# 1-3 Transforming Linear Functions

Warm Up

Lesson Presentation

Lesson Quiz

## Warm Up

**Give the coordinates of each transformation of (2, –3).**

**1.** horizontal translation of 5 (7, –3)

**2.** vertical translation of –1 (2, –4)

**3.** reflection across the $x$-axis (2, 3)

**4.** reflection across the $y$-axis (–2, –3)

**Evaluate $f(-2)$ and $f(1.5)$.**

**5.** $f(x) = 3(x + 5) - 1$ 8; 18.5

**6.** $f(x) = x^2 + 4x$ –4; 8.25

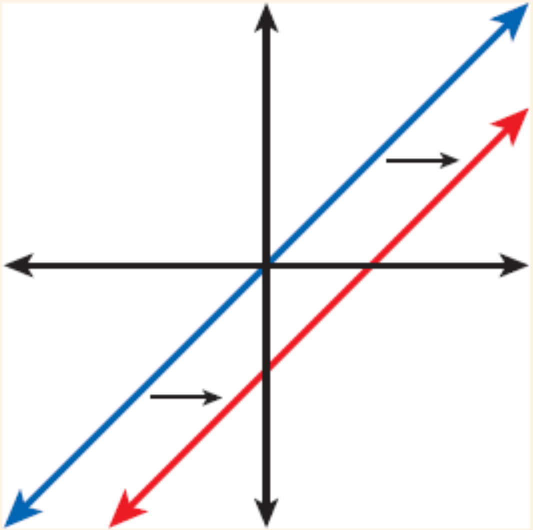
## Objectives

Transform linear functions.

Solve problems involving linear transformations.

In Lesson 1-8, you learned to transform functions by transforming each point. Transformations can also be expressed by using function notation.

## Horizontal Shift of $|h|$ Units

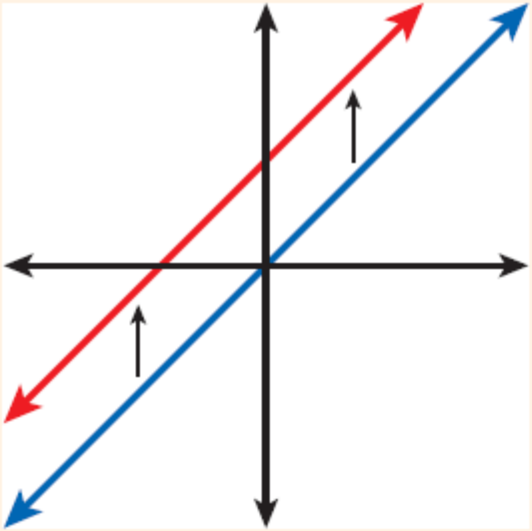


Input value changes.

$f(x) \rightarrow f(x - h)$

$h > 0$ moves right $h < 0$ moves left

### Vertical Shift of $|k|$ Units



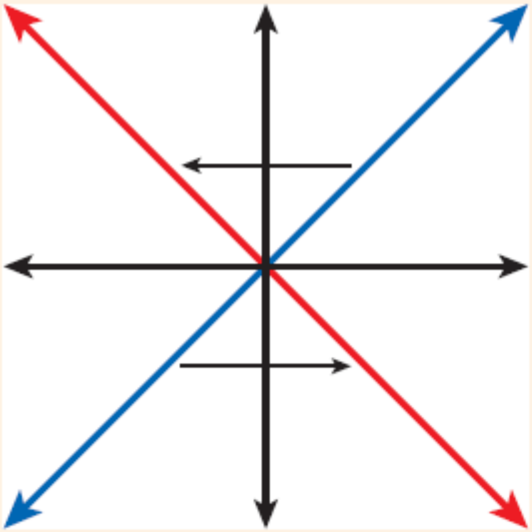
Output value changes.

