Solving Two-Step and Multi-Step Equations

Warm Up

Evaluate each expression.

1. $9 - 3(-2)$ 15

2. $3(-5 + 7)$ 6

3. $12\left(\frac{3+(-7)}{12}\right)$ –4

4. $26 - 4(7 - 5)$ 18

Simplify each expression.

5. $10c + c$ $11c$

6. $8.2b + 3.8b - 12b$ 0

7. $5m + 2(2m - 7)$ $9m - 14$

8. $6x - (2x + 5)$ $4x - 5$

Objective

Solve equations in one variable that contain more than one operation.

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

Cost per CD **Total cost**

$$3.95c + 19.95 = 63.40$$

Cost of discount card

$$3.95c + 19.95 = 63.40$$

Operations in the Equation	**To Solve**
1. First *c* is **multiplied** by 3.95.	1. **Subtract** 19.95 from both sides of the equation.
2. Then 19.95 is **added**.	2. Then **divide** both sides by 3.95.

Example 1A: Solving Two-Step Equations

Solve 18 = 4*a* + 10.

$$18 = 4a + 10$$
$$\underline{-10} \quad \underline{\quad - 10}$$
$$8 = 4a$$

First a is multiplied by 4. Then 10 is added. Work backward: Subtract 10 from both sides.

$$\frac{8}{4} = \frac{4a}{4}$$
$$2 = a$$

Since a is multiplied by 4, divide both sides by 4 to undo the multiplication.

Example 1B: Solving Two-Step Equations

Solve $5t - 2 = -32$.

$$5t - 2 = -32$$
$$\underline{\quad + 2} \quad \underline{\quad + 2}$$
$$5t = -30$$

First t is multiplied by 5. Then 2 is subtracted. Work backward: Add 2 to both sides.

$$\frac{5t}{5} = \frac{-30}{5}$$
$$t = -6$$

Since t is multiplied by 5, divide both sides by 5 to undo the multiplication.

Check it Out! Example 1a

Solve $-4 + 7x = 3$.

$$\begin{aligned} -4 + 7x &= 3 \\ +4 \qquad &\;\; +4 \\ \hline 7x &= 7 \end{aligned}$$

First x is multiplied by 7. Then –4 is added. Work backward: Add 4 to both sides.

$$\frac{7x}{7} = \frac{7}{7}$$

Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.

$$x = 1$$

Check it Out! Example 1b

Solve $1.5 = 1.2y - 5.7$.

$$\begin{aligned} 1.5 &= 1.2y - 5.7 \\ +5.7 & \quad\quad\;\; +5.7 \\ \hline 7.2 &= 1.2y \end{aligned}$$

First y is multiplied by 1.2. Then 5.7 is subtracted. Work backward: Add 5.7 to both sides.

$$\frac{7.2}{1.2} = \frac{1.2y}{1.2}$$

$$6 = y$$

Since y is multiplied by 1.2, divide both sides by 1.2 to undo the multiplication.

Check it Out! Example 1c

Solve $\frac{n}{7} + 2 = 2$.

$$\frac{n}{7} + 2 = 2$$
$$\underline{\quad -2} \quad \underline{-2}$$

First n is divided by 7. Then 2 is added. Work backward: Subtract 2 from both sides.

$$\frac{n}{7} = 0$$

$$(7)\frac{n}{7} = (7)0$$

Since n is divided by 7, multiply both sides by 7 to undo the division.

$$n = 0$$

Example 2A: Solving Two-Step Equations That Contain Fractions

Solve $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$.

Method 1 Use fraction operations.

$$\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$$

$$\underline{\quad + \frac{3}{4} \quad + \frac{3}{4}}$$

Since $\frac{3}{4}$ is subtracted from $\frac{y}{8}$, add $\frac{3}{4}$ to both sides to undo the subtraction.

$$\frac{y}{8} = \frac{16}{12}$$

Since y is divided by 8, multiply both sides by 8 to undo the division.

$$8\left(\frac{y}{8}\right) = 8\left(\frac{16}{12}\right)$$

Example 2A Continued

Solve $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$.

Method 1 Use fraction operations.

$$8\left(\frac{y}{8}\right) = 8\left(\frac{16}{12}\right)$$

$$y = \frac{8 \bullet 16}{12}$$ *Simplify.*

$$y = \frac{32}{3}$$

Example 2A Continued

Solve $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$.

Method 2 Multiply by the LCD to clear the fractions.

$$24\left(\frac{y}{8} - \frac{3}{4}\right) = 24\left(\frac{7}{12}\right)$$

Multiply both sides by 24, the LCD of the fractions.

$$24\left(\frac{y}{8}\right) - 24\left(\frac{3}{4}\right) = 24\left(\frac{7}{12}\right)$$

Distribute 24 on the left side.

$$3y - 18 = 14$$

Simplify.

$$\begin{aligned} 3y - 18 &= 14 \\ +18 &\quad +18 \\ \hline 3y &= 32 \end{aligned}$$

Since 18 is subtracted from 3y, add 18 to both sides to undo the subtraction.

