My Steps for Factoring:

1. **ALWAYS check for a GCF first**
   a. this should be done regardless of how you are factoring or what type of polynomial you have (binomial, trinomial …)
   b. keep in mind that the GCF could be a number, a variable, a quantity, or any combination of the three

2. if the polynomial has two terms (binomial), check to see if both terms are perfect squares or perfect cubes
   a. If the two terms are perfect squares, and they are being subtracted, use the difference of squares formula
      i. $x^2 - y^2 = (x + y)(x - y)$

   b. If the two terms are perfect cubes, and they are being added or subtracted, use the sum or difference of cubes formulas
      i. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
      ii. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

   c. If none of the above apply to a binomial, it is not factorable

3. if the polynomial has three terms (trinomial), use the ac-method
   a. the steps for using the ac-method are available on page 3 of the Factor by Grouping and the ac-method notes

4. if the polynomial has four terms, factor by grouping

Regardless of how you factor, ALWAYS check to see if your factors are factorable and ALWAYS factor completely. Example 1 on the next page shows how to factor completely.

Example 1: Factor the following polynomial completely.

$$w^7 + 8w^4 - 16w^3 - 128$$

We start with a polynomial containing four terms; those four terms have no common factors other than one, so I'll simply to factor by grouping.

$$w^4(w^3 + 8) - 16(w^3 + 8)$$

I remove a factor of w^4 from the first group and a factor of -16 from the second group.

$$(w^3 + 8) \quad (w^4 - 16)$$

I now have two factors, $(w^3 + 8)$ and $(w^4 - 16)$. I have already factored by grouping, but I can factor each of these factors further, since $(w^3 + 8)$ is a sum of cubes and since $(w^4 - 16)$ is a difference of squares.

$$(w \cdot w \cdot w + 2 \cdot 2 \cdot 2)(w^2 \cdot w^2 - 4 \cdot 4)$$

$$(w + 2)(w^2 - 2w + 4)(w^2 - 4)(w^2 + 4)$$

I have already factored the original polynomial by grouping, and I've also factored each of the factors using formulas. However I can still factor further, because $(w^2 - 4)$ is a difference of squares:

$$(w + 2)(w^2 - 2w + 4)(w \cdot w - 2 \cdot 2)(w^2 + 4)$$

$$(w + 2)(w^2 - 2w + 4)(w + 2)(w - 2)(w^2 + 4)$$

At this point all my factors have been factored completely. However since I had two factors of $(w + 2)$, I'll expressed them as $(w + 2)^2$.

$$\mathbf{(w^2 + 4)(w^2 - 2w + 4)(w + 2)^2(w - 2)}$$

Again, regardless of how you factor, **ALWAYS** check to see if your factors are factorable and **ALWAYS** factor completely. It is very likely that you will use more than one of the Steps for Factoring when completing the problems in HW7 and when you see similar problems on Exam 2. On Example 1 I used factor by grouping, sum of cubes, difference of squares, and difference of squares again to factor the polynomial $x^7 + 8x^4 - 16x^3 - 128$ completely.

Example 2: Factor each polynomial completely.

a. $-3x^3 + 48x$

$-3x(x^2 - 16)$

$-3x(x \cdot x - 4 \cdot 4)$

$-3x(x + 4)(x - 4)$

b. $x^4 - 10x^2 + 24$

ac	***b***	**Think about the signs of the product and the sum.**

c. $45x^3 + 90x^2 - 5x - 10$

$5(9x^3 + 18x^2 - x - 2)$

$5\left(9x^2(x + 2) - 1(x + 2)\right)$

$5(x + 2)(9x^2 - 1)$

$5(x + 2)(3x \cdot 3x - 1 \cdot 1)$

$\mathbf{5(x + 2)(3x - 1)(3x + 1)}$

d. $2a^4 + 250a$

e. $x^6 - 625x^2$

f. $10w^6 - 13w^3 + 3$

$$10w^6 - 3w^3 - 10w^3 + 3$$

$$w^3(10w^3 - 3) - 1(10w^3 - 3)$$

$$(10w^3 - 3)(w^3 - 1)$$

$$(10w^3 - 3)(w \cdot w \cdot w - 1 \cdot 1 \cdot 1)$$

$$\boldsymbol{\left(10w^3 - 3\right)(w - 1)\left(w^2 + w + 1\right)}$$

g. $4x^2(x-1)^2 - 18x(x-1)(3x+2)$

h. $64a^3 - b^6$

$$(4a)^3 - (b^2)^2$$

$$(4a - b^2)((4a)^2 - (4a)(b^2) + (b^2)^2)$$

$$\boldsymbol{\left(4a - b^2\right)\left(16a^2 - 4ab^2 + b^4\right)}$$

i. $a^{12} + b^{12}$

Since this is a binomial, and since the two terms are being added together, I am looking for terms that are both perfect cubes in order to factor using the Sum of Cubes formula.

Keep in mind that $a^{12} + b^{12}$ could be expressed as a sum of squares as $(a^6)^2 + (b^6)^2$, but since we do not have a sum of squares factoring formula, we would not be able to factor this expression further.

$$(a^4)^3 + (b^4)^3$$

$$(a^4 + b^4)(a^8 - a^4b^4 + b^8)$$

$a^4 + b^4$ is a sum of squares (two perfect squares being added together), but a sum of squares cannot be factored. Therefore, $(a^4 + b^4)(a^8 - a^4b^4 + b^8)$ is the final answer.

$$\boldsymbol{(a^4 + b^4)(a^8 - a^4b^4 + b^8)}$$

j. $2x^8 - 15x^4 - 27$

Since this is a trinomial, I'll use the ac-method to factor.

Since $ac = -54$ I need to list factors of -54. And since $b = -15$, I need to find two factors of -54 (one negative and one positive) which added together result in a sum of -15. So the two factors are 3 and -18.

$$2x^8 + 3x^4 - 18x^4 - 27$$

$$x^4(2x^4 + 3) - 9(2x^4 + 3)$$

$$(2x^4 + 3)(x^4 - 9)$$

I've factored the trinomial into two binomials, but one of those binomials $(x^4 - 9)$ can be factored further using the Difference of Squares formula.

$$(2x^4 + 3)(x^4 - 9)$$

$$\boldsymbol{(2x^4 + 3)(x^2 - 3)(x^2 + 3)}$$

Answers to Examples:

1. $(w-2)(w+2)^2(w^2+4)(w^2-2w+4)$ *; 2a.* $-3x(x+4)(x-4)$ *;*
2b. $(x+2)(x-2)(x^2-6)$ *; 2c.* $5(x+2)(3x-1)(3x+1)$ *;*
2d. $2a(a+5)(a^2-5a+25)$ *; 2e.* $x^2(x-5)(x+5)(x^2+25)$ *;*
2f. $(w-1)(w^2+w+1)(10w^3-3)$ *;*
2g. $2x(x-1)(2x^2-29x-18)$ *;*
2h. $(4a-b^2)(16a^2+4ab^2+b^4)$ *; 2i.* $(a^4+b^4)(a^8+a^4b^4+b^8)$ *;*
2j. $(x^2-3)(x^2+3)(2x^4+3)$ *;*