2. A vehicle is traveling at 62 km/h when the driver sees a traffic light in an intersection 530 m ahead turn red. The light's red cycle duration is 25 s. The driver wants to enter the intersection without stopping the vehicle, just as the light turns green. If the vehicle decelerates at a constant rate of  $0.35 \text{ m/s}^2$ , what will be its approximate speed when the light turns green?

- (A) 31 km/h
- (B) 43 km/h
- (C) 59 km/h
- (D) 63 km/h

$$v = 62 \frac{\text{km}}{\text{h}} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) = 17.22 \frac{\text{m}}{\text{s}}$$

$$s = 530 \text{ m}$$

$$a = -0.35 \frac{\text{m}}{\text{s}^2}$$

$$a = -0.35 \frac{\text{m}}{\text{s}^2}$$

$$v = 62 \frac{\text{km}}{\text{h}}$$



$$L = 530 \text{ m}$$

$$t = 25 \text{ s}$$

$$v_t = v_0 + a t = 17.22 \frac{\text{m}}{\text{s}} - (0.35)(25)$$

$$= 8.47 \frac{\text{m}}{\text{s}} \left( \frac{\text{km}}{1000 \text{ m}} \right) \left( \frac{3600 \text{ s}}{\text{h}} \right) = 30.5 \frac{\text{km}}{\text{h}}$$

4. A particle's position is defined by

$$s(t) = 2 \sin ti + 4 \cos tj \quad [t \text{ in radians}]$$

What is most nearly the magnitude of the particle's velocity when  $t = 4 \text{ rad}$ ?

- (A) 2.6
- (B) 2.7
- (C) 3.3
- (D) 4.1

$$w(t) = \frac{ds}{dt} = (2 \cos t)_i - (4 \sin t)_j = -1.31i - (-3.03)_j$$

$$|w(4)| = \sqrt{(-1.31)^2 + (3.03)^2} = 3.3$$

5. A roller coaster train climbs a hill with a constant gradient. During a 10 s period, the acceleration is constant at  $0.4 \text{ m/s}^2$ , and the average velocity of the train is  $40 \text{ km/h}$ . What is most nearly the velocity of the train after 10 s?

WHEN AVERAGE VELOCITY IS PROVIDED.

$$t = 10 \text{ s}$$

$$a = 0.4 \frac{\text{m}}{\text{s}^2}$$

$$V = 40 \frac{\text{km}}{\text{h}}$$

$$= 11.1 \frac{\text{m}}{\text{s}}$$

- (A) 9.1 m/s
- (B) 11 m/s
- (C) 13 m/s
- (D) 15 m/s

$$s(t) = V_{\text{ave}} \cdot t = 40 \frac{\text{km}}{\text{h}} \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) t = 11.1 \frac{\text{m}}{\text{s}} (10 \text{ s}) = 111.1 \text{ m}$$

$$\Delta s = v_0 t + \frac{1}{2} a t^2$$

$$111.1 \text{ m} = v_0 (10 \text{ s}) + \frac{1}{2} (0.4 \frac{\text{m}}{\text{s}^2}) (10 \text{ s})^2$$

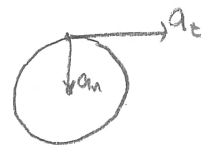
$$v_0 = 9.11 \frac{\text{m}}{\text{s}}$$

$$V_f = v_0 + a t = 9.11 \frac{\text{m}}{\text{s}} + (0.4 \frac{\text{m}}{\text{s}^2}) (10 \text{ s}) = 13.11 \frac{\text{m}}{\text{s}}$$

6. Choose the equation that best represents a rigid body or particle under constant acceleration.

- (A)  $a = 9.81 \text{ m/s}^2 + v_0/t$
- (B)  $v = a_0(t - t_0) + v_0$
- (C)  $v = v_0 + \int_0^t a(t) dt$
- (D)  $a = v_t^2/r$

$$a_n = \frac{v_t}{r}$$



7. A particle's curvilinear motion is represented by the equation  $s(t) = 40t + 5t^2 - 8t^3$ . What is most nearly the initial acceleration of the particle?

- (A) 2 m/s<sup>2</sup>  
 (B) 3 m/s<sup>2</sup>  
 (C) 8 m/s<sup>2</sup>  
 → (D) 10 m/s<sup>2</sup>

$$v(t) = \frac{d}{dt} s(t) = t + 10t - 24t^2$$

$$a(t) = \frac{d}{dt} v(t) = +10 - 48t$$

$$a(0) = +10$$

8. The rotor of a steam turbine is rotating at 7200 rpm when the steam supply is suddenly cut off. The rotor decelerates at a constant rate and comes to rest after 5 min. What is most nearly the angular deceleration of the rotor?

- (A) 0.40 rad/s<sup>2</sup>  
 → (B) 2.5 rad/s<sup>2</sup>  
 (C) 5.8 rad/s<sup>2</sup>  
 (D) 16 rad/s<sup>2</sup>

$$\omega_f = \omega_0 + \alpha t$$

$$0 = 753.98 \frac{\text{rad}}{\text{s}} + \alpha (300\text{s})$$

$$\alpha = -2.51 \frac{\text{rad}}{\text{s}^2}$$

$$7200 \frac{\text{rev}}{\text{min}} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 753.98 \frac{\text{rad}}{\text{s}} = \omega$$

$$t = 5 \text{ min} \left( \frac{60\text{s}}{\text{min}} \right) = 300\text{s}$$

9. The angular position of a car traveling around a curve is described by the following function of time (in seconds).

$$\theta(t) = t^3 - 2t^2 - 4t + 10$$

What is most nearly the angular acceleration of the car at a time of 5 s?

