1.3 Order of Operations

As it turns out, there are more than just 4 basic operations. There are five.

The fifth basic operation is that of repeated multiplication. We call these exponents.

There is a bit of terminology with exponents that we will need throughout the remainder of the textbook. It is given below.

$$\underbrace{a \cdot a \cdot a \cdot a \cdot \cdots a}_{\text{n-times}} = a^{n}$$

n-times   base   exponent

Let's get an idea of working with exponents in our first example. For the sake of reference, when the instructions say "simplify", this means, do everything you possibly can to make the problem as simple as possible.

This means, if we **can** do something, we **must** do it.

Example 1:

Simplify.

    a. 6^2 b. $(-7)^2$ c. -7^2 d. $(-3)^2 2^3$ e. $\left(\frac{2}{3}\right)^3$

Solution:
a. The first thing we should do write the exponent expression out so that we can actually do the multiplication. Remember, exponents mean repeated multiplication. So
$$6^2 = 6 \cdot 6 = 36$$

b. Again, we need to write it out fully. However, one thing we need to always keep in mind with an exponent is, an exponent is only applied to the one item immediately before it.

    So, in this case, the one item immediately before the 2 exponent is a parenthesis. Therefore, the whole expression containing the parenthesis is what is to the second power.

    This means we have $(-7)^2 = (-7)(-7) = 49$.

c. Similar to part b above, we have to be careful of what the exponent is actually applied to. In this case, the exponent is only applied to the "7" not the - part of the 7.

    So we write out $-7^2 = -7 \cdot 7 = -49$.

d. Again we write out the exponents fully then multiply out. We get
$$(-3)^2 2^3 = (-3)(-3) \cdot 2 \cdot 2 \cdot 2 = 72$$

e. Lastly, we work with fractions the same way as integers. Write it out, and multiply it out. We get
$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

Now that we have all of the basic operations, we need to be able to work problems which have all of these operations involved, and more.

To do this, we need to establish the order in which we should always attack a problem. We call this order **The Order of Operations**.

The Order of Operations
1. Simplify inside the grouping symbols. Such as: ( ), [ ], | |, ——
2. Simplify exponents.
3. Multiply and Divide, <u>LEFT to RIGHT</u>
4. Add and Subtract, <u>LEFT to RIGHT</u>

One note about the first step in the order of operations, the absolute value and fraction bar not only get treated first in the order of operations, but they also perform an operation themselves. These operations must be done as well before we can move to the next step in the order of operations.

With this order of operations, we can simplify numerous expressions. Keep in mind, we <u>always</u> follow the order of operations, no matter what.

<u>Example 2</u>:

Simplify.

    a. $2^3 \cdot 3 - 3$ b. $8 - (-3)^2 - (-2)$ c. $14 - 2^2 - |4 - 7|$

Solution:
a. To simplify these expressions, we need to be sure to follow the order of operations. The first thing we notice is that we don't have any grouping symbols. Therefore, we start with simplifying exponents.
We have
$$2^3 \cdot 3 - 3$$
$$= 8 \cdot 3 - 3$$
Now we multiply
$$8 \cdot 3 - 3$$
$$= 24 - 3$$
And lastly we subtract
$$24 - 3$$
$$= 21$$

b. Here, we start by noticing that we do have some grouping symbols. However, there are no operations to perform inside of them. Therefore, we can move to the exponents, and so on. We need to just be very careful of our exponents. As we saw in example 1, we can get into trouble when there are negatives involved.
$$8 - (-3)^2 - (-2) \quad \text{Simplify } (-3)^2 = (-3)(-3) = 9$$
$$= 8 - 9 - (-2) \quad \text{Double switch the negatives}$$
$$= 8 - 9 + 2 \quad \text{Add and subtract to finish}$$
$$= 1$$

c. Again, we follow the order of operations. We start with the grouping symbols, | | in this case.

$$14 - 2^2 - |4 - 7|$$ Simplify inside the | |

$$= 14 - 2^2 - |{-3}|$$ Perform the operation |-3| = 3

$$= 14 - 2^2 - 3$$ Simplify exponents $2^2 = 4$

$$= 14 - 4 - 3$$ Subtract

$$= 7$$

Now that we have the basic idea, let's work on some others that are more complex.

