1 Curved Thin Flange Un-equal I Beams

Draft 2, 3/12/08

1.1 Introduction

Curved beams with the symmetrical shape of an "I" with unequal flange widths are very common in mechanical engineering. In addition to the "I" shape they can include the "T" shape, the inverted "T" shape, the "U" shape, the inverted "U" shape, the rectangle, and the hollow rectangle. Here the general curved beam formulation of Oden [1] is utilized with special simplifications for the restricted symmetric shapes cited above. The loadings are bending moments (M_z and M_y), an axial load N_s, and transverse shear loads (V_y and V_z). The TK Solver implementation reports the circumferential stresses at the inner and outer fibers, and the radial stress and transverse shear stress at the neutral axis. An extension to recover the Von Mises effective stress will be added soon.

In addition, the user can specify a specific (r, z) or (y, z) position for recovering the stress components. By filling a list of r-coordinates plots of the stresses and geometric features can be plotted vs. radial position. Some plots will show the straight beam stresses for comparison purposes. Multiple rules for the same quantity are often used to be consistent with the different definitions used in various references. Many authors like to relate the stresses to the (pure bending) eccentricity, e. However, the eccentricity is often numerically ill-conditioned because it can be the difference in two large numbers. Low accuracy in computing the eccentricity can cause large errors in the stress estimates.

For wide, relatively thin flanges anti-clastic bending of those flanges reduces their effective widths and the member's load bearing capacity. An optional "Bleich" correction is computed for the user's consideration. Its effect will be demonstrated below. For thin webs in "T" or "U" sections you must also check for local buckling, but that relation is not included in the present model.

Usually you want to supply only the minimum amount of data that will uniquely define the curved beam problem. For example, in a rectangular section if you supply the width and the depth (or alternately the inner and outer radii) you do not need to calculate the area. If you did supply the area you may not have given enough digits to satisfy the TK accuracy requirements, so you may want to make the computed area a "Guess" entry. The general instructions for dealing with a curved beam model are given in Figure 1.

INSTRUCTIONS
1. Input geometric data for the section.
2. Set position r (or y) where you want the stresses.
3. If moment is from a simple offset force then:
a. enter offset distance and N_s force value,
b. solve to get MzOffsetN value,
c. copy MzOffsetN output and paste it in M_z input
d. If N_s is parallel to face, cut and paste it into V_y, zero N_s.
4. Enter any non-zero shear forces or line loads.
5. Iff a wide flange correction is desired; solve once, copy and paste
widthin_in and widthin_out to width_in and width_out, respectively.
6. Solve for all variables and stresses at (r, z) or (y, z)
For plots of items throught the depth of the beam:
a. Click on the name 'r', select fill list icon (paint bucket).
b. In fill list window select: Linear, Fill by Section, Last entry = 101,
for First value enter "r_inner", Last value = "r_outer", Fill List.
c. Execute list solve (3 lightbulb icon).
d. Select plots from down arrow by Plots.
Accidental mouse clicks in plot area can cause error
message "No points to plot." To fix this, go to Navigation menu,
scroll down to X-axis, Minimum, Maximum: and clear the two values.
Do the same for the Y-axis, Minimum, Maximum. Exit, replot.
This error also occurs if you forget to fill the r-list for a new problem.

Figure 1 TK User Instructions

This TK Solver model has been validated against several known solutions. A few will be discussed here while others will just be summarized by their Variables Sheet and selected plots in the Appendix. Examples are given in both English and Metric units, depending on the reference cited.

1.2 Seely-Smith Examples

1.2.1 Heavy Clamp (without shear stress)

The first example is a classic thick flanged curve "I" member for use as a heavy clamp. The widths of the inner flange, web, and outer flange are 6, 0.75 and 4 inches, with corresponding radial depths of 2, 3, and 2 inches, respectively. The inner radius is 1.84 inches. The clamp (Figure **2**) is loaded with a force, normal to the cross-section, of 10,000 lb that is offset from the radius center by 3 inches. Those input items are in Figure 3. The goal is to determine the inner fiber normal stress and the shear stress where the reduced web width is encountered.