- (A) 4.0 rad/s<sup>2</sup>  
 (B) 6.0 rad/s<sup>2</sup>  
 → (C) 26 rad/s<sup>2</sup>  
 (D) 30 rad/s<sup>2</sup>

$$\omega(t) = \frac{d}{dt} \theta(t) = 3t^2 - 4t - 4$$

$$\alpha(t) = \frac{d}{dt} \omega(t) = 6t - 4$$

$$\alpha(5) = 26 \frac{\text{rad}}{\text{s}^2}$$

FE Review-Dynamics

10. A vehicle is traveling at 70 km/h when the driver sees a traffic light in the next intersection turn red. The intersection is 250 m away, and the light's red cycle duration is 15 s. What is most nearly the uniform deceleration that will put the vehicle in the intersection the moment the light turns green?

- (A) 0.18 m/s<sup>2</sup>
- (B) 0.25 m/s<sup>2</sup>
- (C) 0.37 m/s<sup>2</sup>
- (D) 1.3 m/s<sup>2</sup>

$$s_f = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$250 \text{ m} = (19.44 \frac{\text{m}}{\text{s}})(15 \text{ s}) + \frac{1}{2} a (15 \text{ s})^2$$

$$a = -0.37 \frac{\text{m}}{\text{s}^2}$$

$$v_0 = 70 \frac{\text{km}}{\text{h}} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{1}{3600 \text{ s}} \right)$$

$$v_0 = 19.44 \frac{\text{m}}{\text{s}}$$

$$s = 250 \text{ m}$$

$$t = 15 \text{ s}$$

11. A projectile has an initial velocity of 85 m/s and a launch angle of 60° from the horizontal. The surrounding terrain is level, and air friction is to be disregarded. What is most nearly the horizontal distance traveled by the projectile?

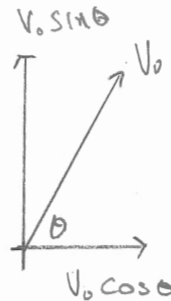
- (A) 80 m
- (B) 400 m
- (C) 640 m
- (D) 1200 m

TIME

$$v_y = -gt + v_0 \sin \theta$$

$$0 = -9.81 \frac{\text{m}}{\text{s}^2} (t) + 85 \frac{\text{m}}{\text{s}} \sin 60^\circ$$

$$\frac{t}{2} = 7.5 \text{ s} \quad t = 15 \text{ s}$$



$$v_0 = 85 \frac{\text{m}}{\text{s}}$$

$$\theta = 60^\circ$$

ALT

$$R = \frac{v_0^2 \sin 2\theta}{g} = 637.82 \text{ m}$$

RANGE

$$x = v_0 \cos \theta (t) = 85 \frac{\text{m}}{\text{s}} \cos 60^\circ (15 \text{ s}) = 637.5 \text{ m}$$

12. A particle's position is defined by

$$s(t) = 15 \sin t \mathbf{i} + 8.5 \cos t \mathbf{j} \quad [t \text{ in radians}]$$

What is most nearly the magnitude of the particle's acceleration when  $t = \pi$ ?

- (A) 6.5
- (B) 8.5
- (C) 15
- (D) 17

$$v(t) = \frac{d}{dt} s(t) = (15 \cos t) \mathbf{i} - (8.5 \sin t) \mathbf{j}$$

$$a(t) = \frac{d}{dt} v(t) = (-15 \sin t) \mathbf{i} - (8.5 \cos t) \mathbf{j}$$

$$a(\pi) = -0.82 \mathbf{i} + (-8.49) \mathbf{j}$$

$$|a(\pi)| = \sqrt{(-0.82)^2 + (-8.49)^2} = 8.53$$

13. A particle's curvilinear motion is represented by the equation  $s(t) = 30t - 8t^2 + 6t^3$ . What is most nearly the maximum speed reached by the particle?

- (A) 26 m/s  
 (B) 30 m/s  
 (C) 35 m/s  
 (D) 48 m/s

$$v(t) = 30 - 16t + 18t^2$$

$$a(t) = \frac{d}{dt} v(t) = -16 + 36t$$

SET TO ZERO

$$36t - 16 = 0 \quad \therefore t = 0.44 \text{ s}$$

$$v(0.44 \text{ s}) = 30 - 16(0.44) + 18(0.44)^2$$

$$v(0.44 \text{ s}) = 26.44 \frac{\text{m}}{\text{s}}$$

14. A projectile has an initial velocity of 80 m/s and a launch angle of  $42^\circ$  from the horizontal. The surrounding terrain is level, and air friction is to be disregarded. What is most nearly the maximum elevation achieved by the projectile?

- (A) 72 m  
 (B) 150 m  
 (C) 350 m  
 (D) 620 m

$$v_y = -gt + v_0 \sin \theta$$

$$0 = -9.81 \frac{\text{m}}{\text{s}^2} \cdot t + 80 \frac{\text{m}}{\text{s}} \sin 42^\circ$$

$$\frac{t}{2} = 5.46 \text{ s}$$

$$y = -\frac{gt^2}{2} + v_0 \sin \theta (t)$$

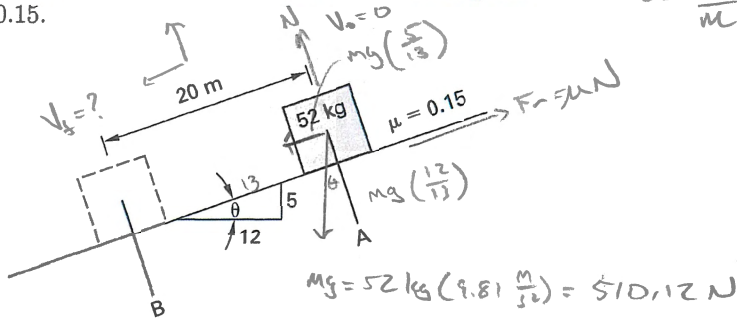
$$= -\frac{9.81 \frac{\text{m}}{\text{s}^2} (5.46 \text{ s})^2}{2} + 80 \frac{\text{m}}{\text{s}} \sin 42^\circ (5.46 \text{ s})$$

$$y = 146 \text{ m}$$

$$v_0 = 80 \frac{\text{m}}{\text{s}}$$

$$\theta = 42^\circ$$

1. The 52 kg block shown starts from rest at position A and slides down the inclined plane to position B. The coefficient of friction between the block and the plane is  $\mu = 0.15$ .



