

1.2.1 Adding Whole Numbers and Applications

Learning Objective(s)

- 1 Add whole numbers without regrouping.
- 2 Add whole numbers with regrouping.
- 3 Find the perimeter of a polygon.
- 4 Solve application problems using addition.

Introduction

Adding is used to find the total number of two or more quantities. The total is called the **sum**, or the number that results from the addition. You use addition to find the total distance that you travel if the first distance is 1,240 miles and the second distance is 530 miles. The two numbers to be added, 1,240 and 530, are called the **addends**. The total distance, 1,770 miles, is the sum.

Adding Whole Numbers, without Regrouping

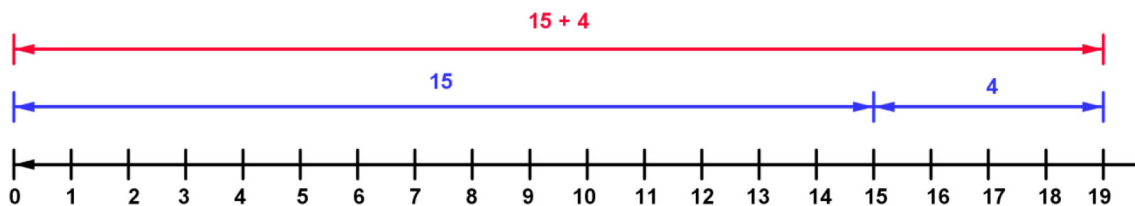
Objective 1

Adding numbers with more than one digit requires an understanding of **place value**. The place value of a digit is the value based on its position within the number. In the number 492, the 4 is in the hundreds place, the 9 is in the tens place, and the 2 is in the ones place. You can use a number line to add. In the example below, the blue lines represent the two quantities, 15 and 4, that are being added together. The red line represents the resulting quantity.

Example

Problem

$$15 + 4 = ?$$



On the number line, the blue line segment stretches across 15 units, representing the number 15. The second blue segment shows that if you add 4 more units, the resulting number is 19.

Answer

$$15 + 4 = 19$$

You can solve the same problem without a number line, by adding vertically. When adding numbers with more than 1 digit, it is important to line up your numbers by place

value, as in the example below. You must add ones to ones, tens to tens, hundreds to hundreds, and so on.

Example	
Problem	$15 + 4 = ?$
$\begin{array}{r} 15 \\ + 4 \\ \hline \end{array}$	Because 5 and 4 have the same place value, make sure they are aligned when you add.
$\begin{array}{r} 15 \\ + 4 \\ \hline 9 \end{array}$	First, add the ones digits (the numbers on the right). The result goes in the ones place for the answer.
$\begin{array}{r} 15 \\ + 4 \\ \hline 19 \end{array}$	Then, add the tens digits and put the result in the tens place of the answer. In this case, there is no tens digit in the second number, so the result is the same as the tens digit of the first number (1).
<i>Answer</i>	$15 + 4 = 19$

This strategy of lining up the numbers is effective for adding a series of numbers as well.

Example	
Problem	$1 + 2 + 3 + 2 = ?$
$\begin{array}{r} 1 \\ 2 \\ 3 \\ + 2 \\ \hline 8 \end{array}$	
<i>Answer</i>	$1 + 2 + 3 + 2 = 8$

Adding Whole Numbers, with Regrouping

When adding whole numbers, a place-value position can have only one digit in it. If the sum of digits in a place value position is more than 10, you have to **regroup** the number of tens to the next greater place value position.

When you add, make sure you line up the digits according to their place values, as in the example below. As you regroup, place the regrouped digit above the appropriate digit in the next higher place value position and add it to the numbers below it.

Example	
Problem	$45 + 15 = ?$
$\begin{array}{r} 1 \\ 45 \\ + 15 \\ \hline 0 \end{array}$	Add the ones. Regroup as needed. The sum of 5 and 5 is 10. This is 1 ten and 0 ones. Write the number of ones (0) in the ones place and the 1 ten in the tens place above the 4.
$\begin{array}{r} 1 \\ 45 \\ + 15 \\ \hline 60 \end{array}$	Add the tens, $1 + 4 + 1$ is 6 tens. The final sum is 60.
Answer	$45 + 15 = 60$

You must add digits in the ones place first, the digits in the tens place next, and so on. Go from right to left.

Example

Problem $4,576 + 698 = ?$

$$\begin{array}{r} 4,576 \\ + 698 \\ \hline \end{array}$$

First, write the problem with one addend on top of the other. Be sure you line up the place values!

$$\begin{array}{r} 1 \\ 4,576 \\ + 698 \\ \hline 4 \end{array}$$

Add the numbers in the ones place. Since the sum is 14, write the ones value (4) in the ones place of the answer. Write the 1 ten in the tens place above the 7.

$$\begin{array}{r} 11 \\ 4,576 \\ + 698 \\ \hline 74 \end{array}$$

Add the numbers in the tens place. Since the sum is 17 tens, regroup 17 tens as 1 hundred, 7 tens. Write 7 in the tens place in the answer and write the 1 hundred in the hundreds place above the 5.

$$\begin{array}{r} 111 \\ 4,576 \\ + 698 \\ \hline 274 \end{array}$$

Add the numbers in the hundreds place, including the 1. Again, the sum is more than one digit. Rename 12 hundreds as 2 hundreds and 1 thousand. Write the 2 in the hundreds place and the 1 above the 4 in the thousands place.

$$\begin{array}{r} 111 \\ 4,576 \\ + 698 \\ \hline 5,274 \end{array}$$

Add the numbers in the thousands place, including the 1. The final sum is 5,274.

