1.1 Identify Points, Lines, and Planes

Objective: Name and sketch geometric figures.

Key Vocabulary

• Undefined terms - These words <u>do not</u> have formal definitions, but there is <u>agreement</u> aboutwhat they mean.

• **Point** - A **point** has no **dimension**. It is represented by a dot.

• Line - A line has <u>one</u> dimension. It is represented by a line with two arrowheads, but it extends <u>without</u> end. Through any two points, there is exactly one line. You can use any two points on a line to name it. Line AB (written as

 \overrightarrow{AB}) and points A and B are used here to define the terms below.

• Plane - A plane has <u>two</u> dimensions. It is represented by a shape that looks like a floor or a wall, but it <u>extends without end</u>. Through any <u>three</u> points not on the same line, there is exactly one plane. You can use <u>three</u> points that are not all on the same line to name a plane.



point A

plane M or plane ABC

endpoint

R

endpoint

A

endpoint A

A C

ray

- Collinear points Collinear points are points that lie on the same line.
- Coplanar points Coplanar points are points that lie in the same plane.

 Defined terms - In geometry, terms that can be described using <u>known</u> words such as <u>point</u> or <u>line</u> are called defined terms.

• Line segment, Endpoints - The line segment *AB*, or segment *AB*, (written as \overline{AB}) consists of the endpoints *A* and *B* and all points on \overline{AB} that are between *A* and *B*. Note that \overline{AB} can also be named \overline{BA} .

• **Ray** - The **ray** *AB* (written as \overrightarrow{AB}) consists of the <u>endpoint</u> *A* and <u>all</u> <u>points</u> on \overrightarrow{AB} that lie on the same side of *A* as *B*. Note that \overrightarrow{AB} and \overrightarrow{BA} are <u>different</u> rays.

• **Opposite rays** - If point <u>C</u> lies on \overrightarrow{AB} <u>between</u> A and B, then \overrightarrow{CA} and \overrightarrow{CB} are opposite rays.

• Intersection - The intersection of the figures is the set of <u>points</u> the figures have in <u>common</u>.

The intersection of two different lines is <u>a point</u>.

Solution:

The intersection of two different planes is a line.



EXAMPLE 1 Name points, lines, and planes

a. Give two other names for \overrightarrow{LN} and for plane Z.

b. Name three points that are collinear. Name four points that are coplanar.

- **a.** Other names for \overrightarrow{LN} are \overrightarrow{LM} and <u>line b</u>. Other names for plane Z are plane <u>LMP</u> and <u>LNP</u>.
- **b.** Points <u>*L*</u>, <u>*M*</u>, and <u>*N*</u> lie on the same line, so they are collinear. Points <u>*L*</u>, <u>*M*</u>, <u>*N*</u>, and <u>*P*</u> lie on the same line, so they are coplanar.



EXAMPLE 2 Name segments, rays, and opposite rays

a. Give another name for \overline{VX} .

b. Name all rays with endpoint *W*. Which of these rays are opposite rays? Solution \overline{V} is \overline{VV} if \overline{VV}

Solution a. Another name for \overline{VX} is \overline{XV} .

b. The rays with endpoint *W* are $\overrightarrow{WV}, \overrightarrow{WY}, \overrightarrow{WX}, \text{ and } \overrightarrow{WZ}$. The opposite rays with endpoint *W* are \overrightarrow{WV} and \overrightarrow{WX} , and \overrightarrow{WV} and \overrightarrow{WZ} .

EXAMPLE 3 Sketch intersections of lines and planes

- a. Sketch a plane and a line that is in the plane.
- b. Sketch a plane and a line that does not intersect the plane.
- c. Sketch a plane and a line that intersects the plane at a point.

Solution



EXAMPLE 4 Sketch intersections of planes Sketch two planes that intersect in a line. Solution

STEP 1 Draw a vertical plane. Shade the plane.

STEP 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden. **STEP 3** Draw the line of intersection.



(1.1 cont.)

1.1 Cont.

Checkpoint Use the diagram in Example 1.

1. Give two other names for \overrightarrow{MQ} . Name a point that is not coplanar with points *L*, *N*, and *P*.

QM and line *a*; point *Q*

Checkpoint Use the diagram in Example 2.

- **2.** Give another name for \overline{YW} . \overline{WY}
- **3.** Are \overrightarrow{VX} and \overrightarrow{XV} the same ray? Are \overrightarrow{VW} and \overrightarrow{VX} the same ray? *Explain*.

No, the rays do not have the same endpoint; Yes, the rays have a common endpoint, are collinear, and consist of the same points.

Checkpoint Complete the following exercises.



2.4 Use Postulates and Diagrams

Obj.: Use postulates involving points, lines, and planes.

Key Vocabulary

• Line perpendicular to a plane - A line is a line perpendicular to a plane if and only if the line <u>intersects</u> the plane in a <u>point</u> and is <u>perpendicular</u> to every line in the <u>plane</u> that intersects it at that point.