$$F = ma$$

$$a = \frac{F}{m}$$

$$\sum F_x = ma = Mg\left(\frac{5}{13}\right) - \mu mg$$

$$g\left(\frac{5}{13}\right) - \mu g = a$$

$$a = 9.81\left(\frac{5}{13}\right) - (0.15)(9.81)$$

$$a = 2.3 \frac{m}{s^2}$$

$$v_f^2 = v_0^2 + 2a\Delta s$$

$$v_f^2 = 0 + 2\left(2.3 \frac{m}{s^2}\right)(20m)$$

$$v_f = 9.59 \frac{m}{s}$$

What is most nearly the velocity of the block at position B?

- (A) 2.4 m/s
- (B) 4.1 m/s
- (C) 7.0 m/s
- (D) 9.8 m/s

2. A 5 kg block begins from rest and slides down an inclined plane. After 4 s, the block has a velocity of 6 m/s. If the angle of inclination of the plane is  $45^\circ$ , approximately how far has the block traveled after 4 s?

- (A) 1.5 m
- (B) 3.0 m
- (C) 6.0 m
- (D) 12 m

$$v_f^2 = v_0^2 + 2a\Delta s$$

$$\left(6 \frac{m}{s}\right)^2 = 0^2 + 2\left(1.5 \frac{m}{s^2}\right)\Delta s$$

$$\Delta s = 12m$$

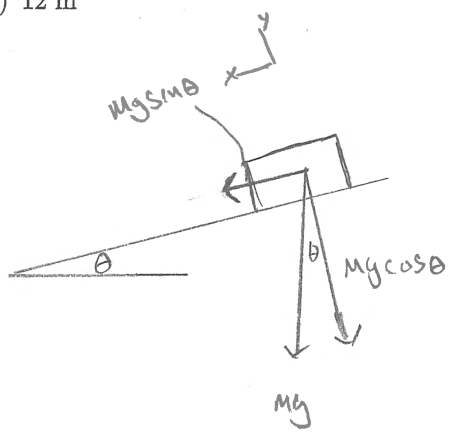
$$v_0 = 0$$

$$v_f = 6 \frac{m}{s}$$

$$t = 4s$$

$$\theta = 45^\circ$$

$$m = 5kg$$



$$v_f = v_0 + at$$

$$6 \frac{m}{s} = 0 + a(4s)$$

$$a = 1.5 \frac{m}{s^2}$$

FE Review-Dynamics

3. The elevator in a 20-story apartment building has a mass of 1800 kg. Its maximum velocity and maximum acceleration are 2.5 m/s and 1.4 m/s<sup>2</sup>, respectively. A passenger weighing 67 kg stands on a scale in the elevator as the elevator ascends at its maximum acceleration. When the elevator reaches its maximum acceleration, the scale most nearly reads

$$\sum F_y = ma = 67 \text{ kg} \left( 9.81 \frac{\text{m}}{\text{s}^2} + 1.4 \frac{\text{m}}{\text{s}^2} \right)$$

$$F = 751 \text{ N}$$

- (A) 67 N
- (B) 560 N
- (C) 660 N
- (D) 750 N

REST-ELEVATOR



$$Mg = 1800 \text{ kg} \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) = 17658 \text{ N}$$



$v = 2.5 \frac{\text{m}}{\text{s}}$   
 $a = 1.4 \frac{\text{m}}{\text{s}^2}$   
 $a = 9.81 \frac{\text{m}}{\text{s}^2}$

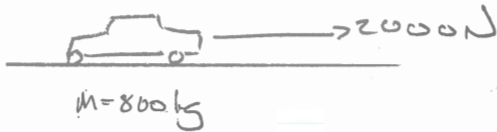
$$Mg = 67 \text{ kg} \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) + Mg_p$$

$$657.27 \text{ N} + 17658 \text{ N}$$

$$18315.27 \text{ N}$$

4. A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is most nearly the greatest acceleration that the car can reach without breaking the rope?

- (A) 1.2 m/s<sup>2</sup>
- (B) 2.5 m/s<sup>2</sup>
- (C) 3.8 m/s<sup>2</sup>
- (D) 4.5 m/s<sup>2</sup>



$$F = ma$$

$$a = \frac{2000 \text{ N}}{800 \text{ kg}} = 2.50 \frac{\text{m}}{\text{s}^2}$$

FE Review-Dynamics

5. An 8 kg block begins from rest and slides down an inclined plane. After 10 s, the block has a velocity of 15 m/s. The plane's angle of inclination is 30°. What is most nearly the coefficient of friction between the plane and the block?

