1.5.1 Understanding Exponents and Square Roots

Learning Objective(s)
1 Evaluate expressions containing exponents.
2 Write repeated factors using exponential notation.
3 Find a square root of a perfect square.

Introduction

Exponents provide a special way of writing repeated multiplication. Numbers written in this way have a specific form, with each part providing important information about the number. Writing numbers using exponents can save a lot of space, too. The inverse operation of multiplication of a number by itself is called finding the square root of a number. This **operation** is helpful for problems about the area of a square.

Understanding Exponential Notation

Objective 1

Exponential notation is a special way of writing repeated factors, for example 7 • 7. Exponential notation has two parts. One part of the notation is called the **base**. The base is the number that is being multiplied by itself. The other part of the notation is the **exponent**, or power. This is the small number written up high to the right of the base. The exponent, or power, tells how many times to use the base as a **factor** in the multiplication. In the example, 7 • 7 can be written as 7^2, 7 is the base and 2 is the exponent. The exponent 2 means there are two factors.

$$7^2 = 7 \cdot 7 = 49$$

You can read 7^2 as "seven squared." This is because multiplying a number by itself is called "**squaring** a number." Similarly, raising a number to a power of 3 is called "**cubing** the number." You can read 7^3 as "seven cubed."

You can read 2^5 as "two to the fifth power" or "two to the power of five." Read 8^4 as "eight to the fourth power" or "eight to the power of four." This format can be used to read *any* number written in exponential notation. In fact, while 6^3 is most commonly read "six cubed," it can also be read "six to the third power" or "six to the power of three."

To find the value of a number written in exponential form, rewrite the number as repeated multiplication and perform the multiplication. Two examples are shown below.

Example	
Problem	**Find the value of 4^2.**
4 is the base. 2 is the exponent.	An exponent means repeated multiplication. The base is 4; 4 is the number being multiplied. The exponent is 2; This means to use two factors of 4 in the multiplication.
$4^2 = 4 \cdot 4$	Rewrite as repeated multiplication.
$4 \cdot 4 = 16$	Multiply.
Answer $4^2 = 16$	

Example	
Problem	**Find the value of 2^5.**
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	Rewrite 2^5 as repeated multiplication. The base is 2, the number being multiplied. The exponent is 5, the number of times to use 2 in the multiplication.
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ $4 \cdot 2 \cdot 2 \cdot 2$ $8 \cdot 2 \cdot 2$ $16 \cdot 2$ 32	Perform multiplication.
Answer $2^5 = 32$	

Self Check A

Find the value of 4^3.

Writing Repeated Multiplication Using Exponents

Objective **2**

Writing repeated multiplication in exponential notation can save time and space. Consider the example 5 • 5 • 5 • 5. We can use exponential notation to write this repeated multiplication as 5^4. Since 5 is being multiplied, it is written as the base. Since the base is used 4 times in the multiplication, the exponent is 4. The expression 5 • 5 • 5 • 5 can be rewritten in shorthand exponential notation as 5^4 and is read, “five to the fourth power” or “five to the power of 4.”

To write repeated multiplication of the same number in exponential notation, first write the number being multiplied as the base. Then count how many times that number is used in the multiplication, and write that number as the exponent. Be sure to count the numbers, *not* the multiplication signs, to determine the exponent.

Example		
Problem	**Write 7 • 7 • 7 in exponential notation.**	
	7 is the base.	The base is the number being multiplied, 7.
	Since 7 is used 3 times, 3 is the exponent.	The exponent tells the number of times the base is multiplied.
Answer	*7 • 7 • 7 =* 7^3	*This is read “seven cubed.”*

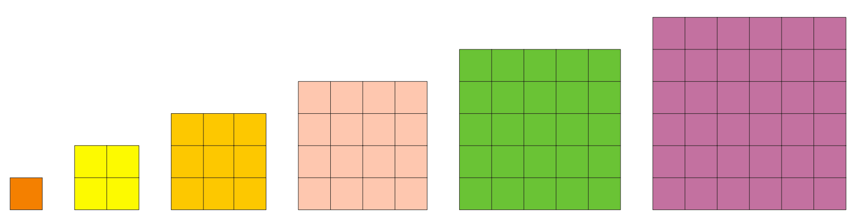
Self Check B

Write 10 • 10 • 10 • 10 • 10 • 10 in exponential notation.

