1-1

1. Plan

Objectives

1 To use inductive reasoning to make conjectures

Examples

1 Finding and Using a Patternn
2 Using Inductive Reasoning
3 Finding a Counterexample
4 Real-World Connection

Professional Development

Math Background

Inductive reasoning assumes that an observed pattern will continue. This may or may not be true. For example, "$x = x \cdot x$" is true for $x = 0$ and $x = 1$, but then the pattern fails. Inductive reasoning can lead to conjectures that seem likely but are unproven. A single counterexample is enough to disprove a conjecture.

More Math Background: p. 2C

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:
Skills Handbook, p. 753.

1-1 Patterns and Inductive Reasoning

What You'll Learn

- To use inductive reasoning to make conjectures

. . . And Why

To predict future sales for a skateboard business, as in Example 4

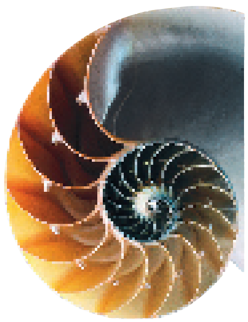
Check Skills You'll Need GO for Help Skills Handbook page 753

Here is a list of the counting numbers: 1, 2, 3, 4, 5, . . . Some are even and some are odd.

1. Make a list of the positive even numbers. **2, 4, 6, 8, 10, . . .**
2. Make a list of the positive odd numbers. **1, 3, 5, 7, 9, . . .**
3. Copy and extend this list to show the first 10 perfect squares.
   $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, \ldots$
   3. $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$
4. Which do you think describes the square of any odd number?
   It is odd. It is even. **It is odd.**

New Vocabulary • inductive reasoning • conjecture • counterexample

1 Using Inductive Reasoning



Real-World Connection

You can predict growth of the chambered nautilus shell by studying patterns in its cross sections.

Inductive reasoning is reasoning that is based on patterns you observe. If you observe a pattern in a sequence, you can use inductive reasoning to tell what the next terms in the sequence will be.

1 EXAMPLE Finding and Using a Pattern

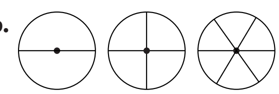
Find a pattern for each sequence. Use the pattern to show the next two terms in the sequence.

a. 3, 6, 12, 24, . . .

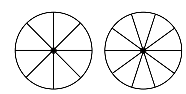
3 → 6 → 12 → 24
× 2 × 2 × 2

Each term is twice the preceding term. The next two terms are $2 \times 24 = 48$ and $2 \times 48 = 96$.

b.



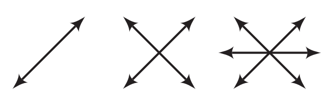
Each circle has one more segment through the center to form equal parts. The next two figures:



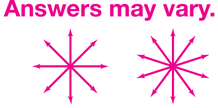
Quick Check

1 Write the next two terms in each sequence.

a. 1, 2, 4, 7, 11, 16, 22, . . . **29, 37**
b. Monday, Tuesday, Wednesday, . . . **Thursday, Friday**
c.



Answers may vary. Sample:



Differentiated Instruction Solutions for All Learners

Special Needs L1	Below Level L2
Help students grasp the role of *examples* and *counterexamples* in proof. A conjecture (statement) cannot be proven true by one example, or any number of examples. However one counterexample can prove that a conjecture is false. **learning style: verbal**	Have students recreate the geometric pattern in Example 1 to reinforce using a pattern. **learning style: visual**

A conclusion you reach using inductive reasoning is called a **conjecture.**

2 EXAMPLE Using Inductive Reasoning

Make a conjecture about the sum of the first 30 odd numbers.

Find the first few sums. Notice that each sum is a perfect square.

$1 = 1 = 1^2$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$

The perfect squares form a pattern.

Using inductive reasoning, you can conclude that the sum of the first 30 odd numbers is 30^2, or 900.