- (A) 0.15
- (B) 0.22
- (C) 0.40
- (D) 0.85

$$\sum F_x = Ma = N - mg \cos \theta$$

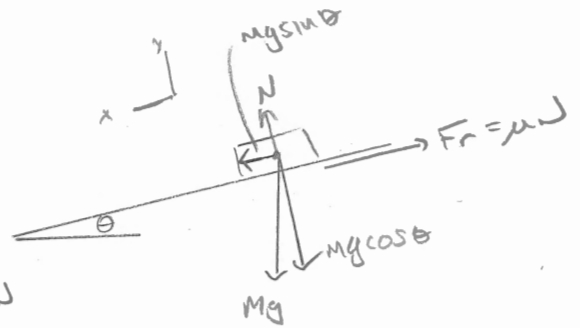
$$N = (8 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) \cos 30^\circ = 67.97 \text{ N}$$

$$\sum F_y = Ma = mg \sin \theta - \mu N$$

$$a = g \sin \theta - \mu g \cos \theta$$

$$1.5 \frac{\text{m}}{\text{s}^2} = 9.81 \frac{\text{m}}{\text{s}^2} \sin 30^\circ - 9.81 \frac{\text{m}}{\text{s}^2} \mu \cos 30^\circ$$

$$\mu = 0.4$$



$$v_f = v_0 + at$$

$$15 = 0 + 10a$$

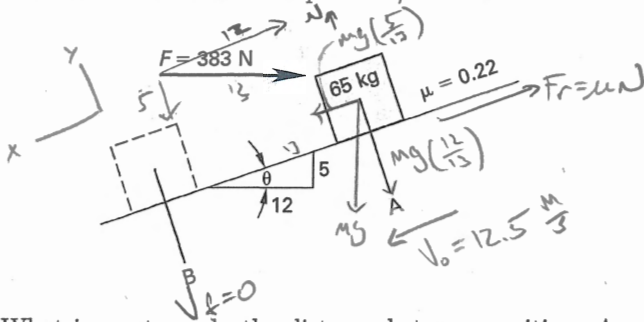
$$a = 1.5 \frac{\text{m}}{\text{s}^2}$$

$m = 8 \text{ kg}$   
 $\theta = 30^\circ$   
 $t = 10$   
 $v_f = 15 \frac{\text{m}}{\text{s}}$   
 $v_0 = 0$

6. If the sum of the forces on a particle is not equal to zero, the particle is

- (A) moving with constant velocity in the direction of the resultant force
- (B) accelerating in a direction opposite to the resultant force
- (C) accelerating in the same direction as the resultant force
- (D) moving with a constant velocity opposite to the direction of the resultant force

7. A 383 N horizontal force is applied to the 65 kg block shown. Beginning at position A, the block moves down the slope at a velocity of 12.5 m/s and comes to a complete stop at position B. The coefficient of friction between the block and the plane is  $\mu = 0.22$ .



$$\sum F_y = 0 = N - \mu Mg \left(\frac{12}{13}\right) - F \left(\frac{5}{13}\right)$$

$$N = (0.22)(65 \text{ kg})(9.81) \left(\frac{12}{13}\right) + 383 \left(\frac{5}{13}\right)$$

$$N = 276.8 \text{ N}$$

$$\sum F_x = ma = mg \left(\frac{5}{13}\right) - F \left(\frac{12}{13}\right) - \mu N$$

$$a = -2.6 \frac{\text{m}}{\text{s}^2}$$

What is most nearly the distance between positions A and B?

- (A) 6.1 m
- (B) 9.1 m
- (C) 15 m
- (D) 19 m

$$v_f^2 = v_0^2 + 2a \Delta s$$

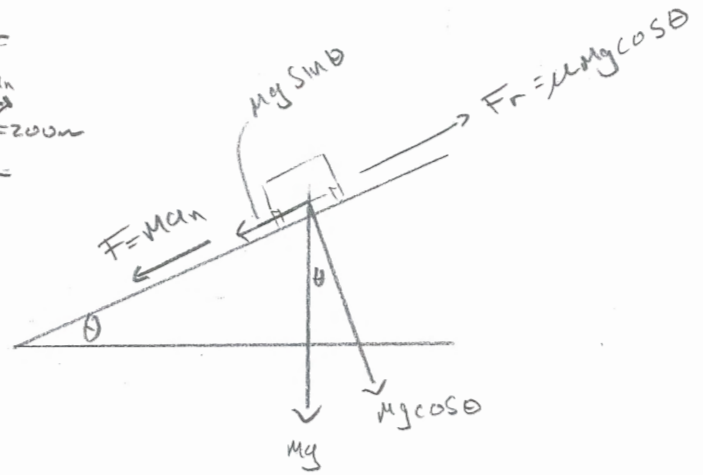
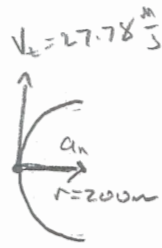
$$0 = 12.5^2 + 2(-2.6) \Delta s$$

$$\Delta s = 30 \text{ m}$$



FE Review-Dynamics

1. A 1530 kg car is towing a 300 kg trailer. The coefficient of friction between all tires and the road is 0.80. The car and trailer are traveling at 100 km/h around a banked curve of radius 200 m. What is most nearly the necessary banking angle such that tire friction will NOT be necessary to prevent skidding?



(A)  $8.0^\circ$   $\Sigma F = ma_n$

→ (B)  $21^\circ$

(C)  $36^\circ$   $Mg \sin \theta - \mu Mg \cos \theta = M \frac{v_t^2}{r}$

(D)  $78^\circ$

$$g \sin \theta = \mu g \cos \theta = \frac{v_t^2}{r}$$

$$9.81 \sin \theta = (0.8)(9.81) \cos \theta = \frac{27.78^2}{200}$$

$$\theta = 20.77^\circ$$

2. Why does a spinning ice skater's angular velocity increase as she brings her arms in toward her body?

(A) Her mass moment of inertia is reduced.

(B) Her angular momentum is constant.

(C) Her radius of gyration is reduced.

→ (D) all of the above

3. A 1 m long uniform rod has a mass of 10 kg. It is pinned at one end to a frictionless pivot. What is most nearly the mass moment of inertia of the rod taken about the pivot point?