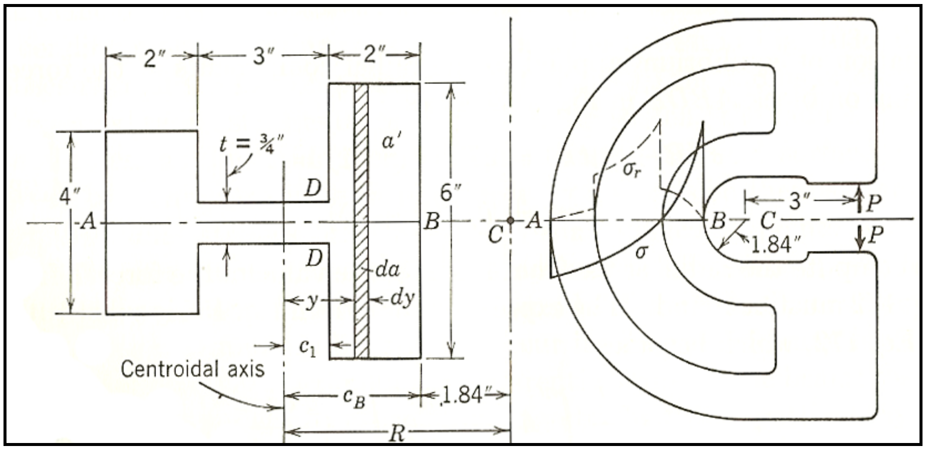

Figure 2 C-clamp

<table>
<tr><th>Input</th><th>Name</th><th>Output</th><th>Unit</th><th>Comment</th></tr>
<tr><td></td><td></td><td></td><td></td><td>Stresses in unsymmetric curved or straigth beams, J.E. Akin 2007</td></tr>
<tr><td></td><td></td><td></td><td></td><td>In Honor of Prof. J.T. Oden's 70-th Year. Ref: Mech. of Elastic Structures,</td></tr>
<tr><td></td><td></td><td></td><td></td><td>cg + ---------------> s-axis</td></tr>
<tr><td></td><td></td><td></td><td></td><td>^ |</td></tr>
<tr><td></td><td></td><td></td><td></td><td>| v y-axis   r = R - y</td></tr>
<tr><td></td><td></td><td></td><td></td><td>| ^ r</td></tr>
<tr><td></td><td></td><td></td><td></td><td>| R | z-axis by R.H.R. from s cross y</td></tr>
<tr><td></td><td></td><td></td><td></td><td>+   +</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[width_out] by   depth_out   ^</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[width_web] by   depth_web | depth</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[width_in]   by   depth_in   v</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>==> Data for: Seely-Smith p. 166 P100 unequal I beam ok</td></tr>
<tr><td></td><td></td><td></td><td></td><td>SECTION GEOMETRY</td></tr>
<tr><td></td><td>A</td><td>22.25</td><td>in^2</td><td>cross-sectional area</td></tr>
<tr><td></td><td>A_m</td><td>5.87322387</td><td>in</td><td>>0, integral over A of 1 / r dA = A / R_na</td></tr>
<tr><td>2</td><td>depth_in</td><td></td><td>in</td><td>unequal I flange depth at r_inner & y_inner, >= 0</td></tr>
<tr><td>3</td><td>depth_web</td><td></td><td>in</td><td>unequal I center web, >= 0</td></tr>
<tr><td>2</td><td>depth_out</td><td></td><td>in</td><td>unequal I flange depth at r_outer & y_outer >= 0</td></tr>
<tr><td></td><td>depth</td><td>7</td><td>in</td><td>total depth, depth_in + depth_web + depth_out</td></tr>
<tr><td></td><td>J_y</td><td>71.5538781</td><td>in^4</td><td>integral over A of z^2 / (1 - ky) dA</td></tr>
<tr><td></td><td>J_z</td><td>154.827304</td><td>in^4</td><td>integral over A of y^2 / (1 - ky) dA</td></tr>
<tr><td>3.85</td><td>r</td><td></td><td>in</td><td>radius to point y (move to input for plot)</td></tr>
<tr><td>1.84</td><td>r_inner</td><td></td><td>in</td><td>inner radius of the section</td></tr>
<tr><td></td><td>r_web</td><td>3.84</td><td>in</td><td>radius at beninning of web</td></tr>
<tr><td></td><td>r_web_top</td><td>6.84</td><td>in</td><td>top radius of the web</td></tr>
<tr><td></td><td>r_outer</td><td>8.84</td><td>in</td><td>outer radius of the section</td></tr>
<tr><td></td><td>R</td><td>4.8905618</td><td>in</td><td>radius to centroid (y = 0 = z)</td></tr>
<tr><td>6</td><td>width_in</td><td></td><td>in</td><td>unequal I width at r_inner & y_inner, can be width_web</td></tr>
<tr><td>.75</td><td>width_web</td><td></td><td>in</td><td>unequal I width at center web section, >0</td></tr>
<tr><td>4</td><td>width_out</td><td></td><td>in</td><td>unequal I width at r_outer & y_outer, can be width_web</td></tr>
<tr><td></td><td>y</td><td>1.0405618</td><td>in</td><td>distance from centroid, in curved plane</td></tr>
<tr><td></td><td>y_inner</td><td>3.0505618</td><td>in</td><td>y at inner radius (beam bottom)</td></tr>
<tr><td></td><td>y_outer</td><td>-3.9494382</td><td>in</td><td>y at outer radius (beam top)</td></tr>
<tr><td>0</td><td>z</td><td></td><td>in</td><td>distance from centroid, normal to curved plane</td></tr>
<tr><td></td><td>z_max</td><td>3</td><td>in</td><td>maximum z coordinate on section</td></tr>
<tr><td></td><td>z_min</td><td>-3</td><td>in</td><td>minmum z coordinate on section</td></tr>
<tr><td></td><td>Z</td><td>.290937722</td><td></td><td>int over A of y^2/(1-ky)dA/AR^2 = int over A of y/(1-ky)dA/A</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 3 C-clamp geometry input and output

Note that other geometric features like the area (A), second moment terms (J_y and J_z), and centroidal radius (R) did not have to be input. Likewise, the optional input non-dimensional measure (Z) of the inertia term J_z was computed. The point of interest (r, z) was set at (3.85, 0) to be just inside the beginning of the thin web. That input was checked against the computed radius to the web (r_web). The loading information on the section is presented in the upper part of Figure 4.