• **Postulate** - In geometry, <u>rules</u> that are accepted <u>without proof</u> are called *postulates* or *axioms*.

POSTULATES

Point, Line, and Plane Postulates

POSTULATE 5 - Through any two points there exists exactly one line.

POSTULATE 6 - A line contains at least two points.

POSTULATE 7 - If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 - Through any <u>three</u> noncollinear <u>points</u> there exists exactly <u>one plane</u>.

POSTULATE 9 - A **plane** contains at least **three** noncollinear **points**.

POSTULATE 10 - If <u>two</u> points lie in a <u>plane</u>, then the <u>line</u> containing them lies in the <u>plane</u>.

POSTULATE 11 - If <u>two planes</u> intersect, then their intersection is a <u>line</u>.

CONCEPT SUMMARY - Interpreting a Diagram

·G

When you interpret a diagram, you can only <u>assume</u> information about size or measure if it is <u>marked</u>.

YOU CAN ASSUME 🚄

All points shown are <u>coplanar</u>. $\angle AHB$ and $\angle BHD$ are a <u>linear pair</u>. $\angle AHF$ and $\angle BHD$ are <u>vertical</u> angles. *A*, *H*, *J*, and *D* are collinear. *AD* and *BF* intersect at *H*. YOU CANNOT ASSUME

G, F, and E are <u>collinear</u>. BF and \overrightarrow{CE} <u>intersect</u>. BF and \overrightarrow{CE} <u>do not</u> intersect. $\angle BHA \cong \angle CJA$

 $\overrightarrow{AD} \perp \overrightarrow{BF}$ or $m \angle AHB = 90^{\circ}$

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.



Solution

Postulate <u>8</u> Through any three <u>noncollinear</u> points there exists exactly one plane.

EXAMPLE 2 Identify postulates from a diagram

Use the diagram to write examples of Postulates 9 and 11. **Solution:**

Postulate 9 Plane \underline{Q} contains at least three noncollinear points, \underline{W} , V, and \underline{Y} .

Postulate 11 The intersection of plane P and plane Q is <u>line b</u>.



EXAMPLE 3 Use given information to sketch a diagram Sketch a diagram showing \overline{RS} perpendicular to \overline{TV} , intersecting at point X. Solution: Step 1 Draw \overline{RS} and label points R and S. Step 2 Draw a point X between R and S. Step 3 Draw \overline{TV} through X so that it is perpendicular to \overline{RS} .

Notice that the picture was drawn so that X does not look like a midpoint of RS.

D

С

EXAMPLE 4 Interpret a diagram in three dimensions Which of the following statements cannot be assumed from the diagram? F



Solution: With no right angles marked, you cannot assume that $\overrightarrow{BD} \perp \overrightarrow{EC}$ or $\overrightarrow{EC} \perp \text{plane } G$.

2.4 Cont.



1. Which postulate allows you to say that the intersection of line *a* and line *b* is a point?

Postulate 7

2. Write examples of Postulates 5 and 6.

Line *a* passes through *X* and *Y*; line *a* contains points *X* and *Y*.

Checkpoint Complete the following exercises.

3. In Example 3, if the given information indicated that *RX* and *XS* are congruent, how would the diagram change?

Point X would be drawn as the midpoint of \overline{RS} and the congruent segments would be marked.

4. In the diagram for Example 4, can you assume that \overrightarrow{BD} is the intersection of plane *F* and plane *G*?

Yes

1.2 Use Segments and Congruence

Obj.: Use segment postulates to identify congruent segments.

Key Vocabulary

• Postulate, axiom - In Geometry, a rule that is <u>accepted</u> without proof is called a postulate or axiom.

• Coordinate - The points on a line can be matched one to one with the real

numbers. The real number that corresponds to a point is the **coordinate** of the point. • **Distance** - The **distance** between points *A* and *B*, written as *AB*, is the **absolute value** of the **difference** of the coordinates of *A* and *B*.

• Between- When three points are collinear, you can say that one point is between the other two.

• Congruent segments - Line segments that have the <u>same length</u> are called congruent segments.



MAPS The cities shown on the map lie approximately in a straight line. Use the given

distances to find the distance from Lubbock, Texas, to

St. Louis, Missouri.

Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate. LS = LT + TS = 380 + 360 = 740The distance from Lubbock to St. Louis is about 740 miles.





EXAMPLE 4 Compare segments for congruence

Plot F(4, 5), G(-1, 5), H(3, 3), and J(3, -2) int a coordinate plane. Then determine whether FG and HJ are congruent.

Solution: Horizontal segment: Subtract the

<u>x-coordinates</u> of the endpoints.

FG = | 4 - (-1) | = 5

Vertical segment: Subtract the <u>y-coordinates</u> of the endpoints.

