1.2 Energy sources and fluxes on planets.

(Recommended reading: Ingersoll (2013), p. 33-53; Pierrehumbert (2010), Chap. 3).

1.2.1 Planetary Energy Sources

Sources of free energy are fundamental for climate and atmos. chemistry and determine the habitability of planets. They include:

(1) radioactive decay, (2) accretion, (3) core formation, (4) tides, and (5) sunlight.

Fluxes in (1)-(4) at planets' exteriors are often negligible compared to solar radiation. Exceptions are Jupiter, Saturn, and Neptune, and the Jovian moon Io.

1.2.2 Radiation from the Sun and other Stars

1.2.2.1 Spectral Types

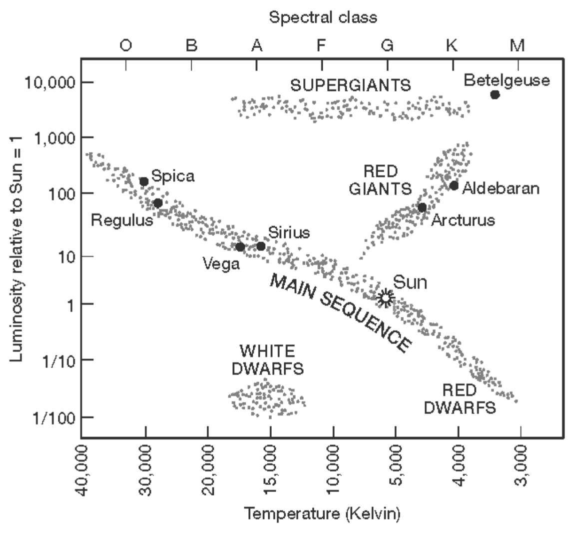
Table 1.3. Effective temperature (T_e) as a function of spectral type

Spectral type	O5	B0	A0	F0	G0	K0	M0	L0
T_e (K)	40000	25000	11000	7600	6000	5100	3600	2200

From hot, blue-white to relatively cold red, the letters OBAFGKMLT[1] designate stellar **spectral types** (ignoring peculiar supergiants). L stars (very low mass red dwarfs and brown dwarfs) and T stars (brown dwarfs) have been added since 1999. These stars are comparatively cool (<2500 K) and emit mostly at IR wavelengths.

Types O-M are divided into 10 subcategories, e.g., the hottest B star is a B0, followed by B1, B2, and so on; B9 is followed by A0. Our Sun is G2.

For historical reasons, the hotter stars are called early types and the cooler stars late types, so that 'B' is earlier than 'F', for example. Each spectral type is characterized by an *effective temperature*, *(T_e)* which applies to a star's photosphere – the region of the star's atmosphere from which we see electromagnetic energy emanating, also called a star's "surface".



[1] Generations of students have remembered stellar spectral types with the mnemonic: 'Oh Be A Fine Girl/Guy Kiss Me'. The addition of LT stars changes the end of the mnemonic to 'My Lips Tonight'.

A plot of stellar luminosity against T_e is the Hertzsprung-Russell (H-R) diagram. The thing to remember is that **temperature is plotted 'backwards'** from hot to cold on the x-axis so that it runs in the same sense as the series of spectral types. Stars lie mainly on a diagonal band, the *main sequence*, from upper left to lower right. For these stars, a *mass-luminosity relation* holds, $L \propto M_s^{3.5}$, where M_s is stellar mass.

1.2.2.2 The Solar Constant: Solar Radiation at the Top of Planetary Atmospheres

The Sun is a gaseous sphere made up of ~73% hydrogen by mass with most of the remainder helium, with gross properties as follows:

$$\left.\begin{aligned} &\text{mass} = M_\odot = 1.99\times10^{30}\ \text{kg}\\ &\text{luminosity} = L_\odot = 3.83\times10^{26}\ \text{W}\\ &\text{radius} = R_\odot = 6.96\times10^{8}\ \text{m}\end{aligned}\right\} \tag{2.1}$$

The *solar constant* is the flux at Earth's mean orbital distance = $(3.83 \times 10^{26}\ \text{W})/(4\pi \times (1.49598 \times 10^{11}\ \text{m})^2) = 1.36 \times 10^3\ \text{W m}^{-2}$. More accurately, satellites have measured the orbit-mean solar constant to be $1360.8\pm0.5\ \text{W m}^{-2}$ (Kopp and Lean, 2011, *GRL*].