				SECTION LOADING
	MzOffsetN	78905.618	in_lb	M_z if due to offset from center by N_z only
78905.618	M_z		in-lb	moment about z-axis, + for tension on +y face
0	M_y		in-lb	moment about y-axis, + in +y-dir
	M_total	78905.618	in-lb	resultant moment
	M_angle	0	deg	resultant moment plane c.w. w.r.t. y-axis
10000	N_s		lb	axial force normal to section
3	offset		in	optional offset of N_s from curve center
0	p_y		lb/in	load per unit length in + y-direction, along s
0	p_z		lb/in	load per unit length in + z-direction, along s
0	V_y		lb	transverse shear force in the + y-direction
0	V_z		lb	transverse shear force in the + z-direction
				NORMAL STRESS
	r_in_bot	0		1 if r is in bottom (inner) section, else 0
	r_in_web	1		1 if r is in web (middle) section, else 0
	r_in_top	0		1 if r is in top (outer) section, else 0
	σ_s	397.940244	psi	normal stress at r,z and y,z in y-s plane
	σ_s_inner	3856.50078	psi	normal stress at inner radius (bottom)
	σ_s_outer	-1389.22795	psi	normal stress at outer radius (top)
	σ_s_ratio	2.77600286		ratio of extreme stresses
	σ_axial	449.438202	psi	axial s-stress
	σ_sMz	-51.4979584	psi	M_z bending stress at r,z and y,z in y-s plane
	σ_y	5262.5172	psi	radial stress through depth at r,z & y,z
	σ_y_na	668.826115	psi	radial stress at neutral axis
	σ_z	0	psi	estimated normal stress through width
				SHEAR STRESS
	A_q	12.0075	in^2	area subjected to σ_s by b
	b	.75	in	beam shear width at y
	b_na	6	in	beam shear width at neutral axis
	Q_z	46.9012542	in^3	shear first moment. Integral over A_q of y/(1-y/R) dA
	τ_ys	0	psi	shear stress at width b(y)
	τ_na	0	psi	shear stress at width b_na

Figure 4 C-clamp loadings and stresses

To avoid likely math errors, the most likely applied moment, due to the offset force (MzOffsetN) was calculated on the first direct solve and then copied as the input moment (M_z) before really computing the stresses. Here, high radial stresses were expected in the web. Its peak value of about 5,260 psi does exceed the inner radius normal stress of about 3,860 psi. Additional data on the neutral axis and the comparison to a straight beam are given in Figure 5. The curved and straight beam axial stress comparison results are shown in Figure 6. Increased radial stresses in the web are clearly seen in Figure 7.

				NEUTRAL AXIS DATA
	e	1.10218245	in	eccentricity for pure bending
	h_in	1.94837934	in	distance from na to inner fiber
	h_out	5.05162066	in	distance from na to outer fiber
	na_in_top	0		1 if NA is in top (outer) section, else 0
	na_in_web	0		1 if NA is in web (middle) section, else 0
	na_in_bot	1		1 if NA is in bottom (inner) section, else 0
	NA_angle	0	deg	neutral axis angle c.w. w.r.t. z-axis
	NA_b	.35105754	in	neutral axis y intercept
	NA_m	0		neutral axis slope
	R_na	3.78837934	in	radius to neutral axis at z = 0
				STRAIGHT BEAM COMPARISONS
	I_z	128.859785	in^4	integral over A of y^2 dA
	I_y	46.7721354	In^4	integral over A of z^2 dA
	st_beam	1086.61272	psi	normal stress in straight beam at y, or r
	st_in	2317.41016	psi	straight beam normal stress at inner radius (bottom)
	st_out	-1968.94906	psi	straight beam normal stress at outer radius (top)
	st_τ	0	psi	shear stress in straight beam