(A)  $0.83 \text{ kg}\cdot\text{m}^2$

(B)  $2.5 \text{ kg}\cdot\text{m}^2$

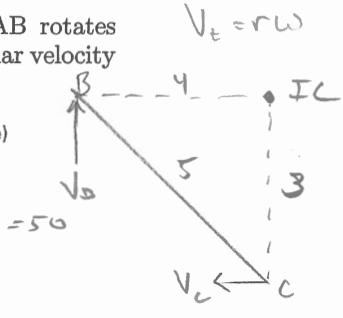
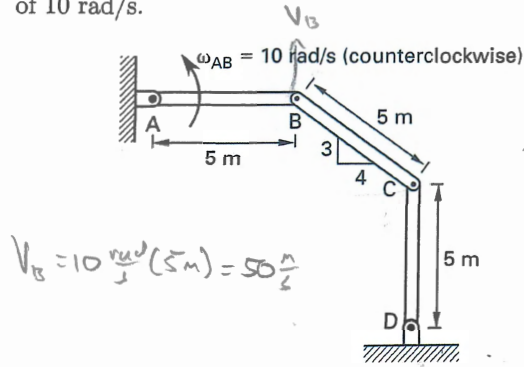
→ (C)  $3.3 \text{ kg}\cdot\text{m}^2$

(D)  $10 \text{ kg}\cdot\text{m}^2$

$$I_{rod} = \frac{1}{3} ML^2 = \frac{1}{3} (10 \text{ kg})(1 \text{ m})^2 = 3.3$$

FE Review-Dynamics

4. In the linkage mechanism shown, link AB rotates with an instantaneous counterclockwise angular velocity of 10 rad/s.

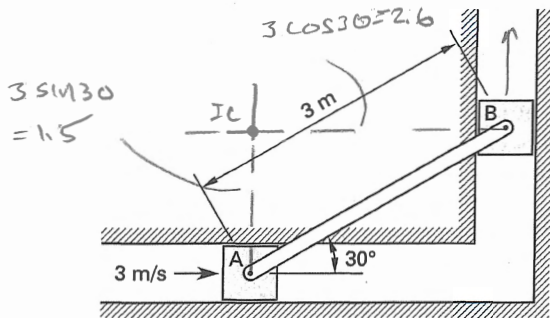


$$\omega_{BC} = \frac{V_B}{r} = \frac{50 \frac{m}{s}}{4m} = 12.5 \frac{rad}{s}$$

What is most nearly the instantaneous angular velocity of link BC when link AB is horizontal and link CD is vertical?

- (A) 2.3 rad/s (clockwise)
- (B) 3.3 rad/s (counterclockwise)
- (C) 5.5 rad/s (clockwise)
- (D) 13 rad/s (clockwise)

5. Two 2 kg blocks are linked as shown.



$$\omega = \frac{V_A}{r_A} = \frac{V_B}{r_B}$$

$$\frac{3 \frac{m}{s}}{1.5m} = \frac{V_B}{2.6m}$$

$$V_B = 5.2 \frac{m}{s}$$

Assuming that the surfaces are frictionless, what is most nearly the velocity of block B if block A is moving at a speed of 3 m/s?

- (A) 0 m/s
- (B) 1.3 m/s
- (C) 1.7 m/s
- (D) 5.2 m/s

FE Review-Dynamics

6. A car travels on a perfectly horizontal, unbanked circular track of radius  $r$ . The coefficient of friction between the tires and the track is 0.3. If the car's velocity is 10 m/s, what is most nearly the smallest radius the car can travel without skidding?

$\mu = 0.3$   
 $v = 10 \frac{m}{s}$

- (A) 10 m
- (B) 34 m
- (C) 50 m
- (D) 68 m

$F_r = m a_n$   
 $\mu m g = m \cdot \frac{v_t^2}{r}$

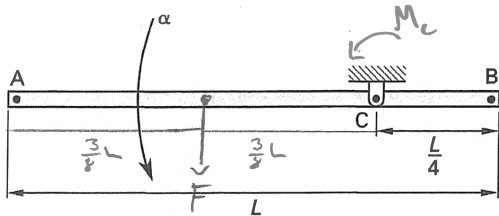
$\mu g = \frac{v_t^2}{r}$

$r = \frac{v_t^2}{\mu g} = \frac{(10 \frac{m}{s})^2}{(0.3)(9.81 \frac{m}{s^2})} = 33.98 m$



7. A uniform rod (AB) of length  $L$  and weight  $W$  is pinned at point C. The rod starts from rest and accelerates with an angular acceleration of  $12g/7L$ .

$\alpha = \frac{12g}{7L}$



What is the instantaneous reaction at point C at the moment rotation begins?

$M = F \cdot r = I \alpha$

- (A)  $\frac{W}{4}$
- (B)  $\frac{W}{3}$
- (C)  $\frac{4W}{7}$
- (D)  $\frac{7W}{12}$

$F \cdot \frac{L}{4} = (\frac{1}{12} M L^2) \frac{12g}{7L} = \frac{12g M L^2}{12 \cdot 7L} = \frac{M g L}{7}$

$F_c \cdot \frac{L}{4} = I \alpha + F_c \cdot r = \frac{M g L}{7}$

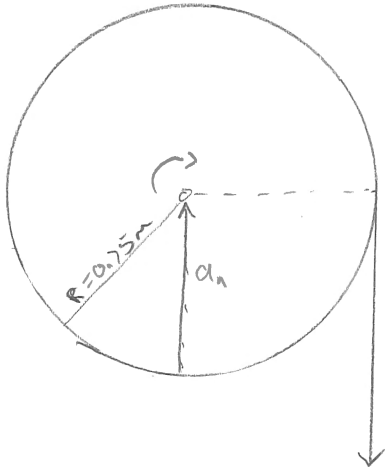
$F_c = \frac{W L}{7} \cdot \frac{4}{L} = \frac{4W}{7}$

FE Review-Dynamics

8. A wheel with a 0.75 m radius has a mass of 200 kg. The wheel is pinned at its center and has a radius of gyration of 0.25 m. A rope is wrapped around the wheel and supports a hanging 100 kg block. When the wheel is released, the rope begins to unwind. What is most nearly the angular acceleration of the wheel as the block descends?