1.2.2.3 The Solar Spectrum

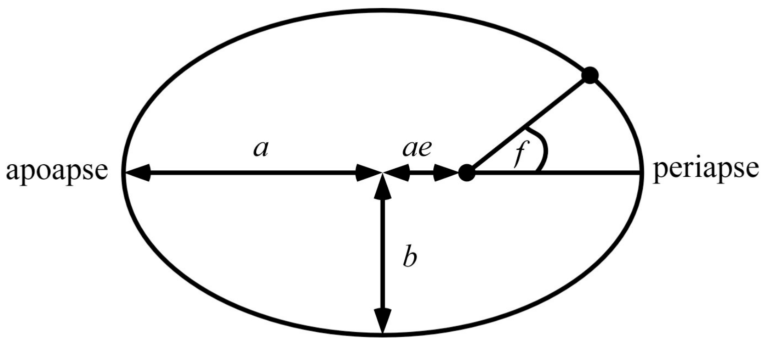
- a continuum from gamma rays to radio waves with a peak flux ~0.5 μm

- In the visible and IR, there are thousands of Fraunhofer absorption lines, characteristic of chemical elements in the photosphere. For UV wavelengths <185 nm, lines tend to be emission.

- A blackbody flux at $T_e = 5780$ K corrected for the distance of a planet is a good approximation to the visible and IR portions of the solar spectrum. At <400 nm, the effective temperature tends to be lower, ~5000 K from 210-300 nm. However, the *Lyman-*α peak in flux at 121 nm, far above the sun's nominal blackbody flux, corresponds to the transition between ground and first excited state of H atoms.

1.2.3 Planetary Energy Balance and the Greenhouse Effect

1.2.3.1 Orbits and Planetary Motion



We need to recap on orbits because insolation varies according to orbital position. The orbital eccentricity e is defined by the ratio of the distance from the center of the ellipse to either focus divided by the semi-major axis (see Fig.). Alternatively, for an orbit with a semi-major axis *a*, and semi-minor axis *b*, the eccentricity is:

$$e^2 = 1 - \left(\frac{b}{a}\right)^2 \tag{2.2}$$

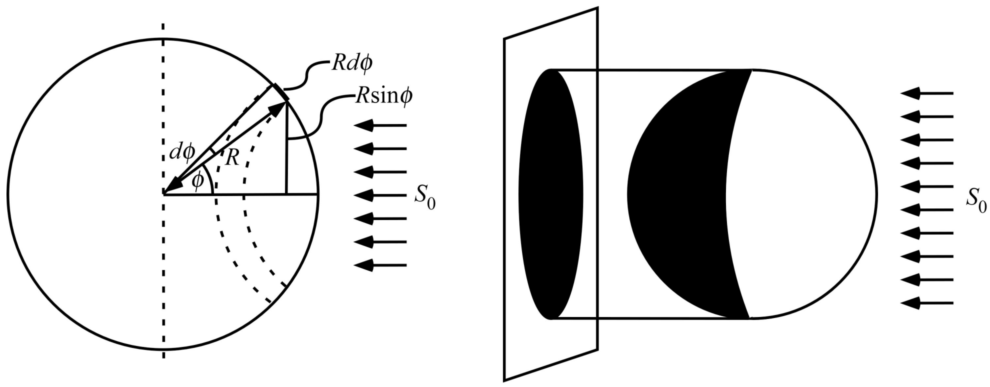
If the orbit is circular, $e = 0$. But by Kepler's first law, planetary orbits are elliptical, which means that $b/a < 1$ so that $0 < e < 1$.

The **true anomaly** *f*, is the angle between perihelion and planetary position. This sets the heliocentric (planet-to-Sun) distance $r_{\odot}$, as

$$\begin{aligned} r_{\odot} &= \frac{a(1-e^2)}{1+e\cos f} \\ \left.\begin{aligned} r_{\odot,perihelion} &= a(1-e) \\ r_{\odot,aphelion} &= a(1+e) \end{aligned}\right\} \end{aligned} \tag{2.3}$$

Pluto has a large eccentricity $e = 0.25$, probably because it is a captured Kuiper Belt Object. Pluto's atmosphere is consequently drastically affected by its orbital position. Mars, with $e = 0.093$, also experiences climatically significant solar flux variation between perihelion and aphelion. In contrast, the effect of Earth's eccentricity of $e = 0.017$ on its climate is one of modest high-latitude ice ages in recent geologic time.