Quick Check 2 Make a conjecture about the sum of the first 35 odd numbers. Use your calculator to verify your conjecture. **The sum of the first 35 odd numbers is 35^2, or 1225.**

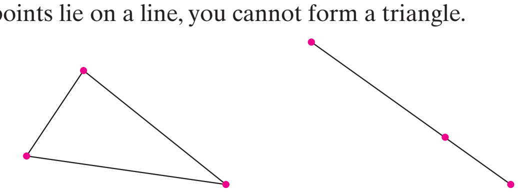
Not all conjectures turn out to be true. You can prove that a conjecture is false by finding one counterexample. A **counterexample** to a conjecture is an example for which the conjecture is incorrect.

3 EXAMPLE Finding a Counterexample

Find a counterexample for each conjecture.

a. The square of any number is greater than the original number.
The number 1 is a counterexample because $1^2 \ngtr 1$.

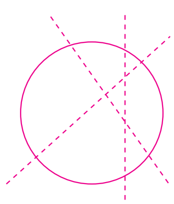
b. You can connect any three points to form a triangle.
If the three points lie on a line, you cannot form a triangle.

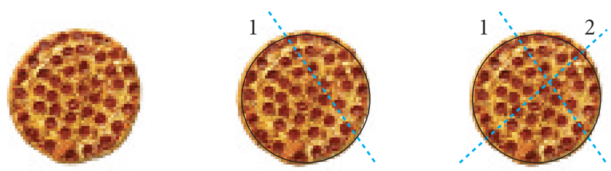


Counterexample

c. Any number and its absolute value are opposites.
The conjecture is true for negative numbers, but not positive numbers.
8 is a counterexample because 8 and $|8|$ are not opposites.

Quick Check 3 Alana makes a conjecture about slicing pizza. She says that if you use only straight cuts, the number of pieces will be twice the number of cuts.





Draw a counterexample that shows you can make 7 pieces using 3 cuts. **See left.**

2. Teach

Guided Instruction

2 EXAMPLE Teaching Tip

Point out that the number that is squared equals the number of terms that are added.

PowerPoint

Additional Examples

1 Find a pattern for the sequence. Use the pattern to show the next two terms in the sequence.

384, 192, 96, 48, … **Each term is half the preceding term; 24, 12.**

2 Make a conjecture about the sum of the cubes of the first 25 counting numbers. **The sum equals $(1 + 2 + 3 + \dots + 25)^2$.**

3 Find a counterexample for each conjecture.

a. A number is always greater than its reciprocal. **Sample: 1 is not greater than $\frac{1}{1} = 1$; $\frac{1}{2}$ and -3 are also counterexamples.**

b. If a number is divisible by 5, then it is divisible by 10. **Sample: 25 is divisible by 5 but not by 10.**

4 The price of overnight shipping was \$8.00 in 2000, \$9.50 in 2001, and \$11.00 in 2002. Make a conjecture about the price in 2003. **Sample: The price will be \$12.50.**

Resources

- Daily Notetaking Guide 1-1 L3
- Daily Notetaking Guide 1-1—Adapted Instruction L1

Closure

Explain how you can use a conjecture to help solve a problem. **Sample: A conjecture can be tested to see whether it is a solution.**

Advanced Learners L4

Have students explore the pattern in Example 2 geometrically by placing 3, then 5, then 7 squares on the top and right sides of the previous square.

learning style: visual

English Language Learners ELL

Exercises 42-46 rely solely on visual clues. Use these exercises to assess ELL students' ability to use inductive reasoning to continue a pattern.

learning style: visual

3. Practice

Assignment Guide

1 A B 1-53

C Challenge 54-55

Test Prep 56-59

Mixed Review 60-62

Homework Quick Check

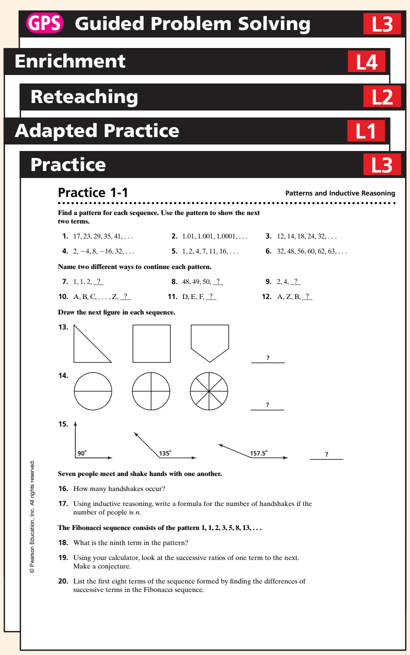
To check students' understanding of key skills and concepts, go over Exercises 4, 26, 40, 44, 48.

