1.3 Proving the Pythagorean Theorem   Geogebra Triangle Unit

Unit	Expressions, Equations and Geometry
Big Idea	Mathematicians apply geometric principles to describe and analyze the physical world around them.
Previously addressed standards	Students know how to find area, perimeter, and volume of shapes.
Standard	**Explain a proof of the Pythagorean Theorem and its converse.** 8.G.B.6
Teaching Point	A proof is a sequence of statements that establish a universal truth. The Pythagorean Theorem must be proved in order to ensure it will always allow us to determine side lengths of right triangles.
Possible Misconceptions and Common Mistakes	- PT works for all triangles, not just right. - PT determines area. - PT has to do with angle measurements (this can be confusing as being a right triangle is a constraint, but the PT does not determine angle measurements). - A proof is the same thing as an example. - A proof must involve pictures (may be a misconception following this lesson).
Materials Needed	- Notes 1.3 - Exit Ticket 1.3 - HW 1.3 - Laptops with internet access (www.geogebra.org)
Agenda	1) Notes (30 min) 2) Exit Ticket (10 min) 3) HW (5 min)

1) **Discussion:** Last night for homework you solved 20 problems that involved the Pythagorean Theorem. Have we proved that the Pythagorean Theorem always works?

   Debate above question as a class. Students will likely bring up the following points:

- **Yes.** We know from yesterday it always works for right triangles because we found examples from class where it didn't work with triangles that were not right.
- **No.** We measured some triangles and it worked for all of these triangles, but we can't be sure it works for all triangles.
- **No**. Yesterday we only showed examples where the triangles were right and had whole number sides. We didn't look at any triangles with rational or irrational side lengths.

Allow students to discuss until there is somewhat of a class consensus and/or until the above points have been made by students. Do not tell the students your opinion at this point.

2) **Notes:** Show students the following definition of a proof from the Harper Collins Dictionary of Mathematics.

proof *n*. a sequence of statements, each of which is either validly derived from those preceding it or is an axiom or assumption, and the final member of which, the conclusion, is the statement of which the truth is thereby established.

Ask students again. Have we proved the Pythagorean Theorem by solving some problems? Discuss/debate until students agree that we have or have not met the requirements of a proof.

Students discuss. Possible points:

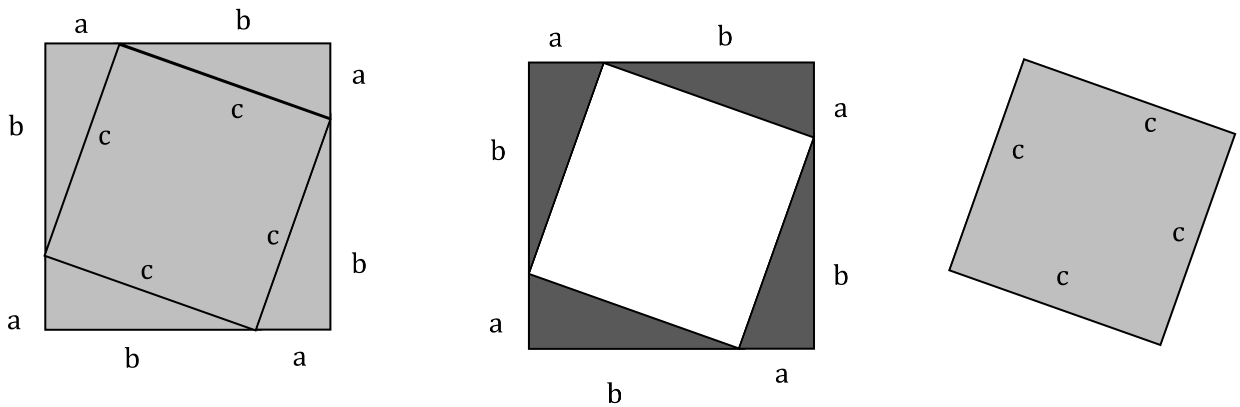
- We know the PT always works because every time we have tried it and measured the angles, it was true. *Solving problems does not tell us that the PT will always work; it only tells us that it works in the situations we have tested by measuring.*
- Counter: We would have to measure every possible right triangle to know it always works. We only tried a handful. *It is impossible to measure every possible right triangle as there are infinite right triangles.*
- Definition of Proof: We did not make a sequence of statements to show that the Pythagorean Theorem works.
- Definition of Proof: We did not "establish truth" by following a sequence of statements.

