### 1.1 Identify Points, Lines, and Planes

## Objective: Name and sketch geometric figures.

## Key Vocabulary

- Undefined terms - These words do not have formal definitions, but there is agreement aboutwhat they mean.
- Point - A point has no dimension. It is represented by a dot.
- Line - A line has one dimension. It is represented by a line with two arrowheads, but it extends without end. Through any two points, there is exactly one line. You can use any two points on a line to name it. Line $A B$ (written as $\overleftrightarrow{\mathbf{A B}}$ ) and points $A$ and $B$ are used here to define the terms below.
- Plane - A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.

plane $M$ or plane $A B C$
- Collinear points - Collinear points are points that lie on the same line.
- Coplanar points - Coplanar points are points that lie in the same plane.
- Defined terms - In geometry, terms that can be described using known words such as point or line are called defined terms.
- Line segment, Endpoints - The line segment $A B$, or segment $A B$, (written as $\underline{\overline{\mathrm{AB}}}$ ) consists of the endpoints $A$ and $B$ and all points on $\overleftrightarrow{A B}$ that are between $A$ and $B$. Note that $\overline{\mathrm{AB}}$ can also be named $\overline{\overline{B A}}$.
- Ray - The ray $A B$ (written as $\overrightarrow{\mathrm{AB}}$ ) consists of the endpoint $A$ and all points on $\overrightarrow{\mathrm{AB}}$ that lie on the same side of $A$ as $B$. Note that $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\overrightarrow{B A}}$ are different rays.


| segment |  |
| :---: | :---: |
| endpoint | endpoint |
| $A$ | $B$ |


 opposite rays.

- Intersection - The intersection of the figures is the set of points the figures have in common.
The intersection of two different lines is a point.


The intersection of two different planes is a line.


EXAMPLE 1 Name points, lines, and planes
a. Give two other names for $\overleftrightarrow{\mathrm{LN}}$ and for plane $Z$.
b. Name three points that are collinear. Name four points that are coplanar.

Solution:
a. Other names for $\overleftrightarrow{L N}$ are $\overleftrightarrow{L M}$ and line $b$. Other names for plane $Z$ are plane $\angle M P$ and $\angle N P$.
b. Points $L, M$, and $N$ lie on the same line, so they are collinear. Points $L, M, N$, and $P$ lie on the sa
 plane, so they are coplanar.
a. Give another name for $\overline{\mathrm{VX}}$.
b. Name all rays with endpoint $W$. Which of these rays are opposite rays?

Solution
a. Another name for $\overline{V X}$ is $\overline{X V}$.
b. The rays with endpoint $W$ are $\overrightarrow{W V}, \overrightarrow{W Y}, \overrightarrow{W X}$, and $\overrightarrow{W Z}$.

The opposite rays with endpoint $W$ are $\overrightarrow{W V}$ and $\overrightarrow{W X}$, and $\overrightarrow{W Y}$ and $\overrightarrow{W Z}$.

## EXAMPLE 3 Sketch intersections of lines and planes

a. Sketch a plane and a line that is in the plane.
b. Sketch a plane and a line that does not intersect the plane.
c. Sketch a plane and a line that intersects the plane at a point.

## Solution

a.

b.

c.


EXAMPLE 4 Sketch intersections of planes Sketch two planes that intersect in a line.

## Solution

STEP 1 Draw a vertical plane. Shade the plane.
STEP 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden. STEP 3 Draw the line of intersection.

### 1.1 Cont.

Checkpoint Use the diagram in Example 1.

1. Give two other names for $\overleftrightarrow{M Q}$. Name a point that is not coplanar with points $L, N$, and $P$.

## $\overleftrightarrow{Q M}$ and line a; point $Q$

## Checkpoint Use the diagram in Example 2.

2. Give another name for $\overline{Y W}$.
$\overline{W Y}$
3. Are $\overrightarrow{V X}$ and $\overrightarrow{X V}$ the same ray? Are $\overrightarrow{V W}$ and $\overrightarrow{V X}$ the same ray? Explain.
No, the rays do not have the same endpoint; Yes, the rays have a common endpoint, are collinear, and consist of the same points.

Checkpoint Complete the following exercises.
4. Sketch two different lines that intersect a plane at different points.

5. Name the intersection of $\overleftrightarrow{M X}$ and line a. point
6. Name the intersection of plane $C$ and plane $D$.
 line a

### 2.4 Use Postulates and Diagrams

Obj.: Use postulates involving points, lines, and planes.

## Key Vocabulary

- Line perpendicular to a plane - A line is a line perpendicular to a plane if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.
- Postulate - In geometry, rules that are accepted without proof are called postulates or axioms.


## POSTULATES

Point, Line, and Plane Postulates
POSTULATE 5 - Through any two points there exists exactly one line.
POSTULATE 6 - A line contains at least two points.
POSTULATE 7 - If two lines intersect, then their intersection is exactly one point.
POSTULATE 8 - Through any three noncollinear points there exists exactly one plane.
POSTULATE 9 - A plane contains at least three noncollinear points.
POSTULATE 10 - If two points lie in a plane, then the line containing them lies in the plane.
POSTULATE 11-If two planes intersect, then their intersection is a line.

## CONCEPT SUMMARY - Interpreting a Diagram

When you interpret a diagram, you can only assume information about size or measure if it is marked.

## YOU CAN ASSUME



All points shown are coplanar.
$\angle A H B$ and $\angle B H D$ are a linear pair.
$\angle A H F$ and $\angle B H D$ are vertical angles.

## $A, H, J$, and $D$ are collinear.

$\overleftrightarrow{A D}$ and $\overleftrightarrow{B F}$ intersect at $H$.

YOU CANNOT ASSUME
$G, F$, and $E$ are collinear.
$\overrightarrow{B F}$ and $\overparen{C E}$ intersect.
$\overleftrightarrow{B F}$ and $\overleftrightarrow{C E}$ do not intersect.
$\angle B H A \cong \angle C J A$
$\overleftrightarrow{A D} \perp \overleftrightarrow{B F}$ or $m \angle A H B=90^{\circ}$

EXAMPLE 1 Identify a postulate illustrated by a diagram State the postulate illustrated by the diagram.


## Solution

## Postulate 8 Through any three noncollinear points there exists exactly one plane.

EXAMPLE 2 Identify postulates from a diagram
Use the diagram to write examples of Postulates 9 and 11.
Solution:
Postulate 9 Plane $Q$ contains at least three noncollinear points, $W, V$, and $Y$.

Postulate 11 The intersection of plane $P$ and plane $Q$ is line $b$.


## EXAMPLE 3 Use given information to sketch a diagram

Sketch a diagram showing $\overline{R S}$ perpendicular to $\widehat{T V}$, intersecting at point X .