Answer $4,576 + 698 = 5,274$

Adding Numbers Using the Partial Sums Method

Another way to add is the partial sums method. In the example below, the sum of $23 + 46$ is found using the partial sums method. In this method, you add together all the numbers with the same place value and record their *values* (not just a single digit). Once you have done this for each place value, add their sums together.

Example		
Problem $23 + 46 = ?$		
<p><u>Step 1: Add Tens</u></p> $\begin{array}{r} 23 \dots\dots\dots 20 \\ 46 \dots\dots\dots \underline{40} \\ 60 \end{array}$	Let's begin by adding the values in the tens position. Notice that the digits in the tens place are highlighted, and on the right, the values are written as 20 and 40.	
<p><u>Step 2: Add Ones</u></p> $\begin{array}{r} 23 \dots\dots\dots 3 \\ 46 \dots\dots\dots \underline{6} \\ 9 \end{array}$	Add the values in the ones place.	
<u>Step 3: Add Parts</u>		
	$\begin{array}{r} 60 \\ + 9 \\ \hline 69 \end{array}$	Finally, add the two sums together.
<i>Answer</i> $23 + 46 = 69$		

The next example adds a series of three numbers. Notice that hundreds is the greatest place value now, so hundreds are added before the tens. (You can add in any order that you prefer.) Also notice that in Step 3, the value in the ones column for 350 is zero, but you still add that in to make sure everything is accounted for.

Example	
Problem $225 + 169 + 350 = ?$	
<p><u>Step 1: Add Hundreds</u></p> $\begin{array}{r} 225 \dots\dots\dots 200 \\ 169 \dots\dots\dots 100 \\ 350 \dots\dots\dots \underline{300} \\ 600 \end{array}$	Add the values represented by the digits in the hundreds place first. This gives a sum of 600.
<u>Step 2: Add Tens</u>	
$\begin{array}{r} 225 \dots\dots\dots 20 \\ 169 \dots\dots\dots 60 \\ 350 \dots\dots\dots \underline{50} \\ 130 \end{array}$	Next, add the values from the tens place. The sum is 130.

Step 3: Add Ones

$$\begin{array}{r} 22\mathbf{5} \dots\dots\dots \mathbf{5} \\ 16\mathbf{9} \dots\dots\dots \mathbf{9} \\ 35\mathbf{0} \dots\dots\dots \mathbf{0} \\ \hline \mathbf{14} \end{array}$$

Add the values from the ones place. The sum is 14.

Step 4: Add Parts

$$\begin{array}{r} 600 \\ 130 \\ + 14 \\ \hline 744 \end{array}$$

At this point, you have a sum for each place value. Add together these three sums, which gives a final value of 744.

Answer $225 + 169 + 350 = 744$

Self Check A

A local company built a playground at a park. It took the company 124 hours to plan out the playground, 243 hours to prepare the site, and 575 hours to build the playground. Find the total number of hours the company spent on the project.

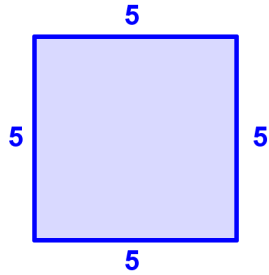
When adding multi-digit numbers, use the partial sums method or any method that works best for you.

Finding the Perimeter of a Polygon

Objective 3

A **polygon** is a many-sided closed figure with sides that are straight line segments. Triangles, rectangles, and pentagons (five-sided figures) are polygons, but a circle or semicircle is not. The **perimeter** of a polygon is the distance around the polygon. To find the perimeter of a polygon, add the lengths of its sides, as in the example below.

Example	
Problem	One side of a square has a length of 5 cm. Find the perimeter.



Draw the polygon and label the lengths of the sides. Since the side lengths of a square are equal, each side is 5 cm.

$$\begin{array}{r} 5 \\ 5 \\ 5 \\ + 5 \\ \hline 20 \end{array}$$

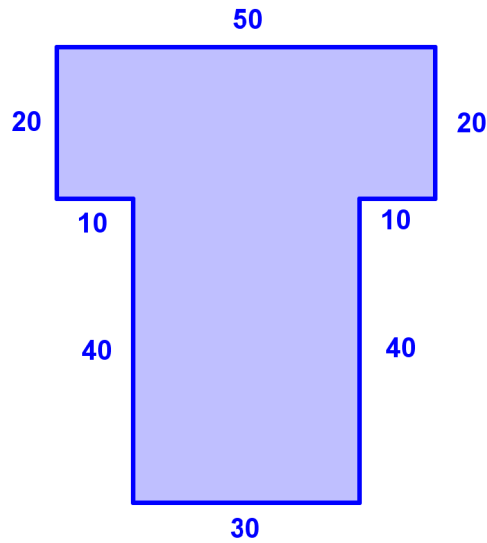
Add the lengths of each side.

Answer The perimeter is 20 cm.

The key part of completing a polygon problem is correctly identifying the side lengths. Once you know the side lengths, you add them as you would in any other addition problem.