Figure 5 C-clamp neutral axis location

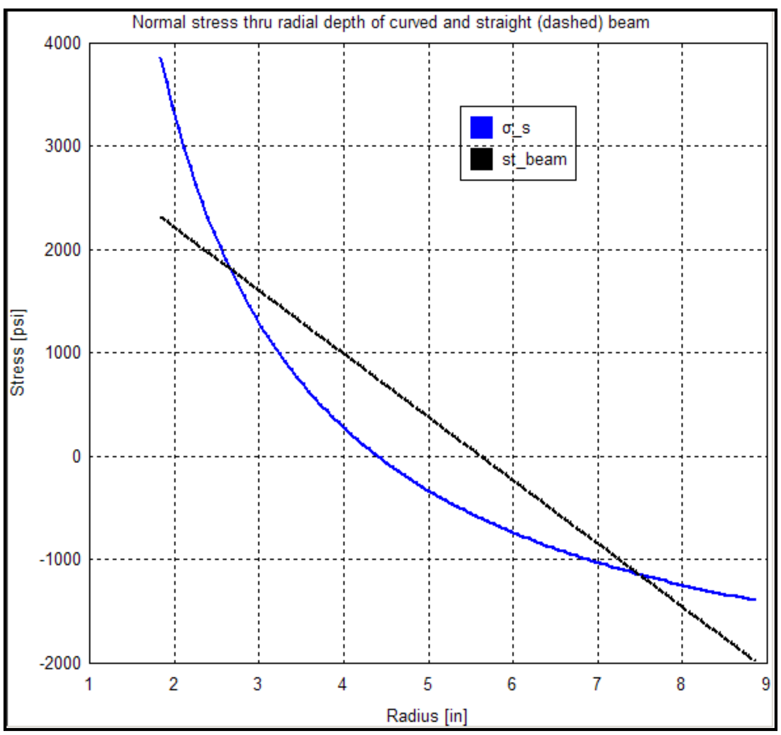

Figure 6 C-clamp axial stress comparisons

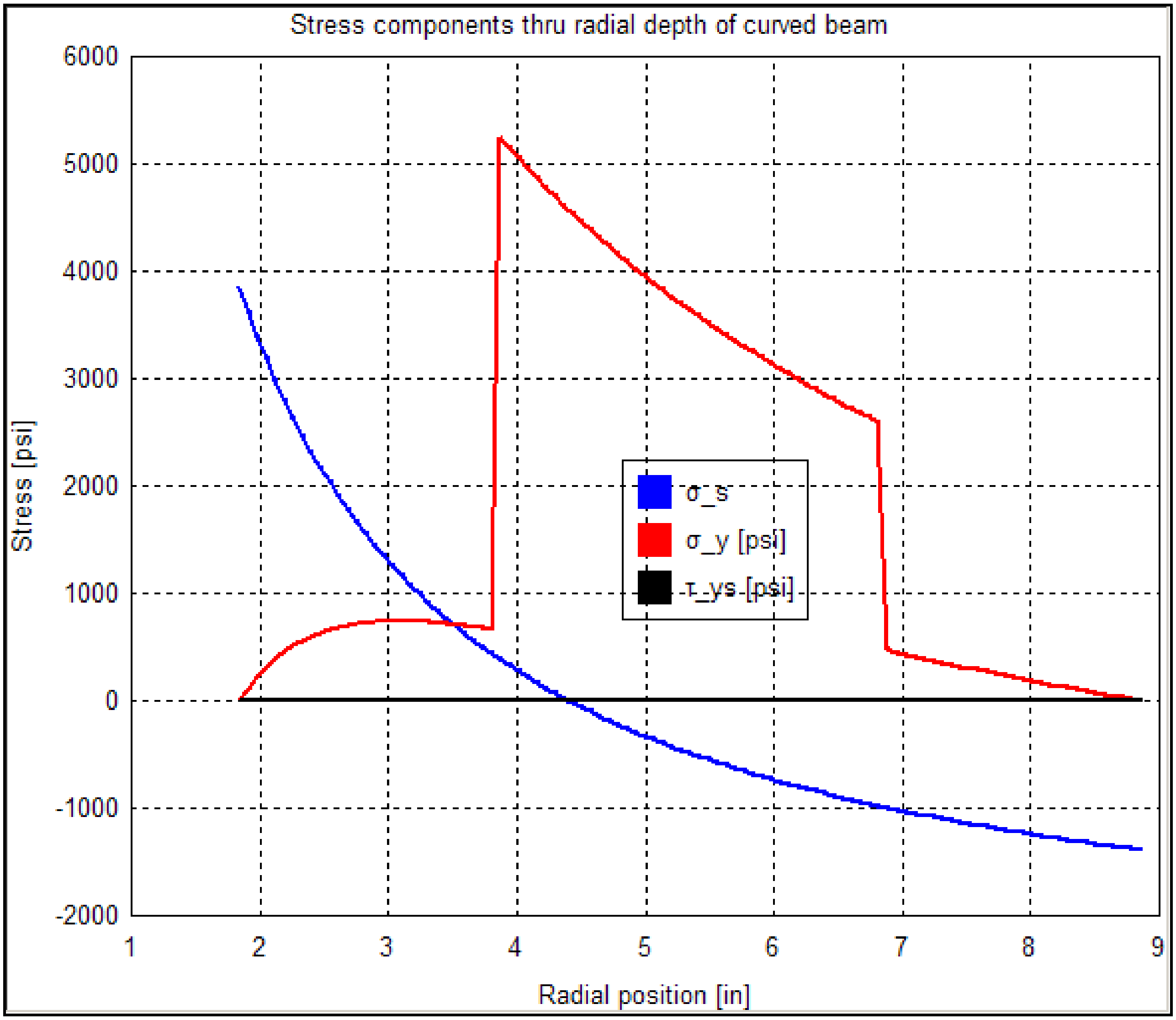

Figure 7 C-clamp radial and normal stress (zero shear stress)

1.2.2 C-clamp with Shear Stresses

The section chosen above was a plane of symmetry normal to the applied load so it did not involve a shear component. Other planes through the axis of rotation of the section would reduce the normal force (N_s) while introducing a transverse shear (V_y) force. To show shear stresses and how they compare to a straight beam consider a small C-clamp with an unequal "I" shape. The part data are shown in Figure 8 while the corresponding inputs are given in Figure 9 for the plane having no transverse shear.