1.2.3.2 Time- and Spatial-Averaged Incident Solar Flux



On a planet of radius *R*, consider an elemental ring of planetary surface defined by angle ϕ, which goes from 0 to $\pi/2$ radians. The elementary surface has area $2\pi(R\sin\phi)Rd\phi$, and the component of sunlight normal to the surface is $S_0\cos\phi$, where S_0 is the solar flux at the top of the atmosphere. Over a sunlit hemisphere,

$$\text{total power} = 2\pi R^2 S_0 \int_0^{\pi/2} \cos\phi \sin\phi \, d\phi \tag{2.4}$$

Substitute $\sin 2\phi = 2\sin\phi\cos\phi$ to solve the integral (2.4), to get power [Watts]:

$$\text{total power} = \pi R^2 S_0 \int_0^{\pi/2} \sin 2\phi \, d\phi, \qquad \text{sub. for } x = 2\phi, d\phi = dx/2$$
$$= \frac{\pi R^2 S_0}{2} \int_0^{\pi} \sin x \, dx = \frac{\pi R^2 S_0}{2} \left[-\cos x \right]_0^{\pi} = \pi R^2 S_0 \tag{2.5}$$

A hemisphere has area $2\pi R^2$. Hence the *area-averaged* incident flux (in W m^{-2}) is $\pi R^2 S_0 / 2\pi R^2 = S_0/2$. But the hemisphere is illuminated only half the day, so

$$\text{time-averaged incident flux} = \frac{S_0}{4} \tag{2.6}$$

Another way to look at eq. (2.6) is to note that the factor of 1/4 represents the ratio of disk to sphere areas ($\pi R^2 / 4\pi R^2$). The solar flux intercepted by the planet is a projected disk of area πR^2 compared with a total planetary area of $4\pi R^2$. For the Earth the globally averaged insolation at the top of the atmosphere given by eq. (2.6) is (1360.8±0.5 W m^{-2})/4 = 340.2±0.1 W m^{-2} using the measured value of S_0.

1.2.3.3 Albedo

The *albedo* is the fraction of incident power that gets reflected. There are 3 kinds:

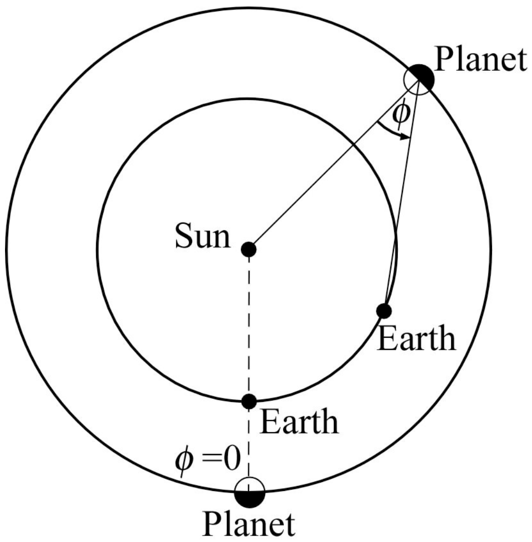
1) *Monochromatic albedo* is the fraction of incident power that gets reflected or scattered back to space at a given frequency of light:

$$A_\nu = \frac{(\text{reflected or scattered power at frequency } \nu)}{(\text{incident radiation power at frequency } \nu)} \tag{2.7}$$

2) *Bond albedo* (synonymous with *planetary albedo*) is where we integrate this over all frequencies:

$$A_b = \frac{(\text{total reflected or scattered radiation power})}{(\text{incident radiation power})} \tag{2.8}$$

For climate calculations, we use the Bond albedo.