Example

Problem A company is planning to construct a building. Below is a diagram illustrating the shape of the building's floor plan. The length of each side is given in the diagram. Measurements for each side are in feet. Find the perimeter of the building.



$$\begin{array}{r} 50 \\ 20 \\ 20 \\ 10 \\ 10 \\ 40 \\ 40 \\ + 30 \\ \hline 220 \end{array}$$

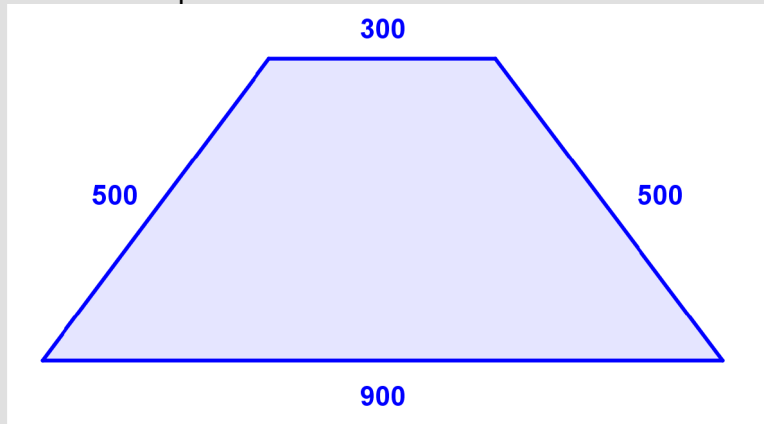
Add the lengths of each side, making sure to align all numbers according to place value.

Answer

The perimeter is 220 ft.

Self Check B

Find the perimeter of the trapezoid in feet.



Solving Application Problems

Objective 4

Addition is useful for many kinds of problems. When you see a problem written in words, look for key words that let you know you need to *add* numbers.

Example	
Problem	A woman preparing an outdoor market is setting up a stand with 321 papayas, 45 peaches, and 213 mangos. How many pieces of fruit in total does the woman have on her stand?
$\begin{array}{r} 321 \\ 45 \\ + 213 \\ \hline \end{array}$	<p>The words “how many... in total” suggest that you need to add the numbers of the different kinds of fruits.</p> <p>Use any method you like to add the numbers. Below, the partial sums method is used.</p>
<p>Step 1: Add Hundreds</p> $\begin{array}{r} 321 \dots\dots\dots 300 \\ 045 \dots\dots\dots 0 \\ 213 \dots\dots\dots \underline{200} \\ 500 \end{array}$	<p>Add the numbers represented by the digits in the hundreds place first. This gives a sum of 600.</p>

Step 2: Add Tens

$$\begin{array}{r}
 321 \dots\dots\dots 20 \\
 145 \dots\dots\dots 40 \\
 213 \dots\dots\dots \underline{10} \\
 70
 \end{array}$$

Next, add the numbers from the tens place. The sum is 70.

Step 3: Add Ones

$$\begin{array}{r}
 321 \dots\dots\dots 1 \\
 145 \dots\dots\dots 5 \\
 213 \dots\dots\dots \underline{3} \\
 9
 \end{array}$$

Add the numbers from the ones.

Step 4: Add Parts

$$\begin{array}{r}
 500 \\
 70 \\
 + 9 \\
 \hline
 579
 \end{array}$$

Add together the three previous sums. The final sum is 579.

Answer The woman has 579 pieces of fruit on her stand.

Example

Problem Lynn has 23 rock CDs, 14 classical music CDs, 8 country and western CDs, and 6 movie soundtracks. How many CDs does she have in all?

$$\begin{array}{r}
 23 \\
 14 \\
 8 \\
 + 6 \\
 \hline
 \end{array}$$

The words “how many... in all” suggest that addition is the way to solve this problem.

To find how many CDs Lynn has, you need to add the number of CDs she has for each music style.

$$\begin{array}{r}
 2 \\
 23 \\
 14 \\
 8 \\
 + 6 \\
 \hline
 51
 \end{array}$$

Use whatever method you prefer to find the sum of the numbers.

Answer Lynn has 51 CDs.

The following phrases also appear in problem situations that require addition.

Phrase	Example problem
Add to	Jonah was planning a trip from Boston to New York City. The distance is 218 miles. His sister wanted him to visit her in Springfield, Massachusetts, on his way. Jonah knew this would add 17 miles to his trip. How long is his trip if he visits his sister?
Plus	Carrie rented a DVD and returned it one day late. The store charged \$5 for a two-day rental, plus a \$3 late fee. How much did Carrie pay for the rental?
Increased by	One statistic that is important for football players in offensive positions is <i>rushing</i> . After four games, one player had rushed 736 yards. After two more games, the number of yards rushed by this player increased by 352 yards. How many yards had he rushed after the six games?
More than	Lavonda posted 38 photos to her social network profile. Chris posted 27 more photos to his than Lavonda. How many photos did Chris post?

