1.3 SCALARS AND VECTORS

Introduction: Physics is the study of natural phenomena. The study of any natural phenomenon involves measurements. For example, the distance between the planet earth and the Sun is finite. The study of speed of light involves the distance traveled by the ray of light and time consumed.

Any thing that is measurable is termed as 'quantity'. The quantities that come across in physics is referred to as a physical quantity.

Example: Mass, length, time, temperature, etc.,

Whenever we measure a physical quantity, the measured value is always a number. This number makes sense only when the relevant unit is specified.

Thus, the result of a measurement has a numerical value and a unit of measure. For example, the mass of a body is 3 kg. Here a quantity having numerical value 3 and the unit of measure kg are used. The numerical value together with the unit is called the 'magnitude'.

To describe certain physical quantities only magnitude is required. Apart from the mass of a body, distance to any place, time, temperature, height, the number of oscillations of a pendulum and the number of books in a bag are some examples of such numbers. They have no direction associated with them.

'Quantities which require only magnitude for their complete specifications and having no direction associated with them are called scalar quantities'.

To describe certain physical quantities like displacement along with the magnitude, the direction is essential. Consider a body moving from X to Y. XY is the displacement. On the contrary, if the body moves from Y to X, the displacement is YX.

'Quantities which require both magnitude and direction for their complete specification are called vectors'.

Example for vector quantities: momentum, force, torque, magnetic field etc.,

Note: The physical quantity like electric current possesses both the magnitude and direction, still they are not vectors, and similarly any form of energy is a scalar.

Representation of a vector: A vector can be conveniently represented by a straight line with an arrow head. The length of the vector represents its magnitude and the arrow head indicates its direction.

Steps involved representing a vector:

1. By choosing a proper scale, draw a line whose length is proportional to the magnitude of the vector.
2. By following the standard convention to show direction, indicate the direction of the vector by marking an arrow head at one end of the line.

Example: 1) To represent the displacement of a body along x-axis.

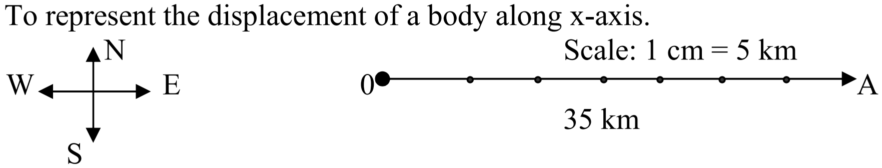


Fig -1 – Graphical representation of a vector

The vector represented by the directed line segment OA in fig (1) is denoted by $\vec{OA}$ (to be read as vector OA) or a simple notation as $\vec{a}$ (to be read as vector a). For vector $\vec{OA}$, O is the initial point and A is the terminal point.

The magnitude of $\vec{OA}$ (or $\vec{a}$) is denoted by $|\vec{OA}|$, (read as modulus of vector OA) or $|\vec{a}|$ (or simply OA or a) and is always positive.

Position vector: To locate the position of any point 'P' in a plane or space, generally a fixed point of reference called the origin 'O' is taken. The vector $\vec{OP}$ is called the position vector of P with respect to O as shown in fig (2).

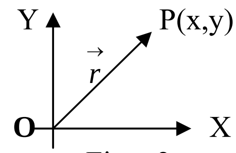


Fig – 2

Note: i) Given a point P, there is one and only one position vector for the point with respect to the origin 'O'.
ii) Position vector of a point 'P' changes if the position of the origin 'O' is changed.

Kinds of vectors:

1) **Unit vector:** A vector having unit magnitude is called unit vector.

If $\vec{A}$ is a vector having magnitude $|\vec{A}| \neq 0$, then $\frac{\vec{A}}{|\vec{A}|}$ is a unit vector having the same direction as $\vec{A}$. It is represented as $\hat{A}$ (read as A cap). $|\hat{A}| = 1$. Thus $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ or $\vec{A} = |\vec{A}|\hat{A} = A\hat{A}$.

The unit vectors along the x, y and z-axis are usually denoted as $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

2) Zero vector or Null vector: A vector having zero magnitude is called a Null vector or Zero vector. It is represented as $\vec{O}$.

Note: a) Zero vector has no specific direction.
b) The position vector of origin is a zero vector.
c) Zero vectors are only of mathematical importance.

3) Equal vectors: Vectors are said to be equal if both vectors have same magnitude and direction.

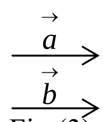


Fig (3)

$\vec{a} = \vec{b}$

4) Parallel vectors (Like vectors): Vectors are said to be parallel if they have the same directions.