Exercises 7, 8 You may want to provide a hint that the letters are the first letters in a sequence of words.

Visual Learners

Exercise 18 Encourage students to draw the first three figures shown to help them see the pattern unfold.

Differentiated Instruction Resources



GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Adapted Practice L1

Practice L3

Practice 1-1 — Patterns and Inductive Reasoning

Find a pattern for each sequence. Use the pattern to show the next two terms.

1. 17, 23, 29, 35, 41, . . .
2. 1.01, 1.001, 1.0001, . . .
3. 12, 14, 18, 24, 32, . . .
4. 2, −4, 8, −16, 32, . . .
5. 1, 2, 4, 7, 11, 16, . . .
6. 32, 48, 56, 60, 62, 63, . . .

Name two different ways to continue each pattern.

7. 1, 1, 2, ?
8. 48, 49, 50, ?
9. 2, 4, ?
10. A, B, C, . . . , Z, ?
11. D, E, F, ?
12. A, Z, B, ?

Draw the next figure in each sequence.

13. ?

14. ?

15. 90° 135° 157.5° ?

Seven people meet and shake hands with one another.

16. How many handshakes occur?
17. Using inductive reasoning, write a formula for the number of handshakes if the number of people is *n*.

The Fibonacci sequence consists of the pattern 1, 1, 2, 3, 5, 8, 13, . . .

18. What is the ninth term in the pattern?
19. Using your calculator, look at the successive ratios of one term to the next. Make a conjecture.
20. List the first eight terms of the sequence formed by finding the differences of successive terms in the Fibonacci sequence.

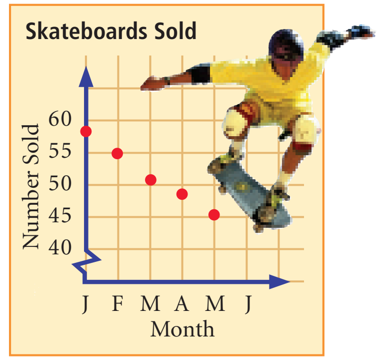
19. The sum of the first 6 positive even numbers is 6 · 7, or 42.

20. The sum of the first 30 positive even numbers is 30 · 31, or 930.

4 EXAMPLE Real-World Connection

Business Sales A skateboard shop finds that over a period of five consecutive months, sales of small-wheeled skateboards decreased.

Use inductive reasoning. Make a conjecture about the number of small-wheeled skateboards the shop will sell in June.



The graph shows that sales of small-wheeled skateboards is decreasing by about 3 skateboards each month. By inductive reasoning you might conclude that the shop will sell 42 skateboards in June.

Quick Check

4 a. Make a conjecture about the number of small-wheeled skateboards the shop will sell in July. **Sample: 39 skateboards**

b. **Critical Thinking** How confident would you be in using the graph to make a conjecture about sales in December? Explain.
Not confident; December is too far away.

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 4)

GO for Help

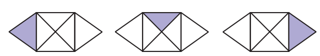
2. 33,333; 333,333

9. 720, 5040

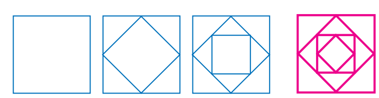
Find a pattern for each sequence. Use the pattern to show the next two terms.

1. 5, 10, 20, 40, . . . **80, 160**
2. 3, 33, 333, 3333, . . .
3. 1, −1, 2, −2, 3, . . . **−3, 4**
4. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ $\mathbf{\frac{1}{16}, \frac{1}{32}}$
5. 15, 12, 9, 6, . . . **3, 0**
6. 81, 27, 9, 3, . . . $\mathbf{1, \frac{1}{3}}$
7. O, T, T, F, F, S, S, E, . . . **N, T**
8. J, F, M, A, M, . . . **J, J**
9. 1, 2, 6, 24, 120, . . .
10. 2, 4, 8, 16, 32, . . . **64, 128**
11. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$ $\mathbf{\frac{1}{36}, \frac{1}{49}}$
12. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ $\mathbf{\frac{1}{5}, \frac{1}{6}}$
13. George, John, Thomas, James, . . . **James, John**
14. Martha, Abigail, Martha, Dolley, . . . **Elizabeth, Louisa**
15. George, Thomas, Abe, Alexander, . . . **Andrew, Ulysses**
16. Aquarius, Pisces, Aries, Taurus, . . . **Gemini, Cancer**

Draw the next figure in each sequence.

