1.3 Representing irrational numbers on Number line

We have learnt that there exist a rational number between any two rational numbers. Therefore, when two rational numbers are represented by points on number line, we can use a point to represent a rational number between them. So there are infinitely many points representing rational numbers. It seems that the number line is consisting of points which represent rational numbers only. Is it true? Can't you represent $\sqrt{2}$ on number line? Let us discuss and locate irrational numbers such as $\sqrt{2}$, $\sqrt{3}$ on the number line.

Example-7. Locate $\sqrt{2}$ on number line

Solution : At O draw a unit square OABC on number line with each side 1 unit in length.

By Pythagoras theorem OB = $\sqrt{1^2+1^2} = \sqrt{2}$

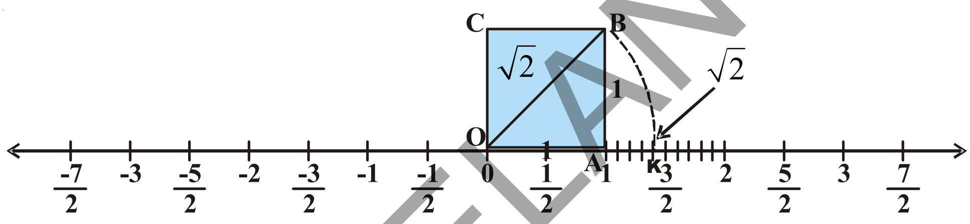


Fig. (i)

We have seen that OB = $\sqrt{2}$. Using a compass with centre O and radius OB, draw an arc on the right side to O intersecting the number line at the point K. Now K corresponds to $\sqrt{2}$ on the number line.

Example-8. Locate $\sqrt{3}$ on the number line.

Solution : Let us return to fig. (i)

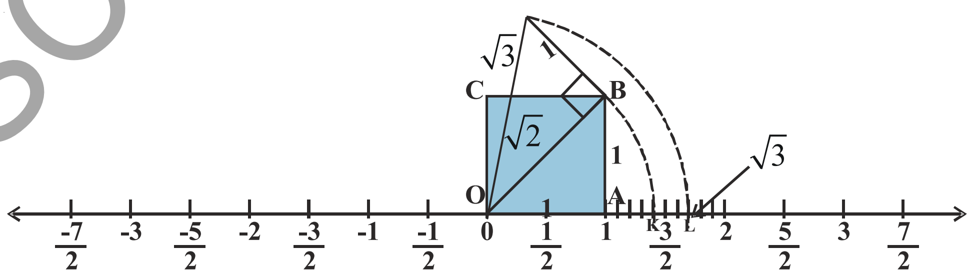


Fig. (ii)

Now construct BD of 1 unit length perpendicular to OB as in Fig. (ii). Join OD

By Pythagoras theorem, OD = $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2+1} = \sqrt{3}$

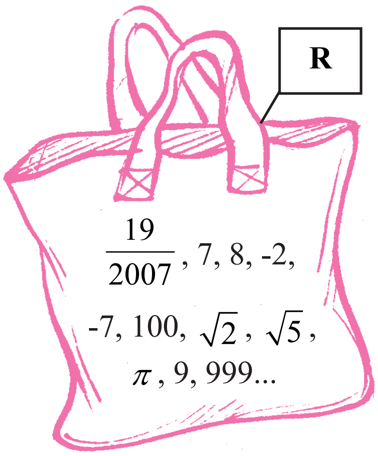
Using a compass, with centre O and radius OD, draw an arc which intersects the number line at the point L right side to 0. Then 'L' corresponds to $\sqrt{3}$. From this we can conclude that many points on the number line can be represented by irrational numbers also. In the same way, we can locate $\sqrt{n}$ for any positive integers n, after $\sqrt{n-1}$ has been located.