Come to the conclusion as a class that we have not yet proved the Pythagorean Theorem. *This is the student's first exposure to proofs, so it is important that students understand that solving problems are examples, but do not prove the Pythagorean Theorem. Give students enough time to come to this conclusion themselves as they will have limited exposure to proofs in this unit and they need to understand the fundamental difference between a proof and an example.*

3) **Point –** We need to find a way to prove that the Pythagorean Theorem always, always works. We want to prove that $a^2 + b^2 = c^2$ is true no matter which right triangle you use.

4) Notes page 1 – Here is one possible proof of the Pythagorean Theorem. Go through with class.

Label the shapes, as follows:



Ask students to determine the area of each shape:

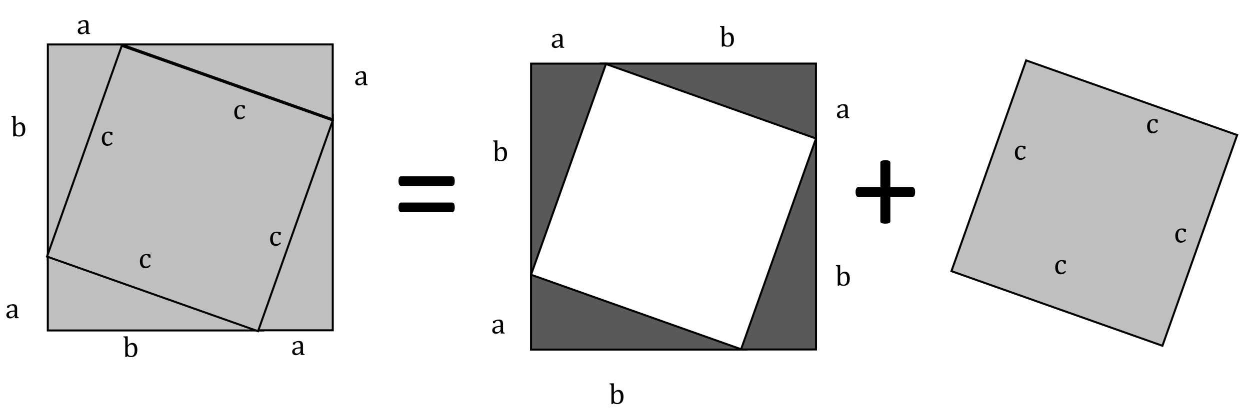
Area of big square = (a + b)(a + b)

Area of triangles = 4 triangles (1/2 ab)

Area of small square = c*c

What equation can you make to show the relationships between the three shaded areas?

The sum of the larger square is equal to the sum of the four triangles and the smaller square.



Discuss: How do we know that this will be a proof, and not just another example? *We are using variables to represent any possible right triangle. We can have "a" or "b" be any size and the picture would still show a relationship between the three pictures.*

Write Equation showing the relationship: $(a+b)(a+b) = 4(1/2ab) + c^2$

Simplify equation:

$(a+b)(a+b) = 4(1/2ab) + c^2$

$a^2 + 2ab + b^2 = 2ab + c^2$

$\boxed{a^2 + b^2 = c^2}$

Show students the dynamic model on Geogebra and discuss how this shows the static model we have gone through:

https://tube.geogebra.org/student/m79364

Note that even though the sizes of *a, b,* and *c* change in the model, they are still not labeled with measurements. With the proof we are showing that the theorem is universal and not dependent on specific measurements (e.g. a 3, 4, 5 triangle).

Ask: Are you convinced? Does this prove the Pythagorean Theorem to you? Discuss how these proofs differ from problem solving or examples with specific numbers (e.g. Pythagorean Triples).

Tell: There are more than 400 proofs of the Pythagorean Theorem. Many of them are geometric and can be modeled by using shapes.

5) **Notes page 2 –** Using Geogebra, find another proof of the Pythagorean Theorem that resonates with you. Draw pictures and explain how the proof illustrates $a^2 + b^2 = c^2$.

Website: www.geogebra.org (not .com!)

Click on "Browse Materials"

Type "Pythagorean Theorem Proof"

Show students how they can sort by relevance, language, rating, etc.