17. 

Sample: 

18. 

Example 2 (page 5)

Use the table and inductive reasoning. Make a conjecture about each value.

19. the sum of the first 6 positive even numbers **19–22. See margin.**
20. the sum of the first 30 positive even numbers
21. the sum of the first 100 positive even numbers

2	=	2 = 1 · 2
2 + 4	=	6 = 2 · 3
2 + 4 + 6	=	12 = 3 · 4
2 + 4 + 6 + 8	=	20 = 4 · 5
2 + 4 + 6 + 8 + 10	=	30 = 5 · 6

22. Use the pattern in Example 2 to make a conjecture about the sum of the first 100 odd numbers.

21. The sum of the first 100 positive even numbers is 100 · 101, or 10,100.

22. The sum of the first 100 odd numbers is 100^2, or 10,000.

25–28. Answers may vary. Samples are given.

25. 8 + (−5) = 3 and 3 ≯ 8

26. $\frac{1}{3} \cdot \frac{1}{2} \ngtr \frac{1}{3}$ and $\frac{1}{3} \cdot \frac{1}{2} \ngtr \frac{1}{2}$

27. −6 − (−4) ≮ −6 and −6 − (−4) ≮ −4

28. $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$ and $\frac{3}{2}$ is improper.

Predict the next term in each sequence. Use your calculator to verify your answer.

23. 12345679 × 9 = 111111111
12345679 × 18 = 222222222
12345679 × 27 = 333333333
12345679 × 36 = 444444444
12345679 × 45 = ■ **555,555,555**

24. 1 × 1 = 1
11 × 11 = 121
111 × 111 = 12321
1111 × 1111 = 1234321
11111 × 11111 = ■ **123,454,321**

Example 3 (page 5)

Find one counterexample to show that each conjecture is false. **25–28. See margin, p. 6.**

25. The sum of two numbers is greater than either number.

26. The product of two positive numbers is greater than either number.

27. The difference of two integers is less than either integer.

28. The quotient of two proper fractions is a proper fraction.

Example 4 (page 6)

29. Weather The speed with which a cricket chirps is affected by the temperature. If you hear 20 cricket chirps in 14 seconds, what is the temperature? **75°F**

Chirps per 14 Seconds

5 chirps	45°F
10 chirps	55°F
15 chirps	65°F

30. Physical Fitness Dino works out regularly. When he first started exercising, he could do 10 push-ups. After the first month he could do 14 push-ups. After the second month he could do 19, and after the third month he could do 25. Predict the number of push-ups Dino will be able to do after the fifth month of working out. How confident are you of your prediction? Explain. **See left.**

30. 40 push-ups; answers may vary. Sample: Not very confident; Dino may reach a limit to the number of push-ups he can do.

33. 0.0001, 0.00001

34. 201, 202

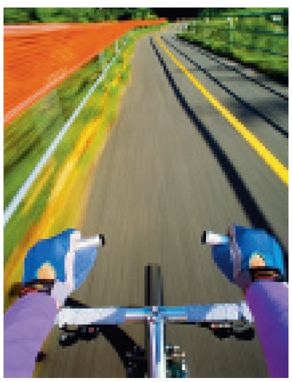
B Apply Your Skills

Find a pattern for each sequence. Use the pattern to show the next two terms.