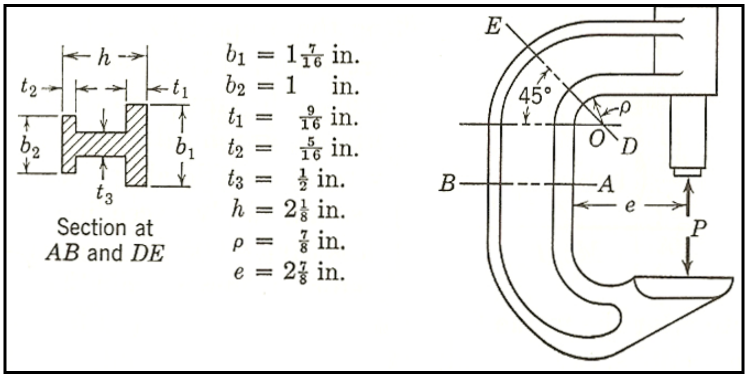

Figure 8 Clamp geometry

Input	Name	Output	Unit	Comment
				==> Data for: Seely-Smith p. 1647 E88a unequal I beam ok
				SECTION GEOMETRY
	A	1.74609375	in^2	cross-sectional area
	A_m	1.13648187	in	>0, integral over A of 1 / r dA = A / R_na
.5625	depth_in		in	unequal I flange depth at r_inner & y_inner, >= 0
	depth_web	1.25	in	unequal I center web, >= 0
.3125	depth_out		in	unequal I flange depth at r_outer & y_outer >= 0
2.125	depth		in	total depth, depth_in + depth_web + depth_out
	J_y	.247024466	in^4	integral over A of z^2 / (1 - ky) dA
	J_z	.889328529	in^4	integral over A of y^2 / (1 - ky) dA
.875	r		in	radius to point y (move to input for plot)
.875	r_inner		in	inner radius of the section
	r_web	1.4375	in	radius at beninning of web
	r_web_top	2.6875	in	top radius of the web
	r_outer	3	in	outer radius of the section
	R	1.78264821	in	radius to centroid (y = 0 = z)
1.4375	width_in		in	unequal I width at r_inner & y_inner, can be width_web
.5	width_web		in	unequal I width at center web section, >0
1	width_out		in	unequal I width at r_outer & y_outer, can be width_web
	y	.90764821	in	distance from centroid, in curved plane
	y_inner	.90764821	in	y at inner radius (beam bottom)
	y_outer	-1.21735179	in	y at outer radius (beam top)
0	z		in	distance from centroid, normal to curved plane
	z_max	.71875	in	maximum z coordinate on section
	z_min	-.71875	in	minmum z coordinate on section
	Z	.160274109		int over A of y^2/(1-ky)dA/AR^2 = int over A of y/(1-ky)dA/A
				SECTION LOADING
	MzOffsetN	13972.9446	in_lb	M_z if due to offset from center by N_z only
13972.945	M_z		in-lb	moment about z-axis, + for tension on +y face
0	M_y		in-lb	moment about y-axis, + in +y-dir
	M_total	13972.9446	in-lb	resultant moment
	M_angle	0	deg	resultant moment plane c.w. w.r.t. y-axis
3000	N_s		lb	axial force normal to section
2.875	offset		in	optional offset of N_s from curve center
0	p_y		lb/in	load per unit length in + y-direction, along s
0	p_z		lb/in	load per unit length in + z-direction, along s
0	V_y		lb	transverse shear force in the + y-direction
0	V_z		lb	transverse shear force in the + z-direction

Figure 9 Clamp data for no shear section

Selected stress outputs are listed in Figure 10 while the combined stresses (with no shear) are seen graphed in Figure 11.

				NORMAL STRESS
	r_in_bot	1		1 if r is in bottom (inner) section, else 0
	r_in_web	0		1 if r is in web (middle) section, else 0
	r_in_top	0		1 if r is in top (outer) section, else 0
	σ_s	26282.7288	psi	normal stress at r,z and y,z in y-s plane
	σ_s_inner	26282.7288	psi	normal stress at inner radius (bottom)
	σ_s_outer	-14136.3709	psi	normal stress at outer radius (top)
	σ_s_ratio	1.85922744		ratio of extreme stresses
	σ_axial	1718.12081	psi	axial s-stress
	σ_sMz	24564.6079	psi	M_z bending stress at r,z and y,z in y-s plane
	σ_y	0	psi	radial stress through depth at r,z & y,z
	σ_y_na	12246.4439	psi	radial stress at neutral axis
	σ_z	0	psi	estimated normal stress through width
				SHEAR STRESS
	A_q	0	in^2	area subjected to σ_s by b
	b	1.4375	in	beam shear width at y
	b_na	.5	in	beam shear width at neutral axis
	Q_z	0	in^3	shear first moment. Integral over A_q of y/(1-y/R) dA
	τ_ys	0	psi	shear stress at width b(y)
	τ_na	0	psi	shear stress at width b_na
				NEUTRAL AXIS DATA
	e	.246245565	in	eccentricity for pure bending
	h_in	.661402645	in	distance from na to inner fiber
	h_out	1.46359735	in	distance from na to outer fiber
	na_in_top	0		1 if NA is in top (outer) section, else 0
	na_in_web	1		1 if NA is in web (middle) section, else 0
	na_in_bot	0		1 if NA is in bottom (inner) section, else 0
	NA_angle	0	deg	neutral axis angle c.w. w.r.t. z-axis
	NA_b	.14992012	in	neutral axis y intercept
	NA_m	0		neutral axis slope
	R_na	1.53640265	in	radius to neutral axis at z = 0
				STRAIGHT BEAM COMPARISONS
	I_z	.823318883	in^4	integral over A of y^2 dA
	I_y	.178302765	In^4	integral over A of z^2 dA
	st_beam	17122.2594	psi	normal stress in straight beam at y, or r
	st_in	17122.2594	psi	straight beam normal stress at inner radius (bottom)
	st_out	-18942.1477	psi	straight beam normal stress at outer radius (top)
	st_τ	0	psi	shear stress in straight beam