## Solution:

Step 1 Draw $\overline{R S}$ and label points $R$ and $S$.
Step 2 Draw a point $X$ between $R$ and $S$.

Step 3 Draw $\overleftrightarrow{T V}$ through $X$ so that it is perpendicular to $\overline{R S}$.


EXAMPLE 4 Interpret a diagram in three dimensions
Which of the following statements cannot be assumed from the diagram?
$E, D$, and $C$ are collinear.
The intersection of $\overleftrightarrow{B D}$ and $\overleftrightarrow{E C}$ is $D$.
$\overleftrightarrow{B D} \perp \overleftrightarrow{E C}$
$\overleftrightarrow{E C} \perp$ plane $G$


Solution: With no right angles marked, you cannot assume that $\overleftrightarrow{B D} \perp \overleftrightarrow{E C}$ or $\overleftrightarrow{E C} \perp$ plane $G$.
2.4 Cont.

Checkpoint Use the diagram In Example 2 to complete the following exercises.

1. Which postulate allows you to say that the intersection of line $a$ and line $b$ is a point?

Postulate 7
2. Write examples of Postulates 5 and 6.

Line a passes through $X$ and $Y$; line a contains points $X$ and $Y$.

Checkpoint Complete the following exercises.
3. In Example 3, if the given information indicated that $\overline{R X}$ and $\overline{X S}$ are congruent, how would the diagram change?
Point $X$ would be drawn as the midpoint of $\overline{R S}$ and the congruent segments would be marked.
4. In the diagram for Example 4, can you assume that $\overleftrightarrow{B D}$ is the intersection of plane $F$ and plane $G$ ?
Yes

### 1.2 Use Segments and Congruence

Obj.: Use segment postulates to identify congruent segments.

## Key Vocabulary

- Postulate, axiom - In Geometry, a rule that is accepted without proof is called a postulate or axiom.
- Coordinate - The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.
- Distance - The distance between points $A$ and $B$, written as $A B$, is the absolute value of the difference of the coordinates of $A$ and $B$.
- Between- When three points are collinear, you can say that one point is between the other two.
- Congruent segments - Line segments that have the same length are called congruent segments.


## POSTULATE 1 Ruler Postulate POSTULATE 2 Segment Addition



If $B$ is between $A$ and $C$, then $A B+B C=A C$. If $A B+B C=A C$, then
 B is between A and C .

Lengths are equal.

$$
\begin{gathered}
A B=C D \\
\pi
\end{gathered}
$$

"is equal to"

Segments are congruent.

$$
\overline{\mathrm{AB}} \underset{\pi}{\cong} \overline{\mathbf{C B}}
$$

"is congruent to"

## EXAMPLE 1 Apply the Ruler Postulate

Measure the length of $\overline{C D}$ to the nearest tenth of a centimeter.


Solution
Align one mark of a metric ruler with $C$. Then estimate
the coordinate of $D$. For example, if you align $C$ with 1 ,
$D$ appears to align with 4.7 . D appears to align with 4.7 .

$C D=|\underline{4.7}-\underline{1}|=\underline{3.7} \quad$ Ruler postulate
The length of $\overline{C D}$ is about 3.7 centimeters.
EXAMPLE 2 Apply the Segment Addition Postulate
MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to
St. Louis, Missouri.

## Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.
$L S=L T+T S=380+360=740$
The distance from Lubbock to St. Louis is about 740 miles.


EXAMPLE 3 Find a length
Use the diagram to find $K L$.
Solution

(1.2 cont.)

Use the Segment Addition Postulate to write an equation. Then solve the equation to find $K L$.

$$
\begin{aligned}
& \frac{J L}{38}=\frac{J K}{23}=\frac{15}{23}+K L \\
& \underline{23}
\end{aligned}
$$

Segment Addition Postulate
Substitute for $J L$ and $J K$.
Subtract 15 from each side.

## EXAMPLE 4 Compare segments for congruence

Plot $F(4,5), G(-1,5), H(3,3)$, and $J(3,-2)$ int a coordinate plane. Then determine whether FG and HJ are congruent.
Solution: Horizontal segment: Subtract the $x$-coordinates of the endpoints.
$F G=|\underline{4-(-1)}|=\underline{5}$
Vertical segment: Subtract the $y$-coordinates of the endpoints.
$H J=|\underline{3-(-2)}|=\underline{5}$
$\overline{F G}$ and $\overline{H J}$ have the same length.


Checkpoint Complete the following exercises.

1. Find the length of $\overline{A B}$ to the nearest $\frac{1}{8}$ inch.

$1 \frac{7}{8}$ inches
2. Find $Q S$ and $P Q$.

61; 24

3. Consider the points $A(-2,-1), B(4,-1), C(3,0)$, and $D(3,5)$. Are $\overline{A B}$ and $\overline{C D}$ congruent?
No

### 1.3 Use Midpoint and Distance Formulas

## Obj.: Find lengths of segments in the coordinate plane.

## Key Vocabulary

- Midpoint - The midpoint of a segment is the point that divides the segment into two congruent segments.
- Segment bisector - A segment bisector is a point, ray, line, line segment, or plane the at intersects the segment at its midpoint.

$M$ is the midpoint of $\overline{\boldsymbol{A B}}$.

$\overleftrightarrow{C D}$ is a segment bisector of $\overline{A B}$. .
So, $\overline{A B} \cong \overline{A B}$ and $\underline{A M}=M B$.
So, $\overline{A B} \cong \overline{A B}$ and $A M=M B$.


## The Midpoint Formula KEY CONCEPT

The coordinates of the midpoint of a segment are the averages of the $\underline{x}$-coordinates and of the $\mathbf{y}$-coordinates of the endpoints. If $A\left(x_{1}, y_{1}\right)$ and $B\left(x_{2}, y_{2}\right)$ are points in a coordinate plane, then the midpoint M of $\overline{\boldsymbol{A B}}$ has coordinates

$$
\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)
$$



The Distance Formula KEY CONCEPT
If $A\left(x_{1}, y_{1}\right)$ and $B\left(x_{2}, y_{2}\right)$ are points is a coordinate plane, then the distance between $A$ and $B$ is

$$
A B=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$



## EXAMPLE 1 Find segment lengths

Find RS.
Solution: Point $T$ is the midpoint of $\overline{R S}$. So, $R T=\underline{T S}=21.7$.

$$
\begin{array}{rlrl}
R S & =\frac{R T}{}+\underline{T S} & & \text { Segment Addition Postulate } \\
& =21.7 \\
& =43.4 & & \text { Substitute. }
\end{array}
$$



The length of $\overline{R S}$ is 43.4 .

EXAMPLE 2 Use algebra with segment lengths
ALGEBRA Point C is the midpoint of $\overline{\boldsymbol{B D}}$.
Find the length of $\overline{\boldsymbol{B C}}$.
Solutigef:1 Write and solve an equation.

$$
\begin{aligned}
B C & =C D \\
3 x-2 & =2 x+3 \\
\underline{x-2} & =3 \\
x & =5
\end{aligned}
$$

Write equation.
Substitute.
Subtract $2 x$ from each side.