$f(x) \rightarrow f(x) + k$

$k > 0$ moves up

$k < 0$ moves down

## Reflection Across *y*-axis

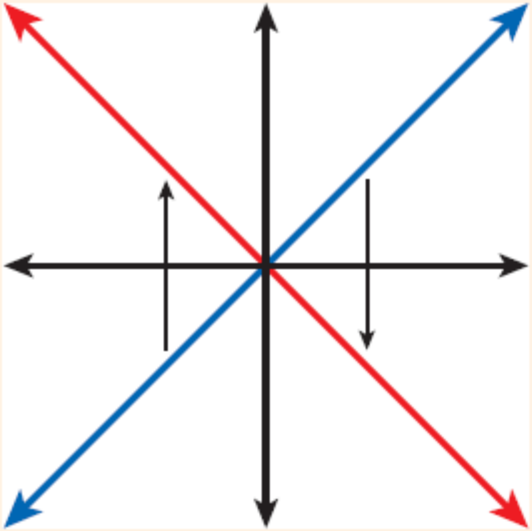


Input value changes.

$f(x) \rightarrow f(-x)$

The lines are symmetric about the *y*-axis.

## Reflection Across *x*-axis



Output value changes.

$f(x) \rightarrow -f(x)$

The lines are symmetric about the *x*-axis.

**Helpful Hint**

To remember the difference between vertical and horizontal translations, think:

"Add to *y*, go high."

"Add to *x*, go left."

## Example 1A: Translating and Reflecting Functions

**Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.**

**$f(x) = x - 2$ , horizontal translation right 3 units**

Translating $f(x)$ 3 units right subtracts 3 from each input value.

$g(x) = f(x - 3)$ *Subtract 3 from the input of f(x).*

$g(x) = (x - 3) - 2$ *Evaluate f at x – 3.*

$g(x) = x - 5$ *Simplify.*

## Example 1B: Translating Reflecting Functions

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

| $x$ | –2 | 0 | 2 |
|---|---|---|---|
| $f(x)$ | 0 | 1 | 2 |

linear function defined in the table; reflection across $x$-axis

## Example 1B Continued

**Step 1** Write the rule for $f(x)$ in slope-intercept form.

| $x$ | –2 | 0 | 2 |
|---|---|---|---|
| $f(x)$ | 0 | 1 | 2 |

The $y$-intercept is 1. *The table contains (0, 1).*

Find the slope: $m = \frac{2-1}{2-0} = \frac{1}{2}$ *Use (0, 1) and (2, 2).*

$y = mx + b$ *Slope-intercept form.*

$y = \frac{1}{2}x + 1$ *Substitute $\frac{1}{2}$ for m and 1 for b.*

$f(x) = \frac{1}{2}x + 1$ *Replace y with f(x).*

## Example 1B Continued

**Step 2** Write the rule for $g(x)$. Reflecting $f(x)$ across the $x$-axis replaces each $y$ with $-y$.

$$g(x) = -\left(\frac{1}{2}x + 1\right) \qquad g(x) = -f(x)$$

$$g(x) = -\frac{1}{2}x - 1$$

## Check It Out! Example 1a

**Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.**

**$f(x) = 3x + 1$; translation 2 units right**

Translating $f(x)$ 2 units right subtracts 2 from each input value.

$g(x) = f(x - 2)$ *Subtract 2 from the input of f(x).*

$g(x) = 3(x - 2) + 1$ *Evaluate f at x – 2.*

$g(x) = 3x - 5$ *Simplify.*

## Check It Out! Example 1b

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

| $x$ | –1 | 0 | 1 |
|---|---|---|---|
| $f(x)$ | 1 | 2 | 3 |

linear function defined in the table; reflection across the $x$-axis

## Check It Out! Example 1b Continued

**Step 1** Write the rule for $f(x)$ in slope-intercept form.

| $x$ | –1 | 0 | 1 |
|---|---|---|---|
| $f(x)$ | 1 | 2 | 3 |

The $y$-intercept is 2. *The table contains (0, 2).*

Find the slope: $m = \frac{3-2}{1-0} = 1$ *Use (0, 1) and (2, 2).*

$y = mx + b$ *Slope-intercept form*

$y = x + 2$ *Substitute 1 for m and 2 for b.*

$f(x) = x + 2$ *Replace y with f(x).*

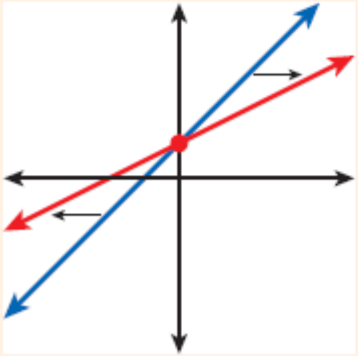
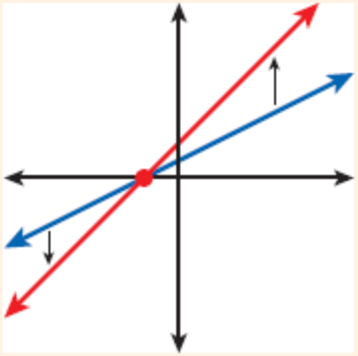
## Check It Out! Example 1b Continued

