1.12.1 Introduction Go back to lesson 9 and provide bullet #3

In today's lesson we will consider two examples of non-inertial reference frames:

- a reference frame that accelerates in a straight line
- a reference frame that moves along a circular path

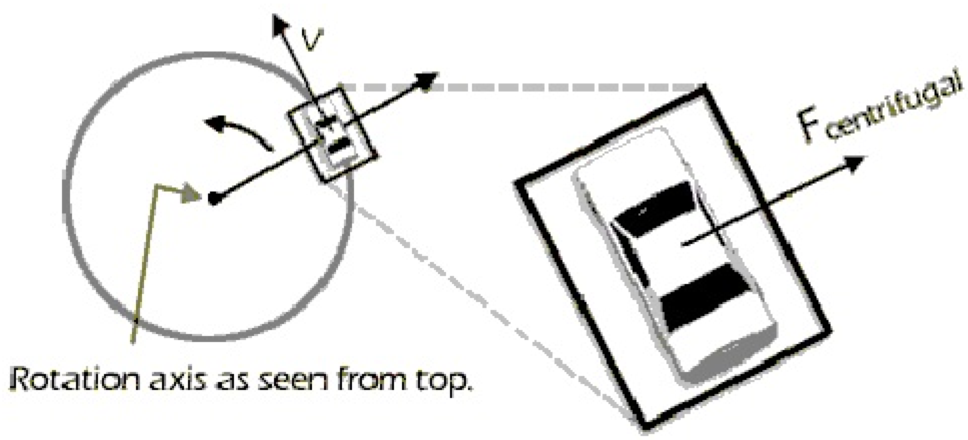
1.12.2 Straight Line Accelerated Motion

Imagine you are sitting in a car that is at rest. If the driver stomps on the gas pedal. Of course, the car will accelerate forward and your body, resisting the motion will feel as though it is being pushed back into the seat. What force is responsible for your body "accelerating" backward?

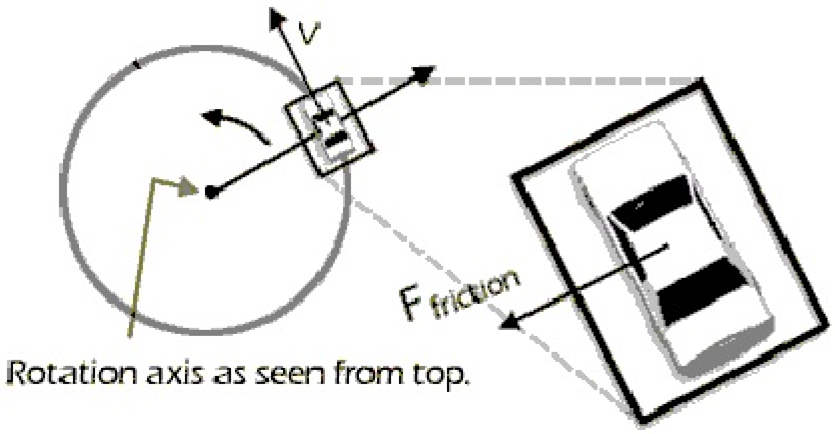
You might think that a force is pushing your body backward into the seat however, the sensation of being pushed back is due to Newton's first law, the law of inertia. This is the first example of a fictitious force acting on your body which we use to explain the sensation we experience in a rotating non-inertial reference frame.

1.12.3 Circular Motion

Imagine that you are driving a car at a constant velocity down a straight, level road. If you now turn the car to follow a curve to the left in the road at a constant speed, you will feel a force that pulls you towards the right (toward the outside of the curve). You might try to explain the force you are experiencing and call it a centrifugal (center-fleeing) force, however we must remember that Newton's laws do not hold in accelerating reference frames such as this one.



An observer standing by the road would analyze your motion quite differently. To a good approximation, they observe the car from an inertial reference frame. They deduce that the friction between the car's tires and the road force the car around the curve. To them this “forcing around the curve” is a center-seeking, or centripetal, force (a force toward the center of the curve) and the resulting acceleration is a centripetal acceleration. They "feel" no centrifugal force. It does not exist as far as they are concerned.



The centrifugal force does not exist in an inertial reference frame. It exists only in rotating reference frames, such as the reference frame of the car going around a curve.