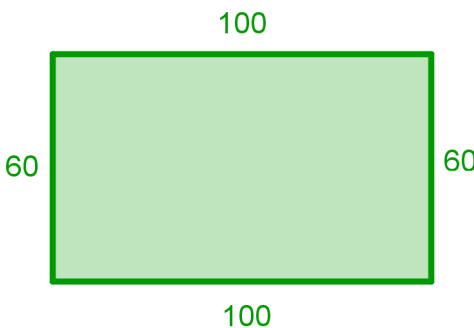
Example	
Problem	Lena was planning a trip from her home in Amherst to the Museum of Science in Boston. The trip is 91 miles. She had to take a detour on the way, which added 13 miles to her trip. What is the total distance she traveled?
<p>The word “added” suggests that addition is the way to solve this problem.</p> <p>To find the total distance, you need to add the two distances.</p> $\begin{array}{r} 91 \\ + 13 \\ \hline 104 \end{array}$ <p>Answer The total distance is 104 miles.</p>	

It can help to seek out words in a problem that imply what operation to use. See if you can find the key word(s) in the following problem that provide you clues on how to solve it.

Self Check C

A city was struck by an outbreak of a new flu strain in December. To prevent another outbreak, 3,462 people were vaccinated against the new strain in January. In February, 1,298 additional people were vaccinated. How many people in total received vaccinations over these two months?

Drawing a diagram to solve problems is very useful in fields such as engineering, sports, and architecture.

Example	
Problem	A coach tells her athletes to run one lap around a soccer field. The length of the soccer field is 100 yards, while the width of the field is 60 yards. Find the total distance that each athlete will have run after completing one lap around the perimeter of the field.
	<p>The words “total distance” and “perimeter” both tell you to add.</p> <p>Draw the soccer field and label the various sides so you can see the numbers you are working with to find the perimeter.</p>
$ \begin{array}{r} 1 \\ 100 \\ 100 \\ 60 \\ + 60 \\ \hline 20 \end{array} $	<p>There is a zero in the ones place, and the sum of 6 and 6 in the tens place is 12 tens. Place 2 tens in the tens place in the answer, and regroup 10 tens as 1 hundred.</p>
$ \begin{array}{r} 1 \\ 100 \\ 100 \\ 60 \\ + 60 \\ \hline 320 \end{array} $	<p>By adding the 1 hundred to the other digits in the hundreds place, you end up with a 3 in the hundreds place of the answer.</p>
<i>Answer</i>	Each athlete will have run 320 yards.

Summary

You can add numbers with more than one digit using any method, including the partial sums method. Sometimes when adding, you may need to regroup to the next greater place value position. Regrouping involves grouping ones into groups of tens, grouping tens into groups of hundreds, and so on. The perimeter of a polygon is found by adding the lengths of each of its sides.

1.2.1 Self Check Solutions

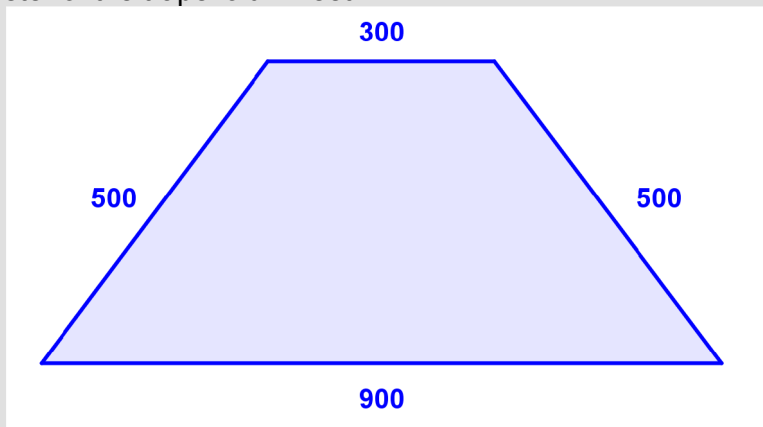
Self Check A

A local company built a playground at a park. It took the company 124 hours to plan out the playground, 243 hours to prepare the site, and 575 hours to build the playground. Find the total number of hours the company spent on the project.

$$800 + 130 + 12 = 942 \text{ hours}$$

Self Check B

Find the perimeter of the trapezoid in feet.



$$300 + 500 + 500 + 900 = 2,200 \text{ ft}$$

Self Check C

A city was struck by an outbreak of a new flu strain in December. To prevent another outbreak, 3,462 people were vaccinated against the new strain in January. In February, 1,298 additional people were vaccinated. How many people in total received vaccinations over these two months?

$$3462 + 1298 = 4,760$$

1.2.2 Subtracting Whole Numbers and Applications

Learning Objective(s)

- 1 Subtract whole numbers without regrouping.
- 2 Subtract whole numbers with regrouping.
- 3 Solve application problems using subtraction.

Introduction

Subtracting involves finding the difference between two or more numbers. It is a method that can be used for a variety of applications, such as balancing a checkbook, planning a schedule, cooking, or travel. Suppose a government official is out of the U.S. on business for 142 days a year, including travel time. The number of days per year she is in the U.S. is the difference of 365 days and 142 days. Subtraction is one way of calculating the number of days she would be in the U.S. during the year.

When subtracting numbers, it is important to line up your numbers, just as with addition. The **minuend** is the greater number from which the lesser number is subtracted. The **subtrahend** is the number that is subtracted from the minuend. A good way to keep minuend and subtrahend straight is that since subtrahend has “subtra” in its beginning, it goes next to the subtraction sign and is the number being subtracted. The **difference** is the quantity that results from subtracting the subtrahend from the minuend. In $86 - 52 = 34$, 86 is the *minuend*, 52 is the *subtrahend*, and 34 is the *difference*.

Subtracting Whole Numbers

Objective 1

When writing a subtraction problem, the minuend is placed above the subtrahend. This can be seen in the example below, where the minuend is 10 and the subtrahend is 7.