**Step 2** Write the rule for $g(x)$. Reflecting $f(x)$ across the $x$-axis replaces each $y$ with $-y$.

$g(x) = -(x - 2)$ $\quad g(x) = -f(x)$

$g(x) = -x + 2$

Stretches and compressions change the slope of a linear function. If the line becomes steeper, the function has been stretched vertically or compressed horizontally. If the line becomes flatter, the function has been compressed vertically or stretched horizontally.

| Stretches and Compressions | |
|---|---|
| **Horizontal** | **Vertical** |
| **Horizontal Stretch/Compression by a Factor of *b*** | **Vertical Stretch/Compression by a Factor of *a*** |
|  Input value changes. $f(x) \rightarrow f\left(\frac{1}{b}x\right)$ |  Output value changes. $f(x) \rightarrow a \cdot f(x)$ |
| $b > 1$ stretches away from the $y$-axis. $0 < \lvert b \rvert < 1$ compresses toward the $y$-axis. | $a > 1$ stretches away from the $x$-axis. $0 < \lvert a \rvert < 1$ compresses toward the $x$-axis. |

**Helpful Hint**

These don't change!

- *y*–intercepts in a horizontal stretch or compression
- *x*–intercepts in a vertical stretch or compression

## Example 2: Stretching and Compressing Linear Functions

**Let $g(x)$ be a horizontal compression of $f(x) = -x + 4$ by a factor of $\frac{1}{2}$. Write the rule for $g(x)$, and graph the function.**

Horizontally compressing $f(x)$ by a factor of $\frac{1}{2}$ replaces each $x$ with $\frac{1}{b}x$ where $b = \frac{1}{2}$ .

## Example 2A Continued

$$g(x) = -\left(\frac{1}{b}\right)x + 4$$

*For horizontal compression, use $\frac{1}{b}$.*

$$= -\left(\frac{1}{\frac{1}{2}}\right)x + 4$$

*Substitute $\frac{1}{2}$ for b.*

$$= -(2x) + 4$$

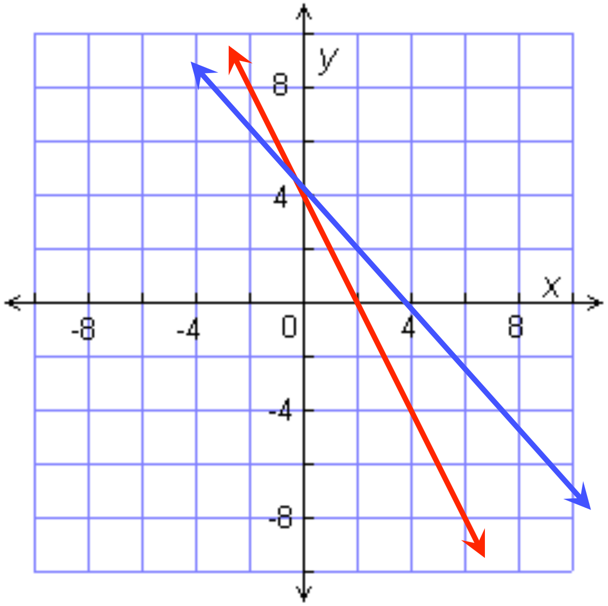
*Replace x with 2x.*

$$g(x) = -2x + 4$$

*Simplify.*

## Example 2A Continued

***Check*** Graph both functions on the same coordinate plane. The graph of $g(x)$ is steeper than $f(x)$, which indicates that $g(x)$ has been horizontally compressed from $f(x)$, or pushed toward the $y$-axis.