Have students work in groups for 10-15 minutes to find a proof they understand and like. Stop the class at times to have groups show a proof they like and explain their thinking.

Write: Ask students to write out their favorite Geogebra proof of the Pythagorean Theorem. They should have drawings that show how the Geogebra proof changed to show that a^2 + b^2 = c^2 always holds true.

Share: If time, have students switch groups (jigsaw style) to share proofs with different groups.

6) Point: The Pythagorean Theorem (a^2 + b^2 = c^2) is always true because we can prove it geometrically without using specific numbers. A proof allows us to be sure that the Pythagorean Theorem will work to find the side lengths of any right triangle.

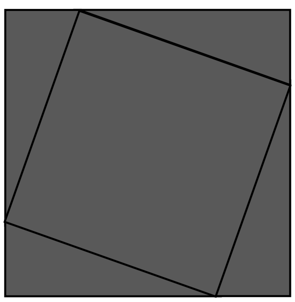
Proving the Pythagorean Theorem

Name: ____

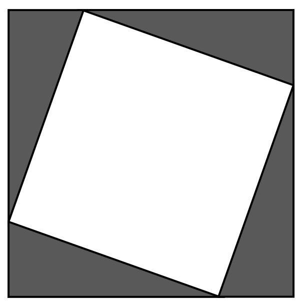
Point

proof *n*. a sequence of statements, each of which is either validly derived from those preceding it or is an axiom or assumption, and the final member of which, the conclusion, is the statement of which the truth is thereby established.

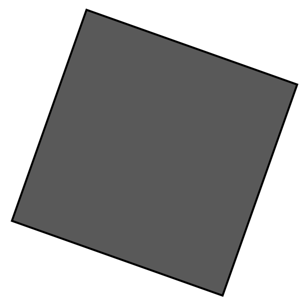
Harper Collins Dictionary of Mathematics



Area of the big square:



Area of the 4 triangles:



Area of the small square:

There are more than 400 proofs of the Pythagorean Theorem! Use www.geogebra.org to find another proof of the Pythagorean Theorem that resonates with you.

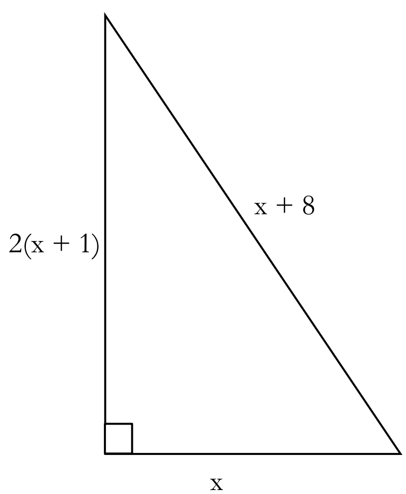
A few hints:
www.geogebra.org (not .com!)
Click on “Browse Materials”
Type “Pythagorean Theorem Proof”
Sort results by relevance, language, and rating to find the worksheets you’d like to use.

Write (and draw) a different and favorite proof of the Pythagorean Theorem:

- ☐ **Above grade level** = awesome!
- ☐ **On track** = great!
- ☐ **Tutoring** = we need to meet and talk about this
- ☐ **Extra work** = you will have additional HW tonight to make sure you master this material.

1.3 Exit Ticket Name: ______________________________

1) What is the perimeter and area of the following triangle? The triangle is <u>not</u> drawn to scale!



2) Agree or disagree with the following statement. Explain your thinking.
Question #1 is a proof of the Pythagorean Theorem because it uses the variable x, which could stand for any possible value. Question #1 shows us that the Pythagorean Theorem <u>always</u> works.

HW 1.3 – Proving the Pythagorean Theorem

Make sure you have the proper heading:

First Name and Last Name Subject Assignment Number Level

Make sure you leave a blank line between each of your answers.

Choose one problem set. You may complete both for extra credit.

Level 3	**Level 4**
What is the difference between a proof and an example? P6, questions 1, 3, 5 (This is not page 6, it is page P6. You should see lesson 0-2 Real Numbers on the next page) Lesson 10 - 5 (starts on page 648) 10 – 12, 19, 20, 22	What is the difference between a proof and an example? P6, questions 1 – 6 (This is not page 6, it is page P6. You should see lesson 0-2 Real Numbers on the next page) Lesson 10- 5 (starts on page 648) 16 – 18, 19, 21, 25