 $HJ = |\underline{3} - (-2)| = \underline{5}$

FG and HJ have the <u>same</u> length.

		y				
G (-	1, 5)			F	(4,	5)
			Н	(3,	3)	
_						_
			1			x
			J(3	, -	2)	

Checkpoint Complete the following exercises.



No

1.3 Use Midpoint and Distance Formulas

Obj.: <u>Find lengths of segments in the coordinate plane</u>.

Key Vocabulary

• Midpoint - The <u>midpoint</u> of a segment is the point that divides the segment into <u>two</u> <u>congruent</u> segments.

• Segment bisector - A segment bisector is a point, ray, line, line segment, or plane the at intersects the segment at its midpoint.

M is the midpoint of AB.

So, $\overline{AB} \cong \overline{AB}$ and $\underline{AM} = \underline{MB}$.

$$\overrightarrow{c}$$
 is a segment bisector of \overline{AB} ...

So, $\overline{AB} \cong AB$ and $\overline{AM} = MB$.

The coordinates of the midpoint of a segment are the <u>averages</u> of the <u>x</u>-coordinates and of the <u>y</u>-coordinates of the endpoints. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are <u>points</u> in a coordinate plane, then

the **<u>midpoint M</u>** of \overline{AB} has coordinates

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

The Distance Formula <u>KEY CONCEPT</u>

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points is a coordinate plane, then the <u>distance</u> between A and B is

AB =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 1 Find segment lengths

RS = RT + TS

= 43.4

Find RS. **Solution:** Point <u>T</u> is the midpoint of \overline{RS} . So, $RT = \underline{TS} = 21.7$.

Segment Addition Postulate Substitute.

Add.

The length of \overline{RS} is <u>43.4</u>.

= 21.7 + 21.7

EXAMPLE 2 Use algebra with segment lengths

ALGEBRA Point C is the midpoint of **BD**.

Find the length of \overline{BC} .

Solution: 1 Write and solve an equation.

BC = CDWrite equation.3x - 2 = 2x + 3Substitute.x - 2 = 3Subtract 2x from
each side.x = 5Add 2 to each side.











EXAMPLE 3 Use the Midpoint Formula

(1.3 cont.)

a. FIND MIDPOINT The endpoints of *PR* are P(-2, 5) and R(4, 3). Find the coordinates of the midpoint M.

Solution:

a. Use the Midpoint Formula.



= M(1, 4)

The coordinates of the midpoint of \overline{PR} are M(1, 4).

b. FIND ENDPOINT The midpoint of AC is M(3, 4). One endpoint is A(1, 6). Find the coordinates of endpoint C. **Solution:**

Step 1 Find x.Step 2 Find y.The coordinates of endpoint C are (5, 2).1 + x36 + y41 + x = 66 + y = 8x = 5y = 2

The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles in Chapter 7.



EXAMPLE 4 Use the Distance Formula

What is the approximate length of **TR**, with endpoints T(-4, 3) and R(3, 2)? Use the Distance Formula. Use the Distance Formula. $RT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula.

$$= \sqrt{(-4 - 3)^{2} + (3 - 2)^{2}}$$

$$= \sqrt{(-7)^{2} + (1)^{2}}$$

$$= \sqrt{49 + 1}$$

$$= \sqrt{50}$$

$$= \approx 7.07$$



Pythagorean Theorem $c^2 = a^2 + b^2$

T(-4.3)

R(3, 2)

x

Evaluate powers.

Add.

Use a calculator.

The length of \overline{RT} is about 7.07.





1.3 Cont.



(13, -19)

Checkpoint Complete the following exercise.

5. What is the approximate length of \overline{GH} , with endpoints G(5, -1) and H(-3, 6)? about 10.63

1.4 Measure and Classify Angles

Obj.: Name, measure, and classify angles.

Key Vocabulary

• Angle - An angle consists of two different rays with the same endpoint.

• Sides. vertex of an angle - The rays are the sides of the angle. The endpoint is the vertex of the angle.

• Measure of an angle - A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called degrees (⁰).

> **Words** The measure of $\angle WXZ$ is 32°. Symbols $m \angle WXZ = 32^{\circ}$

vertex

80 90 100 90

200 20

ନ୍ଦିନ 28

28

100 80 110 70

130 180

28

宮宮

12

120

sides

• Congruent angles - Two angles are congruent angles if they have the same measure.

• Angle bisector - An angle bisector is a ray that divides an angle into two angles that are **congruent**.

POSTULATE 3 - Protractor Postulate

Consider **OB** and a **point** A on **one** side of **OB**. The rays of the form *OA* can be matched one to one with the real numbers from 0 to 180. The measure of $\angle AOB$ is equal 22 to the **absolute** value of the difference between the real **numbers** for \overrightarrow{OA} and \overrightarrow{OB} .

CLASSIFYING ANGLES Angles can be classified as acute, right, obtuse, and straight, as shown below.