Example 2A Continued

Solve $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$.

Method 2 Multiply by the LCD to clear the fractions.

$$\frac{3y}{3} = \frac{32}{3}$$

Since y is multiplied by 3, divide both sides by 3 to undo the multiplication.

$$y = \frac{32}{3}$$

Example 2B: Solving Two-Step Equations That Contain Fractions

Solve $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$.

Method 1 Use fraction operations.

$$\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$$

Since $\frac{3}{4}$ is added to $\frac{2}{3}r$, subtract $\frac{3}{4}$ from both sides to undo the addition.

$$\underline{\quad -\frac{3}{4} \quad -\frac{3}{4}}$$

$$\frac{2}{3}r = -\frac{1}{6}$$

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Since r is multiplied by $\frac{2}{3}$, multiply both sides by $\frac{3}{2}$.

$$\left(\frac{3}{2}\right)\frac{2}{3}r = \left(\frac{3}{2}\right)\left(-\frac{1}{6}\right)$$

Example 2B Continued

Solve $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$.

Method 1 Use fraction operations.

$$\left(\frac{3}{2}\right)\frac{2}{3}r = \left(\frac{3}{2}\right)\left(-\frac{1}{6}\right)$$

$$r = -\frac{3 \bullet 1}{2 \bullet 6}$$

$$r = -\frac{1}{4}$$

Example 2B Continued

Solve $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$.

Method 2 Multiply by the LCD to clear the fractions.

$$12\left(\frac{2}{3}r + \frac{3}{4}\right) = 12\left(\frac{7}{12}\right)$$

Multiply both sides by 12, the LCD of the fractions.

$$12\left(\frac{2}{3}r\right) + 12\left(\frac{3}{4}\right) = 12\left(\frac{7}{12}\right)$$

Distribute 12 on the left side.

$$8r + 9 = 7$$
$$\underline{\quad -9 \quad -9}$$
$$8r = -2$$

Simplify. Since 9 is added to 8r, subtract 9 from both sides to undo the addition.

Example 2B Continued

Solve $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$.

Method 2 Multiply by the LCD to clear the fractions.

$$\frac{8r}{8} = \frac{-2}{8}$$

Since r is multiplied by 8, divide both sides by 8 to undo the multiplication.

$$r = -\frac{1}{4}$$

Check It Out! Example 2a

Solve $\frac{2x}{5} - \frac{1}{2} = 5$.

Method 2 Multiply by the LCD to clear the fractions.

$$10\left(\frac{2x}{5} - \frac{1}{2}\right) = 10(5)$$

Multiply both sides by 10, the LCD of the fractions.

$$10\left(\frac{2x}{5}\right) - 10\left(\frac{1}{2}\right) = 10(5)$$

Distribute 10 on the left side.

$$4x - 5 = 50$$

Simplify.

$$\underline{\quad + 5 \quad + 5}$$

Since 5 is subtracted from 4x, add 5 to both sides to undo the subtraction.

$$4x = 55$$

Check It Out! Example 2a

Solve $\frac{2x}{5} - \frac{1}{2} = 5$.

Method 2 Multiply by the LCD to clear the fractions.

$$\frac{4x}{4} = \frac{55}{4}$$

Simplify. Since 4 is multiplied by x, divide both sides by 4 to undo the multiplication.

$$x = \frac{55}{4}$$

Check It Out! Example 2b

Solve $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$.

Method 2 Multiply by the LCD to clear the fractions.

$$8\left(\frac{3}{4}u + \frac{1}{2}\right) = 8\left(\frac{7}{8}\right)$$

Multiply both sides by 8, the LCD of the fractions.

$$8\left(\frac{3}{4}u\right) + 8\left(\frac{1}{2}\right) = 8\left(\frac{7}{8}\right)$$

Distribute 8 on the left side.

$$6u + 4 = 7$$

Simplify.

$$\underline{\quad -4 \quad -4}$$

$$6u = 3$$

Since 4 is added to 6u, subtract 4 from both sides to undo the addition.

Check It Out! Example 2b Continued

Solve $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$.

Method 2 Multiply by the LCD to clear the fractions.

$$\frac{6u}{6} = \frac{3}{6}$$

Since u is multiplied by 6, divide both sides by 6 to undo the multiplication.

$$u = \frac{1}{2}$$

Check It Out! Example 2c

Solve $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$.