31. 1, 3, 7, 13, 21, . . . **31, 43**
32. 1, 2, 5, 6, 9, . . . **10, 13**
33. 0.1, 0.01, 0.001, . . .
34. 2, 6, 7, 21, 22, 66, 67, . . .
35. 1, 3, 7, 15, 31, . . . **63, 127**
36. 0, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, . . . **$\frac{31}{32}$, $\frac{63}{64}$**
37. M, V, E, M, . . . **J, S**
38. AL, AK, AZ, AR, . . . **CA, CO**
39. H, He, Li, Be, . . . **B, C**

40. Writing Choose two of the sequences in Exercises 31–36 and describe the patterns. **See margin.**

41. Draw two parallel lines on your paper. Locate four points on the paper, each an equal distance from both lines. Describe the figure you get if you continue to locate points, each an equal distance from both lines. **See margin.**

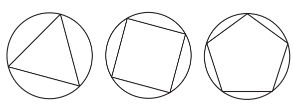


Real-World Connection

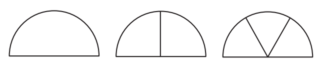
Points along the yellow line are equal distances from both sides of the bike trail (Exercise 41).

Draw the next figure in each sequence. **42–45. See margin.**

42.



43.



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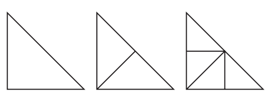
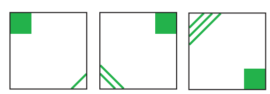
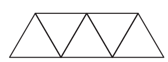


45.



46. Multiple Choice Find the perimeter when 100 triangles are put together in the pattern shown. Assume that all triangle sides are 1 cm long. **B**



Ⓐ 100 cm Ⓑ 102 cm Ⓒ 202 cm Ⓓ 300 cm

Error Prevention!

Exercise 30 Because the problem contains many words, urge students to organize the data in a table. Then point out that the problem asks for the number of push-ups the *fifth* month, not the *next* month.

Exercise 41 You may need to define *parallel* for some students. In addition, students may think the answer is a segment instead of a line. Discuss ways to distinguish segments from lines. The formal treatment of the distance from a point to a line occurs in Chapter 5.

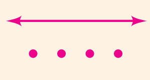
Exercise 46 Students may find it difficult to apply inductive reasoning to this problem. Encourage them to make a table that relates the number of triangles to the perimeter.

Exercise 51 Students may need to review how to use ordered pairs to make a line graph.

Exercise 53 Students may find that the pattern is not as simple as they originally thought. Use this exercise to illustrate that straightforward conjectures may be incorrect.

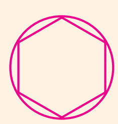
40. Answers may vary. Sample: In Exercise 31, each number increases by increasing multiples of 2. In Exercise 33, to get the next term, divide by 10.

41.



You would get points on a third line between and parallel to the first two lines.

42.



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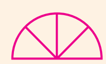

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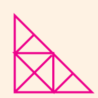
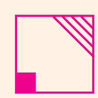


45.

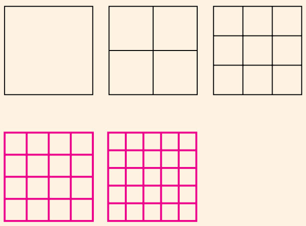


4. Assess & Reteach

PowerPoint

Lesson Quiz

Find a pattern for each sequence. Use the pattern to show the next two terms or figures.

1. 3, −6, 18, −72, 360 **−2160; 15,120**
2. 

Use the table and inductive reasoning. Make a conjecture about each value.

1	= 1	$= \frac{1 \cdot 2}{2}$
1 + 2	= 3	$= \frac{2 \cdot 3}{2}$
1 + 2 + 3	= 6	$= \frac{3 \cdot 4}{2}$
1 + 2 + 3 + 4	= 10	$= \frac{4 \cdot 5}{2}$

3. the sum of the first 10 counting numbers **55**
4. the sum of the first 1000 counting numbers **500,500**

Show that the conjecture is false by finding one counterexample.

5. The sum of two prime numbers is an even number. **Sample: 2 + 3 = 5, and 5 is not even.**

Alternative Assessment

Have each student write two conjectures, one true and one false; exchange conjectures with a partner; and determine whether the partner's conjectures are true or false. Have partners compare their findings.

47. Answers may vary. Samples are given.
a. Women may soon outrun men in running competitions.
b. The conclusion was based on continuing the trend shown in past records.
c. The conclusions are based on fairly recent records for women, and those rates of improvement may not continue. The conclusion about the marathon is most suspect because records date only from 1955.