Step 2 Evaluate the expression for $B C$ when $x=5$.

$$
B C=3 x-2=3(5)-2=13
$$

So, the length of $\overline{B C}$ is 13 .
a. FIND MIDPOINT The endpoints of $\overline{\boldsymbol{P R}}$ are $\mathrm{P}(-2,5)$ and $\mathrm{R}(4,3)$. Find the coordinates of the midpoint M.

## Solution:

a. Use the Midpoint Formula.
$M\left(\frac{\boxed{-2}+\boxed{4}}{\frac{\boxed{2}}{2}}, \frac{\boxed{5}+\boxed{3}}{\boxed{2}}\right)$
$=M(1,4)$


The coordinates of the midpoint of $\overline{P R}$ are $M(1,4)$.
b. FIND ENDPOINT The midpoint of $\overline{\boldsymbol{A C}}$ is $\mathrm{M}(3,4)$. One endpoint is $A(1,6)$. Find the coordinates of endpoint $C$.

## Solution:

Step 1 Find $x$. Step 2 Find $y$. The coordinates of endpoint $C$ are

$$
\begin{array}{rlrl}
\frac{\boxed{1}+x}{2} & =\frac{3}{2} & \frac{\boxed{6}+y}{2} & =4 \\
\frac{1}{2}+x & =\frac{6}{5} & \boxed{6}+y & =8 \\
x & =\boxed{8} & y & =2
\end{array}
$$



The Distance Formula is based on the Pythagorean Theorem, which you will see again when you work with right triangles in Chapter 7.

## Distance Formula

$$
(A B)^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}
$$



Pythagorean Theorem

$$
c^{2}=a^{2}+b^{2}
$$



## EXAMPLE 4 Use the Distance Formula

What is the approximate length of $\overline{\boldsymbol{T R}}$, with endpoints $\mathrm{T}(-4,3)$ and
$\mathrm{R}(3,2)$ ?
Solution:

## Use the Distance Formula.

$$
\begin{aligned}
R T & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(-4-3)^{2}+(3-2)^{2}} \\
& =\sqrt{(-7)^{2}+(1)^{2}} \\
& =\sqrt{49}+\underline{1} \\
& =\sqrt{50}
\end{aligned}
$$

Distance Formula Substitute.

Subtract.


Evaluate powers.
Add.

$$
=\approx 7.07
$$

Use a calculator.

The length of $\overline{R T}$ is about

### 1.3 Cont.

Checkpoint Complete the following exercise.

1. Find $A B$.

$4 \frac{1}{2}$
Checkpoint Complete the following exercise.
2. Point $K$ is the midpoint of $\overline{J L}$. Find the length of $\overline{K L}$.

$6 \frac{1}{5}$

Checkpoint Complete the following exercises.
3. The endpoints of $\overline{C D}$ are $C(-8,-1)$ and $D(2,4)$. Find the coordinates of the midpoint $M$.
$M\left(-3, \frac{3}{2}\right)$
4. The midpoint of $\overline{X Z}$ is $M(5,-6)$. One endpoint is $X(-3,7)$. Find the coordinates of endpoint $Z$. $(13,-19)$

## ( Checkpoint Complete the following exercise.

5. What is the approximate length of $\overline{G H}$, with endpoints $G(5,-1)$ and $H(-3,6) ?$
about 10.63

### 1.4 Measure and Classify Angles

Obj.: Name, measure, and classify angles.

## Key Vocabulary

- Angle - An angle consists of two different rays with the same endpoint.
- Sides, vertex of an angle - The rays are the sides of the angle.

The endpoint is the vertex of the angle.


- Measure of an angle - A protractor can be used to approximate the measure of an angle. An angle is measured in units called degrees $\left({ }^{\circ}\right)$.

Words The measure of $\angle W X Z$ is $32^{\circ}$. Symbols $m \angle W X Z=32^{\circ}$

- Congruent angles - Two angles are congruent angles if they have the same measure.
- Angle bisector - An angle bisector is a ray that divides an angle into two angles that are congruent.


## POSTULATE 3 - Protractor Postulate

Consider $\overrightarrow{O B}$ and a point $A$ on one side of $\overrightarrow{O B}$. The rays of the form $\overrightarrow{O A}$ can be matched one to one with the real numbers from 0 to 180. The measure of $\angle A O B$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{O A}$ and $\overrightarrow{O B}$.


CLASSIFYING ANGLES Angles can be classified as acute, right, obtuse, and straight, as shown below.


POSTULATE 4 - Angle Addition Postulate (AAP)
Words If $\underline{P}$ is in the interior of $\angle R S T$, then the measure of $\angle R S T$ is equal to the sum of the measures of $\angle R S P$ and $\angle P S T$.
Symbols If $P$ is in the interior of $\angle R S T$, then

$$
\underline{\mathrm{m}} \angle \mathrm{RST}=\mathrm{m} \angle \mathrm{RSP}+\mathrm{m} \angle \mathrm{PST} .
$$


 angels are congruent

Angle measures are equal.
$m \angle A=m \angle B$ "equal to"

Angles are congruent.
$\angle A \cong \angle B$
"is congruent to"

EXAMPLE 1 Name angles
Name the three angles in the diagram. Solution:
$\angle A B C$, or $\angle C B A$
$\angle \angle C B D$, or $\angle D B C$
$\angle A B D$, or $\angle D B A$

EXAMPLE 2 Measure and classify angles Use the diagram to find the measure of the indicated angle. Then classify the angle.
a. $\angle \mathrm{WSR}$
b. $\angle \mathrm{TSW}$
c. $\angle R S T$
d. $\angle \mathrm{VST}$

## Solution:

a. $\overrightarrow{S R}$ is lined up with the $0^{\circ}$ on the $\qquad$ outer scale of the protractor. $\overrightarrow{S W}$ passes through $65^{\circ}$ on the outer scale. $\mathrm{So}, m \angle \mathrm{WSR}=65^{\circ}$. It is an acute angle.
b. $\overrightarrow{S T}$ is lined up with the $0^{\circ}$ on the inner scale of the protractor. $\overrightarrow{S W}$ passes through $1 \overline{15^{\circ}}$ on the inner scale. So, $m \angle T S W=115^{\circ}$. It is an obtuse angle.

## EXAMPLE 3 Find angle measures

ALGEBRA Given that $\mathrm{m} \angle \mathrm{GFJ}=155^{\circ}$, find $\mathrm{m} \angle \mathrm{GFH}$ and $\mathrm{m} \angle \mathrm{HFJ}$.