3) Astronomers also use *geometric albedo*. For planets viewed from the Earth, there is an angle between incident sunlight and radiation received on Earth called the phase angle ϕ, When $\phi = 0$, the sunlight is observed in pure backscatter. The geometric albedo is defined by

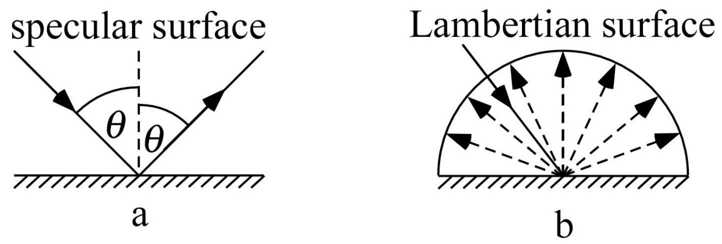
$$A_g = \frac{F(\phi = 0)}{F_{\text{Lambert-disk}}} = \frac{\text{(reflected flux at zero phase angle)}}{\text{(flux reflected from a Lambertian disk of the same cross-section)}}$$

(2.9)

Here, $F_{\text{Lambert-disk}}$ is the flux reflected by a disk with a Lambertian surface of the same cross-section as the planet at the same distance from the Sun. A Lambertian surface is one that reflects all incident radiation isotropically, as illustrated, such that its brightness is the same in all directions of view. (For example, a wall painted in mat white is very roughly Lambertian). Thus, the geometric albedo is the fraction of incident light reflected in the direction of the observer of an outer planet measured at opposition, meaning when the Sun, Earth and planet form a line. Geometric albedo is related to the Bond albedo by the expression:

$$A_b = qA_g \tag{2.10}$$

where q is a "phase integral" which reflects the variation of the intensity of radiation over the phase angle. (A derivation of the phase function can be found in Kartuttnen, (2007, "Fundamental Astronomy"), p.149-151 or in Seager (2010) *Exoplanet Atmospheres*). We merely note that if the Bond albedo A_b is 1, the phase integral q is 1.5 and the geometric albedo is 2/3. A useful thing to remember is: even if the Bond albedo is not unity, the geometric albedo is 2/3 of the Bond albedo if the planet is a Lambertian scatterer. Also Bond and geometric albedos are equal when q =1, which means that reflection from the planet behaves like a Lambertian disk of the same diameter.



1.2.3.4 Planetary equilibrium temperature

Table 1.4 A comparison of equilibrium temperatures calculated using eq. (2.11) and the measured mean global surface temperatures for the inner planets.

Planet	Bond albedo (dimensionless)	Equilibrium temperature (K)	Mean global surface temperature (K)	Greenhouse effect (K)
Mercury	0.058	440*	440	0
Venus	0.77	228	735	507
Earth	0.3	255	288	33
Mars	0.25	210*	218	8

*Caution: for an airless or nearly airless world, the actual mean temperature is not the effective temperature. This requires some thought.

If the bond albedo is $A = A_b$, then, for a planet with negligible internal heat:

absorbed solar radiation flux = outgoing infrared radiation flux

$$(1-A)\frac{S_p}{4} = \sigma T_{\mathrm{eq}}^4 \tag{2.11}$$

where $\sigma = 5.6697 \times 10^{-8}$ W m^{-2} K^{-4} is Stefan's constant. For the Earth, Bond albedo is 0.3, so the LHS of eq. (2.11) is ~238 W m^{-2} = (1-0.3)×(1361 W m^{-2} /4).

At equilibrium, this energy flux must be emitted to space by radiation from the Earth. Solving eq. (2.11) for the equilibrium temperature gives T_{eq} = 255 K, an *effective temperature*, i.e. blackbody equivalent, which is clearly not equal to Earth's actual surface temperature.

1.2.3.5 The Greenhouse Effect

The difference of T_s and T_{eq} is a way to express the magnitude of the greenhouse effect

$$\Delta T_g \equiv T_s - T_{\mathrm{eq}} \tag{2.12}$$

So for the Earth, ΔT_g = 288 – 255 = 33 K.
(Note: the eqm temperature is not really that of an airless Earth because then the albedo would be very different and the temperature very spatially variable. So 33 K is a convention reflecting the fact that 255 K is characteristic of outgoing flux for an isothermal Earth with albedo of 0.3. Thus, the "33 K" really measures a flux difference. The global mean upwelling longwave (LW) flux at the surface is ~390 W m^{-2} (for 288 K), and the outgoing LW flux at the top of the atmosphere (TOA) is ~238 W m^{-2} (255 K equivalent). The LW flux difference that exists between the surface and TOA of 150 W m^{-2} (Δ 33 K equivalent) measures the greenhouse effect).