Understanding and Computing Square Roots

Objective **3**

As you saw earlier, 5^2 is called “five squared.” “Five squared” means to multiply five by itself. In mathematics, we call multiplying a number by itself “squaring” the number. We call the result of squaring a whole number a square or a **perfect square**. A perfect square is any number that can be written as a whole number **raised to the power** of 2. For example, 9 is a perfect square. A perfect square number can be represented as a square shape, as shown below. We see that 1, 4, 9, 16, 25, and 36 are examples of perfect squares.



Shape number	1	2	3	4	5	6
Number of small squares	1x1=1	2x2=4	3x3=9	4x4=16	5x5=25	6x6=36

To square a number, multiply the number by itself. 3 squared = 3^2 = 3 • 3 = 9.

Below are some more examples of perfect squares.

1 squared	1^2	1 • 1	1
2 squared	2^2	2 • 2	4
3 squared	3^2	3 • 3	9
4 squared	4^2	4 • 4	16
5 squared	5^2	5 • 5	25
6 squared	6^2	6 • 6	36
7 squared	7^2	7 • 7	49
8 squared	8^2	8 • 8	64
9 squared	9^2	9 • 9	81
10 squared	10^2	10 • 10	100

The **inverse operation** of squaring a number is called finding the **square root** of a number. Finding a square root is like asking, "what number multiplied by itself will give me this number?" The square root of 25 is 5, because 5 multiplied by itself is equal to 25. Square roots are written with the mathematical symbol, called a **radical sign**, that looks like this: $\sqrt{\ }$. The "square root of 25" is written $\sqrt{25}$.

Example	
Problem	**Find $\sqrt{81}$.**
	$\sqrt{81} = 9$
Answer	$\sqrt{81} = 9$

Self Check C

Find $\sqrt{36}$.

Summary

Exponential notation is a shorthand way of writing repeated multiplication of the same number. A number written in exponential notation has a base and an exponent, and each of these parts provides information for finding the value of the expression. The base tells what number is being repeatedly multiplied, and the exponent tells how many times the base is used in the multiplication. Exponents 2 and 3 have special names. Raising a base to a power of 2 is called "squaring" a number. Raising a base to a power of 3 is called "cubing" a number. The inverse of squaring a number is finding the square root of a number. To find the square root of a number, ask yourself, "What number can I multiply by itself to get this number?"

1.5.1 Self Check Solutions

Self Check A

Find the value of 4^3.

$4 \cdot 4 \cdot 4 = 64$.

Self Check B

Write $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ in exponential notation.

10^6

The base is 10 since that is the number that is being multiplied by itself. The exponent is 6 since there are six 10s in the multiplication.

Self Check C

Find $\sqrt{36}$.

Since $6 \cdot 6 = 36$, $\sqrt{36} = 6$.

1.5.2 Order of Operations

Learning Objective(s)

1 Use the order of operations to simplify expressions, including those with parentheses.
2 Use the order of operations to simplify expressions containing exponents and square roots.

Introduction

People need a common set of rules for performing computation. Many years ago, mathematicians developed a standard **order of operations** that tells you which calculations to make first in an expression with more than one **operation**. Without a standard procedure for making calculations, two people could get two different answers to the same problem. For example, 3 + 5 • 2 has only one correct answer. Is it 13 or 16?

The Order of Addition, Subtraction, Multiplication & Division Operations

Objective 1

First, consider expressions that include one or more of the arithmetic operations: addition, subtraction, multiplication, and division. The order of operations requires that all multiplication and division be performed first, going from left to right in the **expression**. The order in which you compute multiplication and division is determined by which one comes first, reading from left to right.

After multiplication and division has been completed, add or subtract in order from left to right. The order of addition and subtraction is also determined by which one comes first when reading from left to right.

Below, are three examples showing the proper order of operations for expressions with addition, subtraction, multiplication, and/or division.