## Check It Out! Example 2

**Let $g(x)$ be a vertical compression of $f(x) = 3x + 2$ by a factor of $\frac{1}{4}$. Write the rule for $g(x)$ and graph the function.**

Vertically compressing $f(x)$ by a factor of $\frac{1}{4}$ replaces each $f(x)$ with $a \cdot f(x)$ where $a = \frac{1}{4}$.

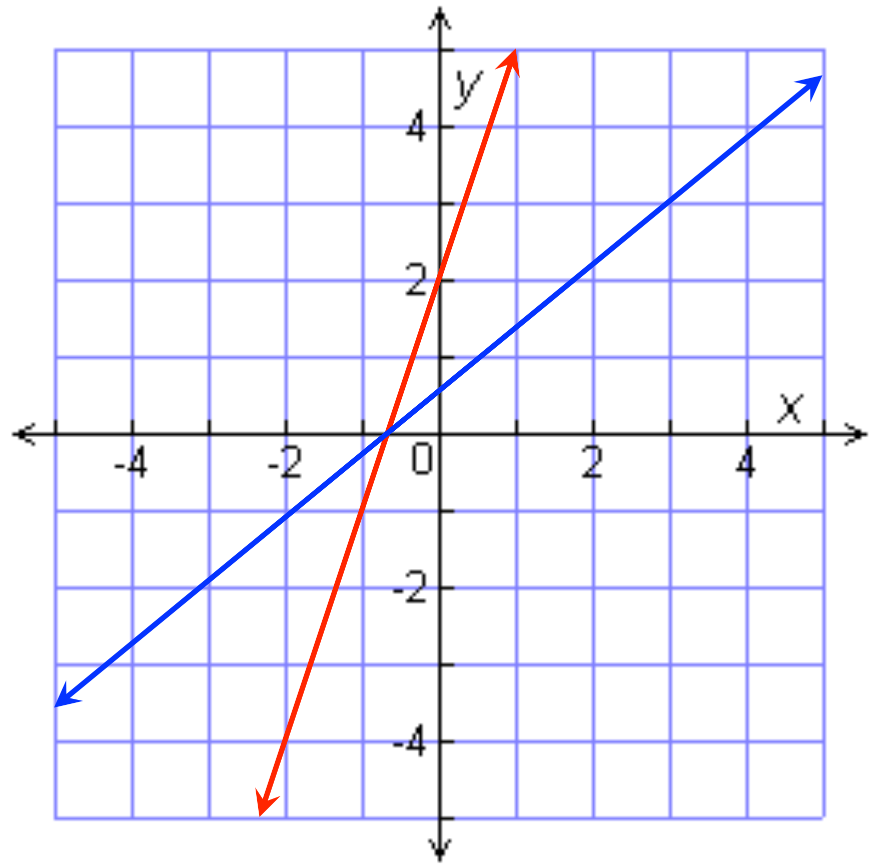
## Check It Out! Example 2 Continued

$g(x) = a(3x + 2)$ *For vertical compression, use a.*

$= \frac{1}{4}(3x + 2)$ *Substitute $\frac{1}{4}$ for a.*

$g(x) = \frac{3}{4}x + \frac{1}{2}$ *Simplify.*

Graph both functions on the same coordinate plane. The graph of $g(x)$ is less steep than $f(x)$, which indicates that $g(x)$ has been vertically compressed from $f(x)$, or compressed towards the $x$-axis.



Some linear functions involve more than one transformation by applying individual transformations one at a time in the order in which they are given.

For multiple transformations, create a temporary function—such as $h(x)$ in Example 3 below—to represent the first transformation, and then transform it to find the combined transformation.

## Example 3: Combining Transformations of Linear Functions

**Let $g(x)$ be a horizontal shift of $f(x) = 3x$ left 6 units followed by a horizontal stretch by a factor of 4. Write the rule for $g(x)$.**

**Step 1** First perform the translation.

$h(x) = f(x + 6)$ — *Add 6 to the input value.* ıt

$h(x) = 3(x + 6)$ — *Evaluate f at x + 6.*

$h(x) = 3x + 18$ — *Distribute.*

## Example 3 Continued

**Step 2** Then perform the stretch.

Stretching $h(x)$ horizontally by a factor of 4 replaces each $x$ with $\frac{1}{b}x$ where $b = 4$.