Example	
Problem	$10 - 7 = ?$
	$\begin{array}{r} 10 \\ - 7 \\ \hline 3 \end{array}$
Answer	$10 - 7 = 3$

When both numbers have more than one digit, be sure to work with one place value at a time, as in the example below.

Example	
Problem 689 – 353 = ?	
$\begin{array}{r} 689 \\ - 353 \\ \hline \end{array}$	First, set up the problem and align the numbers by place value.
$\begin{array}{r} 689 \\ - 353 \\ \hline 6 \end{array}$	Then, subtract the ones.
$\begin{array}{r} 689 \\ - 353 \\ \hline 36 \end{array}$	Next, subtract the tens.
$\begin{array}{r} 689 \\ - 353 \\ \hline 336 \end{array}$	Finally, subtract the hundreds.
<i>Answer</i> 689 – 353 = 336	

Lining up numbers by place value becomes especially important when you are working with larger numbers that have more digits, as in the example below.

Example	
Problem 9,864 – 743 = ?	
$\begin{array}{r} 9864 \\ - 743 \\ \hline \end{array}$	First, set up the problem and align the numbers by place value.
$\begin{array}{r} 9864 \\ - 743 \\ \hline 1 \end{array}$	Then, subtract the ones.
$\begin{array}{r} 9864 \\ - 743 \\ \hline 21 \end{array}$	Next, subtract the tens.

$$\begin{array}{r} 9\ 8\ 6\ 4 \\ - 7\ 4\ 3 \\ \hline 1\ 2\ 1 \end{array}$$

Now, subtract the hundreds.

$$\begin{array}{r} 9\ 8\ 6\ 4 \\ - 7\ 4\ 3 \\ \hline 9\ 1\ 2\ 1 \end{array}$$

There is no digit to subtract in the thousands place, so keep the 9.

Answer $9,864 - 743 = 9,121$

Self Check A

Subtract: $2,489 - 345$.

Subtracting Whole Numbers, with Regrouping

Objective 2

You may need to regroup when you subtract. When you **regroup**, you rewrite the number so you can subtract a greater digit from a lesser one.

When you're subtracting, just regroup to the next greater place-value position in the minuend and add 10 to the digit you're working with. As you regroup, cross out the regrouped digit in the minuend and place the new digit above it. This method is demonstrated in the example below.

Example	
Problem	$3,225 - 476 = ?$
$\begin{array}{r} 3\ 2\ 2\ 5 \\ - 4\ 7\ 6 \\ \hline \end{array}$	First, set up the problem and align the digits by place value.
$\begin{array}{r} 1\ 15 \\ 3\ 2\ 2\ 5 \\ - 4\ 7\ 6 \\ \hline 9 \end{array}$	Since you can't subtract 6 from 5, regroup, so 2 tens and 5 ones become 1 ten and 15 ones. Now you can subtract 6 from 15 to get 9.
$\begin{array}{r} 1\ 11\ 15 \\ 3\ 2\ 2\ 5 \\ - 4\ 7\ 6 \\ \hline 4\ 9 \end{array}$	Next, you need to subtract 7 tens from 1 ten. Regroup 2 hundreds as 1 hundred, 10 tens and add the 10 tens to 1 ten to get 11 tens. Now you can subtract 7 from 11 to get 4.
$\begin{array}{r} 2\ 11\ 11\ 15 \\ 3\ 2\ 2\ 5 \\ - 4\ 7\ 6 \\ \hline 7\ 4\ 9 \end{array}$	To subtract the digits in the hundreds place, regroup 3 thousands as 2 thousands, 10 hundreds and add the 10 hundreds to the 1 hundred that is already in the hundreds place. Now, subtract 4 from 11 to get 7.

$$\begin{array}{r}
 2 \text{ 11 11 15} \\
 3 \text{ 2 2 5} \\
 - 476 \\
 \hline
 2749
 \end{array}$$

Since there is no digit in the thousands place of the subtrahend, bring down the 2 in the thousands place into the answer.

Answer $3,225 - 476 = 2,749$

Self Check B

Subtract: $1,610 - 880$.

Checking Your Work

You can check subtraction by adding the difference and the subtrahend. The sum should be the same as the minuend.

Example	
Problem	Check to make sure that 7 subtracted from 12 is equal to 5.
$12 - 7 = 5$	Here, write out the original equation. The minuend is 12, the subtrahend is 7, and the difference is 5.
$ \begin{array}{r} 5 \\ + 7 \\ \hline 12 \end{array} $	Here, add the difference to the subtrahend, which results in the number 12. This confirms that your answer is correct.
<i>Answer</i> The answer of 5 is correct.	

Checking your work is very important and should always be performed when time permits.

Subtracting Numbers, Using the Expanded Form

An alternative method to subtract involves writing numbers in expanded form, as shown in the examples below. If you have 4 tens and want to subtract 1 ten, you can just think $(4 - 1)$ tens and get 3 tens. Let's see how that works.