Example		
Problem	**Simplify 3 + 5 • 2.**	
	3 + **5 • 2**	Order of operations tells you to perform multiplication before addition.
	3 + 10	Then add.
Answer	3 + 5 • 2 = 13	

Example		
Problem	**Simplify 20 – 16 ÷ 4.**	
	20 – **16 ÷ 4**	Order of operations tells you to perform division before subtraction.
	20 – 4 16	Then subtract.
Answer	20 – 16 ÷ 4 = 16	

Example		
Problem	**Simplify 60 – 30 ÷ 3 • 5 + 7.**	
	60 – **30 ÷ 3 • 5** + 7	Order of operations tells you to perform multiplication and division first, working from left to right, before doing addition and subtraction.
	60 – **10 • 5** + 7 60 – 50 + 7	Continue to perform multiplication and division from left to right.
	10 + 7 17	Next, add and subtract from left to right. (Note that addition is not necessarily performed before subtraction.)
Answer	60 – 30 ÷ 3 • 5 + 7 = 17	

Grouping Symbols and the Order of Operations

Grouping symbols such as parentheses ( ), brackets [ ], braces $\{\ \}$, and fraction bars can be used to further control the order of the four basic arithmetic operations. The rules of the order of operations require computation within grouping symbols to be completed first, even if you are adding or subtracting within the grouping symbols and you have multiplication outside the grouping symbols. After computing within the grouping symbols, divide or multiply from left to right and then subtract or add from left to right.

Example	
Problem	**Simplify 900 ÷ (6 + 3 • 8) – 10.**
900 ÷ **(6 + 3 • 8)** – 10	Order of operations tells you to perform what is inside the parentheses first.
900 ÷ **(**6 + **3 • 8)** – 10 900 ÷ **(6 + 24)** – 10	Simplify the expression in the parentheses. Multiply first.
900 ÷ 30 – 10	Then add 6 + 24.
900 ÷ 30 – 10 30 – 10 20	Now perform division; then subtract.
Answer	900 ÷ (6 + 3 • 8) – 10 = 20

When there are grouping symbols within grouping symbols, compute from the inside to the outside. That is, begin simplifying the innermost grouping symbols first. Two examples are shown.

Example	
Problem	**Simplify 4 – 3[20 – 3 • 4 – (2 + 4)] ÷ 2.**
4 – 3**[20 – 3 • 4 – (2 + 4)]** ÷ 2	There are brackets and parentheses in this problem. Compute inside the innermost grouping symbols first.
4 – 3[**20 – 3 • 4 – (2 + 4)]** ÷ 2 4 – 3[**20 – 3 • 4 – 6]** ÷ 2	Simplify within parentheses.
4 – 3[**20 – 3 • 4 – 6]** ÷ 2 4 – 3**[20 – 12 – 6]** ÷ 2 4 – 3**[8 – 6]** ÷ 2 4 – 3(2) ÷ 2	Then, simplify within the brackets by multiplying and then subtracting from left to right.
4 – **3(2) ÷ 2** 4 – **6 ÷ 2** 4 **–** 3	Multiply and divide from left to right.
4 **–** 3 1	Subtract.
Answer	4 – 3[20 – 3 • 4 – (2 + 4)] ÷ 2 = 1

Remember that parentheses can also be used to show multiplication. In the example that follows, the parentheses are not a grouping symbol; they are a multiplication symbol. In this case, since the problem only has multiplication and division, we compute from left to right. Be careful to determine what parentheses mean in any given problem. Are they a grouping symbol or a multiplication sign?

Example

Problem **Simplify 6 ÷ (3)(2).**

$6 \div 3 \cdot 2$	This expression has multiplication and division only. The multiplication operation can be shown with a dot.
$6 \div 3 \cdot 2$ $2 \cdot 2$ 4	Since this expression has only division and multiplication, compute from left to right.

Answer $6 \div (3)(2) = 4$

Consider what happens if braces are added to the problem above: $6 \div \{(3)(2)\}$. The parentheses still mean multiplication; the additional braces are a grouping symbol. According to the order of operations, compute what is inside the braces first. This problem is now evaluated as $6 \div 6 = 1$. Notice that the braces caused the answer to change from 1 to 4.

Self Check A
Simplify $40 – (4 + 6) \div 2 + 3$.