1.12.4 Is the Centrifugal Force a Real Force?

The rotation of the Earth gives rise to it’s equatorial bulge. It makes the equatorial radius 21.3 kilometres greater than the polar radius. This difference is more than twice the height of Mount. Everest (29,028 ft. or 8.847 kilometres). In addition, engineer’s face problems when designing turbine blades of jet engines that have to stay together at rotation rates of up to 100,000 revolutions per minute. So what do you think? Is the centrifugal force a real force?

1.12.5 Deriving a Mathematical Relationship for Centripetal Acceleration

To determine the value of the acceleration, we begin with the definition of acceleration:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \quad (1.3)$$

During uniform circular motion, acceleration occurs because the direction of the velocity changes, even though the magnitude of the velocity remains constant. For this reason, in the relation above,

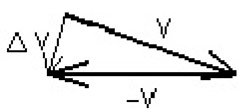
$$|\vec{v}_1| = |\vec{v}_2| = v \ .$$

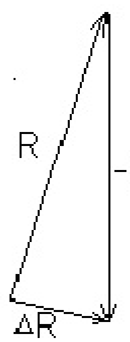
The radius of the circle subtended is also constant, so we can also say that

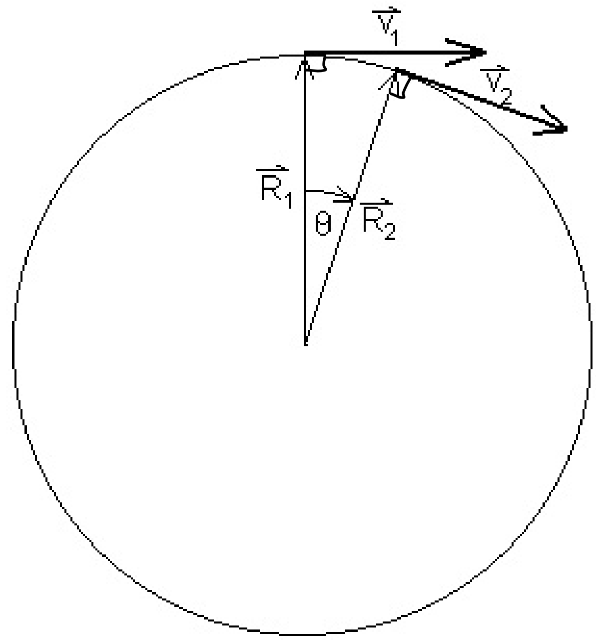
$$|\vec{R}_1| = |\vec{R}_2| = R$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

To determine, mathematically, the value of the acceleration, we subtract the velocity vectors for the situation we are given. For example:







and we compare this triangle with the triangle we get when we consider the change in position of the object. These two triangles are both isosceles and the small angle in both of them is the same. They are therefore similar triangles and the ratios of their corresponding sides are equal. i.e.:

$$\frac{\Delta v}{v} = \frac{\Delta R}{R}$$

$$\therefore \ \Delta v = \frac{v \Delta R}{R}$$

Substituting this into our expression for the instantaneous acceleration:

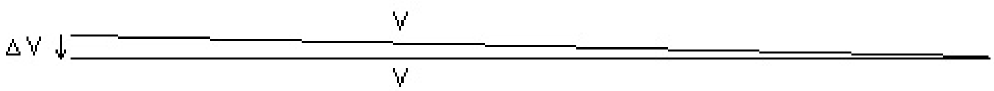
$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \quad \text{or}$$

$$a_{inst} = \lim_{\Delta t \to 0} \frac{v \Delta R}{R \Delta t}$$

$$a_{inst} = \frac{v}{R}\lim_{\Delta t \to 0}\frac{\Delta R}{\Delta t}$$

$$a_{inst} = \frac{v^2}{R}$$

Now that we have an expression for the magnitude of the acceleration, we need to determine its direction. For very small time intervals, (which is what we are interested in after all), we get the situation shown below:



We can see then that the direction of the change in velocity (and therefore the direction of the acceleration) is perpendicular to the velocity vectors and therefore towards the centre of the circle. For this reason, acceleration of an object that is traveling around a circle is called centripetal (centre seeking) acceleration. Since the direction is always known, we can also drop vector notation when considering centripetal acceleration and use the equation:

$$a_c = \frac{v^2}{R} \quad (1.10)$$

Eg.#1 A fly hanging on for dear life swings around on an hour hand that is 30 cm. long.

a) What is the tangential velocity of the fly?