## Solution:

Step 1 Write and solve an equation to find the value of $x$.

$$
\begin{aligned}
& m \angle G F J=m \angle \underline{G F H}+m \angle \underline{H F J} \\
& \begin{aligned}
155^{\circ} & =(4 x+4)^{\circ}+(4 x-1)^{\circ} \\
\frac{155}{152} & =8 x+3 \\
\frac{15 x}{19} & =x
\end{aligned} \\
& \\
&
\end{aligned}
$$

Angle Addition Postulate
Substitute.
Combine like terms.
Subtract 3 from each side.
Divide each side by 8 .

## EXAMPLE 4 Identify congruent angles

Identify all pairs of congruent angles in the diagram.
If $m \angle P=120^{\circ}$, what is $m \angle N$ ? There are two pairs of congruent angles:
Solution:

$$
\begin{aligned}
& \angle P \cong \angle N \text { and } \angle L \cong \angle M \\
& \text { Because } \angle P \cong \angle N, m \angle P=m \angle N \\
& \text { So, } m \angle N=120^{\circ} .
\end{aligned}
$$

c. $m \angle R S T=180^{\circ}$. It is a straight angle.
d. $m \angle V S T=90^{\circ}$. It is a right angle.


Step 2 Evaluate the given expressions when $x=19$.

$$
\begin{aligned}
& m \angle \mathrm{GFH}=(4 x+4)^{\circ}=(4 \cdot 19+4)^{\circ}=80^{\circ} . \\
& m \angle H F J=(4 x-1)^{\circ}=(4 \cdot 19-1)^{\circ}=75^{\circ} .
\end{aligned}
$$

So, $m \angle \mathrm{GFH}=\underline{80^{\circ}}$ and $m \angle \mathrm{HFJ}=75^{\circ}$.

(1.4 cont.)


### 1.4 Cont.

Checkpoint Complete the following exercises.

1. Name all the angles in the diagram at the right.

$\angle F G H$ or $\angle H G F, \angle F G J$ or $\angle J G F, \angle J G H$ or $\angle H G J$
2. What type of angles do the $x$-axis and $y$-axis form in a coordinate plane?
right angles

Checkpoint Complete the following exercise.
3. Given that $\angle V R S$ is a right angle, find $m \angle V R T$ and $\angle T R S$. $m \angle V R T=19^{\circ}, m \angle T R S=71^{\circ}$


Checkpoint Complete the following exercises.
4. Identify all pairs of congruent angles in the diagram. If $m \angle B=135^{\circ}$, what is $m \angle D$ ?

$\angle B \cong \angle D$ and $\angle A \cong \angle C ; 135^{\circ}$
5. In the diagram below, $\overrightarrow{K M}$ bisects $\angle L K N$ and $m \angle L K M=78^{\circ}$. Find $m \angle L K N$.
$156^{\circ}$


### 2.6 Prove Statements about Segments and Angles

Obj.: Write proofs using geometric theorems.

## Key Vocabulary

- Proof - A proof is a logical argument that shows a statement is true. There are several formats for proofs.
- Two-column proof - A two-column proof has numbered statements and
corresponding reasons that show an argument in a logical order.
- Theorem - The reasons used in a proof can include definitions, properties,
postulates, and theorems. A theorem is a statement that can be proven.


## THEOREMS

## Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.
Reflexive For any segment $\mathrm{AB}, \overline{\mathrm{AB}} \cong \overline{\mathrm{AB}}$.
Symmetric If $\overline{A B} \cong \overline{C D}$, then $\overline{C D} \cong \overline{A B}$.
Transitive If $\overline{\overline{A B}} \cong \overline{\mathrm{CD}}$ and $\overline{\mathrm{CD}} \cong \overline{\overline{E F}}$, then $\overline{\overline{A B}} \cong \overline{\mathrm{EF}}$.

## Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

| Reflexive | For any angle $A, \angle A \cong \angle A$ |
| :--- | :--- |
| Symmetric | If $\angle A \cong \angle B$, then $\angle B \cong \angle A$ |
| Transitive | If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. |

EXAMPLE 1 Write a two-column proof
Use the diagram to prove $\mathrm{m} \angle 1=\mathrm{m} \angle 4$.
Given: $m \angle 2=m \angle 3, m \angle A X D=m \angle A X C$
Prove: $m \angle 1=m \angle 4$

| $>$ Statements | Reasons |
| :--- | :--- |
| 1. $m \angle A X C=m \angle A X D$ | 1. Given |

2. $m \angle A X D$

$$
=m \angle \underline{1}+m \angle \underline{2}
$$

3. $m \angle A X C$
4. Angle Addition Postulate


Writing a twocolumn proof is a formal way of organizing your reasons to show a statement is true.
$=m \angle \underline{3}+m \angle \underline{4}$
4. $m \angle 1+m \angle 2$
$=m \angle 3+m \angle 4$
5. $m \angle 2=m \angle 3$
6. $m \angle 1+m \angle 3$
$=m \angle 3+m \angle 4$
7. $m \angle 1=m \angle 4$
3. Angle Addition Postulate
4. Substitution Property of Equality
5. Given
6. Substitution Property of Equality
7. Subtraction Property of Equality

## EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.
If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$

## Solution: Symmetric Property of Angle Congruence

If you know that $\overrightarrow{B D}$ bisects $\angle A B C$, prove that $m \angle A B C$ is two times $m \angle 1$.

Given: $\overrightarrow{B D}$ bisects $\angle A B C$.
Prove: $m \angle A B C=2{ }^{\circ} \angle 1$
\(\left.$$
\begin{array}{l|l}\text { Statements } & \text { Reasons } \\
\hline \text { 1. } \overrightarrow{B D} \text { bisects } \angle A B C . & \text { 1. } \frac{\text { Given }}{\text { 2. } \angle 1 \cong \angle 2}\end{array}
$$ \quad $$
\begin{array}{l}\text { 2. } \begin{array}{l}\text { Definition of angle } \\
\text { bisector }\end{array} \\
\text { 3. } m \angle 1=m \angle 2\end{array}
$$ \quad \begin{array}{l}3. Definition of <br>

congruent angles\end{array}\right]\)| 4. $\frac{\text { Angle Addition }}{\frac{\text { Postulate }}{}}$ |  |
| :--- | :--- |
| 4. $m \angle 1+m \angle 2=m \angle A B C$ | 5. Substitution Property |
| of Equality |  |

## EXAMPLE 4 Solve a multi-step problem

Interstate There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and Mason exit. The distance between rest area B and Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

## Solution:

Solution
Step 1 Draw a diagram.


Step 2 Draw diagrams showing relationships.


Step 3 Write a proof.
Given $R$ is the midpoint of $\overline{A M}, M B=A R$.
Prove $M$ is the midpoint of $\overline{R B}$.
4. $M B=R M$
5. $\overline{M B} \cong \overline{R M}$
6. $M$ is the midpoint of $\overline{R B}$.