$$g(x) = 3\left(\frac{1}{b}\right)x + 18$$ *For horizontal compression, use $\frac{1}{b}$.*

$$g(x) = 3\left(\frac{1}{4}\right)x + 18$$ *Substitute 4 for b.*

$$g(x) = \frac{3}{4}x + 18$$ *Simplify.*

## Check It Out! Example 3

**Let $g(x)$ be a vertical compression of $f(x) = x$ by a factor of $\frac{1}{2}$ followed by a horizontal shift 8 left units. Write the rule for $g(x)$.**

**Step 1** First perform the translation.

| | |
|---|---|
| $h(x) = f(x + 8)$ | *Add 8 to the input value.* |
| $h(x) = x + 8$ | *Evaluate f at x + 8.* |
| $h(x) = x + 8$ | *Distribute.* |

## Check It Out! Example 3

**Step 2** Then perform the stretch.

Stretching $h(x)$ vertically by a factor of $\frac{1}{2}$ multiplies the function by $\frac{1}{2}$.

$$g(x) = \left(\frac{1}{2}\right)(x + 8)$$

*Multiply the function by $\frac{1}{2}$.*

$$g(x) = \frac{1}{2}(x + 8)$$

*Simplify.*

## Homework!

**Page 28: 1-6, 8-14, 16-20**

## Example 4A: Fund-raising Application

**The golf team is selling T-shirts as a fund-raiser. The function R(*n*) = 7.5*n* represents the team's revenue in dollars, and *n* is the number of t-shirts sold.**

**The team paid $60 for the T-shirts. Write a new function *P*(*n*) for the team's profit.**

The initial costs must be subtracted from the revenue.

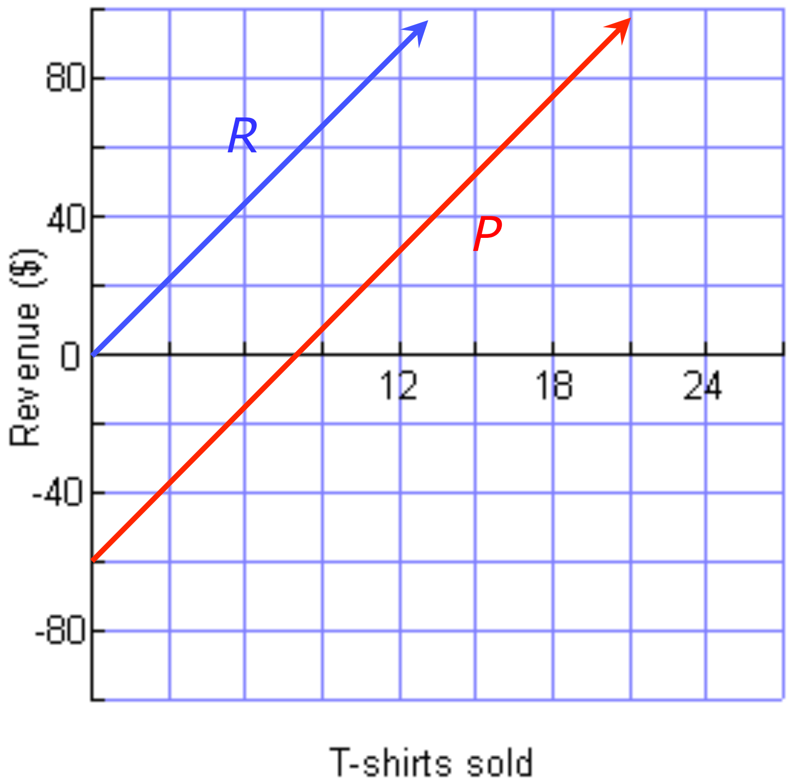
$R(n) = 7.5n$ *Original function*

$P(n) = 7.5n - 60$ *Subtract the expenses.*

## Example 4B: Fund-raising Application

**Graph both *P*(*n*) and *R*(*n*) on the same coordinate plane.**

Graph both functions. The lines have the same slope but different *y*-intercepts.




Note that the profit can be negative but the number of T-shirts sold cannot be less than 0.