$$v = \frac{2\pi r}{T}$$

$$= \frac{2 \times 3.14 \times 0.30m.}{12hr. \times 3600s.}$$

$$= 4.36 \times 10^{-5} m/s$$

$\vec{v} = 4.36 \times 10^{-5}$ m/s [⊥ R]

b) What is the fly's acceleration?

$$a_c = \frac{v^2}{r}$$

$$= \frac{(4.36 \times 10^{-5})^2}{0.30m.}$$

$$= 6.34 \times 10^{-9} m/s^2$$

$\vec{a} = 6.34 \times 10^{-9}$ m/s² [towards centre of the circle]

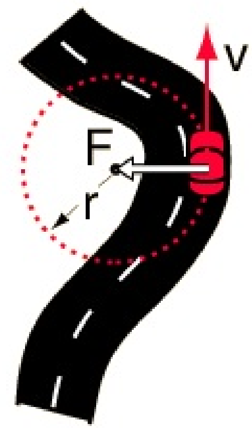
Eg.#2 What is the centripetal acceleration of a stone on the end of a string 0.30 m. long if it rotates at a rotational frequency of 20.0 Hz?

$$\frac{1}{T} = f \qquad v = \frac{2\pi r}{T} = 2\pi r \times \frac{1}{T} = 2\pi r f \qquad a_c = \frac{v^2}{r} = \frac{4\pi^2 r^2 f^2}{r} = 4\pi^2 r f^2$$

$$a_c = 4\pi^2 r f^2$$
$$= 4 \times 3.14^2 \times 0.30m.\times(20.0Hz.)^2$$
$$= 4.7 \times 10^3 m/s^2$$

$\vec{a}_c = 4.7 \times 10^3 m/s^2$ [towards the centre of the circle]

1.12.6 Forces and Circular Motion



When an object travels in a circular path, a centre-seeking force keeps the object following the circular path. This force can be caused by tension, gravity, friction, the Normal force or the vector sum of these forces.

When solving dynamics problems involving centripetal acceleration, we must be careful not to substitute the centripetal force as a force on the left side of the equation. The product of the mass and the centripetal acceleration belongs on the right side of the equation (since it is an acceleration after all). An example should help illustrate this. Consider the situation shown above. The friction force is the only horizontal force. Therefore

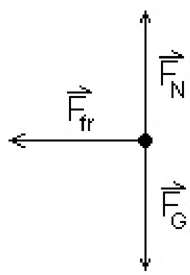
$$\vec{F}_{NET} = m\vec{a}$$
$$F_{fr} = m\frac{v^2}{r}$$
$$\mu F_N = m\frac{v^2}{r}$$

etc...

Notice that the centripetal force is the net force. It never appears on the left side of Newton's third law.

Eg.#3 A 1.2 × 10^3 kg. car negotiates a 50.0 m. curve at 22 m/s.

a) Draw a FBD and label the forces.



b) Determine the minimum static coefficient of friction needed to keep the car on the road.

$$\vec{F}_{NET} = m\vec{a}$$

$$\mu mg = m\frac{v^2}{r}$$

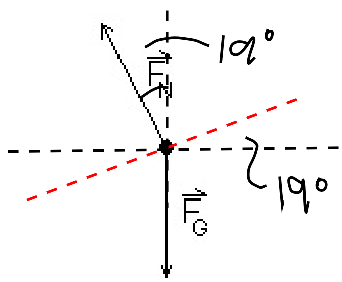
$$\mu = \frac{v^2}{rg}$$

$$= \frac{(22m./s.)^2}{50.0m.\times 9.8m./s.^2}$$

$$= 0.99$$

Eg.#4 A 1.2 × 10^3 kg. car negotiates a frictionless 50.0 m. curve that is banked at 19° to the horizontal at 22 m/s.

a) Draw a FBD and label the forces



b) What force is responsible for the centripetal acceleration?