<u>Example 3</u>:

Simplify.

a. $-2^2 + 4[16 \div (3 - 5)]$ b. $24 \div \dfrac{3^2}{8-5} - (-5)$ c. $\dfrac{-10-2}{6^2-30} \div |2 - 4|$

Solution:
a. As in example 2, we follow the order of operations.

$$-2^2 + 4[16 \div (3 - 5)]$$ Simplify inside the grouping symbols working from the inside out

$$= -2^2 + 4[16 \div (-2)]$$

$$= -2^2 + 4 \cdot [-8]$$ Simplify exponents $-2^2 = -2 \cdot 2 = 4$

$$= -4 + 4 \cdot [-8]$$ Multiply

$$= -4 + (-32)$$ Add

$$= -36$$

b. Again, follow the order of operations

$$24 \div \dfrac{3^2}{8 - 5} - (-5)$$ Simplify top and bottom of the fraction (grouping symbol)

$$= 24 \div \dfrac{9}{3} - (-5)$$ Divide to fully simplify the fraction bar

$$= 24 \div 3 - (-5)$$ Divide

$$= 8 - (-5)$$ Double switch

$$= 8 + 5$$ Add

$$= 13$$

c. Lastly, we follow the order of operations as seen below

$$\frac{-10-2}{6^2-30} \div |2-4|$$ Simplify the fraction bar and absolute values (grouping symbols)

$$= \frac{-12}{36-30} \div |-2|$$ Continue to simplify the fraction bar

$$= \frac{-12}{6} \div |-2|$$ Perform the operations of the fraction bar and absolute value

$$= -2 \div 2$$ Divide to finish

$$-1$$

The last thing we want to do with the order of operations is we want to be able to work problems containing rational numbers by using the order of operations. We will look at this in our final example.

Example 4:

Simplify.

a. $\frac{3}{8} \div \left(\frac{5}{6} + \frac{2}{3}\right)$ b. $\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^3 \div \frac{3}{5}$ c. $\frac{2}{5}\left(1 - \frac{1}{2}\right)^2 - \frac{1}{2^3 + 3 \cdot 4}$

Solution:
a. We still use the same order of operations, even though we are working with rational numbers.

We have

$$\frac{3}{8} \div \left(\frac{5}{6} + \frac{2}{3}\right)$$ Simplify inside parenthesis by adding as before (over the LCD, 6)

$$= \frac{3}{8} \div \left(\frac{5}{6} + \frac{2 \cdot 2}{3 \cdot 2}\right)$$

$$= \frac{3}{8} \div \left(\frac{5}{6} + \frac{4}{6}\right)$$ Add the numerators

$$= \frac{3}{8} \div \frac{9}{6}$$ Reduce the fraction

$$= \frac{3}{8} \div \frac{3}{2}$$ Flip and multiply

$$= \frac{3}{8} \cdot \frac{2}{3}$$ Cancel and finish

$$= \frac{1}{4}$$

b. Again, follow the order of operations as below

$$\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^3 \div \frac{3}{5}$$ Simplify exponents $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ and

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$= \frac{9}{16} - \frac{1}{8} \div \frac{3}{5}$$

Divide by flipping and multiplying

$$= \frac{9}{16} - \frac{1}{8} \cdot \frac{5}{3}$$

$$= \frac{9}{16} - \frac{5}{24}$$

$$= \frac{9 \cdot 3}{16 \cdot 3} - \frac{5 \cdot 2}{24 \cdot 2}$$

$$= \frac{27}{48} - \frac{10}{48}$$

$$= \frac{17}{48}$$

c. Lastly, we follow the order of operations as we have in all examples thus far

$$\frac{2}{5}\left(1 - \frac{1}{2}\right)^2 - \frac{1}{2^3 + 3 \cdot 4}$$ Simplify inside ( ) and fraction bar

$$= \frac{2}{5}\left(\frac{1}{2}\right)^2 - \frac{1}{8 + 3 \cdot 4}$$ Continue simplifying the bottom of the fraction bar by following the order of operations ($\cdot$ before +)

$$= \frac{2}{5}\left(\frac{1}{2}\right)^2 - \frac{1}{8 + 12}$$

$$= \frac{2}{5}\left(\frac{1}{2}\right)^2 - \frac{1}{20}$$ Simplify exponents $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$$= \frac{2}{5} \cdot \frac{1}{4} - \frac{1}{20}$$ Multiply

$$= \frac{2}{20} - \frac{1}{20}$$ Subtract over the LCD

$$= \frac{1}{20}$$

1.3 Exercises

Simplify.