POSTULATE 4 - Angle Addition Postulate (AAP) Words If **P** is in the interior of $\angle RST$, then the measure of $\angle RST$ $\underline{m} \angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$. *m∠RSP* Interio **Symbols** If *P* is in the interior of $\angle RST$, then $m \angle RST = m \angle RSP + m \angle PST$. Angle measures are equal. Angles are congruent. the arcs show that the4. $m \angle A = m \angle B$ $\angle A \cong \angle B$ "equal to" angels are congruent "is congruent to" B∠

EXAMPLE 1 Name angles

Name the three angles in the diagram. Solution:

EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. ∠WSR	b. ∠TSW
c. ∠RST	d. ∠VST

- C. ∠RSI Solution:
- a. \overrightarrow{SR} is lined up with the 0° on the outer scale of the protractor. \overline{SW} passes through 65° on the outer scale. So, $m \angle WSR = 65^{\circ}$. It is an acute angle.
- **b.** \overrightarrow{ST} is lined up with the 0° on the inner scale of the protractor. \overrightarrow{SW} passes through 115° on the inner scale. So, $m \angle TSW = 115^\circ$. It is an obtuse angle.

EXAMPLE 3 Find angle measures

ALGEBRA Given that $m \angle GFJ = 155^{\circ}$, find

Angle Addition

$m \angle GFH$ and $m \angle HFJ$.

 $m \angle GFJ = m \angle GFH + m \angle HFJ$

Solution:

Step 1 Write and solve an equation to find the value of x.

	Postulate
<u>155°</u> = $(4x + 4)^{\circ} + (4x - 1)^{\circ}$	Substitute.
<u>155</u> = $8x + 3$	Combine like terms.
<u>152</u> = <u>8x</u>	Subtract <u>3</u> from each side.
<u>19</u> = x	Divide each side by 8.

EXAMPLE 4 Identify congruent angles

Identify all pairs of congruent angles in the diagram. If $m \angle P = 120^\circ$, what is $m \angle N$? There are two pairs of congruent angles: Solution: $\angle P \cong \angle N$ and $\angle L \cong \angle M$ Because $\angle P \cong \angle N$, $m \angle P = \underline{m \angle N}$. So, $m \angle N = 120^\circ$. **EXAMPLE 5** Double an angle measure In the diagram at the right, \overline{WY} bisects $\angle XWZ$, and $m \angle XWY = 29^{\circ}$. Find m∠XWZ By the Angle Addition Postulate, $m \angle XWZ = m \angle XWY + m \angle YWZ$. Solution:

Because \overrightarrow{WY} bisects $\angle XWZ$, you know $\angle XWY \cong \angle YWZ$. So, $m \angle XWY = m \angle YWZ$, and you can write $m \angle XWZ = \underline{m} \angle XWY + \underline{m} \angle YWZ$ $= 29^{\circ} + 29^{\circ} = 58^{\circ}$.

vertex.

c. $m \angle RST = 180^{\circ}$. It is <u>a straight</u> angle. **d.** $m \angle VST = 90^\circ$. It is a right angle.

Step 2 Evaluate the given expressions when x = 19. $m \angle GFH = (4x + 4)^\circ = (4 \cdot 19 + 4)^\circ = 80^\circ$. $m \angle HFJ = (4x - 1)^\circ = (4 \cdot 19 - 1)^\circ = 75^\circ$. So, $m \angle GFH = 80^\circ$ and $m \angle HFJ = 75^\circ$.

(1.4 cont.)

1.4 Cont.

Checkpoint Complete the following exercises.

right angles

Checkpoint Complete the following exercise.

Checkpoint Complete the following exercises.

2.6 Prove Statements about Segments and Angles

Obj.: Write proofs using geometric theorems.

Key Vocabulary

• **Proof** - A **proof** is a **logical argument** that shows a statement is **true**. There are several formats for proofs.

• **Two-column proof** - A **two-column proof** has numbered <u>statements</u> and corresponding <u>reasons</u> that show an argument in a <u>logical</u> order.

• Theorem - The reasons used in a proof can include definitions, properties,

postulates, and *theorems*. A theorem is a statement that can be proven.

THEOREMS

Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive	For any segment AB, AB ≅ AB.
<u>Symmetric</u>	If AB ≅ CD , then CD ≅ AB .
<u>Transitive</u>	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

<u>Reflexive</u>	For any angle A, $\angle A \cong \angle A$.
Symmetric	If $\angle A \cong \angle B$, then $\underline{\angle B} \cong \underline{\angle A}$.
Transitive	If $\angle A \cong \underline{\angle B}$ and $\underline{\angle B} \cong \angle C$, then $\underline{\angle A \cong \angle C}$.