Earth's greenhouse contributions *in clear skies* are (Kiehl & Trenberth,1997, *BAMS*):
- ~2/3 of the warming from H_2O
- most of the remaining 1/3 from CO_2
- 2-3 K from CH_4, O_3, N_2O, and various chlorofluorocarbons (CFCs)

Greenhouse contributions *taking into account clouds* are (Schmidt et al. 2010, *GRL*):
~50% from water vapor
~25% from clouds
~20% from CO_2, and the rest from other gases

Note: clouds enhance the greenhouse by 30 W m^{-2}, but reduce Earth's absorbed radiation by -48 W m^{-2} by albedo, and so greatly *cool* the Earth in net by -18 W m^{-2}.

H_2O acts in a different manner than CO_2 because it's near its condensation temperature. Effectively, H_2O is a slave to CO_2 or other greenhouse gases: if CO_2 levels increase and warm the Earth, then the vapor pressure of H_2O gets bigger, which amplifies the greenhouse effect. We call H_2O a (positive) *feedback*. We call CO_2 a *forcing* = a persistent disturbance that changes the climate system's energy balance.

Qu.) Look at Eqs (2.11) and (2.12). What 3 factors does the mean surface temperature depend upon?

When we consider the evolution of climates, any changes in T_s (e.g., low-latitude glaciation) can only be understood by appealing to changes in one or more of these factors. The most difficult factor is the albedo, A. Much of the albedo on Earth, Venus, etc., is caused by clouds. Earth's clouds account for ~0.15 out of the 0.3.

1.2.3.6 Giant Planets, Internal Heat and Equilibrium Temperature

Earth's geothermal heat flow is ~87 mW m^{-2} compared to a net solar flux 238 W m^{-2}.

But Jupiter, Saturn and Neptune radiate significantly more energy than they absorb. e.g., Jupiter radiates ~13.6 W m^{-2} while it absorbs only ~8.2 W m^{-2} solar. An internal energy flux of ~5.4 W m^{-2} makes up the difference. The source of internal energy in the gas giants is gravitational P.E. released from contraction and accretion. In Saturn's case, it is believed to have been in state of internal differentiation for billions of years, in which immiscible helium "rains out" of a deep interior layer of metallic hydrogen towards Saturn's core, with release of gravitational energy. Because Saturn is smaller and cooler in its interior than Jupiter, the phase separation of helium has been going for far longer than inside Jupiter, where it began more recently. Uranus has little internal heat flow (<42 mW m^{-2}). This puzzle might be due to different internal structure or if Uranus is out of equilibrium for some reason.

For giant planets with internal heat fluxes, measured *effective temperature* T_e, exceeds predicted *equilibrium temperature*, T_{eq} with solar flux. The internal heat flux, F_i, given by

$$F_i = \sigma(T_e^{\,4} - T_{\text{eq}}^{\,4}) \tag{2.13}$$

The total emitted flux modified to include this internal heat is given by:

outgoing radiation flux = absorbed solar flux + internal heat flux

$$\sigma T_e^4 = (1-A)\frac{S_p}{4} + F_i \tag{2.14}$$

(Note differing definitions in the literature: sometimes people define equilibrium temperature as being with all energy sources, including internal energy fluxes).

1.2.4 Climate Feedbacks in the Earth System

Feedbacks in the Earth system amplify the mean temperature of the planet in positive feedback or stabilize it in negative feedback. There are many potential feedbacks for climate, of which four important ones are:

(1) positive feedback from atmospheric water vapor
(2) positive feedback from ice-albedo – critical in advances and retreats of the *Pleistocene* (2.6 Ma-10 ka) ice sheets and modern global warming.
(3) negative feedback from outgoing radiation on short timescales; and
(4) negative feedback from the carbonate-silicate cycle on geological timescales