Figure 10 Clamp stresses in no shear section

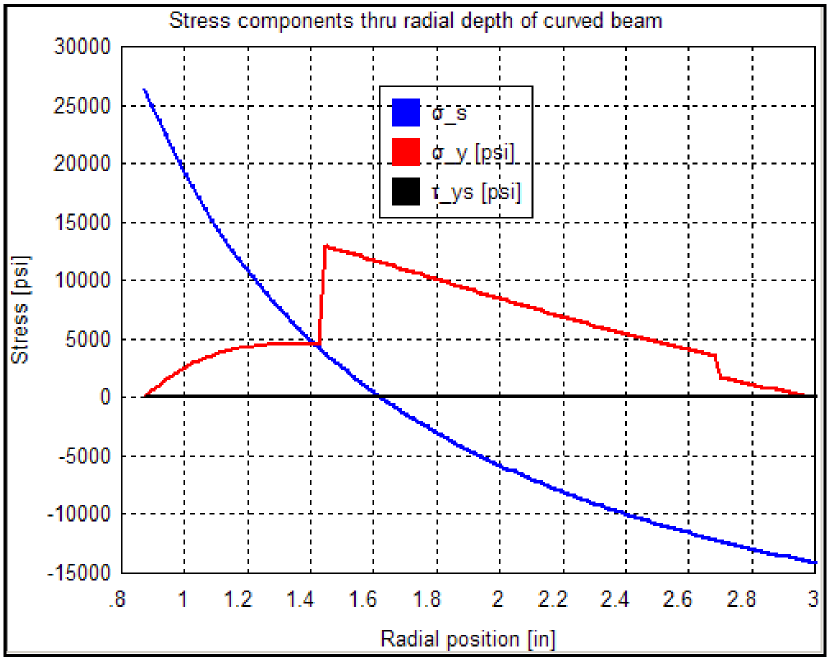

Figure 11 Stress graphs in no shear region

1.2.3 Shear plane results

On the plane at 45 degrees from the horizontal the load reduces to equal normal and shear components. The lever arm for the bending moment is also slightly reduced. Changing those loads gives shear and normal stress results that are compared to a straight beam in Figure 12, and shown combined in Figure 13. The shear stress is much lower than the radial stress in this example.