TRY THESE

Locate $\sqrt{5}$ and $-\sqrt{5}$ on number line. [**Hint** : $5^2 = (2)^2 + (1)^2$]

1.3 REAL NUMBERS

All rational numbers can be written in the form of $\frac{p}{q}$, where p and q are integers and q $\neq$ 0. There are also other numbers that cannot be written in the form $\frac{p}{q}$, where p and q are integers and are called irrational numbers. If we represent all rational numbers and all irrational numbers and put these on the number line, would there be any point on the number line that is not covered?



The answer is no! The collection of all rational and irrational numbers completely covers the line. This combination makes a new collection called Real Numbers, denoted by R. Real numbers cover all the points on the number line. We can say that every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number. So we call this as the real number line.

Here are some examples of Real numbers

$-5.6, \sqrt{21}, -2, 0, 1, \frac{1}{5}, \frac{22}{7}, \pi, \sqrt{2}, \sqrt{7}, \sqrt{9}$, 12.5, 12.5123..... etc. You may find that both rational and Irrationals are included in this collection.

Example-9. Find any two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$.

Solution : We know that $\frac{1}{5} = 0.20$

$\frac{2}{7} = 0.\overline{285714}$

To find two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$, we need to look at the decimal form of the two numbers and then proceed. We can find infinitely many such irrational numbers.

Examples of two such irrational numbers are

0.201201120111..., 0.24114111411114…, 0.25231617181912..., 0.267812147512 …

Can you find four more irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$?

Example-10. Find an irrational number between 3 and 4.

Solution :

> If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between *a* and *b*.

$\therefore$ An irrational number between 3 and 4 is $\sqrt{3 \times 4} = \sqrt{3} \times \sqrt{4}$

$= \sqrt{3} \times 2 = 2\sqrt{3}$

Example-11. Examine, whether the following numbers are rational or irrational :

(i) $\left(3+\sqrt{3}\right)+\left(3-\sqrt{3}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\frac{10}{2\sqrt{5}}$ (iv) $\left(\sqrt{2}+2\right)^2$

Solution :

(i) $\left(3+\sqrt{3}\right)+\left(3-\sqrt{3}\right) = 3+\sqrt{3}+3-\sqrt{3}$

= 6, which is a rational number.

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

We know that $(a+b)(a-b) \equiv a^2 - b^2$ is an identity.

Thus $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = 3^2 - \left(\sqrt{3}\right)^2 = 9 - 3 = 6$ which is a rational number.

(iii) $\dfrac{10}{2\sqrt{5}} = \dfrac{10 \div 2}{2\sqrt{5} \div 2} = \dfrac{5}{\sqrt{5}} = \dfrac{\sqrt{5} \times \sqrt{5}}{\sqrt{5}} = \sqrt{5}$, which is an irrational number.

(iv) $\left(\sqrt{2}+2\right)^2 = \left(\sqrt{2}\right)^2 + 2.\sqrt{2}\ .2 + 2^2 = 2 + 4\sqrt{2} + 4$

$= 6 + 4\sqrt{2}$, which is an irrational number.

EXERCISE - 1.2

1. Classify the following numbers as rational or irrational.

   (i) $\sqrt{27}$ (ii) $\sqrt{441}$ (iii) 30.232342345…

   (iv) 7.484848… (v) 11.2132435465 (vi) 0.3030030003.....

2. Give four examples for rational and irrational numbers?
3. Find an irrational number between $\frac{5}{7}$ and $\frac{7}{9}$. How many more there may be?
4. Find two irrational numbers between 0.7 and 0.77
5. Find the value of $\sqrt{5}$ upto 3 decimal places.
6. Find the value of $\sqrt{7}$ up to six decimal places by long division method.
7. Locate $\sqrt{10}$ on the number line.
8. Find atleast two irrational numbers between 2 and 3.
9. State whether the following statements are true or false. Justify your answers.

   (i) Every irrational number is a real number.

   (ii) Every rational number is a real number.