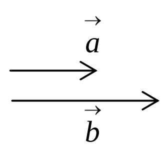


Fig (4)

The vectors $\vec{a}$ and $\vec{b}$ represent parallel vectors.

Note: Two equal vectors are always parallel but, two parallel vectors may or may not be equal vectors.

5) Anti parallel vectors (Unlike vectors): Vectors are said to be anti parallel if they acts in opposite direction.

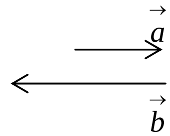


Fig (5)

The vectors $\vec{a}$ and $\vec{b}$ are anti parallel vectors.

6) Negative vector : The negative vector of any vector is a vector having equal magnitude but acts in opposite direction.

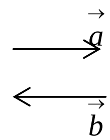
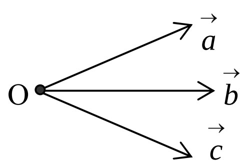


Fig (6) $\vec{a} = -\vec{b}$ **OR** $\vec{b} = -\vec{a}$

7) Concurrent vectors (Co initial vectors): vectors having the same initial point are called concurrent vectors or co initial vectors.



$\vec{a}$, $\vec{b}$ and $\vec{c}$ are concurrent at point 'O'.

Fig (7)

8) Coplanar vectors: The vectors in the same plane are called coplanar vectors.

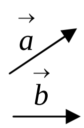


Fig (8a)

The vector $\vec{a}$ and $\vec{b}$ are coplanar vectors

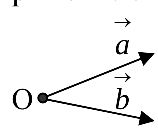
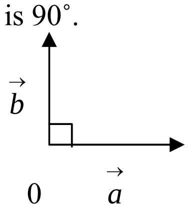


Fig (8b)

The vectors $\vec{a}$ and $\vec{b}$ are concurrent coplanar vectors.

9) Orthogonal vectors: Two vectors are said to be orthogonal to one another if the angle between them is 90˚.



The vector $\vec{a}$ and $\vec{b}$ are orthogonal to one another.

Fig (9)

VECTOR ALGEBRA:

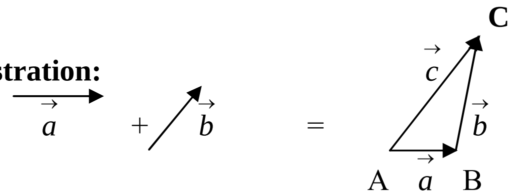
Addition of vectors: The addition of scalars involves only the addition of their magnitudes. But, when a vector is added with another vector we have to consider their direction also.

A vector can be added with another vector provided both the vectors represents the same physical quantity. For example, the addition of a vector representing displacement of a body with another vector representing velocity of the body is meaningless.

METHODS OF VECTOR ADDITION:

I .Triangle method of vector addition OR Tail to tip method of vector addition:

Illustration:



$\Rightarrow \vec{a} + \vec{b} = \vec{c}$

Fig (10)

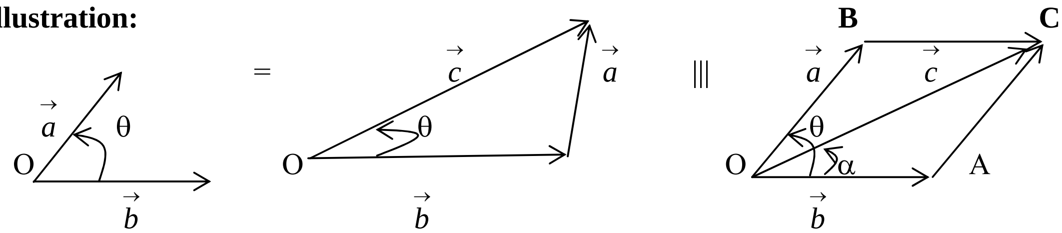
Explanation: To add $\vec{a}$ with $\vec{b}$, translate $\vec{b}$, by drawing parallel to itself so that the origin or initial point of $\vec{b}$ is at the tip of vector a. $\vec{a}$ and $\vec{b}$ are two vectors represented by two sides of a triangle taken in the same sense (direction). The vector sum of $\vec{a}$ and $\vec{b}$ (also called resultant of $\vec{a}$ and $\vec{b}$) is represented by the third side of the triangle taken in opposite sense(direction).

Statement: Triangle law of vector addition states that if two vectors can be represented in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented completely by the third side of the triangle taken in opposite order.

II. Parallelogram method of vector addition:

To add two vectors placed with common initial point, the parallelogram method of vector is used.