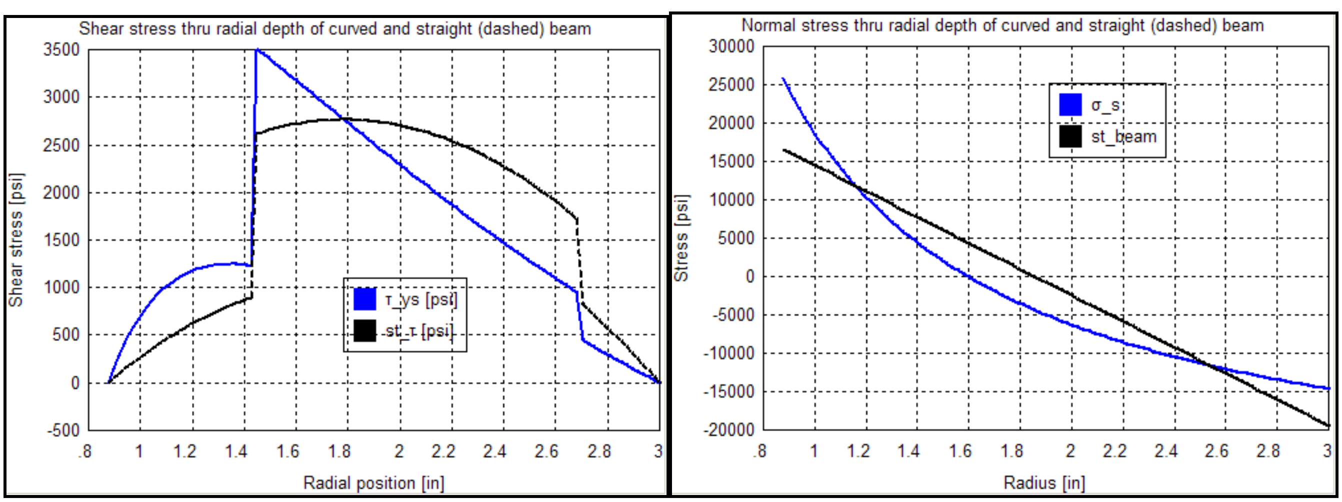

Figure 12 Clamp section with transverse shear stress

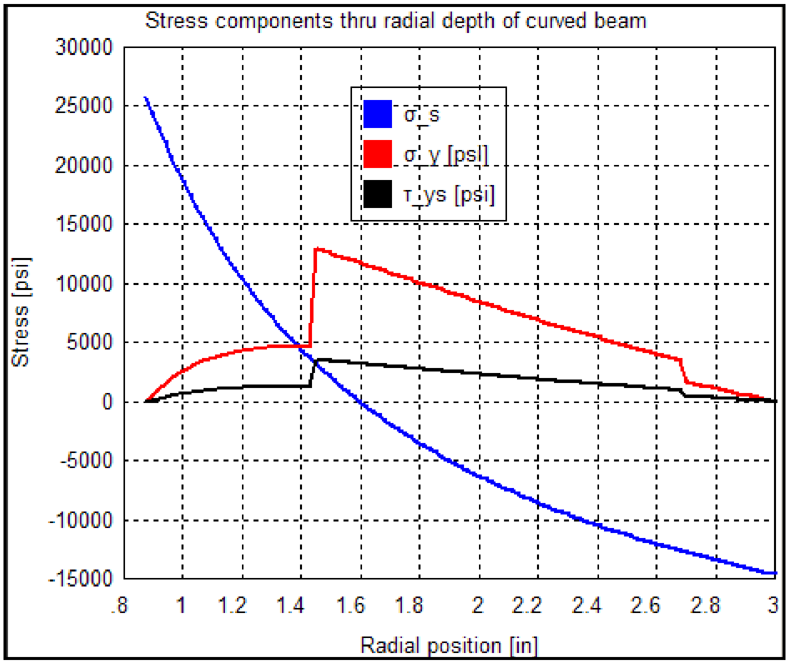

Figure 13 Clamp stresses in a plane with transverse shear force

1.2.4 Equal Flange I-Beam

The final discussion from Seely and Smith involves a doubly symmetric curved I-beam (Figure 14). The widths of the inner flange, web, and outer flange are 3.5, 0.5 and 3.5 inches, with corresponding radial depths of 0.5, 3, and 0.5 inches, respectively. The inner radius is 1.75 inches.

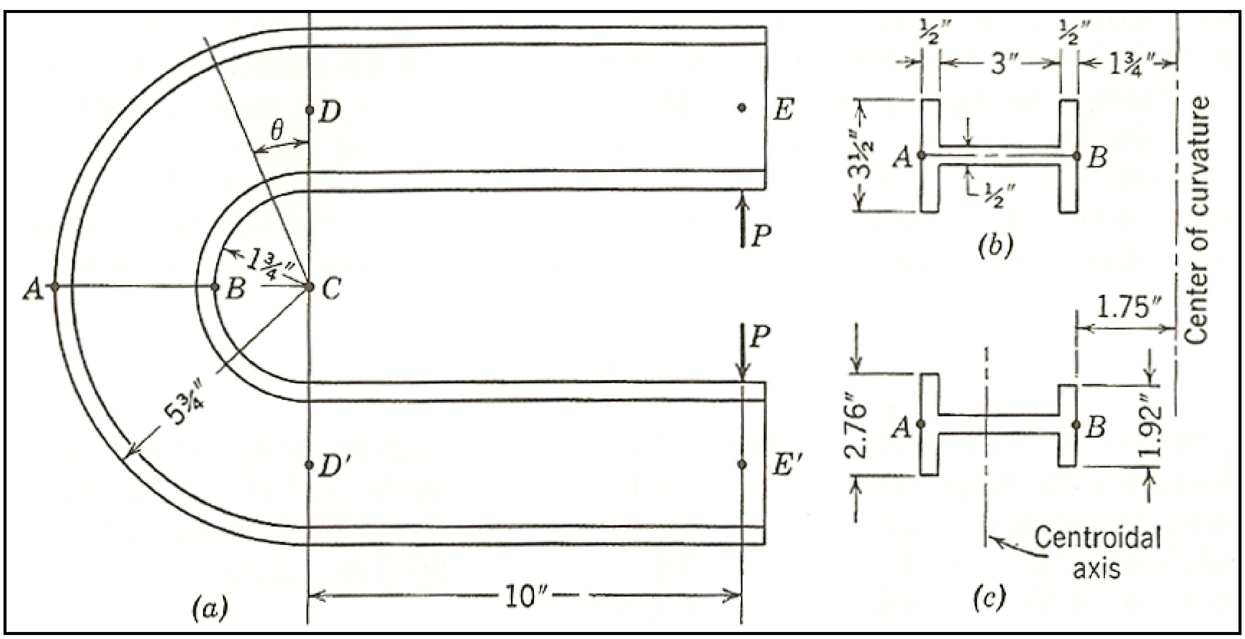

Figure 14 Relatively thin flanged curved beam

The flange width-to-thickness ratio suggests that they should be checked for reduced effectiveness. The first solve gives the usual geometric data along with suggested effective flange widths, as seen in Figure 15. The first direct solve was used to calculate the total bending moment, MzOffsetN, which was copied into the actual moment field, M_z (as seen in Figure 16), and then the initial stress states (Figure 17) were obtained from the next direct solve. The initial neutral axis data and the initial Bleich flange correction factors are in Figure 18.