1. 2^6 2. 3^8 3. $(-10)^4$ 4. -2^6

5. -5^2 6. $(-1)^4$ 7. $(-2)^2 3^3$ 8. $3^4 5^2$

9. $2^2(-5)^3$ 10. $(-2)^4 4^2$ 11. $(-2)^3 2^3$ 12. $(-3)^3(-5)^2$

13. $\left(\frac{2}{5}\right)^3$ 14. $\left(-\frac{7}{8}\right)^2$ 15. $\left(-\frac{1}{2}\right)^4$ 16. $\left(\frac{2}{3}\right)^5$

17. $34 + 9 \cdot 2$ 18. $6 \div 3 \cdot 4^2$

19. $15 \div 5 \cdot 2^2$ 20. $9^2 + 6^3$

21. $-5 - 3(7 - 3 \cdot 8)$ 22. $(-4)^2 - 4(-6)(-3)$

23. $3 - [8 - (7 - 6 \cdot 4)]$ 24. $|4^2 - 8| - (-2)^3$

25. $(-2 + 1)^3 - 3(2 - 3)^2 - 3$ 26. $[5 \cdot (3 - 5)^2] - (-1)^3$

27. $6(-8 + 10)^2 - 5(4 - 7) - 39$ 28. $3^2 - 4^2 + (3 - 4)^2$

29. $5 - |2^2 + 3^2 - 8 \cdot 7|$ 30. $-|2 - 3^2 + 4^2 \cdot (-1)^4|$

31. $-21(|5 - 3 \cdot 8| + |9(11 - 5)|)$ 32. $-4 \cdot 8 - |-5(3 + 6)| - 3^2$

33. $(-2)^2 - 3^2 \div |1 - 4|^2 \cdot 2 - 6 + 2^2$ 34. $-6^2 - |7 - 5 \cdot 3| - 6 \div 2 \cdot 3^2$

35. $7(-18 + 16) - 3(6 - 7)^3 - 3 \cdot 9$ 36. $(3 \cdot 5^2 - 2 \cdot 37)^4 - (7 \cdot (-2) - 5 \cdot 3)^2$

37. $(-2 - 4)^2 |6 - 4|^2 - (-12) \div 4 \cdot (1 - 2)^2$ 38. $-5|(3 - 5 \cdot 3^2) + 2^3| \cdot |4 - 2^3|$

39. $32 \div \left(\frac{16-8}{3-7}\right)^2$ 40. $\frac{2^2-15}{3^3-16} \cdot (-2)^2$

41. $\left(\frac{12-6}{7-4}\right)^2 \left(\frac{4+2\cdot3}{-10}\right)^4 - 4$ 42. $\frac{(5+2^2)\cdot6-2\cdot3^2}{3+3^2+6\cdot3} - 1$

43. $(2 + (-5)^2)\left(\frac{4^2+2(-3)}{2}\right)$ 44. $\frac{3^2-11}{2^3-4} \div \left(\frac{6-3^3\cdot2}{2^2\cdot3^2-2\cdot3}\right)^2$

45. $(4 - 3^2) \cdot \left(\frac{-1(-2)^2+(-1)(3)}{(-1)(-2)}\right)$ 46. $\frac{(8+3^2)\cdot3-16}{3+2^2+20} - 1^6$

47. $\left(\frac{-1-3\cdot2}{-3}\right) \div \left(2 + \frac{(-3)^2}{(-1)(-3)}\right)$ 48. $\frac{(9+7)\cdot4-24}{1+2^2+20}$

49. $\frac{5}{6} + \frac{1}{6} \cdot \frac{5}{7}$ 50. $\frac{5}{8} \div \frac{1}{6} - \frac{3}{8}$

51. $\frac{4}{81} \div \frac{1}{9} \cdot \frac{7}{9}$ 52. $\frac{6}{11}\left(\frac{3}{7} - \frac{3}{42}\right)$

53. $-\frac{1}{2} + \frac{3}{8} \div \left(-\frac{9}{12}\right)$ 54. $\frac{1}{3} + \left(\frac{3}{8} + \left(\frac{2}{3}\right)^2\right)$

55. $\left(-\frac{3}{4}\right)^2 - \frac{1}{8} \div \frac{1}{3} - \frac{3}{16}$ 56. $\frac{1}{2} \div \left(\frac{1}{3} + \frac{1}{2}\right) - \frac{2}{3}$

57. $\dfrac{(6-3)^2 \cdot 2}{2^2-2} \div (5-8)^2$

58. $\dfrac{3^2-10}{4^3-20} - \dfrac{2}{11}$

59. $\left(\dfrac{3}{8} - \dfrac{1}{2}\right)^2 \div \left(\dfrac{1}{2} - \dfrac{2}{3}\right)^2$

60. $\left(\dfrac{3}{4} - 1\right)^3 + \left(\dfrac{5}{2} \div \dfrac{15}{4}\right)^2$