Example	
Problem	$45 - 12 = ?$
$45 = 40 + 5$ $\underline{12 = 10 + 2}$	Let's write the numbers in expanded form so you can see what they really mean.
$45 = 40 + 5$ $\underline{12 = 10 + 2}$ 30	Look at the tens. The minuend is 40, or 4 tens. The subtrahend is 10, or 1 ten. Since $4 - 1 = 3$, 4 tens - 1 ten = 3 tens, or 30.
$45 = 40 + 5$ $\underline{-12 = 10 + 2}$ 30 + 3	Look at the ones. $5 - 2 = 3$. So, $30 + 3 = 33$.
<i>Answer</i> $45 - 12 = 33$	

Now let's use this method in the example below, which asks for the difference of 467 and 284. In the tens place of this problem, you need to subtract 8 from 6. What can you do?

Example	
Problem	$467 - 284 = ?$
<u>Step 1: Separate by place value</u> 4 hundreds + 6 tens + 7 ones 2 hundreds + 8 tens + 4 ones	Write both the minuend and the subtrahend in expanded form.
<u>Step 2: Identify impossible differences</u> $6 - 8 = []$	Here, we identify differences that are not whole numbers. Since 8 is greater than 6, you won't get a whole number difference.
<u>Step 3: Regroup</u> $3 \text{ hundreds} + 16 \text{ tens} + 7 \text{ ones}$ $\underline{- 2 \text{ hundreds} + 8 \text{ tens} + 4 \text{ ones}}$ $1 \text{ hundred} + 8 \text{ tens} + 3 \text{ ones}$	Regroup one of the hundreds from the 4 hundreds into 10 tens and add it to the 6 tens. Now you have 16 tens. Subtracting 8 tens from 16 tens yields a difference of 8 tens.
<u>Step 4: Combine the parts</u> $1 \text{ hundred} + 8 \text{ tens} + 3 \text{ ones} = 183$	Combining the resulting differences for each place value yields a final answer of 183.
<i>Answer</i> $467 - 284 = 183$	

Self Check C

A woman who owns a music store starts her week with 965 CDs. She sells 452 by the end of the week. How many CDs does she have remaining?

Example	
Problem	$45 - 17 = ?$
	When you try to subtract 17 from 45, you would first try to subtract 7 from 5. But 5 is less than 7.
$45 = 40 + 5$ $17 = 10 + 7$	Let's write the numbers in expanded form so you can see what they really mean.
$45 = 30 + 15$ $17 = 10 + 7$	Now, regroup 4 tens as 3 tens and 10 ones. Add the 10 ones to 5 ones to get 15 ones, which is greater than 7 ones, so you can subtract.
$45 = 30 + 15$ $- 17 = 10 + 7$ <hr/> $20 + 8$	Finally, subtract 7 from 15, and 10 from 30 and add the results: $20 + 8 = 28$.
<i>Answer</i> $45 - 17 = 28$	

Solve Application Problems Using Subtraction

Objective 3

You are likely to run into subtraction problems in every day life, and it helps to identify key phrases in a problem that indicate that subtraction is either used or required. The following phrases appear in problem situations that require subtraction.

Phrase or word	Example problem
Less than	The cost of gas is 42 cents per gallon less than it was last month. The cost last month was 280 cents per gallon. How much is the cost of gas this month?
Take away	Howard made 84 cupcakes for a neighborhood picnic. People took away 67 cupcakes. How many did Howard have left?
Decreased by	The temperature was 84°F in the early evening. It decreased by 15° overnight. What was the temperature in the morning?

Subtracted from	Jeannie works in a specialty store on commission. When she sells something for \$75, she subtracts \$15 from the \$75 and gives the rest to the store. How much of the sale goes to the store?
The difference	What is the difference between this year's rent of \$1,530 and last year's rent of \$1,450?
Fewer than	The number of pies sold at this year's bake sale was 15 fewer than the number sold at the same event last year. Last year, 32 pies were sold. How many pies were sold this year?

When translating a phrase such as "5 fewer than 39" into a mathematical expression, the order in which the numbers appears is critical. Writing $5 - 39$ would not be the correct translation. The correct way to write the expression is $39 - 5$. This results in the number 34, which is 5 fewer than 39. The chart below shows how phrases with the key words above can be written as mathematical expressions.

Phrase	Expression
three subtracted from six	$6 - 3$
the difference of ten and eight	$10 - 8$
Nine fewer than 40	$40 - 9$
Thirty-nine decreased by fourteen	$39 - 14$
Eighty-five take away twelve	$85 - 12$
Four less than one hundred eight	$108 - 4$

Example	
Problem Each year, John is out of the U.S. on business for 142 days, including travel time. The number of days per year he is in the U.S. is the difference of 365 days and 142 days. How many days during the year is John in the U.S.?	
$\begin{array}{r} 365 \\ - 142 \\ \hline \end{array}$	<p>The words "the difference of" suggest that you need to subtract to answer the problem.</p> <p>First, write out the problem based on the information given and align numbers by place value.</p>

$$\begin{array}{r} 365 \\ -142 \\ \hline 223 \end{array}$$

Then, subtract numbers in the ones place.

$$\begin{array}{r} 365 \\ -142 \\ \hline 223 \end{array}$$

Subtract numbers in the tens place.

$$\begin{array}{r} 365 \\ -142 \\ \hline 223 \end{array}$$

Finally, subtract numbers in the hundreds place.