The horizontal component of the Normal force

c) Determine the constant speed that the car must travel in order to stay on the road surface.

$$F_N \cos\Theta = mg \ (i) \ \text{also} \quad F_N \sin\Theta = m\frac{v^2}{r} \ (ii)$$

$$\therefore \tan\Theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\Theta}$$

$$= \sqrt{50.0m.\times 9.81m/s^2 \times \tan 19^\circ}$$

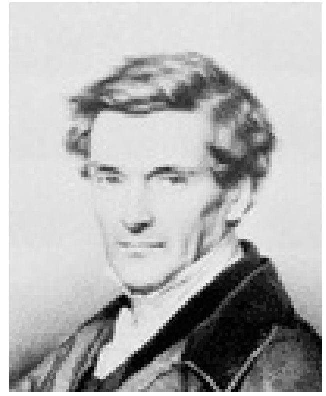
$$= 13m/s$$

(ii) ÷ (i)

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{mv^2}{r} \div mg$$

$$\tan\theta = \frac{v^2}{rg}$$

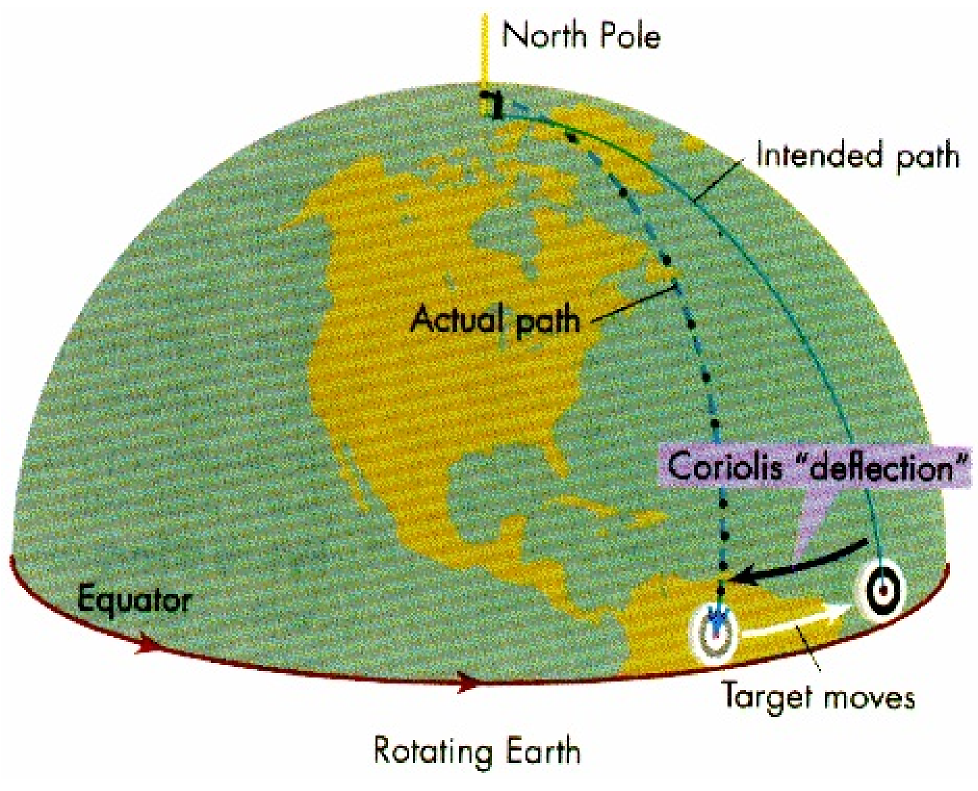
1.12.7 The Coriolis Effect



Gustave Coriolis (1792 – 1843)

The Coriolis effect is an inertial force described by the 19th-century French engineer-mathematician Gustave-Gaspard Coriolis in 1835. Coriolis showed that, in order to use Newtonian laws of motion in a rotating reference frame, an inertial force must be included in the equations of motion. This force, the Coriolis force, acts to the right of the direction of body motion for counterclockwise rotation of the reference frame or to the left for clockwise rotation.

This Coriolis force causes a deflection of the path of an object that moves within the rotating coordinate system. The object does not actually deviate from its path, but it appears to do so because of the motion of the coordinate system.



The Coriolis effect is most apparent in the path of an object moving longitudinally. On the Earth an object that moves along a north-south path, or longitudinal line, will undergo apparent deflection to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. There are two reasons for this phenomenon: first, the Earth rotates eastward; and second, the tangential velocity of a point on the Earth is a function of latitude (the velocity is essentially zero at the poles and it attains a maximum value at the Equator). Thus, if a cannon were fired northward from a point on the Equator, the projectile would land to the east of its due north path. This variation would occur because the projectile was moving eastward faster at the Equator than was its target farther north. Similarly, if the weapon were fired toward the Equator from the North Pole, the projectile would again land to the right of its true path. In this case, the target area would have moved eastward before the shell reached it because of its greater eastward velocity. An exactly similar displacement occurs if the projectile is fired in any direction.