Method 1 Use fraction operations.

$$\frac{n}{5} - \frac{1}{3} = \frac{8}{3}$$

Since $\frac{1}{3}$ is subtracted from $\frac{n}{5}$, add $\frac{1}{3}$ to both sides to undo the subtraction.

$$\underline{\quad + \frac{1}{3} \quad + \frac{1}{3}}$$

$$\frac{n}{5} = \frac{9}{3}$$

$$\frac{n}{5} = 3$$

Simplify.

Check It Out! Example 2c Continued

Solve $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$.

Method 1 Use fraction operations.

$$\frac{n}{5} = 3$$

$$5\left(\frac{n}{5}\right) = 5(3)$$

Since n is divided by 5, multiply both sides by 5 to undo the division.

$n = 15$

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

Example 3A: Simplifying Before Solving Equations

Solve $8x - 21 - 5x = -15$.

$8x - 21 - 5x = -15$

$8x - 5x - 21 = -15$ *Use the Commutative Property of Addition.*

$3x - 21 = -15$ *Combine like terms.*

$\underline{\quad + 21 \quad +21}$ *Since 21 is subtracted from 3x, add 21 to both sides to undo the subtraction.*

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$ *Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$x = 2$

Example 3B: Simplifying Before Solving Equations

Solve $10y - (4y + 8) = -20$

$10y + (-1)(4y + 8) = -20$ — *Write subtraction as addition of the opposite.*

$10y + (-1)(4y) + (-1)(8) = -20$ — *Distribute –1 on the left side.*

$10y - 4y - 8 = -20$ — *Simplify.*

$6y - 8 = -20$ — *Combine like terms.*

$\underline{\quad + 8 \quad} \quad \underline{+ 8}$ — *Since 8 is subtracted from 6y, add 8 to both sides to undo the subtraction.*

$6y = -12$

$\frac{6y}{6} = \frac{-12}{6}$ — *Since y is multiplied by 6, divide both sides by 6 to undo the multiplication.*

$y = -2$

Check It Out! Example 3a

Solve $2a + 3 - 8a = 8$.

$$2a + 3 - 8a = 8$$

$$2a - 8a + 3 = 8$$ *Use the Commutative Property of Addition.*

$$-6a + 3 = 8$$ *Combine like terms.*

$$\underline{\quad - 3 \quad - 3}$$ *Since 3 is added to –6a, subtract 3 from both sides to undo the addition.*

$$-6a = 5$$

$$\frac{-6a}{-6} = \frac{5}{-6}$$ *Since a is multiplied by –6, divide both sides by –6 to undo the multiplication.*

$$a = -\frac{5}{6}$$

Check It Out! Example 3b

Solve $-2(3 - d) = 4$

$$-2(3 - d) = 4$$

$$(-2)(3) + (-2)(-d) = 4$$ *Distribute –2 on the left side.*

$$-6 + 2d = 4$$ *Simplify.*

$$\begin{aligned} -6 + 2d &= 4 \\ \underline{+6 \qquad\quad} &\; \underline{+6} \end{aligned}$$ *Add 6 to both sides.*

$$2d = 10$$

$$\frac{2d}{2} = \frac{10}{2}$$ *Since d is multiplied by 2, divide both sides by 2 to undo the multiplication.*

$$d = 5$$

Check It Out! Example 3c

Solve $4(x - 2) + 2x = 40$

$$4(x - 2) + 2x = 40$$

$$(4)(x) + (4)(-2) + 2x = 40$$ *Distribute 4 on the left side.*

$$4x - 8 + 2x = 40$$ *Simplify.*

$$4x + 2x - 8 = 40$$ *Commutative Property of Addition.*

$$6x - 8 = 40$$ *Combine like terms.*

$$\underline{\quad +8 \quad} \quad \underline{+8}$$ *Since 8 is subtracted from 6x, add 8 to both sides to undo the subtraction.*

$$6x = 48$$

$$\frac{6x}{6} = \frac{48}{6}$$ *Since x is multiplied by 6, divide both sides by 6 to undo the multiplication.*

$$x = 8$$

Example 4: Application

Jan joined the dining club at the local café for a fee of $29.95. Being a member entitles her to save $2.50 every time she buys lunch. So far, Jan calculates that she has saved a total of $12.55 by joining the club. Write and solve an equation to find how many time Jan has eaten lunch at the café.

Example 4: Application Continued

1 Understand the Problem

The **answer** will be the number of times Jan has eaten lunch at the café.

List the important information:

- Jan paid a $29.95 dining club fee.
- Jan saves $2.50 on every lunch meal.
- After one year, Jan has saved $12.55.