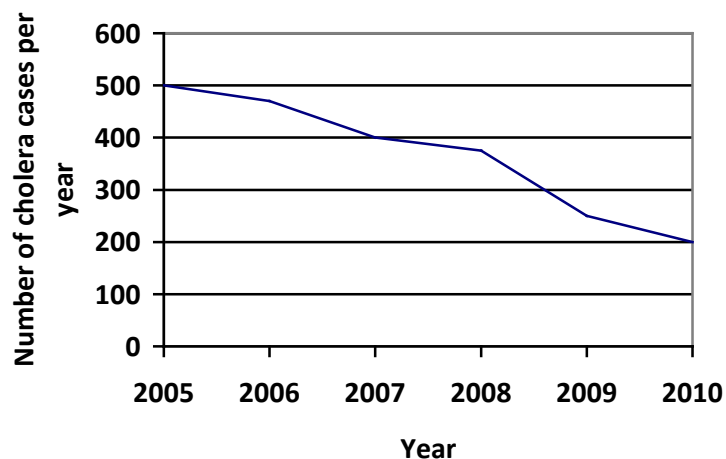
Answer John is in the U.S. 223 days during the year.

Self Check D

To make sure he was paid up for the month on his car insurance, Dave had to pay the difference of the amount on his monthly bill, which was \$289, and what he had paid earlier this month, which was \$132. Write the difference of \$289 and \$132 as a mathematical expression.

Example

Problem An African village is now getting cleaner water than it used to get. The number of cholera cases in the village has declined over the past five years. Using the graph below, determine the difference between the number of cholera cases in 2005 and the number of cases in 2010.



The words “the difference” suggest that you need to subtract to answer the problem.

First, use the graph to find the number of cholera cases per year for the two years: 500 in 2005 and 200 in 2010.

$$\begin{array}{r} 500 \\ -200 \\ \hline 300 \end{array}$$

Then write the subtraction problem and align numbers by place value. Subtract the numbers as you usually would.

Answer $500 - 200 = 300$ cases

Summary

Subtraction is used in countless areas of life, such as finances, sports, statistics, and travel. You can identify situations that require subtraction by looking for key phrases, such as *difference* and *fewer than*. Some subtraction problems require regrouping to the next greater place value, so that the digit in the minuend becomes greater than the corresponding digit in the subtrahend. Subtraction problems can be solved without regrouping, if each digit in the minuend is greater than the corresponding digit in the subtrahend.

In addition to subtracting using the standard algorithm, subtraction can also be accomplished by writing the numbers in expanded form so that both the minuend and the subtrahend are written as the sums of their place values.

1.2.2 Self Check Solutions

Self Check A

Subtract: $2,489 - 345$.

2,144

Self Check B

Subtract: $1,610 - 880$.

730

Self Check C

A woman who owns a music store starts her week with 965 CDs. She sells 452 by the end of the week. How many CDs does she have remaining?

$965 - 452 = 513$

Self Check D

To make sure he was paid up for the month on his car insurance, Dave had to pay the difference of the amount on his monthly bill, which was \$289, and what he had paid earlier this month, which was \$132. Write the difference of \$289 and \$132 as a mathematical expression.

The difference of 289 and 132 can be written as $289 - 132$.

1.2.3 Estimation

Learning Objective(s)

- 1 Use rounding to estimate sums and differences.
- 2 Use rounding to estimate the solutions for application problems.

Introduction

An **estimate** is an answer to a problem that is close to the solution, but not necessarily exact. Estimating can come in handy in a variety of situations, such as buying a computer. You may have to purchase numerous devices: a computer tower and keyboard for \$1,295, a monitor for \$679, the printer for \$486, the warranty for \$196, and software for \$374. Estimating can help you know *about* how much you'll spend without actually adding those numbers exactly.

Estimation usually requires **rounding**. When you round a number, you find a new number that's close to the original one. A rounded number uses zeros for some of the place values. If you round to the nearest ten, you will have a zero in the ones place. If you round to the nearest hundred, you will have zeros in the ones and tens places. Because these place values are zero, adding or subtracting is easier, so you can find an estimate to an exact answer quickly.

It is often helpful to estimate answers before calculating them. Then if your answer is not close to your estimate, you know something in your problem-solving process is wrong.

Using Rounding to Estimate Sums and Differences

Objective 1

Suppose you must add a series of numbers. You can round each addend to the nearest hundred to estimate the sum.

Example	
Problem	Estimate the sum $1,472 + 398 + 772 + 164$ by rounding each number to the nearest hundred.
1,472.....1,500	First, round each number to the nearest hundred.
398..... 400	
772.....800	
164.....200	

$\begin{array}{r} 1,500 \\ 400 \\ 800 \\ + \underline{200} \\ 2,900 \end{array}$	<p>Then, add the rounded numbers together.</p>
<p>Answer The estimate is 2,900.</p>	

In the example above, the exact sum is 2,806. Note how close this is to the estimate, which is 94 greater.

In the example below, notice that rounding to the nearest ten produces a far more accurate estimate than rounding to the nearest hundred. In general, rounding to the lesser place value is more accurate, but it takes more steps.