The Coriolis deflection is therefore related to the motion of the object, the motion of the Earth, and the latitude.

1.12.8 Is it Possible to Detect the Earth's Rotation in a Draining Sink?

Yes, but it is very difficult. Because the Coriolis force is so small, you must go to extraordinary lengths to detect it. But, it has been done. You cannot use an ordinary sink since it is not circularly symmetric: its oval shape and off-center drain render any results suspect. You need to use a smooth pan of about one meter in diameter with a very small hole in the center. A stopper (which could be removed from below so as to not introduce any spurious motion) blocks the hole while the pan is being filled with water. The water is then allowed to sit undisturbed for perhaps a week to let all of the motion die out which was introduced during filling. Then, the stopper is removed (from below). Because the hole is very small, the pan drains slowly indeed. This is necessary, because it takes hours before the tiny Coriolis force could develop sufficient deviation in the draining water for it to produce a circular flow. With these procedures, the rotation is always cyclonic.

The direction of rotation in draining sinks and toilets is NOT determined by the rotation of the Earth, but by rotation that was introduced earlier when it was being filled or subsequently being disturbed (say by washing). The rotation of the Earth does influence the direction of rotation of large weather systems and large vortices in the oceans, for these are very long-lived phenomena and so allow the very weak Coriolis force to produce a significant effect, with time.

Worksheet 1.12

1. You are whirling a ball on the end of a string in a horizontal circle around your head. What is the effect on the magnitude of the centripetal acceleration of the ball if

(a) the speed of the ball remains constant, but the radius of the circle doubles?
The centripetal acceleration will drop to half it's previous value.

(b) the radius of the circle remains constant, but the speed doubles?
The centripetal acceleration will be four times greater

2. At a distance of 25 km from the eye of a hurricane, the wind is moving at 180 km/h in a circle. What is the magnitude of the centripetal acceleration, in metres per second squared, of the particles that make up the wind?

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{(50m./s.)^2}{25000m.} \\ &= 0.1m./s.^2 \end{aligned}$$

3. Calculate the magnitude of the centripetal acceleration in the following situations:
a) An electron is moving around a nucleus with a speed of 2.18×10^6 m/s. The diameter of the electron's orbit is 1.06×10^{-10} m.

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{\left(2.18 \times 10^6 m./s.\right)^2}{5.30 \times 10^{-11} m.} \\ &= 8.97 \times 10^{22} m./s.^2 \end{aligned}$$

b) A coin is placed flat on a vinyl record, turning at 33 rpm. The coin is 13 cm. from the centre of the record.

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi \times 0.13m. \times 33}{60} \\ &= 0.45m./s. \end{aligned}$$

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{(0.45m./s.)^2}{0.13m.} \\ &= 1.6m./s.^2 \end{aligned}$$

4. A ball on a string, moving in a horizontal circle of radius 2.0 m, undergoes a centripetal acceleration of magnitude 15 m/s^2. What is the speed of the ball?

$$v = \sqrt{r \times a}$$
$$= \sqrt{2.0m. \times 15m./s.^2}$$
$$= 5.5m./s.$$

5. Mercury orbits the Sun in an approximately circular path, at an average distance of 5.79 x 10^{10} m, with a centripetal acceleration of magnitude 4.0 x 10^{-2} m/s^2. What is its period of revolution around the Sun,

a) in seconds?

$$v = \sqrt{r \times a} = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{\sqrt{r \times a}}$$
$$= \frac{2\pi \times 5.79 \times 10^{10} m.}{\sqrt{5.79 \times 10^{10} m. \times 4.0 \times 10^{-2} m/s^2}}$$
$$= 7.6 \times 10^6 s.$$

b) in “Earth” days?

$$T = \frac{T(\text{sec})}{86400s/day}$$
$$= \frac{7.6 \times 10^6 s.}{86400s/day}$$
$$= 88d$$

6. Patrons on an amusement park ride called the Rotor stand with their backs against the wall of a rotating cylinder while the floor drops away beneath them. To keep from sliding downward, they require a centripetal acceleration in excess of about 25 m/s^2. The Rotor has a diameter of 5.0 m. What is the minimum frequency for its rotation?

$$v = \sqrt{r \times a} = 2\pi r f$$
$$f = \frac{\sqrt{r \times a}}{2\pi r}$$
$$= \frac{\sqrt{2.5m. \times 25m./s.^2}}{2\pi \times 2.5m.}$$
$$= 0.50Hz.$$