   (iii) Every real number need not be a rational number

   (iv) $\sqrt{n}$ is not irrational if n is a perfect square.

   (v) $\sqrt{n}$ is irrational if n is not a perfect square.

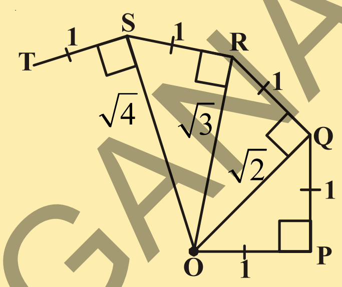
   (vi) All real numbers are irrational.

Activity

Constructing the 'Square root spiral'.

Take a large sheet of paper and construct the 'Square root spiral' in the following manner.

Step 1 : Start with point 'O' and draw a line segment $\overline{OP}$ of 1 unit length.



Step 2 : Draw a line segment $\overline{PQ}$ perpendicular to $\overline{OP}$ of unit length (where OP = PQ = 1) (see Fig)

Step 3 : Join O, Q. (OQ = $\sqrt{2}$)

Step 4 : Draw a line segment $\overline{QR}$ of unit length perpendicular to $\overline{OQ}$.

Step 5 : Join O, R. (OR = $\sqrt{3}$)

Step 6 : Draw a line segment RS of unit length perpendicular to $\overline{OR}$.

Step 7 : Continue in this manner for some more number of steps, you will create a beautiful spiral made of line segments $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{ST}$, $\overline{TU}$ ... etc. Note that the line segments $\overline{OQ}$, $\overline{OR}$, $\overline{OS}$, $\overline{OT}$, $\overline{OU}$ ... etc. denote the lengths $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}$ respectively.

1.4 Representing Real numbers on the Number line through Successive magnification

In the previous section, we have seen that any real number has a decimal expansion. Now first let us see how to represent terminating decimal on the number line. Suppose we want to locate 2.776 on the number line. We know that this is a terminating decimal and this lies between 2 and 3.

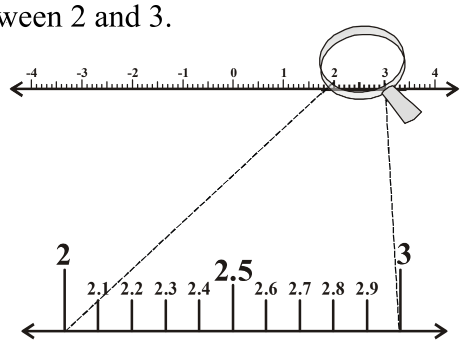
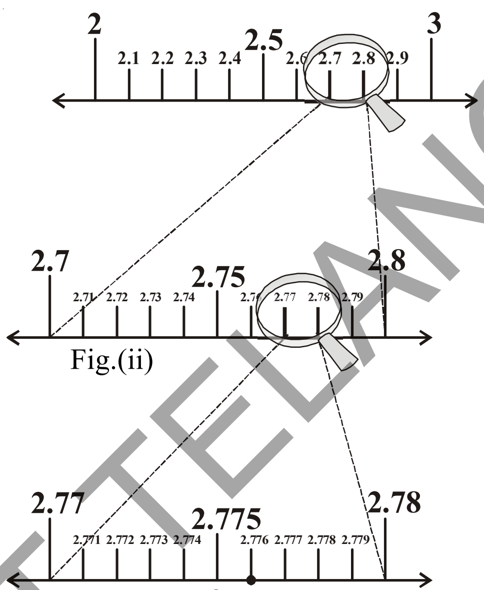


Fig.(i)

So, let us look closely at the portion of the number line between 2 and 3. Suppose we divide this into 10 equal parts as in Fig.(i). Then the markings will be like 2.1, 2.2, 2.3 and so on. To have a clear view, let us assume that we have a magnifying glass in our hand and look at the portion between 2 and 3. It will look like what you see in figure (i).