EXAMPLE 1 Write a two-column proof

Use the diagram to prove	m∠1 = m∠4.		2 1/
Given: $m \angle 2 = m \angle 3$, r	m∠AXD = m∠AXC	A	
Prove: $m \angle 1 = m \angle 4$	[►] Statements	Reasons	4
	1. $m \angle AXC = m \angle AXD$	1. Given	
	2. $m \angle AXD$ = $m \angle \underline{1} + m \angle \underline{2}$	2. Angle Addition Postulate	вс
	3. $m \angle AXC$ = $m \angle 3$ + $m \angle 4$	3. Angle Addition Postulate	
Writing a two- column proof is	4. $m \angle 1 + m \angle 2$ = $m \angle 3 + m \angle 4$	4. <u>Substitution Property</u> of Equality	
a formal way of organizing your reasons to show a statement is true.	5. <i>m</i> ∠2 = <i>m</i> ∠3	5. Given	
	$6. m \angle 1 + m \angle \underline{3} \\ = m \angle 3 + m \angle 4$	 Substitution Property of Equality 	
	7. $m \angle 1 = m \angle 4$	7. <u>Subtraction Property</u> of Equality	

<<u>►</u> / D

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$

Solution: Symmetric Property of Angle Congruence

EXAMPLE 3 Use properties of equality

If you know that \overline{BD} bisects $\angle ABC$, prove that $m \angle ABC$ is two times $m \angle 1$.

Given: BD bisects $\angle ABC$.	Statements	Reasons	
Prove: $m \angle ABC = 2 m \angle 1$	1. \overrightarrow{BD} bisects $\angle ABC$.	1. Given	*
	2. ∠1 ≅ ∠2	2. Definition of angle bisector	A D
	3. <u>m∠1 = m∠2</u>	3. Definition of congruent angles	1
	4. $m \angle 1 + m \angle 2 = m \angle ABC$	4. <u>Angle Addition</u> Postulate	B C
	5. $m \angle 1 + m \angle \underline{1} = m \angle ABC$	5. Substitution Property of Equality	
	6. $2 \cdot m \angle 1 = m \angle ABC$	6. Distributive Property	

EXAMPLE 4 Solve a multi-step problem

Interstate There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and Mason exit. The distance between rest area B and Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

Sol	ution:	Statements	Reasons
Solut i Step 1	ion Rest Rice Mason Rest area A exit exit area B Draw a diagram.	1. <i>R</i> is the midpoint of \overline{AM} , MB = AR.	1. <u>Given</u>
Step 2	Draw diagrams showing relationships.	2. $\overline{AR} \cong \overline{RM}$	2. Definition of midpoint
Step 3	A R M B A R M B Write a proof.	3. <i>AR</i> = <i>RM</i>	3. Definition of congruent segments
	Given R is the midpoint of \overline{AM} , $MB = AR$. Prove M is the midpoint of \overline{RB} .	4. <i>MB</i> = <i>RM</i>	4. Transitive Property of Congruence
		5. <u>MB</u> ≅ <u>RM</u>	5. Definition of congruent segments
		6. <i>M</i> is the midpoint of \overline{RB} .	6. Definition of midpoint

CONCEPT SUMMARY - Writing a Two-Column Proof

In a proof, you make <u>one</u> statement at a time, until you reach the <u>conclusion</u>. Because you make statements based on <u>facts</u>, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

Proof of the Symmetric Property of Angle Congruence

GIVEN ♦	Ζ	1 ≅	∠ 2
PROVE ♦	Ζ	2 ≅	∠ 1

STATEMENTS	REASONS
$(1, \angle 1 \cong \angle 2)$	1. Given
2. $m \angle 1 = m \angle 2$	2. Definition of congruent angles
3. $m \angle 2 = m \angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

The <u>number</u> of statements will <u>vary</u>

Remember to give a <u>reason</u> for the <u>last</u> statement.

Copy or <u>draw diagrams</u> and <u>label</u> given information to help develop proofs.

(2.6 cont.)

Definitions, postulates

Theorems Statements based on facts that you know or on conclusions from deductive reasoning 2.6 Cont.

Checkpoint Complete the following exercises.

 Three steps of a proof are shown. Give the reasons for the last two steps. 		
Given $BC = AB$ Prove $AC = AB + AB$	A B C	
Statements	Reasons	
1 PC - AP	1 Civon	

1 . $BC = AB$	1. Given
2. $AC = AB + BC$	2. <u>Segment Addition</u> Postulate
3. $AC = AB + AB$	3. Substitution Property of
	Equality

2. Name the property illustrated by the statement. If $\angle H \cong \angle T$ and $\angle T \cong \angle B$, then $\angle H \cong \angle B$.

Transitive Property of Angle Congruence

Checkpoint Complete the following exercise.

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

```
Rest area A and rest area B
```

1.5 Describe Angle Pair Relationships

Obj.: Use special angle relationships to find angle measure.

Key Vocabulary

• Complementary angles - Two angles are complementary angles if the sum of their measures is 90°.