1.2.4.1 Climate Sensitivity and Energy Balance

Earth's climate feedback is illustrated by a simple energy balance, following Budyko [1969]. Consider the Earth as a blackbody with a surface temperature $T = T_0 + \Delta T$. If T_0 is 273.15 K, then the surface temperature T_s in °C equals ΔT. Thus, the emitted flux is

$$
\begin{aligned}
F_{IR} &= \sigma T^4 = \sigma(T_0 + T_s)^4 \\
&= \sigma(T_0 + T_s)^2(T_0 + T_s)^2 = \sigma(T_0^2 + 2T_sT_0 + T_s^2)(T_0^2 + 2T_sT_0 + T_s^2) \\
&= (\sigma T_0^4) + (4\sigma T_0^3 T_s) + \ldots
\end{aligned}
$$

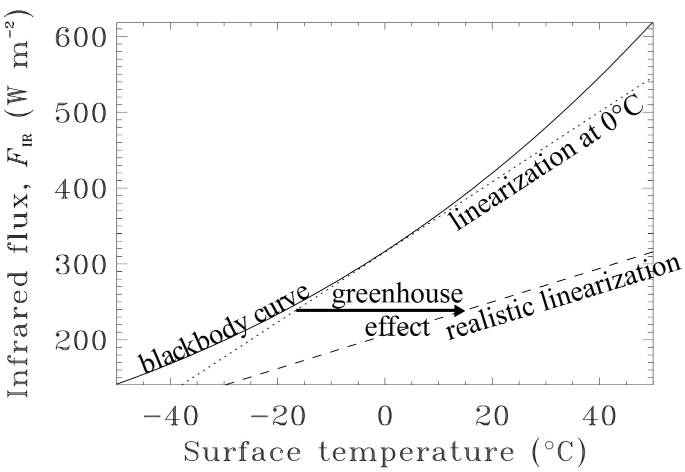
Taking only the first two terms, we can linearize the emitted power as

$$F_{IR} = a + bT_s; \qquad T_s \text{ in °C} \tag{2.15}$$

If the Earth behaved truly like a blackbody, then this linearization would give $a = \sigma(273.15)^4 = 315.58$ W m^{-2} and $b = 4\sigma(273.15)^3 = 4.62$ W m^{-2} °C^{-1}. But for the actual climate, the values of a and b are found to be $a = 206$ W m^{-2} and $b = 2.2$ W m^{-2} °C^{-1}. Inserting these numbers in eq. (2.15):

239 W m^{-2} = 206 W m^{-2} + (2.2)(T_s), from which we calculate T_s = 15°C.

Because B is smaller in the real climate than for a blackbody, the surface temperature T_s, is *more sensitive* to changes in F_{IR} (or the solar constant S_0, since F_{IR} is proportional to S_0), i.e., T_s increases more for a given change in F_{IR} than the blackbody case. Qu.) Why?



Above: Graph showing the outgoing IR at the top of the terrestrial atmosphere F_{IR}, as a function of surface temperature. The solid line is the blackbody case. A linearization to this curve about 0°C gives a surface temperature of -16.6°C for an outgoing flux of 239 W m^{-2}. A more realistic linear model has a surface temperature of 15°C and a significantly different slope.

Climate sensitivity λ, is the ratio of the change in global mean surface temperature ΔT_s at equilibrium to the change in climate forcing ΔQ (in W m^{-2}):

$$\Delta T_s = \lambda \Delta Q \tag{2.16}$$

The change in forcing could be a change in the solar constant or a change in the greenhouse effect. Let us evaluate the climate sensitivity for a blackbody and compare our simple but more realistic linearized model (ignoring T-dependence of albedo):

1) **Blackbody case**

$$F_{IR} = \sigma T^4 = \frac{(1-A)}{4} S_0$$

So

$$\frac{d(F_{IR})}{dS_0} = \frac{(1-A)}{4} = \left(\frac{d(F_{IR})}{dT}\right)\left(\frac{dT}{dS_0}\right) \tag{2.17}$$

Hence,

$$\lambda = \frac{dT}{dS_0} = \frac{(1-A)}{4}\left(\frac{1}{d(F_{IR})/dT}\right) = \frac{(1-A)}{4(4\sigma T^3)} = \frac{T}{4}\left(\frac{(1-A)}{4\sigma T^4}\right) = \frac{T}{4S_0} \tag{2.18}$$

Qu.) What is λ for the Earth in this blackbody case? What temperature change would be caused by a 1% change in solar flux?