Illustration:



$\vec{b} + \vec{a}$ = $\vec{c}$ — $\vec{a} + \vec{b} = \vec{c}$

Fig (11a) Fig (11b) Fig (11c)

Explanation: To add vector $\vec{b}$ with $\vec{a}$ inclined at an angleθ, draw equal vector of $\vec{a}$ at the tip of $\vec{b}$. By law of triangle method of vector addition $\vec{c} = \vec{b} + \vec{a}$ (fig-11b). Repeat the process, by drawing equal vector of $\vec{b}$ at the tip of $\vec{a}$ (fig-11c). Again by law of triangle method of vector addition $\vec{c} = \vec{a} + \vec{b}$. Note that $\vec{a} + \vec{b} = \vec{b} + \vec{a}$, that is vector addition follows commutative rule. $\vec{c}$, the diagonal of the completed parallelogram represents the vector sum of $\vec{a}$ and $\vec{b}$ completely both in magnitude and direction.

Statement of parallelogram law of vector addition:

“ *It states that if two vectors acting at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from that point, the resultant is represented completely by the diagonal of the parallelogram passing through that point*”.

In fig(11a), if $\vec{a}$ and $\vec{b}$ are two vectors and θ is the angle between them, then their vector sum is represented by the diagonal $\vec{c}$.

It can be shown that the magnitude of $\vec{c}$ is, c = $\sqrt{a^2 + b^2 + 2ab\cos\theta}$

If α is the angle made by the direction of $\vec{c}$ with that of $\vec{b}$, then

$$\tan\alpha = \frac{a\sin\theta}{b + a\cos\theta}$$

Note: It is a common mistake to draw the sum vector as the diagonal running between the tips of the two vectors as shown in fig (12).

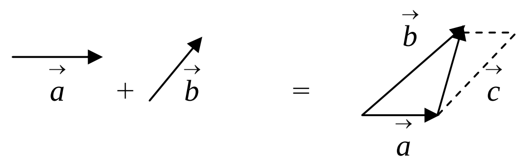


Fig (12)

> **Think**: In fact, the diagonal represents the difference between the two vectors, not their sum!

Note: 1) The advantage of the parallelogram method is that one can get both the sum and the difference of two vectors if one knows how to identify the appropriate directions.

2) The resultant of two vectors does not depend on the order in which the vectors are added. This fact leads to

a) Commutative law of vector addition:

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

b) Associative law of vector addition: If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vector ,then

$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

III. Law of polygon of vector addition:

Statement: It states that if a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then their sum (resultant) is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ represent the sides OA, AB, BC and CD of the polygon OABCDO.

Their resultant $\vec{R}$ is represented by the closing side OD of the polygon taken in the opposite order.

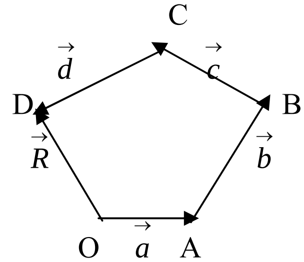


Fig (13)

Subtraction of two vectors:

Subtraction of one vector $\vec{b}$ from another vector $\vec{a}$ can be realized using the definition of a negative of a vector as follows.

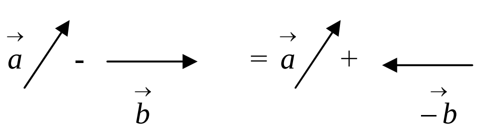


Fig (14)

Triangle method:

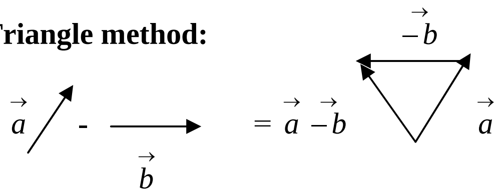


Fig (15)

Parallelogram method:

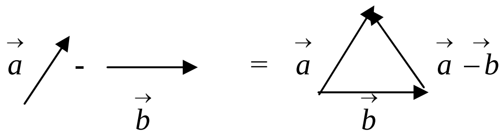


Fig (16)

Note: 1) Subtraction of one vector with another vector is regarded as the addition of one vector with a negative of another vector.

2) The knowledge of subtraction of vectors is useful in understanding the concept of 'relative velocity'.

RESOLUTION OF VECTORS: Resolution of a vector means the process of splitting of a vector into components. If a vector is resolved into two components along the two mutually perpendicular directions, they are called '**rectangular components'**.

Consider a vector R represented by OC both in magnitude and direction as shown in fig (17). Draw OX and OY which are mutually perpendicular to each other from O.