• Supplementary angles - Two angles are supplementary angles if the sum of their measures is 180°.

• Adjacent angles - Adjacent angles are two angles that share a common vertex and side, but have no common interior points.

• Linear pair - Two adjacent angles are a linear pair if their noncommon sides are opposite rays.

• Vertical angles - Two angles are vertical angles if their sides form two pairs of opposite rays.

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 2 = 57^{\circ}$, find $m \angle 1$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4 = 41^{\circ}$, find $m \angle 3$. Solution:

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

- •AC, BD, and BE intersect at point B.
- $\angle DBE$ and $\angle EBC$ are <u>adjacent</u> angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the <u>interior</u> of \angle *DBC*.

In the diagram above, you <u>cannot conclude</u> that $AB \cong BC$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a <u>right</u> angle. This information must be indicated <u>by marks</u>, as shown at the right.

1.5 Cont.

2.7 Prove Angle Pair Relationships

Obj.: Use properties of special pairs of angles.

Key Vocabulary

- Complementary angles Two angles whose measures have the sum 90°.
- Supplementary angles Two angles whose measures have the sum 180°.
- Linear pair Two adjacent angles whose noncommon sides are opposite rays.
- Vertical angles Two angles whose sides form <u>two pairs</u> of opposite <u>rays</u>.

Right Angles Congruence Theorem

All right angles are **congruent**.

Congruent Supplements Theorem

If <u>two angles</u> are supplementary to the <u>same</u> angle (or to congruent angles), then they are <u>congruent</u>. If $\angle 1$ and $\underline{\angle 2}$ are supplementary and $\angle 3$ and $\underline{\angle 2}$ are supplementary, then $\underline{\angle 1} \cong \underline{\angle 3}$.

Congruent Complements Theorem

If <u>two angles</u> are complementary to the <u>same</u> angle (or to congruent angles), then they are <u>congruent</u>. If $\angle 4$ and $\underline{\angle 5}$ are complementary and $\angle 6$ and $\underline{\angle 5}$ are complementary, then $\underline{\angle 4} \cong \underline{\angle 6}$.

Linear Pair Postulate

If two angles form a linear pair, then they are **supplementary**. $\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $\underline{m} \angle 1 + \underline{m} \angle 2 = 180^{\circ}$.

Vertical Angles Congruence Theorem Vertical angles are congruent.

 $\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$

EXAMPLE 1 Use right angle congruence

Write a proof. **GIVEN:** $JK \perp KL$, $ML \perp KL$ **PROVE:** $\angle K \cong \angle L$

Statements

- **1.** $\overline{JK} \perp \overline{KL}, \overline{ML} \perp \overline{KL}$
- **2.** $\angle K$ and $\angle L$ are right angles.
- **3.** ∠K ≅ ∠L

Abbreviation All Rt. ∠'s ≅

<mark>≅ Supp. Th</mark>

2. Definition of perpendicular lines

Reasons

1. Given

3. Right Angles Congruence Theorem

The measure of $\angle FKG$ is 67°.

2.7 Cont.

Checkpoint Complete the following exercises.

1.6 Classify Polygons

Obj.: Classify polygons.

Key Vocabulary

#

• **Polygon** - In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a <u>closed</u> plane figure with the following properties:

Side - 1. It is formed by three or more line <u>segments</u> called sides.
2. Each side intersects exactly <u>two sides</u>, one at each <u>endpoint</u>, so that no two sides with a common endpoint are collinear.

Vertex - Each endpoint of a side is a vertex of the polygon. The plural of vertex is vertices.

• **Convex** - A polygon is **convex** if <u>no line</u> that contains a side of the polygon contains a point in the <u>interior</u> of the polygon.

- Concave A polygon that is not convex is called nonconvex or concave.
- *n*-gon The term *n*-gon, where *n* is the <u>number</u> of a polygon's <u>sides</u>, can also be used to <u>name</u> a polygon. For example, a polygon with 14 sides is a 14-gon.
- Equilateral In an equilateral polygon, all sides are congruent.
- Equiangular In an equiangular polygon, <u>all angles</u> in the interior of the polygon are <u>congruent</u>.

• Regular - A regular polygon is a convex polygon that is <u>both</u> equilateral and equiangular.

В

A polygon can be named by listing the vertices in consecutive <u>order</u>. For example, ABCDE and <u>CDEAB</u> are both <u>correct</u> names for the polygon at the right.

CLASSIFYING POLYGONS A polygon is named by the number of its sides.

of sides	Type of polygon	# of sides	Type of polygon
<u>3</u>	Triangle	<u>8</u>	Octagon
4	Quadrilateral	9	<u>Nonagon</u>
<u>5</u>	Pentagon	<u>10</u>	Decagon
6	Hexagon	<u>12</u>	Dodecagon
7	Heptagon	n	<u>n-gon</u>

EXAMPLE 1 Identify polygons

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

Solution:

a. Some segments intersect more than two segments, so it is <u>not a polygon</u>.