## Example 4C: Fund-raising Application

**Describe the transformation(s) that have been applied.**

The graph indicates that $P(n)$ is a translation of $R(n)$. Because 60 was subtracted, $P(n) = R(n) - 60$. This indicates a vertical shift 60 units down.

## Check It Out! Example 4a

**The Dance Club is selling beaded purses as a fund-raiser. The function R(*n*) = 12.5*n* represents the club's revenue in dollars where *n* is the number of purses sold.**

**The club paid $75 for the materials needed to make the purses. Write a new function *P*(*n*) for the club's profit.**

**What if …?** The club members decided to double the price of each purse

The initial costs must be subtracted from the revenue.

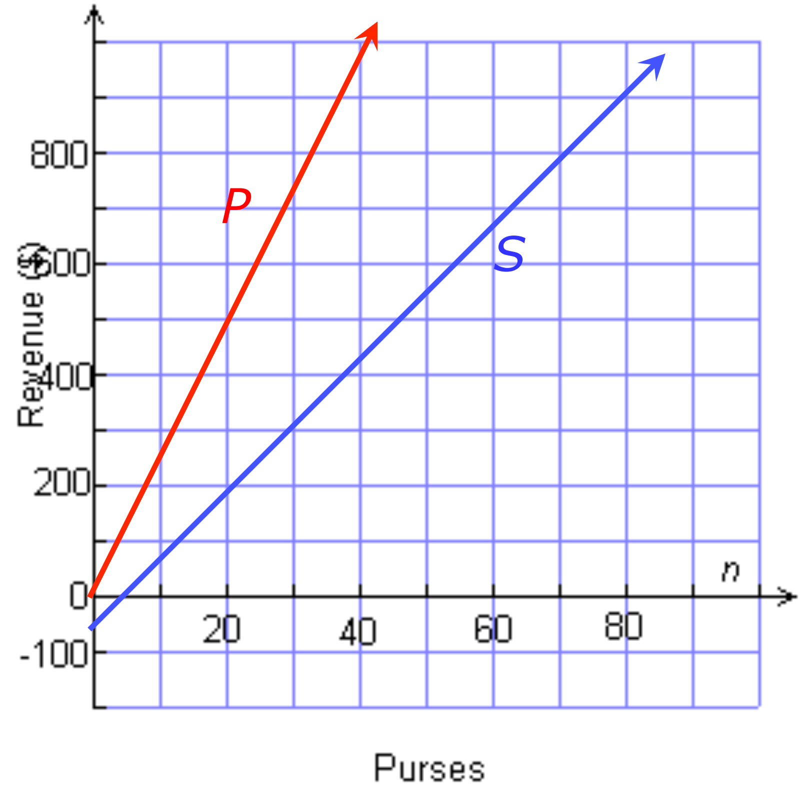
$S(n) = 25n - 75$ *Subtract the expenses.*

## Check It Out! Example 4b

**Graph both *S*(*n*) and *P*(*n*) on the same coordinate plane.**

Graph both functions. The lines have the same slope but different *y*-intercepts.

Note that the profit can be negative but the number of purses sold cannot be less than 0.



## Check It Out! Example 4c

**Describe the transformation(s) that have been applied.**

The graph indicates that $P(n)$ is a compression of $S(n)$.

Because the price was doubled, $S(n) = 2R(n) - 75$.

This indicates a horizontal compression by a factor of $\frac{1}{2}$.

## Lesson Quiz: Part I

**Let $g(x)$ be the indicated transformation of $f(x) = 3x + 1$. Write the rule for $g(x)$.**

**1.** horizontal translation 3 units right $g(x) = 3x - 8$

**2.** reflection across the $x$-axis $g(x) = -3x - 1$

**3.** vertical stretch by a factor of 2. $g(x) = 6x + 2$

**4.** vertical shift up 4 units followed by a horizontal compression of $\frac{1}{3}$. $g(x) = 9x + 5$

## Lesson Quiz: Part II

**5. The cost of a classified ad is represented by $C(l) = 1.50l + 4.00$ where $l$ is the number of lines in the ad. The cost is increased by $3.00 when color is used.**

Write a new function $H(l)$ for the cost of a classified ad in color, and describe the transformation(s) that have been applied.

$H(l) = 1.50l + 7.00$; shift 3 units up