Example	
<p>Problem Estimate the sum $1,472 + 398 + 772 + 164$ by first rounding each number to the nearest ten.</p>	
<p>1,472....1,470 398..... 400 772.....770 164.....160</p>	<p>First, round each number to the nearest ten.</p>
$\begin{array}{r} 12 \\ 1470 \\ 400 \\ 770 \\ + \underline{160} \\ 00 \end{array}$	<p>Next, add the ones and then the tens. Here, the sum of 7, 7, and 6 is 20. Regroup.</p>
$\begin{array}{r} 12 \\ 1470 \\ 400 \\ 770 \\ + \underline{160} \\ 800 \end{array}$	<p>Now, add the hundreds. The sum of the digits in the hundreds place is 18. Regroup.</p>
$\begin{array}{r} 12 \\ 1470 \\ 400 \\ 770 \\ + \underline{160} \\ 2800 \end{array}$	<p>Finally, add the thousands. The sum in the thousands place is 2.</p>
<p>Answer The estimate is 2,800.</p>	

Note that the estimate is 2,800, which is only 6 less than the actual sum of 2,806.

Self Check A

In three months, a freelance graphic artist earns \$1,290 for illustrating comic books, \$2,612 for designing logos, and \$4,175 for designing web sites. Estimate how much she earned in total by first rounding each number to the nearest hundred.

You can also estimate when you subtract, as in the example below. Because you round, you do not need to subtract in the tens or hundreds places.

Example	
Problem	Estimate the difference of 5,876 and 4,792 by first rounding each number to the nearest hundred.
5,876...5,900 4,792...4,800	First, round each number to the nearest hundred.
$\begin{array}{r} 5,900 \\ - 4,800 \\ \hline 1,100 \end{array}$	Subtract. No regrouping is needed since each number in the minuend is greater than or equal to the corresponding number in the subtrahend.
Answer	The estimate is 1,100.

The estimate is 1,100, which is 16 greater than the actual difference of 1,084.

Self Check B

Estimate the difference of 474,128 and 262,767 by rounding to the nearest thousand.

Solving Application Problems by Estimating

Objective 2

Estimating is handy when you want to be sure you have enough money to buy several things.

Example	
Problem	When buying a new computer, you find that the computer tower and keyboard cost \$1,295, the monitor costs \$679, the printer costs \$486, the 2-year warranty costs \$196, and a software package costs \$374. Estimate the total cost by first rounding each number to the nearest hundred.

1,295.....1,300	First, round each number to the nearest hundred.
679..... 700	
486.....500	
196.....200	
374.....400	
 2	
1 3 0 0	Add.
7 0 0	
5 0 0	
2 0 0	
+ 4 0 0	After adding all of the rounded values, the estimated answer is \$3,100.
3,1 0 0	
Answer	The total cost is approximately \$3,100.

Estimating can also be useful when calculating the total distance one travels over several trips.

Example	
Problem	James travels 3,247 m to the park, then 582 m to the store. He then travels 1,634 m back to his house. Find the total distance traveled by first rounding each number to the nearest ten.
3247.....3,250	First, round each number to the nearest ten.
582.....580	
1634.... 1,630	
 1	
3 2 5 0	Adding the numbers in the tens place gives 16, so you need to regroup.
5 8 0	
+ 1 6 3 0	
6 0	
 1 1	
3 2 5 0	Adding the numbers in the hundreds place gives 14, so regroup.
5 8 0	
+ 1 6 3 0	
4 6 0	

$$\begin{array}{r}
 11 \\
 3250 \\
 580 \\
 + 1630 \\
 \hline
 5,460
 \end{array}$$

Adding the numbers in the thousands place gives 5.

Answer The total distance traveled was approximately 5,460 meters.

In the example above, the final estimate is 5,460 meters, which is 3 less than the actual sum of 5,463 meters.

Estimating is also effective when you are trying to find the difference between two numbers. Problems dealing with mountains like the example below may be important to a meteorologist, a pilot, or someone who is creating a map of a given region. As in other problems, estimating beforehand can help you find an answer that is close to the exact value, preventing potential errors in your calculations.

Example	
Problem	One mountain is 10,496 feet high and another mountain is 7,421 feet high. Find the difference in height by first rounding each number to the nearest 100.
10,496...10,500 7,421... ..7,400	First, round each number to the nearest hundred.
$ \begin{array}{r} 10500 \\ - 7400 \\ \hline 3100 \end{array} $	Then, align the numbers and subtract. The final estimate is 3,100, which is 25 greater than the actual value of 3,075.
Answer The estimated difference in height between the two mountains is 3,100 feet.	

Self Check C

A space shuttle traveling at 17,581 miles per hour decreases its speed by 7,412 miles per hour. Estimate the speed of the space shuttle after it has slowed down by rounding each number to the nearest hundred.

Summary

Estimation is very useful when an exact answer is not required. You can use estimation for problems related to travel, finances, and data analysis. Estimating is often done before adding or subtracting by rounding to numbers that are easier to think about. Following the rules of rounding is essential to the practice of accurate estimation.

1.2.3 Self Check Solutions

Self Check A

In three months, a freelance graphic artist earns \$1,290 for illustrating comic books, \$2,612 for designing logos, and \$4,175 for designing web sites. Estimate how much she earned in total by first rounding each number to the nearest hundred.

Round the numbers to \$1,300, \$2,600, and \$4,200 and added them together to get the estimate: \$8,100

Self Check B

Estimate the difference of 474,128 and 262,767 by rounding to the nearest thousand.

Round to 474,000 and 263,000 and subtract to get 211,000

Self Check C

A space shuttle traveling at 17,581 miles per hour decreases its speed by 7,412 miles per hour. Estimate the speed of the space shuttle after it has slowed down by rounding each number to the nearest hundred.

$17,600 - 7,400 = 10,200$ mi/h