Let the vector R makes an angle θ with X-axis. Drop Perpendiculars from the tip of vector C to X and Y axes. $\vec{OA} = \vec{P}$ is the component of $\vec{R}$ along X-axis and is called **horizontal component of** $\vec{R}$. $\vec{OB} = \vec{Q}$ is the component of $\vec{R}$ along Y-axis and is called **the vertical component of** $\vec{R}$.

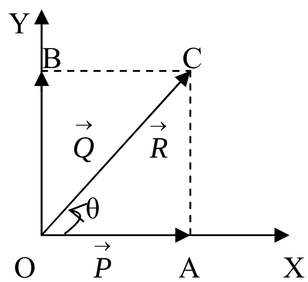


Fig (17)

From fig (17), $\vec{R} = \vec{P} + \vec{Q} = \hat{i}P + \hat{j}Q$

Where $\hat{i}$ and $\hat{j}$ are the unit vectors acting along X and Y axes respectively.

From fig(17), the magnitude of OA is

$$\frac{OA}{OC} = \cos\theta \quad \Rightarrow \quad OA = OC\cos\theta \qquad \therefore P = R\cos\theta$$

The magnitude of OB is

$$\frac{AC}{OC} = \sin\theta$$

$$AC = OC\sin\theta \Rightarrow Q = R\sin\theta$$

Thus, $\vec{R} = (R\cos\theta)\,\hat{i} + (R\sin\theta)\,\hat{j}$

Also, $\frac{AC}{OA} = \tan\theta$ **or** $\tan\theta = \frac{Q}{P}$, where θ gives the resultant direction.

From geometry of the figure, it can be shown that, $R^2 = P^2 + Q^2$

****

QUESTIONS:

1. What is a scalar?

2. Identify whether the following quantities are scalars or vectors?

(i) Mass (ii) weight (iii)speed (iv)velocity (v)energy (vi)work (vii)force (ix)temperature (x)pressure (xi)angular momentum (xii)wavelength.

3. What is a vector?

4. Find the magnitude of $4\hat{i} + 3\hat{j}$

5. **Two vectors** $\vec{P}$ and $\vec{Q}$ act at an angle 60^0 with each other. If P = 20 units and Q = 8 units find the magnitude of resultant $\vec{R}$.

6. **If** $\vec{A} = 6\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 2\hat{j}$ find $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

7. **If** $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, find the unit vector C

8. When will be $\vec{P} + \vec{Q} = \vec{O}$

9. A vector of magnitude 10 units makes an angle of 30^0 with the X-axis. Find its X and Y components.

10. For what angle the magnitude of X and Y components of a vector become equal?

ANSWERS :

1)A physical quantity which requires only magnitude for their complete description are called scalars.

2)SCALARS: mass, speed, energy, work, temperature and pressure.

VECTORS: Weight, velocity, force, angular momentum and wavelength.

3)A Physical quantity which requires both the magnitude and direction for their complete description are called vectors.

4)Given vector is $4\hat{i} + 3\hat{j}$

The magnitude of the vector, $r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2}$ = 25 units.

5)Given: P = 20 units and Q = 8 units and $\theta = 60^0$ R = ?

Magnitude of the resultant vector $\vec{R}$ is

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{20^2 + 8^2 + 2(20)(8)\cos 60^\circ}$

$= \sqrt{400 + 64 + 2(20)(8)(0.5000)}$ = 24.98 units

6)Given: $6\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 2\hat{j}$

i) $\vec{A} + \vec{B} = (6\hat{i} + 3\hat{j}) + (3\hat{i} + 2\hat{j}) = 9\hat{i} + 5\hat{j}$

ii) $\vec{A} - \vec{B} = (6\hat{i} + 3\hat{j}) - (3\hat{i} + 2\hat{j}) = 3\hat{i} + \hat{j}$

7) Given: $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$ then $\hat{A}$=? We know that, $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$$\hat{A} = \frac{3\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{3^2 + (-2)^2 + 1^2}} = \frac{3\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{14}}$$

8) $\vec{P} + \vec{Q} = \vec{O}$ if $\vec{Q} = -\vec{P}$ i.e., the magnitude of $\vec{Q}$ is equal to the magnitude of $\vec{P}$ and acts in opposite direction.

(Note: Two equal vectors acting in opposite direction cancel each other)

9) Given: **$\theta = 30^0$ with X-axis**

X-component=$10\cos 30^0$

= 10(0.8660) =8.660 units

Y-component =$10\sin 30^0$

= 10(0.5000) =5.000 units.

10) Let the vector be R

Its X-component is $R\cos\theta$ and Y- component is $R\sin\theta$

Given: $R\cos\theta = R\sin\theta$

$\cos\theta = \sin\theta$ this is possible only when **$\theta = 45^0$**