Example 4: Application Continued

2 Make a Plan

Let m represent the number of meals that Jan has paid for at the café. That means that Jan has saved $2.50m$. However, Jan must also add the amount she spent to join the dining club.

total amount saved	=	amount saved on each meal	−	dining club fee
12.55	=	$2.50m$	−	29.95

Example 4: Application Continued

3 Solve

$$12.55 = 2.50m - 29.95$$
$$\underline{+\,29.95} \qquad \underline{\qquad\qquad + 29.95}$$
$$42.50 = 2.50m$$

Since 29.95 is subtracted from 2.50m, add 29.95 to both sides to undo the subtraction.

$$\frac{42.50}{2.50} = \frac{2.50m}{2.50}$$

Since m is multiplied by 2.50, divide both sides by 2.50 to undo the multiplication.

$$17 = m$$

Example 4: Application Continued

4 Look Back

Check that the answer is reasonable. Jan saves \$2.50 every time she buys lunch, so if she has lunch 17 times at the café, the amount saved is 17(2.50) = 42.50.

Subtract the cost of the dining club fee, which is about \$30. So the total saved is about \$12.50, which is close to the amount given in the problem, \$12.55.

Check It Out! Example 4

Sara paid $15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was $735.95. How much was the monthly fee?

Check It Out! Example 4 Continued

1 Understand the Problem

The **answer** will the monthly membership fee.

List the important information:

- Sara paid $15.95 to become a gym member.
- Sara pays a monthly membership fee.
- Her total cost for 12 months was $735.95.

Check It Out! Example 4 Continued

2 Make a Plan

Let m represent the monthly membership fee that Sara must pay. That means that Sara must pay $12m$. However, Sara must also add the amount she spent to become a gym member.

total cost	=	monthly fee	+	initial membership
735.95	=	$12m$	+	15.95

Check It Out! Example 4 Continued

3 Solve

$$735.95 = 12m + 15.95$$
$$\underline{-15.95} \qquad \underline{\qquad -15.95}$$
$$720 = 12m$$

Since 15.95 is added to 12m, subtract 15.95 from both sides to undo the addition.

$$\frac{720}{12} = \frac{12m}{12}$$

$$60 = m$$

Since m is multiplied by 12, divide both sides by 12 to undo the multiplication.

Check It Out! Example 4 Continued

4 Look Back

Check that the answer is reasonable. Sara pays $60 a month, so after 12 months Sara has paid 12(60) = 720.

Add the cost of the initial membership fee, which is about $16. So the total paid is about $736, which is close to the amount given in the problem, $735.95.

Example 5A: Solving Equations to Find an Indicated Value

If $4a + 0.2 = 5$, find the value of $a - 1$.

Step 1 Find the value of *a*.

$$\begin{aligned} 4a + 0.2 &= 5 \\ \underline{\quad -0.2} &\ \ \underline{-0.2} \\ 4a &= 4.8 \end{aligned}$$

Since 0.2 is added to 4a, subtract 0.2 from both sides to undo the addition.

$$\frac{4a}{4} = \frac{4.8}{4}$$

Since a is multiplied by 4, divide both sides by 4 to undo the multiplication.

$$a = 1.2$$

Step 2 Find the value of $a - 1$.

$1.2 - 1$ *To find the value of a – 1, substitute 1.2 for a.*

0.2 *Simplify.*

Example 5B: Solving Equations to Find an Indicated Value

If $3d - (9 - 2d) = 51$, find the value of $3d$.

Step 1 Find the value of d.

$$3d - (9 - 2d) = 51$$

$$3d - 9 + 2d = 51$$

$$5d - 9 = 51$$

$$\underline{\quad +9 \quad} \quad \underline{+9}$$

Since 9 is subtracted from 5d, add 9 to both sides to undo the subtraction.

$$5d = 60$$

$$\frac{5d}{5} = \frac{60}{5}$$

Since d is multiplied by 5, divide both sides by 5 to undo the multiplication.

$$d = 12$$

Example 5B Continued

If $3d - (9 - 2d) = 51$, find the value of $3d$.

Step 2 Find the value of $3d$.

$d = 12$

$3(12)$ *To find the value of 3d, substitute 12 for d.*

36 *Simplify.*

Lesson Quiz: Part 1

Solve each equation.

1. $4y + 8 = 2$ $-\frac{3}{2}$

2. $\frac{3}{4}a + 14 = 8$ -8

3. $2y + 29 - 8y = 5$ 4

4. $3(x - 9) = 30$ 19

5. $x - (12 - x) = 38$ 25

6. $\frac{z}{6} - \frac{5}{8} = \frac{7}{8}$ 9

Lesson Quiz: Part 2

7. If $3b - (6 - b) = -22$, find the value of $7b$. -28

8. Josie bought 4 cases of sports drinks for an upcoming meet. After talking to her coach, she bought 3 more cases and spent an additional \$6.95 on other items. Her receipts totaled \$74.15. Write and solve an equation to find how much each case of sports drinks cost.

$4c + 3c + 6.95 = 74.15$; \$9.60