- **b.** The figure is <u>a convex polygon</u>.
- c. The figure is <u>a concave polygon</u>.

EXAMPLE 2 Classify polygons

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

Solution: The polygon has <u>8</u> sides. It is equilateral and equiangular, so it is a <u>regular octagon</u>.

EXAMPLE 3 Find side lengths

ALGEBRA A head of a bolt is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal bolt. Find the length of a side.

Solution:

First, write and solve an equation to find the value of *x*. Use the fact that the sides of a regular hexagon are congruent.

4x + 3 = 5x - 1 Write an equation.

 $\underline{4} = \underline{x}$ Simplify.

Then evaluate one of the expressions to find a side length when x = 4. 4x + 3 = 4(4) + 3 = 19

The length of a side is <u>19</u> millimeters.

Hexagonal means "shaped like a hexagon."

1.6 Cont. (Write these on your paper)

Checkpoint Tell whether the figure is a polygon and whether it is convex or concave.

3. Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular.

quadrilateral

4. The expressions $(4x + 8)^{\circ}$ and $(5x - 5)^{\circ}$ represent the measures of two of the congruent angles in Example 3. Find the measure of an angle. 60°

2.1 Use Inductive Reasoning

Obj.: Describe patterns and use inductive reasoning.

Key Vocabulary

• Conjecture - A conjecture is an unproven statement that is based on observations.

• Inductive reasoning - You use inductive reasoning when you find a pattern in specific cases and then write a **conjecture** for the general case.

• Counterexample - A counterexample is a specific case for which the conjecture is false. You can show that a conjecture is false, however, by simply finding one counterexample.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Solution:

Figure 3		

Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eighths . Shade the section just below the horizontal segment at the left .

Figure 1

EXAMPLE 2 Describe a number pattern

Describe the pattern in the numbers -1, -4, -16, -64, ... and write the next three numbers in the pattern. Notice that each number in the pattern is four times

Solution:

Three dots (. . .) tell you that the pattern continues.

the previous number.

The next three numbers are -256, -1024, and -4096.

EXAMPLE 3 Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution:

Make a table and look for a pattern. Notice the pattern in how the number of connections increases . You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture	•	••	\triangleleft	\square	
Number of connections	0	_1_	3	6	?

Conjecture You can connect five noncollinear points 6 + 4, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 1, 3, and 5 are called *consecutive odd numbers*. Make and test a conjecture about the sum of any three consecutive odd numbers.

Solution:

Step 1 Find a pattern using groups of small numbers.

Conjecture The sum of any three consecutive odd numbers is three times <u>the second number</u>.

Step 2 Test your conjecture using other numbers.

 $-1 + 1 + 3 = \underline{3} = \underline{1} \cdot 3 \checkmark$ 103 + 105 + 107 = 315 = 105 \cdot 3 \sqrt{

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the

larger number. Solution:

To find a counterexample, you need to find a difference that is <u>greater</u> than the <u>larger</u> number.

8 - (-4) = 12

Because <u>12</u> \ll <u>8</u>, a counterexample exists. The conjecture is false.

EXAMPLE 6 Real world application

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.
Solution:
NFL Average

The scatter plot shows that the values <u>increased</u> each year. So, one possible conjecture is that the average player in the NFL is earning <u>more</u> money today than in 1999.

2.1 Cont.

Checkpoint Complete the following exercise.

2.3 Apply Deductive Reasoning

Obj.: Use deductive reasoning to form a logical argument.

Key Vocabulary

• **Deductive reasoning - Deductive reasoning** uses facts, <u>definitions</u>, accepted properties, and the laws of logic to form a <u>logical argument</u>.

**** This is different from *inductive reasoning*, which uses specific examples and **patterns** to form a conjecture.****

Laws of Logic KEY CONCEPT

Law of **Detachment**

If the hypothesis of a true conditional statement is true, then the <u>conclusion</u> is also <u>true</u>.

Law of Syllogism

If hypothesis *p*, then conclusion *q*.

 \nearrow If these statements are <u>true</u>,

If hypothesis <u>p</u>, then conclusion <u>r</u>.

then this statement is <u>true</u>.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation. **a.** If two angles have the same measure, then they are congruent. You

know that $m \angle A = m \angle B$.

Solution:

a. Because $m \angle A = m \angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

b. Jesse goes to the gym every weekday. Today is Monday. **Solution:**

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "<u>If it is a</u> <u>weekday</u>," and the conclusion is "<u>then Jesse goes</u> to the gym."

"Today is Monday" satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.

EXAMPLE 2 Use the Law of Syllogism

(2.3 cont.)

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

Solution: a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the

following.

If Ron eats lunch today, then <u>he will drink a glass of</u> <u>milk</u>.

b. If $x^2 > 36$, then $x^2 > 30$. If x > 6, then $x^2 > 36$.