- (A) 5.9 rad/s<sup>2</sup>
- (B) 6.5 rad/s<sup>2</sup>
- (C) 11 rad/s<sup>2</sup>
- (D) 14 rad/s<sup>2</sup>

$M_w = 200 \text{ kg}, M_B = 100 \text{ kg}$   
 $r = 0.75 \text{ m}$   
 $r_g = 0.25 \text{ m}$



$r_g = \sqrt{\frac{I}{M}} = 0.25 \text{ m} \quad I = M r_g^2$   
 $I = (0.25 \text{ m})^2 (200 \text{ kg}) = 12.5 \text{ kg m}^2$

$M = I \alpha = F \cdot r$

$(M_B - M_B a) R = M_w r_g^2 \alpha$   
 $M_B R (g - a) = M_w r_g^2 \alpha \quad a = r \alpha$   
 $M_B R (g - R \alpha) = M_w r_g^2 \alpha$

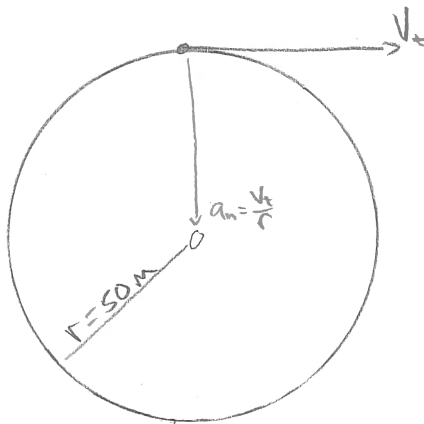
$(100 \text{ kg})(0.75 \text{ m})(9.81 \frac{\text{m}}{\text{s}^2} - (0.75 \text{ m}) \alpha) = 200 \text{ kg} (0.25 \text{ m})^2 \alpha$   
 $\alpha = 10.70 \frac{\text{rad}}{\text{s}^2}$

$Mg = (100 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) = 981 \text{ N}$

9. A car travels around an unbanked 50 m radius curve without skidding. The coefficient of friction between the tires and road is 0.3. What is most nearly the car's maximum velocity?

- (A) 14 km/h
- (B) 25 km/h
- (C) 44 km/h
- (D) 54 km/h

$\mu = 0.3$



$F = M a_n$

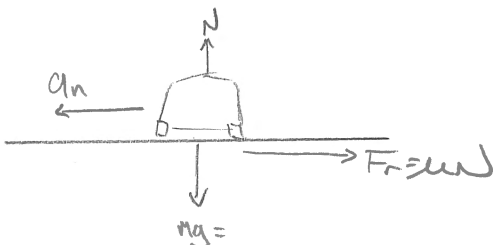
$\mu N = M \frac{V_t^2}{r}$

$\mu Mg = M \frac{V_t^2}{r}$

$\mu g = \frac{V_t^2}{r}$

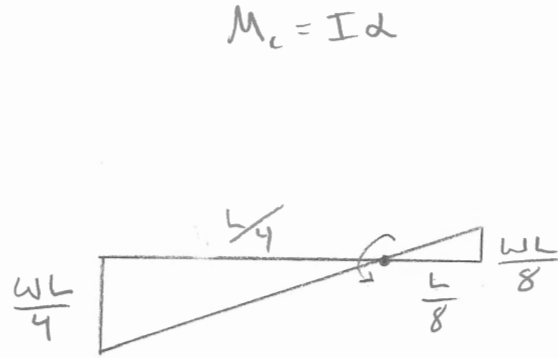
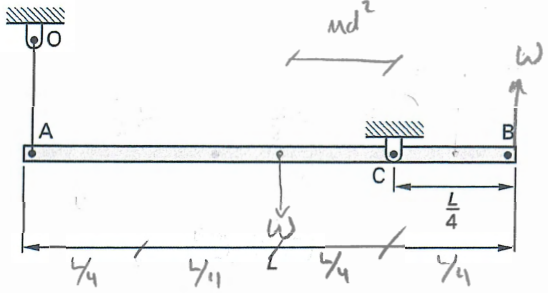
$V = \sqrt{\mu g r} = \sqrt{(0.3)(9.81 \frac{\text{m}}{\text{s}^2})(50 \text{ m})}$

$V_t = 12.13 \frac{\text{m}}{\text{s}} \left( \frac{\text{km}}{1000 \text{ m}} \right) \left( \frac{3600 \text{ s}}{\text{h}} \right) = 43.6 \frac{\text{km}}{\text{h}}$



FE Review-Dynamics

10. A uniform rod (AB) of length  $L$  and weight  $W$  is pinned at point C and restrained by cable OA. The cable is suddenly cut. The rod starts to rotate about point C, with point A moving down and point B moving up.



The instantaneous linear acceleration of point B is

- (A)  $\frac{3g}{16}$
- (B)  $\frac{g}{4}$
- (C)  $\frac{3g}{7}$
- (D)  $\frac{3g}{4}$

PARALLEL AXIS THEOREM

$$I = I_c + md^2 = \frac{1}{12} ML^2 + M\left(\frac{L}{4}\right)^2 = \frac{7}{48} ML^2$$

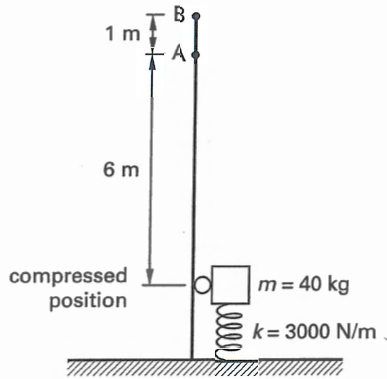
$$M_c = \frac{WL}{4} = \frac{mgL}{4}$$

$$\alpha = \frac{M_c}{I} = \frac{\frac{mgL}{4}}{\frac{7}{48} ML^2} = \frac{mgL}{4} \cdot \frac{48}{7ML^2}$$

$$\alpha = \frac{12g}{7L}$$

$$a_t = r \cdot \alpha = \frac{L}{4} \cdot \frac{12g}{7L} = \frac{3g}{7}$$

1. The 40 kg mass,  $m$ , in the illustration shown is guided by a frictionless rail. The spring constant,  $k$ , is 3000 N/m. The spring is compressed sufficiently and released, such that the mass barely reaches point B.