## Reasons

1. Given
2. Definition of midpoint
3. Definition of congruent segments
4. Transitive Property of Congruence
5. Definition of congruent segments
6. Definition of midpoint

## CONCEPT SUMMARY - Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-andreason pair you write is given information.

## Proof of the Symmetric Property of Angle Congruence

GIVEN $\leqslant 1 \cong \angle 2$
PROVE $\leqslant 2 \cong \angle 1$
STATEMENTS
$\left.\begin{array}{ll}\text { 1. } \angle 1 \cong \angle 2 & \text { REASONS } \\ \text { 2. } m \angle 1=m \angle 2 & \text { 1. Given } \\ \text { 3. } m \angle 2=m \angle 1 & \text { 2. Definition of congruent angles } \\ \text { 4. Symmetric Property of Equality } \\ \text { 3. } \angle 2 \cong \angle 1 & \text { 4. Definition of congruent angles }\end{array}\right\}$
The number of
statements will vary $\quad$ Remember to give a reason

Copy or draw diagrams and label given information to help develop proofs.


Theorems Statements based on facts that you know or on conclusions from deductive reasoning 2.6 Cont.

## Checkpoint Complete the following exercises.

1. Three steps of a proof are shown. Give the reasons for the last two steps.
Given $B C=A B$


Prove $A C=A B+A B$

| Statements | Reasons |
| :--- | :--- |
| 1. $B C=A B$ | 1. Given |

2. $A C=A B+B C$
3. $A C=A B+A B$
4. Segment Addition Postulate
5. Substitution Property of Equality
6. Name the property illustrated by the statement. If $\angle H \cong \angle T$ and $\angle T \cong \angle B$, then $\angle H \cong \angle B$.
Transitive Property of Angle Congruence
Checkpoint Complete the following exercise.
7. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?
Rest area $A$ and rest area B

### 1.5 Describe Angle Pair Relationships

Obj.: Use special angle relationships to find angle measure.

## Key Vocabulary

- Complementary angles - Two angles are complementary angles if the sum of their measures is $90^{\circ}$.
- Supplementary angles - Two angles are supplementary angles if the sum of their measures is $180^{\circ}$.
- Adjacent angles - Adjacent angles are two angles that share a common vertex and side, but have no common interior points.
- Linear pair - Two adjacent angles are a linear pair if their noncommon sides are opposite rays.
- Vertical angles - Two angles are vertical angles if their sides form two pairs of opposite rays.

$\angle 1$ and $\angle 2$ are a linear pair.
$\angle 3$ and $\angle 6$ are vertical angles. $\angle \underline{4}$ and $\angle \underline{5}$ are vertical angles.



## EXAMPLE 1 Identify complements and supplements

 In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.Solution: Because $52^{\circ}+38^{\circ}=90^{\circ}, \angle A B D$ and $\angle C D B$ are complementary angles.

Because $52^{\circ}+128^{\circ}=180^{\circ}, \angle A B D$ and $\angle E D B$ are supplementary angles.

Because $\angle C D B$ and $\angle B D E$ share a common vertex and side, they are adjacent angles.


EXAMPLE 2 Find measures of a complement and a supplement
a. Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 2=57^{\circ}$, find $m \angle 1$.
b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4=41^{\circ}$, find $m \angle 3$.

## Solution:

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.


$$
m \angle 3=\underline{180^{\circ}}-\underline{m \angle 4}=\underline{180^{\circ}}-41^{\circ}=139^{\circ}
$$

EXAMPLE 3 Find angle measures
SPORTS The basketball pole forms a pair of supplementary angles with the ground. Find $m \angle B C A$ and $m \angle D C A$.

## Solution:

Step 1 Use the fact that $180^{\circ}$
is the sum of the measures
of supplementary angles.

$$
\begin{array}{rlrl}
m \angle B C A+m \angle D C A & =180^{\circ} & \text { Write equation. } \\
\left(\underline{3 x+8)^{\circ}+\left(\frac{4 x-3)^{\circ}}{}\right.}=\underline{180^{\circ}}\right. & & \text { Substitute. } \\
\frac{7 x+5}{7 x} & =180 & & \text { Combine like terms. } \\
\boxed{x} & =175 & & \text { Subtract. } \\
& & \text { Divide. }
\end{array}
$$



Step 2 Evaluate the original expressions when $x=\underline{25}$.

$$
\begin{aligned}
& m \angle B C A=(3 x+8)^{\circ}=(3 \cdot 25+8)^{\circ}=83^{\circ} . \\
& m \angle D C A=(4 x-3)^{\circ}=(4 \cdot 25-3)^{\circ}=97^{\circ} .
\end{aligned}
$$

The angle measures are $83^{\circ}$ and $97^{\circ}$.

## EXAMPLE 4 Identify angle pairs

Identify all of the linear pairs and all of the vertical angles
in the figure at the right.
Solution:
In the diagram, one side of $\angle 1$ and one side of $\angle 4$ are opposite rays. But the angles are not a linear pair because they are not adjacent

To find vertical angles, look for angles formed by intersecting lines.
$\qquad$ and $\qquad$ are vertical angles.
To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.
 $\frac{\angle 1}{\text { linear pair }}$ $\angle 2$ are a linear pair. $\qquad$ $\angle 2$ and $\qquad$ $\angle 3$ are a linear pair.

## EXAMPLE 5 Find angle measures in a linear pair

ALGEBRA Two angles form a linear pair. The measure of one angle is
4 times the measure of the other. Find the measure of each angle.

## Solution:

Let $x^{\circ}$ be the measure of one angle. The measure of the other angle is $4 x^{\circ}$. Then use the fact that the angles of a linear pair are supplementary to write an equation.
$\begin{aligned} \underline{x^{\circ}}+\underline{4 x^{\circ}} & =180^{\circ} \\ \underline{5 x} & =180\end{aligned} \quad$ Write an equation..
$\underline{x}=36 \quad$ Divide each side by 5
The measures of the angles are $36^{\circ}$ and $4\left(36^{\circ}\right)=144^{\circ}$.

## CONCEPT SUMMARY Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right:


- All points shown are coplanar.
- Points $A, B$, and $C$ are collinear, and $B$ is between $A$ and $C$.
$\cdot \overrightarrow{A C}, \overrightarrow{B D}$, and $B E$ intersect at point $B$.
- $\angle D B E$ and $\angle E B C$ are adjacent angles, and $\angle A B C$ is a straight angle.
- Point $E$ lies in the interior of $\angle D B C$.

In the diagram above, you cannot conclude that $\overline{A B} \cong \overline{B C}$, that $\angle D B E \cong \angle E B C$, or that $\angle A B D$ is a right angle. This information must be indicated by marks, as shown at the right.