47. Math in the Media Read this exerpt from a news article.

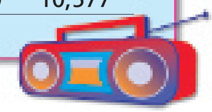
> **Top female runners** have been improving about twice as quickly as the fastest men, a new study says. If this pattern continues, women may soon outrun men in competition!
>
> The study is based on world records collected at 10-year intervals, starting in 1905 for men and in the 1920s for women. If the trend continues, the top female and male runners in races ranging from 200 m to 1500 m might attain the same speeds sometime between 2015 and 2055.
>
> Women's marathon records date from 1955 but their rapid fall suggests that the women's record will equal that of men even more quickly.

a. What conclusion was reached in the study? **a–c. See left.**
b. How was inductive reasoning used to reach the conclusion?
c. Explain why the conclusion that women may soon be outrunning men may be incorrect. For which race is the conclusion most suspect? For what reason?

48. Communications GPS The table shows the number of commercial radio stations in the United States for a 50-year period. **See**
a. Make a line graph of the data. **back of book.**
b. Use the graph and inductive reasoning to make a conjecture about the number of radio stations in the United States in the year 2010. **about 12,000 radio stations**
c. How confident are you about your conjecture? Explain. **See back of book.**

Number of Radio Stations

1950	2,835
1960	4,224
1970	6,519
1980	7,871
1990	9,379
2000	10,577



SOURCE: Federal Communications Commission

49. Answers may vary. Sample: 1, 3, 9, 27, 81, . . . 1, 3, 5, 7, 9, . . .

49. Open-Ended Write two different number-pattern sequences that begin with the same two numbers. **See left.**

50. Error Analysis For each of the past four years, Paulo has grown 2 in. every year. He is now 16 years old and is 5 ft 10 in. tall. He figures that when he is 22 years old he will be 6 ft 10 in. tall. What would you tell Paulo about his conjecture? **See margin.**

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For Exercise 51, you may want to review "Coordinates of a point" in the Glossary.

51. Coordinate Geometry You are given x- and y-coordinates for 14 points.
$A(1, 5)$ $B(2, 2)$ $C(2, 8)$ $D(3, 1)$ $E(3, 9)$ $F(6, 0)$ $G(6, 10)$
$H(7, -1)$ $I(7, 11)$ $J(9, 1)$ $K(9, 9)$ $L(10, 2)$ $M(10, 8)$ $N(11, 5)$
a. Graph each point. **See margin.**
b. Most of the points fit a pattern. Which points do not? ***H* and *I***
c. Describe the figure that fits the pattern. **a circle**

52. History Leonardo of Pisa (about 1175–1258), also known as Fibonacci (fee buh NAH chee), was born in Italy and educated in North Africa. He was one of the first Europeans to use modern numerals instead of Roman numerals. The special sequence 1, 1, 2, 3, 5, 8, 13, . . . is known as the Fibonacci sequence. Find the next three terms of this sequence. **21, 34, 55**

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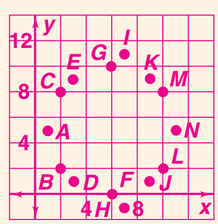
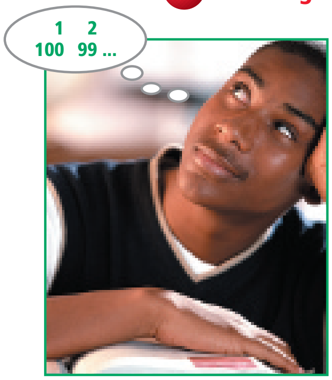
53. Time Measurement Leap years have 366 days. **See back of book.**
a. The years 1984, 1988, 1992, 1996, and 2000 are consecutive leap years. Look for a pattern in their dates. Then, make a conjecture about leap years.
b. Of the years 2010, 2020, 2100, and 2400, which do you think will be leap years?
c. **Research** Find out whether your conjecture for part (a) and your answer for part (b) are correct. How are leap years determined?

50. His conjecture is probably false because most people's growth slows by 18 until they stop growing somewhere between 18 and 22 years.