The Order of Operations

1) Perform all operations within grouping symbols first. Grouping symbols include parentheses ( ), braces { }, brackets [ ], and fraction bars.
2) Multiply and Divide, from left to right.
3) Add and Subtract, from left to right.

Performing the Order of Operations with Exponents and Square Roots

Objective 2

So far, our rules allow us to simplify expressions that have multiplication, division, addition, subtraction or grouping symbols in them. What happens if a problem has **exponents** or **square roots** in it? We need to expand our order of operation rules to include exponents and square roots.

If the expression has exponents or square roots, they are to be performed *after* parentheses and other grouping symbols have been simplified and *before* any multiplication, division, subtraction and addition that are outside the parentheses or other grouping symbols.

Note that you compute from more complex operations to more basic operations. Addition and subtraction are the most basic of the operations. You probably learned these first. Multiplication and division, often thought of as repeated addition and subtraction, are more complex and come before addition and subtraction in the order of operations. Exponents and square roots are repeated multiplication and division, and because they're even more complex, they are performed before multiplication and division. Some examples that show the order of operations involving exponents and square roots are shown below.

Example		
Problem	**Simplify $14 + 28 \div 2^2$.**	
	$14 + 28 \div 2^2$	This problem has addition, division, and exponents in it. Use the order of operations.
	$14 + 28 \div 4$	Simplify 2^2.
	$14 + 7$	Perform division before addition.
	21	Add.
Answer $14 + 28 \div 2^{2} = 21$		

Example		
Problem	**Simplify $3^2 \cdot 2^3$.**	
	$3^2 \cdot 2^3$	This problem has exponents and multiplication in it.
	$9 \cdot 8$	Simplify 3^2 and 2^3.
	72	Perform multiplication.
Answer $3^2 \cdot 2^3 = 72$		

Example		
Problem	**Simplify $(3 + 4)^2 + (8)(4)$.**	
	$\mathbf{(3 + 4)}^2 + (8)(4)$	This problem has parentheses, exponents, and multiplication in it. The first set of parentheses is a grouping symbol. The second set indicates multiplication. Grouping symbols are handled first.
	$7^2 + (8)(4)$ $49 + (8)(4)$	Add the numbers inside the parentheses that are serving as grouping symbols. Simplify 7^2.
	$49 + 32$	Perform multiplication.
	81	Add.
Answer	$(3 + 4)^2 + (8)(4) = 81$	

Self Check B

Simplify $77 – (1 + 4 – 2)^2$.

The Order of Operations

1) Perform all operations within grouping symbols first. Grouping symbols include parentheses ( ), braces { }, brackets [ ], and fraction bars.
2) Evaluate exponents and roots of numbers, such as square roots.
3) Multiply and Divide, from left to right.
4) Add and Subtract, from left to right.

Some people use a saying to help them remember the order of operations. This saying is called PEMDAS or "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally." The first letter of each word begins with the same letter of an arithmetic operation.

Please $\Rightarrow$ **P**arentheses (and other grouping symbols)

Excuse $\Rightarrow$ **E**xponents

My **D**ear $\Rightarrow$ **M**ultiplication and **D**ivision (from left to right)

Aunt **S**ally $\Rightarrow$ **A**ddition and **S**ubtraction (from left to right)

Note: Even though multiplication comes before division in the saying, division could be performed first. Which is performed first, between multiplication and division, is determined by which comes first when reading from left to right. The same is true of addition and subtraction. Don't let the saying confuse you about this!

Summary

The order of operations gives us a consistent sequence to use in computation. Without the order of operations, you could come up with different answers to the same computation problem. (Some of the early calculators, and some inexpensive ones, do NOT use the order of operations. In order to use these calculators, the user has to input the numbers in the correct order.)

1.5.2 Self Check Solutions

Self Check A

Simplify $40 - (4 + 6) \div 2 + 3$.

38

Compute the addition in parentheses first. $40 - 10 \div 2 + 3$.
Then, perform division. $40 - 5 + 3$. Finally, add and subtract from left to right.

Self Check B

Simplify $77 - (1 + 4 - 2)^2$.

68

Correct. $77 - (1 + 4 - 2)^2 = 77 - (3)^2 = 77 - 9 = 68$