### 1.5 Cont.

## - Checkpoint Complete the following exercises.

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

complementary: $\angle D E F$ and $\angle A B C$;
supplementary; $\angle F E G$ and $\angle A B C$;
adjacent: $\angle D E F$ and $\angle F E G$
2. Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 1=73^{\circ}$, find $m \angle 2$.
$17^{\circ}$
3. Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4=37^{\circ}$, find $m \angle 3$.
$143^{\circ}$

Checkpoint Complete the following exercise.
4. In Example 3, suppose the angle measures are $(5 x+1)^{\circ}$ and $(6 x+3)^{\circ}$. Find $m \angle B C A$ and $m \angle D C A$.
$81^{\circ}$ and $99^{\circ}$

Checkpoint Complete the following exercise.
5. Identify all of the linear pairs and all of the vertical angles in the figure.

linear pairs: none; vertical angles: $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5, \angle 3$ and $\angle 6$

## Checkpoint Complete the following exercise.

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the meaure of each angle.

### 2.7 Prove Angle Pair Relationships

## Obj.: Use properties of special pairs of angles.

## Key Vocabulary

- Complementary angles - Two angles whose measures have the sum $90^{\circ}$.
- Supplementary angles - Two angles whose measures have the sum $180^{\circ}$.
- Linear pair - Two adjacent angles whose noncommon sides are opposite rays.
- Vertical angles - Two angles whose sides form two pairs of opposite rays.


## Right Angles Congruence Theorem

All right angles are congruent.

## Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent. If $\angle 1$ and $\underline{\angle 2}$ are supplementary and
$\angle 3$ and $\underline{\angle 2}$ are supplementary, then $\angle 1 \cong \angle 3$.
Congruent Complements Theorem
If two angles are complementary to the same angle (or to congruent angles), then they are congruent.
If $\angle 4$ and $\angle 5$ are complementary and
$\angle 6$ and $\underline{\angle 5}$ are complementary, then $\angle 4 \cong \angle 6$.

## Linear Pair Postulate

If two angles form a linear pair, then they are supplementary. $\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m \angle 1+m \angle 2=180^{\circ}$.

## Vertical Angles Congruence Theorem

Vertical angles are congruent.
$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$
EXAMPLE 1 Use right angle congruence Write a proof.
GIVEN: $\overline{\mathrm{JK}} \perp \overline{\mathrm{KL}}, \overline{\mathrm{ML}} \perp \overline{\mathrm{KL}}$
PROVE: $\angle \mathrm{K} \cong \angle \mathrm{L}$
Statements

1. $\overline{J K} \perp \overline{K L}, \overline{M L} \perp \overline{K L}$
2. $\angle K$ and $\angle L$ are
right angles.
3. $\angle K \cong \angle L$

Reasons
$\cong$ Comp. Th


Abbreviation
All Rt. $\angle$ 's $\cong$
$\cong$ Supp. Th


1. Given
2. Definition of perpendicular lines
3. Right Angles Congruence Theorem

Write a proof.
GIVEN: $\angle 1$ and $\angle 2$ are supplements. $\angle 1$ and $\angle 4$ are supplements.

$$
\mathrm{m} \angle 2=45^{\circ}
$$



PROVE: $\mathrm{m} \angle 4=45^{\circ}$
Statements

1. $\angle 1$ and $\angle 2$ are
supplements. $\angle 1$ and
$\angle 4$ are supplements.
2. $\angle 2 \cong \angle 4$
3. $m \angle 2=m \angle 4$
4. $m \angle 2=45^{\circ}$
5. $m \angle 4=45^{\circ}$

## Reasons

1. $\qquad$
2. Congruent Supplements Theorem
3. Definition of congruent angles
4. $\qquad$
5. Substitution Property of Equality

## EXAMPLE 3 Use the Vertical Angles Congruence Theorem Prove vertical angles are congruent.

GIVEN: $\angle 4$ is a right angle.
PROVE: $\angle 2$ and $\angle 4$ are supplementary.

## Reasons

1. $\angle 4$ is a right angle.
2. $m \angle 4=90^{\circ}$
3. $\angle 2 \cong \angle 4$
4. $m \angle 2=m \angle 4$
5. $m \angle 2=90^{\circ}$
6. $\frac{\angle 2 \text { and } \angle 4 \text { are }}{\text { supplementary. }}$
7. Given
8. Definition of a right angle
9. Vertical Angles
Congruence Theorem
10. Definition of congruent angles
11. Substitution Property of Equality
12. $m \angle 2+m \angle 4=180^{\circ}$

## EXAMPLE 4 Find angle measures

Write and solve an equation to find $x$. Use $x$ to find the $m \angle F K G$.

## Solution:

Because $m \angle F K G$ and $m \angle G K H$ form a linear pair, the sum of their measures is $180^{\circ}$.


$$
(4 x-1)^{\circ}+113^{\circ}=180^{\circ} \quad \text { Write equation. }
$$

$$
\begin{aligned}
4 x+\frac{112}{4 x} & =\frac{180}{68} \\
x & =17
\end{aligned}
$$

Simplify.
Subtract 112 from each side.

Divide each side by 4.

Use $x=\underline{17}$ to find $m \angle F K G$.
$m \angle F K G=(4 x-1)^{\circ} \quad$ Write equation.
$=[4(\underline{17})-1]^{\circ} \quad$ Substitute 17 for $x$.
$=[68-1]^{\circ} \quad$ Multiply.
$=67^{\circ}$ Simplify.

The measure of $\angle F K G$ is $\qquad$ $67^{\circ}$ .
2.7 Cont.

Checkpoint Complete the following exercises.

1. In Example 1, suppose you are given that $\angle K \cong \angle L$.

Can you use the Right Angles Congruence Theorem to prove that $\angle K$ and $\angle L$ are right angles? Explain.
No, you cannot prove that $\angle K$ and $\angle L$ are right angles, because the converse of the Right Angles Congruence Theorem is not always true.
2. Suppose $\angle A$ and $\angle B$ are complements, and $\angle A$ and $\angle C$ are complements. Can $\angle B$ and $\angle C$ be supplements? Explain.
No, $\angle B$ and $\angle C$ are complements by the Congruent Complements Theorem, so they cannot be supplements.
Checkpoint In Exercises 3 and 4, use the diagram.
3. If $m \angle 4=63^{\circ}$, find $m \angle 1$ and $m \angle 2$.
$m \angle 1=117^{\circ}, m \angle 2=63^{\circ}$
4. If $m \angle 3=121^{\circ}$, find $m \angle 1, m \angle 2$, and $m \angle 4$.
$m \angle 1=121^{\circ}, m \angle 2=59^{\circ}, m \angle 4=59^{\circ}$
Checkpoint Complete the following exercise.
5. Find $m \angle A E B$.
$m \angle A E B=70^{\circ}$


### 1.6 Classify Polygons

Obj.: Classify polygons.