2) **Linearized model**. The purely Stefan-Boltzmann case is unrealistic because it neglects climate feedbacks. If we consider our linearized model, we have

$$F_{IR} = a + bT = \frac{(1-A)}{4} S_0 \tag{2.19}$$

If we now apply eq. (2.17), we obtain

$$\lambda = \frac{dT}{dS_0} = \frac{(1-A)}{4}\left(\frac{1}{d(F_{IR})/dT}\right) = \frac{(1-A)}{4b} \tag{2.20}$$

Qu.) What is λ for the Earth in this case? How does it compare with the previous case? What temperature change would be caused by a 1% change in solar flux?

Qu.) Radiative calculations suggest that a doubling of CO_2 from 300 ppmv to 600 ppmv causes an extra 4 W m^{-2} in Earth's greenhouse effect and warms the surface. What surface temperature increase do we get in our linearized model? How does it compare with sophisticated 3D climate models?

1.2.5 Radiative Time Constants

For planetary thermal equilibrium, we can think of some height, z_e, where the temperature T, equals the effective temperature T_e. We call this level the *emission level*. It's analogous to a star's photosphere. Given a lapse rate Γ, we have:

$$T_{\text{surface}} = T_e + \Gamma z_e \tag{2.21}$$

Qu.) What is the emission level, given that the Earth's troposphere has a typical lapse rate $\Gamma \sim 6$ K km^{-1}, $T_{\text{surface}} = 288$ K, and $T_e = 255$ K.

We can think of energy absorbed from the Sun as being balanced by emission at this level at temperature T_e. The column mass at this level, M_{ce}, is given by $M_{ce} = p_e/g$, where p_e is the pressure at the emission level. If a temperature change ΔT_{el} is forced by the absorption of solar radiation, the heat change is given by

$$\text{heat change per m}^2 = M_{ce} c_p (\Delta T) = \frac{p_e c_p (\Delta T_e)}{g}$$

where c_p is the specific heat capacity and g is gravitational acceleration. For sunlight,

$$\text{power absorbed per m}^2 = \frac{(1-A)S_p}{4}$$

Consequently, we can define a time constant as follows:

$$\tau_{\text{e}} = \left(\frac{p_e c_p (\Delta T_e)}{g} \right) \Bigg/ \left(\frac{(1-A)S_p}{4} \right) = \frac{4 p_e c_p (\Delta T_e)}{(1-A) S_p g} \tag{2.22}$$

This equation is also often used in the literature for any pressure level p with temperature T, replacing ΔT_e with T.
If we impose $\tau_{\text{e}} = 1$ day, we obtain a diurnal variation in temperature at the emission level of

$$\Delta T_e = \frac{(1-A) S_p g}{4 p_e c_p} \tag{2.23}$$

ΔT_e are 2 K (Venus), 2 K (Earth), 80 K (Mars), and 0.001 K (Jupiter). Diurnal radiative effects are clearly important on Mars. Also from (2.23), the pressure level where $\Delta T_e = T_e$ is: 700 mb (Venus), 8 mb (Earth), 2 mb (Mars), 0.05 mb (Jupiter).

Note that the radiative time constant is sometimes defined differently. If the pressure in eq. (2.22) is taken as surface pressure, with $\Delta T_e = T_e$ and using eq. (2.11) to substitute for $\sigma T_e^4 = S(1-A)/4$, we get

$$\tau_{\text{rad1}} = \frac{M_c c_p}{\sigma T_e^3} \tag{2.24}$$

where M_{c} is the total columnar mass p/g; this applies only to planets with rocky surfaces.

Note the radiative time constant is used in simple models of atmospheric dynamics to simulate diabatic heating or cooling. So we'll come back to it later in that context.

Example: Qu.) What is the radiative time constant in days for hot Jupiter HD209458b at 0.1 bar where T ~ 1300 K? Given that this exoplanet's orbital period is 3.5 days, do you expect to see day-night temperature differences? (Data: assume Bond albedo = 0.3; g = 18.5 m/s^2, c_p = 14000 J kg^{-1} K^{-1} for H_2, the stellar luminosity is 1.6 times the Sun; the orbital distance of the planet is 0.045 AU; the solar constant at Earth = 1360 W m^{-2}). (See Iro et al (2005) Astronomy & Astrophysics for more detailed radiative calculations for this exoplanet).