Now, 2.776 lies between 2.7 and 2.8. So, let us focus on the portion between 2.7 and 2.8 (See Fig. (ii). We imagine that this portion has been divided into ten equal parts. The first mark will represent 2.71, the second is 2.72, and so on. To see this clearly, we magnify this as shown in Fig(ii).



चित्र

(iii)

Again 2.776 lies between 2.77 and 2.78. So, let us focus on this portion of the number line see Fig. (iii) and imagine that it has been divided again into ten equal parts. We magnify it to see it better, as in Fig.(iii).

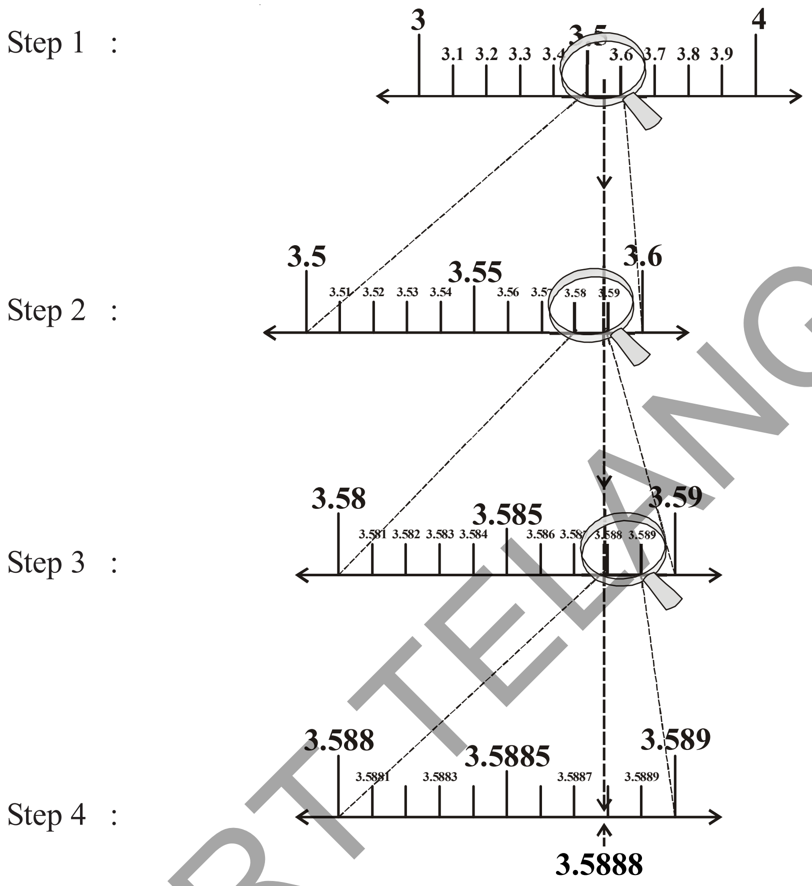
The first mark represents 2.771, second mark 2.772 and so on, 2.776 is the 6^{th} mark in these subdivisions.

We call this process of visualization of presentation of numbers on the number line through a magnifying glass, as the process of successive magnification.

Now let us try and visualize the position of a real number with a non-terminating recurring decimal expansion on the number line by the process of successive magnification with the following example.

Example-12. Visualise the representation of $3.5\overline{8}$ on the number line through successive magnification upto 4 decimal places.

Solution: Once again we proceed with the method of successive magnification to represent 3.5888 on number line.



EXERCISE - 1.3

1. Visualise 2.874 on the number line, using successive magnification.
2. Visualilse $5.\overline{28}$ on the number line, upto 3 decimal places.

1.5 OPERATIONS ON REAL NUMBERS

We have learnt, in previous class, that rational numbers satisfy the commutative, associative and distributive laws under addition and multiplication. And also, we learnt that rational numbers are closed with respect to addition, subtraction, multiplication. Can you say irrational numbers are also closed under four fundamental operations?