## Key Vocabulary

- Polygon - In geometry, a figure that lies in a plane is called a plane figure. A polygon is a closed plane figure with the following properties:
- Side - 1. It is formed by three or more line segments called sides.

2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

- Vertex - Each endpoint of a side is a vertex of the polygon. The plural of vertex is vertices.
- Convex - A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.
- Concave - A polygon that is not convex is called nonconvex or concave.
- $n$-gon - The term $n$-gon, where $n$ is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.
- Equilateral - In an equilateral polygon, all sides are congruent.
- Equiangular - In an equiangular polygon, all angles in the interior of the polygon are congruent.
- Regular - A regular polygon is a convex polygon that is both equilateral and equiangular.

A polygon can be named by listing the vertices in consecutive order. For example, ABCDE and CDEAB are both correct names for the polygon at the right.

regular pentagon


concave polygon

CLASSIFYING POLYGONS A polygon is named by the number of its sides.

| \# of sides | Type of polygon | \# of sides | Type of polygon |
| :---: | :---: | :---: | :---: |
| $\frac{3}{4}$ | Triangle | $\underline{8}$ | Octagon |
| $\frac{\text { Quadrilateral }}{}$ | $\frac{9}{\text { Pentagon }}$ | $\underline{10}$ | $\underline{\text { Nonagon }}$ |
| $\frac{\text { Decagon }}{6}$ | $\underline{\text { Hexagon }}$ | $\underline{12}$ | Dodecagon |
| 7 | $\underline{\text { Heptagon }}$ | $\underline{n}$ | $\underline{n-g o n}$ |

Tell whether the figure is a polygon and whether it is convex or concave.
a.

b.

C.


## Solution:

a. some segments intersect more than two segments, so it is not a polygon.
b. The figure is $\qquad$ a convex polygon
c. The figure is a concave polygon .

## EXAMPLE 2 Classify polygons

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.
Solution: The polygon has 8 sides. It is equilateral and equiangular, so it is a regular octagon.


## EXAMPLE 3 Find side lengths

ALGEBRA A head of a bolt is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal bolt. Find the length of a side.

## Solution:

First, write and solve an equation to find the value of $x$. Use the fact that the sides of a regular hexagon are congruent.

$$
\begin{array}{rlrl}
\frac{4 x+3}{4} & =\frac{5 x-1}{x} & & \text { Write an equation. } \\
& & \text { Simplify. }
\end{array}
$$

$(5 x-1) \mathrm{mm}$
$\left(4 x^{\circ}+3\right) \mathrm{mm}$


Hexagonal means
"shaped like a hexagon."

Then evaluate one of the expressions to find a side length when $x=4$.

$$
4 x+3=4(4)+3=19
$$

The length of a side is $\qquad$ 19 millimeters.
1.6 Cont. (Write these on your paper)

- checkpoint Tell whether the figure is a polygon and whether it is convex or concave.

1. 


convex polygon
2.

not a polygon

Checkpoint Complete the following exercises.
3. Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular.
 quadrilateral
4. The expressions $(4 x+8)^{\circ}$ and $(5 x-5)^{\circ}$ represent the measures of two of the congruent angles in Example 3. Find the measure of an angle. $60^{\circ}$

### 2.1 Use Inductive Reasoning

Obj.: Describe patterns and use inductive reasoning.

## Key Vocabulary

- Conjecture - A conjecture is an unproven statement that is based on observations.
- Inductive reasoning - You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.
- Counterexample - A counterexample is a specific case for which the conjecture is false. You can show that a conjecture is false, however, by simply finding one counterexample.


## EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.
Solution:


Figure 2

Figure


Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eighths. Shade the section just below the horizontal segment at the left .


## EXAMPLE 2 Describe a number pattern

Describe the pattern in the numbers $-1,-4,-16,-64, \ldots$ and write the next three numbers in the pattern.

## Solution:

Three dots (. . .) tell you that the pattern continues. Notice that each number in the pattern is four times the previous number.


The next three numbers are $-256,-1024$, and -4096 .

## EXAMPLE 3 Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

## Solution:

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

| Number of points | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Picture | $\bullet$ | - | $\square$ | - | 4 |
| Number of <br> connections | 0 | -1 | 3 | 6 | $?$ |

Conjecture You can connect five noncollinear points $6+4$, or 10 different ways.

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

## Solution:

Step 1 Find a pattern using groups of small numbers.

$$
\begin{aligned}
& 1+3+5=9 \\
& =3 \cdot 3 \\
& \begin{aligned}
3+5+7 & =15 \\
& =5 \cdot 3
\end{aligned}
\end{aligned}
$$

$\begin{aligned} 5+7+9 & =\underline{21} \\ & =\underline{7} \cdot 3\end{aligned}$
$\begin{aligned} 7+9+11 & =27 \\ & =\underline{9} \cdot 3\end{aligned}$

Conjecture The sum of any three consecutive odd numbers is three times the second number.

Step 2 Test your conjecture using other numbers.

$$
\begin{aligned}
& -1+1+3=3=1 \cdot 3 \\
& 103+105+107=315=105 \cdot 3
\end{aligned}
$$

## EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.
Conjecture The difference of any two numbers is always smaller than the
larger number.
Solution:

To find a counterexample, you need to find a difference that is greater than the larger number.

Because 12 大 8 , a counterexample exists. The conjecture is false.

## EXAMPLE 6 Real world application

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.

## Solution:

The scatter plot shows that the values increased each year. So, one possible conjecture is that the average player in the NFL is earning more money today than in 1999.


### 2.1 Cont.

- Checkpoint Complete the following exercise.

1. Sketch the fifth figure in the pattern in Example 1.


Figure 5
Checkpoint Complete the following exercises.
2. Describe the pattern in the numbers $1,2.5,4,5.5, \ldots$ and write the next three numbers in the pattern.
The numbers are increasing by $1.5 ; 7,8.5,10$.
3. Rework Example 3 if you are given six noncollinear points.

15 different wavs
Checkpoint Complete the following exercise.
4. Make and test a conjecture about the sign of the product of any four negative numbers.

The result of the product of four negative numbers is a positive number; $(-1)(-2)(-5)(-1)=10$.

## Checkpoint Complete the following exercises.

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.
$\frac{4}{\frac{1}{2}}=8$
6. Use the graph in Example 6 to make a conjecture that could be true. Give an explanation that supports your reasoning.

The average salary of an NFL player in future years will be higher than the previous year; the average salary of an NFL player increased for the 5 years from 1999 to 2003.

### 2.3 Apply Deductive Reasoning

## Obj.: Use deductive reasoning to form a logical argument.

## Key Vocabulary

- Deductive reasoning - Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.
**** This is different from inductive reasoning, which uses specific examples and patterns to form a conjecture.*


## Laws of Logic KEY CONCEPT

## Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

## Law of Syllogism

If hypothesis $\underline{p}$, then conclusion $\underline{q}$.
If hypothesis $\underline{q}$, then conclusion $\underline{r}$.