51. a.





Challenge

54. History When he was in the third grade, German mathematician Karl Gauss (1777–1855) took ten seconds to sum the integers from 1 to 100. Now it's your turn. Find a fast way to sum the integers from 1 to 100; from 1 to n. (*Hint:* Use patterns.) **See margin.**

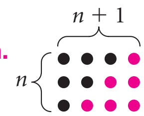
55a. 1, 3, 6, 10, 15, 21

55. a. Algebra Write the first six terms of the sequence that starts with 1, and for which the difference between consecutive terms is first 2, and then 3, 4, 5, and 6.

b. Evaluate $\frac{n^2 + n}{2}$ for $n = 1, 2, 3, 4, 5,$ and 6. Compare the sequence you get with your answer for part (a). **They are the same.**

c. Examine the diagram at the right and explain how it illustrates a value of $\frac{n^2 + n}{2}$. **See margin.**

d. Draw a similar diagram to represent $\frac{n^2 + n}{2}$ for $n = 5$. **See margin.**



Test Prep

Multiple Choice

56. The sum of the numbers from 1 to 10 is 55. The sum of the numbers from 11 to 20 is 155. The sum of the numbers from 21 to 30 is 255. Based on this pattern, what is the sum of numbers from 91 to 100? **B**

A. 855 **B.** 955 **C.** 1055 **D.** 1155

57. Which of the following conjectures is false? **J**

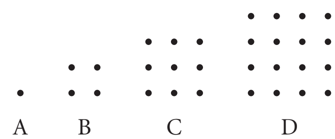
F. The product of two even numbers is even.
G. The sum of two even numbers is even.
H. The product of two odd numbers is odd.
J. The sum of two odd numbers is odd.

58. [2] a. 25, 36, 49
b. n^2
[1] one part correct

Short Response

58. a. How many dots would be in each of the next three figures? **a–b. See left.**

b. Write an expression for the number of dots in the nth figure.



Extended Response

59. a. Describe the pattern. List the next two equations in the pattern.

b. Guess what the product of 181 and 11 is. Test your conjecture.

c. State whether the pattern can continue forever. Explain. **a–c. See margin.**

(101)(11) = 1111
(111)(11) = 1221
(121)(11) = 1331
(131)(11) = 1441
(141)(11) = 1551

Mixed Review

Skills Handbook

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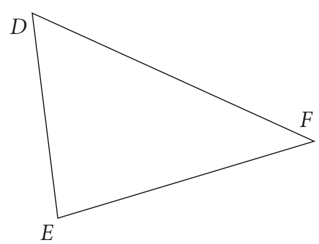
60. Measure the sides DE and EF to the nearest millimeter. **30 mm; 40 mm**

61. Measure each angle of DEF to the nearest degree. **∠D: 59°; ∠E: 60°; ∠F: 40°**

62. Draw a triangle that has sides of length 6 cm and 5 cm with a 90° angle between those two sides. **Check students' work.**

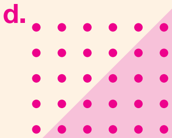


Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 75
- Test-Taking Strategies, p.70
- Test-Taking Strategies with Transparencies

55. c. The diagram shows the product of n and $n + 1$ divided by 2 when $n = 3$. The result is 6.

d.



59. [4] a. The product of 11 and a three-digit number that begins and ends in 1 is a four-digit number that begins and ends in 1 and has middle digits that are each one greater than the middle digit of the three-digit number.
(151)(11) = 1661
(161)(11) = 1771

b. 1991

c. No; (191)(11) = 2101

[3] minor error in explanation

[2] incorrect description in part (a)

[1] correct products for (151)(11), (161)(11), and (181)(11)

54. Answers may vary. Sample:

$$\begin{array}{r} 100 + 99 + 98 + \ldots + 3 + 2 + 1 \\ \underline{1 + 2 + 3 + \ldots + 98 + 99 + 100} \\ 101 + 101 + 101 + \ldots + 101 + 101 + 101 \end{array}$$

The sum of the first 100 numbers is $\frac{100 \cdot 101}{2}$, or 5050.

The sum of the first n numbers is $\frac{n(n + 1)}{2}$.