Solution: b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If x > 6, then $x^2 > 30$.

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

Solution: c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the

Law of Syllogism to write a new conditional statement.

EXAMPLE 3 Use inductive and deductive reasoning

What conclusion can you make about the sum of an odd integer and an odd integer? **Solution:**

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$-3 + 5 = \underline{2}, -1 + 5 = \underline{4}, 3 + 5 = \underline{8}$$

$$-3 + (-5) = \underline{-8}, 1 + (-5) = \underline{-4},$$

$$3 + (-5) = \underline{-2}$$

Conjecture: Odd integer + Odd integer = <u>Even</u> intege

Step 2 Let *n* and *m* each be any integer. Use deductive reasoning to show the conjecture is true.

2*n* and 2*m* are <u>even</u> integers because any integer multiplied by 2 is <u>even</u>.

2n - 1 and 2m + 1 are odd integers because 2n and 2m are even integers.

(2n - 1) + (2m + 1) represents the sum of an <u>odd</u> integer 2n - 1 and an <u>odd</u> integer 2m + 1.

 $(2n - \underline{1}) + (2m + \underline{1}) = \underline{2}(n + m)$

The result is the product of 2 and an integer n + m. So, 2(n + m) is an <u>even</u> integer.

The sum of an odd integer and an odd integer is an <u>even</u> integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of inductive reasoning or deductive reasoning. Explain your choice.

a. The runner's average speed decreases as time spent running increases.

Solution: a. <u>Inductive</u> reasoning, because it is based on a pattern in the data

b. The runner's average speed is slower when running for 40 minutes than when running for 10 minutes.

Solution: b. <u>Deductive</u> reasoning, because you are comparing values that are given on the graph

2.3 Cont.

2.2 Analyze Conditional Statements

Obj.: <u>Write definitions as conditional statements.</u>

Key Vocabulary

• Conditional statement - A conditional statement is a logical statement that has two parts, a hypothesis and a conclusion.

• **Converse** - To write the **converse** of a conditional statement, **exchange** the **hypothesis** and **conclusion**.

• Inverse - To write the inverse of a conditional statement, <u>negate (not)</u> both the hypothesis and the conclusion.

• Contrapositive - To write the contrapositive, first write the <u>converse</u> and then <u>negate both</u> the hypothesis and the conclusion.

• If-then form, Hypothesis, Conclusion - When a <u>conditional</u> statement is written in if-then form, the <u>"if"</u> part contains the hypothesis and the "<u>then</u>" part contains the conclusion. Here is an example:

If it is raining, then there are clouds in the sky.

hypothesis

conclusion

• Negation - The negation of a statement is the <u>opposite</u> of the <u>original</u> statement.

• Equivalent statements - When two statements are <u>both true</u> or both <u>false</u>, they are called equivalent statements.

• Perpendicular lines - If two lines intersect to form a right angle, then they are perpendicular lines. You can write "line 1 is perpendicular to line *m*" as $1 \perp m$. to line *m*" as $1 \perp m$.

• **Biconditional statement –** A **biconditional statement** is a statement that contains the **converse** and the phrase <u>"if and only if."</u>

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form. All vertebrates have a backbone. **Solution:**

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If <u>an animal is a vertebrate</u>, then <u>it has a</u> backbone .

EXAMPLE 2 Write four related conditional statements

(2.2 cont.)

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is *true* or *false*. **Solution:**

lf-then form	If-then form If you are an Olympian, then you are an athlete. <i>True</i> , Olympians are athletes.
Converse	Converse If you are an athlete, then you are an Olympian. <i>False</i> , not all athletes are Olympians.
Inverse	Inverse <u>If you are not an Olympian, then you are not</u> an athlete. <i>False</i> , even if you are not an Olympian, you can still be an athlete.
Contrapositive	Contrapositive <u>If you are not an athlete, then you</u> are not an Olympian. <i>True</i> , a person who is not an athlete cannot be an Olympian.

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\overrightarrow{AC} \perp \overrightarrow{BD}$

b. $\angle AED$ and $\angle BEC$ are a linear pair. **Solution**:

DEC

- a. The statement is <u>true</u>. The right angle symbol indicates that the lines intersect to form a <u>right</u> angle. So you can say the lines are <u>perpendicular</u>.
- **b.** The statement is <u>false</u>. Because $\angle AED$ and $\angle BEC$ are not <u>adjacent</u> angles, $\angle AED$ and $\angle BEC$ are not a <u>linear pair</u>.

EXAMPLE 4 Write a biconditional

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel. **Solution:**

Converse: If two lines are parallel, then they lie in the same plane and do not intersect.

Biconditional: <u>Two lines are parallel if and only if they</u> lie in the same plane and do not intersect.

2.2 Cont.

A student is in group A if and only if the student is a boy.