$$V = V$$

$$\frac{1}{2} k s^2 = mgh$$

$$\frac{1}{2} (3000 \frac{N}{m}) s^2 = 40 \text{ kg} (9.81 \frac{m}{s^2}) (7m)$$

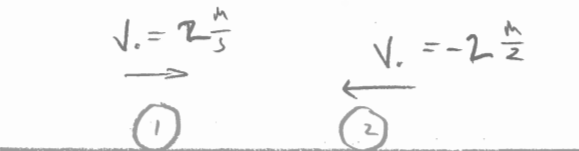
$$s = 1.35 \text{ m}$$

What is most nearly the initial spring compression?

- (A) 0.96 m
- (B) 1.3 m
- (C) 1.4 m
- (D) 1.8 m

2. Two balls, both of mass 2 kg, collide head on. The velocity of each ball at the time of the collision is 2 m/s. The coefficient of restitution is 0.5. Most nearly, what are the final velocities of the balls?

- (A) 1 m/s and -1 m/s
- (B) 2 m/s and -2 m/s
- (C) 3 m/s and -3 m/s
- (D) 4 m/s and -4 m/s



$$m_1 = m_2 = 2 \text{ kg}$$

$e = 0.5$  ∴  $\frac{1}{2}$  THE ENERGY IS RESTORED  
VELOCITIES ARE CUT IN  
HALF AFTER REBOUND  
NO NEED TO PERFORM CALLS  
VELOCITY ONLY

$$(v_1 - v_2)e = v_1' - v_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

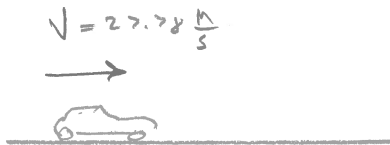
FE Review-Dynamics

3. A 1500 kg car traveling at 100 km/h is towing a 250 kg trailer. The coefficient of friction between the tires and the road is 0.8 for both the car and trailer. Approximately what energy is dissipated by the brakes if the car and trailer are braked to a complete stop?

- (A) 96 kJ  
 (B) 390 kJ  
 (C) 580 kJ  
 → (D) 680 kJ
- $M = 1750 \text{ kg}$   
 $V = 100 \frac{\text{km}}{\text{h}} = 27.78 \frac{\text{m}}{\text{s}}$   
 $\mu = 0.8$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} (1750 \text{ kg}) \left( 27.78 \frac{\text{m}}{\text{s}} \right)^2$$

$$= 675262.35 \text{ N}\cdot\text{m}$$



4. A 3500 kg car traveling at 65 km/h skids and hits a wall 3 s later. The coefficient of friction between the tires and the road is 0.60. What is most nearly the speed of the car when it hits the wall?

- (A) 0.14 m/s  
 (B) 0.40 m/s  
 (C) 5.1 m/s  
 (D) 6.2 m/s
- $M = 3500 \text{ kg}$   
 $V_0 = 65 \frac{\text{km}}{\text{h}} = 18.06 \frac{\text{m}}{\text{s}}$   
 $\mu = 0.6$

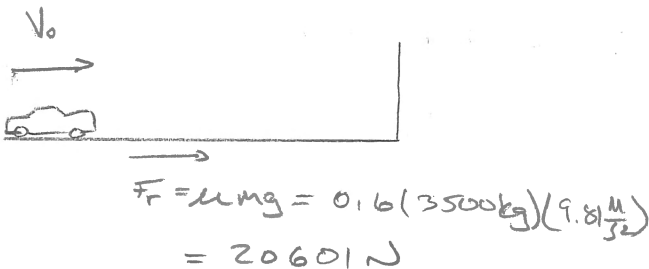
$$a = \frac{dv}{dt} = \frac{980 \frac{\text{m}}{\text{s}}}{3 \text{ s}} = 326.67 \frac{\text{m}}{\text{s}^2}$$

$$V_f = V_0 + at = 18.06 - (326.67)(3)$$

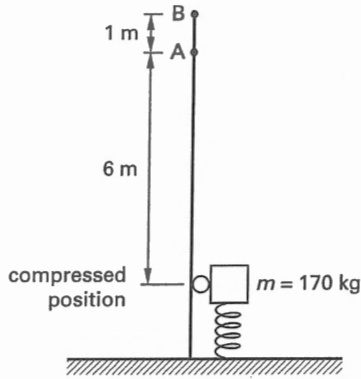
IMPULSE MOMENTUM

$$m \Delta v = F \Delta t$$

$$v = \frac{F \cdot t}{m} = \frac{(20601 \text{ N})(3 \text{ s})}{3500 \text{ kg}}$$



5. In the illustration shown, the 170 kg mass,  $m$ , is guided by a frictionless rail. The spring is compressed sufficiently and released, such that the mass barely reaches point B.



$$T_A + U_A = U_B$$

$$T_A + mgh_A = mgh_B$$

$$T_A = (170 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(6 \text{ m}) = (170 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(7 \text{ m})$$

$$T_A = 1670 \text{ J}$$

What is most nearly the kinetic energy of the mass at point A?

- (A) 20 J
- (B) 220 J
- (C) 390 J
- (D) 1700 J

6. A pickup truck is traveling forward at 25 m/s. The bed is loaded with boxes whose coefficient of friction with the bed is 0.40. What is most nearly the shortest time that the truck can be brought to a stop such that the boxes do not shift?