If these statements are true,

If hypothesis $\underline{p}$, then conclusion $\underline{r}$.
then this statement is true.

## EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.
a. If two angles have the same measure, then they are congruent. You know that $m \angle A=m \angle B$.

## Solution:

a. Because $m \angle A=m \angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.
b. Jesse goes to the gym every weekday. Today is Monday.

Solution:
b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is a weekday ," and the conclusion is "then Jesse goes to the gym ."
"Today is Monday" satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.

## EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.
a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

Solution: a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.
If Ron eats lunch today, then he will drink a glass of milk.
b. If $x^{2}>36$, then $x^{2}>30$. If $x>6$, then $x^{2}>36$.

## Solution:

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.
If $x>6$, then $x^{2}>30$.
c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.
Solution:
c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

## EXAMPLE 3 Use inductive and deductive reasoning

What conclusion can you make about the sum of an odd integer and an odd integer?

## Solution:

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.
$-3+5=\underline{2},-1+5=\underline{4}, 3+5=\underline{8}$
$-3+(-5)=\frac{-8}{-2}, 1+(-5)=-4$,
$3+(-5)=$
$3+(-5)=-2$
Conjecture: Odd integer + Odd integer $=$ Even intege
Step 2 Let $n$ and $m$ each be any integer. Use deductive reasoning to show the conjecture is true.
$2 n$ and $2 m$ are even integers because any integer multiplied by 2 is even.
$2 n-1$ and $2 m+1$ are odd integers because $2 n$ and $2 m$ are even integers.

$$
\begin{aligned}
& (2 n-1)+(2 m+1) \text { represents the sum of } \\
& \text { an odd integer } 2 n-1 \text { and an odd integer } \\
& 2 m+1 . \\
& (2 n-1)+(2 m+1)=2(n+m)
\end{aligned}
$$

The result is the product of 2 and an integer $n+m$. So, $2(n+m)$ is an even integer.
The sum of an odd integer and an odd integer is an even integer.

## EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of inductive reasoning or deductive reasoning.
Explain your choice.
a. The runner's average speed decreases as time spent running increases.
Solution: a. Inductive reasoning, because it is based on a
b. The runner's average speed is slower when running for 40 minutes than when running for 10 minutes.


Solution: b. Deductive reasoning, because you are comparing values that are given on the graph

### 2.3 Cont.

## Checkpoint Complete the following exercises.

1. If $0^{\circ}<m \angle A<90^{\circ}$, then $A$ is acute. The measure of $\angle A$ is $38^{\circ}$. Using the Law of Detachment, what statement can you make?
$\angle A$ is acute.
2. State the law of logic that is illustrated below.

If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.
If you do your homework, then you can watch your favorite show.

Law of Syllogism
Checkpoint Complete the following exercise.
3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.
The sum of a negative integer and itself is twice the integer; $-n+(-n)=-2 n=2(-n)$.
(V) Checkpoint Complete the following exercises.
4. Use inductive reasoning to write another statement about the graph in Example 4.
Sample answer: The faster the average speed of the runner, the less time he or she is running.
5. Use deductive reasoning to write another statement about the graph in Example 4.

Sample answer: The runner's average speed is faster when running for 10 minutes than when running for 40 minutes.

### 2.2 Analyze Conditional Statements

Obj.: Write definitions as conditional statements.

## Key Vocabulary

- Conditional statement - A conditional statement is a logical statement that has two parts, a hypothesis and a conclusion.
- Converse - To write the converse of a conditional statement, exchange the hypothesis and conclusion.
- Inverse - To write the inverse of a conditional statement, negate (not) both the hypothesis and the conclusion.
- Contrapositive - To write the contrapositive, first write the converse and then negate both the hypothesis and the conclusion.
- If-then form, Hypothesis, Conclusion - When a conditional statement is written in if-then form, the "if" part contains the hypothesis and the "then" part contains the conclusion. Here is an example:

If it is raining, then there are clouds in the sky.
$\underbrace{}_{\text {hypothesis }}$
conclusion

- Negation - The negation of a statement is the opposite of the original statement.
- Equivalent statements - When two statements are both true or both false, they are called equivalent statements.
- Perpendicular lines - If two lines intersect to form a right angle, then they are perpendicular lines. You can write "line 1 is perpendicular to line $m$ " as $1 \perp m$.
- Biconditional statement - A biconditional statement is a
 statement that contains the converse and the phrase "if and only if."


## EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form. All vertebrates have a backbone.

## Solution:

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

## If an animal is a vertebrate, then it has a

backbone.

EXAMPLE 2 Write four related conditional statements
Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is true or false.
Solution:

If-then form

Converse
If-then form If you are an Olympian, then you are an athlete. True, Olympians are athletes.
Converse If you are an athlete, then you are an Olympian. False, not all athletes are Olympians.

Inverse

Contrapositive

Inverse If you are not an Olympian, then you are not an athlete. False, even if you are not an Olympian, you can still be an athlete.
Contrapositive If you are not an athlete, then you are not an Olympian. True, a person who is not an athlete cannot be an Olympian.

## EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.
a. $\overleftrightarrow{A C} \perp \overleftrightarrow{B D}$
b. $\angle A E D$ and $\angle B E C$ are a linear pair.

## Solution:


a. The statement is true. The right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.
b. The statement is false. Because $\angle A E D$ and $\angle B E C$ are not adjacent angles, $\angle A E D$ and $\angle B E C$ are not a linear pair.

## EXAMPLE 4 Write a biconditional

Write the definition of parallel lines as a biconditional.
Definition: If two lines lie in the same plane and do not intersect, then they are parallel. Solution:
Converse: If two lines are parallel, then they lie in the same plane and do not intersect.
Biconditional: Two lines are parallel if and only if they lie in the same plane and do not intersect.

### 2.2 Cont.

(V) checkpoint Write the conditional statement in if-then form.

1. All triangles have 3 sides.

If a figure is a triangle, then it has 3 sides.
2. When $x=2, x^{2}=4$.

If $x=2$, then $x^{2}=4$.

Checkpoint Complete the following exercises.
3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Squares are rectangles." Decide whether each statement is true or false.
If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.
Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.
Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.
Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.
4. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.
a. $\angle \mathrm{GLK}$ and $\angle J L K$ are supplementary.

b. $\overleftrightarrow{G J} \perp \overleftrightarrow{H K}$
(a) True; linear pairs of angles are supplementary.
(b) False; it is not known that the lines intersect at right angles.
5. Write the statement below as a biconditional.

Statement: If a student is a boy, he will be in group A. If a student is in group $A$, the student must be a boy. A student is in group A if and only if the student is a boy.