- (A) 2.3 s
- (B) 4.7 s
- (C) 5.9 s
- (D) 6.4 s

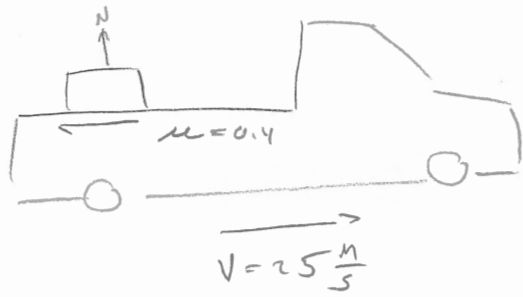
IMPULSE

$$F_f = \mu mg$$

$$m \Delta v = F_f \Delta t$$

$$\Delta t = \frac{m \Delta v}{F} = \frac{m \Delta v}{\mu mg} = \frac{\Delta v}{\mu g}$$

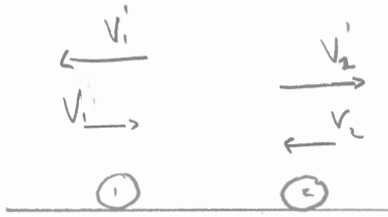
$$t = \frac{25 \frac{\text{m}}{\text{s}}}{(0.4)(9.81 \frac{\text{m}}{\text{s}^2})} = 6.37 \text{ s}$$





7. Two balls both have a mass of 8 kg and collide head on. The velocity of each ball at the time of collision is 18 m/s. The velocity of each ball decreases to 10 m/s in opposite directions after the collision. Approximately how much energy is lost in the collision?

- (A) 0.57 kJ
  - (B) 0.91 kJ
  - (C) 1.8 kJ
  - (D) 2.3 kJ
- $v_1 = v_2 = 18 \frac{m}{s}$   
 $v_1' = v_2' = 10 \frac{m}{s}$   
 $m = 8 \text{ kg}$



$$E_T =$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{10 - (-10)}{+18 - (-18)} = 0.56$$

56% OF ENERGY IS CONSERVED

$$E + T_1 = T_2$$

$$E + \frac{1}{2} M v_1^2 + \frac{1}{2} M v_2^2 = \frac{1}{2} M v_1'^2 + \frac{1}{2} M v_2'^2$$

$$E = 2 \cdot \frac{1}{2} M v_1'^2 - 2 \cdot \frac{1}{2} M v_1^2$$

$$= (8 \text{ kg})(18)^2 - (8 \text{ kg})(10)^2 = 1792 \text{ J}$$

8. The impulse-momentum principle is mostly useful for solving problems involving

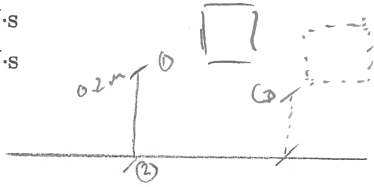
- (A) force, velocity, and time
- (B) force, acceleration, and time
- (C) velocity, acceleration, and time
- (D) force, velocity, and acceleration

$$m \Delta v = F \cdot t$$

FE Review-Dynamics

9. A 12 kg aluminum box is dropped from rest onto a large wooden beam. The box travels 0.2 m before contacting the beam. After impact, the box bounces 0.05 m above the beam's surface. Approximately what impulse does the beam impart on the box?

- (A) 8.6 N-s
- (B) 12 N-s
- (C) 36 N-s
- (D) 42 N-s



$$1-2 \quad T_1 + V_1 = T_2 + V_2$$

$$mgh_1 = \frac{1}{2} m v_2^2$$

$$gh_1 = \frac{v_2^2}{2}$$

$$v_2 = \sqrt{2gh_1} = 1.98 \frac{m}{s} \downarrow$$

IMPULSE

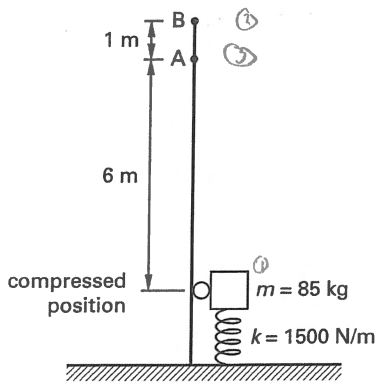
$$m \Delta v = F \cdot t$$

$$IMP = m \Delta v$$

$$= (12 \text{ kg}) (1.98 \frac{m}{s} + 0.99 \frac{m}{s})$$

$$IMP = 35.64 \text{ N-s}$$

10. The 85 kg mass,  $m$ , shown is guided by a frictionless rail. The spring is compressed sufficiently and released, such that the mass barely reaches point B. The spring constant,  $k$ , is 1500 N/m.



$$1-2 \quad T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} k s^2 = mgh$$

$$\frac{1}{2} (1500 \frac{N}{m}) s^2 = (85 \text{ kg}) (9.81 \frac{m}{s^2}) (7 \text{ m})$$

$$s = 2.79 \text{ m}$$

1-3

$$\frac{1}{2} k s^2 = \frac{1}{2} m v_3^2 + mgh_3$$

$$\frac{1}{2} (1500) (2.79)^2 = \frac{1}{2} (85) v_3^2 + (85) (9.81) (6)$$

$$v_3 = 4.43 \frac{m}{s}$$

What is most nearly the velocity of the mass at point A?

- (A) 3.1 m/s
- (B) 4.4 m/s *THINK OF IT LIKE A BOUNCING BALL.*
- (C) 9.8 m/s
- (D) 20 m/s

ALT

$$T_1 + V_1 = T_2 + V_2$$

$$mgh_1 = \frac{1}{2} m v_2^2 + mgh_2$$

$$gh_1 = \frac{v_2^2}{2} + gh_2$$

$$(9.81)(7) = \frac{v_2^2}{2} + (9.81)(6)$$

$$v_2 = 4.43 \frac{m}{s}$$

MUCH EASIER