Input	Name	Output	Unit	Comment
				==> Data for: Seely-Smith p162 P97 thin unequal I beam ok
				SECTION GEOMETRY
	A	5	in^2	cross-sectional area
	A_m	1.62165065	in	>0, integral over A of 1 / r dA = A / R_na
.5	depth_in		in	unequal I flange depth at r_inner & y_inner
3	depth_web		in	unequal I center web, >= 0
.5	depth_out		in	unequal I flange depth at r_outer & y_outer
	depth	4	in	total depth, depth_in + depth_web + depth_out
	J_y	4.61919792	in^4	integral over A of z^2 / (1 - ky) dA
	J_z	15.2042336	in^4	integral over A of y^2 / (1 - ky) dA
1.75	r		in	radius to point y (fill the r list and move to input for plot)
1.75	r_inner		in	inner radius of the section
	r_web	2.25	in	radius at beninning of web
	r_web_top	5.25	in	top radius of the web
	r_outer	5.75	in	outer radius of the section
	R	3.75	in	radius to centroid (y = 0 = z)
3.5	width_in		in	unequal I width at r_inner & y_inner, can be width_web
	widthin_in	1.93587285	in	copy to width_in for thin flange correction
.5	width_web		in	unequal I width at center web section, >0
3.5	width_out		in	unequal I width at r_outer & y_outer, can be width_web
	widthin_out	2.80774381	in	copy to width_out for thin flange correction
	y	2	in	distance from centroid, in curved plane
	y_inner	2	in	y at inner radius (beam bottom)
	y_outer	-2	in	y at outer radius (beam top)
0	z		in	distance from centroid, normal to curved plane
	z_max	1.75	in	maximum z coordinate on section
	z_min	-1.75	in	minmum z coordinate on section
	Z	.21623799		int over A of y^2/(1-ky)dA/AR^2 = int over A of y/(1-ky)dA/A

Figure 15 Actual geometry of a thin flange section

Input	Name	Output	Unit	Comment
				SECTION LOADING
	MzOffsetN	41250	in-lb	M_z if due to offset from center by N_z only
41250	M_z		in-lb	moment about z-axis, + for tension on +y face
0	M_y		in-lb	moment about y-axis, + in +y-dir
	M_total	41250	in-lb	resultant moment
	M_angle	0	deg	resultant moment plane c.w. w.r.t. y-axis
3000	N_s		lb	axial force normal to section
10	offset		in	optional offset of N_s from curve center
0	p_y		lb/in	load per unit length in + y-direction, along s
0	p_z		lb/in	load per unit length in + z-direction, along s
0	V_y		lb	transverse shear force in the + y-direction
0	V_z		lb	transverse shear force in the + z-direction

Figure 16 Loading of the thin flange symmetric section

			NORMAL STRESS
σ_s	10027.4005	psi	normal stress at r,z and y,z in y-s plane
σ_s_inner	10027.4005	psi	normal stress at inner radius (bottom)
σ_s_outer	-5138.7741	psi	normal stress at outer radius (top)
σ_s_ratio	1.95132154		ratio of extreme stresses
σs_B_in	17026.6358	psi	final stress in inner flange for thin flange CORRECTION
σs_B_out	-8035.1518	psi	final stress in outer flange for thin flange CORRECTION
σ_axial	600	psi	axial s-stress
σ_sMz	9427.40052	psi	M_z bending stress at r,z and y,z in y-s plane
σ_y	0	psi	radial stress through depth at r,z & y,z
σ_y_na	8276.35179	psi	radial stress at neutral axis
σ_z	0	psi	estimated normal stress through width
			SHEAR STRESS
A_q	0	in^2	area subjected to σ_s by b
b	3.5	in	beam shear width at y
b_na	.5	in	beam shear width at neutral axis
Q_z	0	in^3	shear first moment. Integral over A_q of y/(1-y/R) dA
τ_ys	0	psi	shear stress at width b(y)
τ_na	0	psi	shear stress at width b_na

Figure 17 First stress estimate on the actual section

			NEUTRAL AXIS DATA
e	.666721865	in	eccentricity for pure bending
h_in	1.33327813	in	distance from na to inner fiber
h_out	2.66672187	in	distance from na to outer fiber
na_in_top	0		1 if NA is in top (outer) section, else 0
na_in_web	1		1 if NA is in web (middle) section, else 0
na_in_bot	0		1 if NA is in bottom (inner) section, else 0
NA_angle	0	deg	neutral axis angle c.w. w.r.t. z-axis
NA_b	.37096631	in	neutral axis y intercept
NA_m	0		neutral axis slope
R_na	3.08327813	in	radius to neutral axis at z = 0
			THIN FLANGES CORRECTION via Bleich
α_in	.478624282		inner flange length reduction factor, <1
α_out	.769247935		outer flange length reduction factor, <1
β_in	1.69801094		inner flange stress increase factor, > 1, < 1.732
β_out	1.56363204		outer flange stress increase factor, > 1, < 1.732
ν	.3		Poisson's ratio, default = 0.3

Figure 18 Neutral axis results and suggested thin flange corrections

The Bleich correction factors are based on a thin shell analogy and agree well with experimental results showing increased stresses (around the web) resulting from less effective flanges. In the past,the correction factors were applied only once. That is, the reduced section geometry was used to find all new section properties and then a single corrected set of stress results were obtained. Since the complicated correction factors are automated here, you can continue the corrections until the flange size change is less that some reasonable value, such as 10% of the flange width.

In the first correction estimate at the inner fiber (Figure 18) the suggested flange length was reduced by more than half (0.48 of the original) and the stress increase factor of about 1.7 was near the theoretical maximum change of √3 = 1.732. Thus, it was decided to utilize a small series of Bleich corrections and see how the effective flange widths and stresses converged. The first cross-sectional area produced circumferential and radial stresses near 10,000 psi. Since the plane of interest was normal to the load there were no shear stresses due to flexure.

Figure 19 shows the initial cross-section (top) and its corresponding stress distributions, along with the final shape computed (bottom) and its stresses. The radial stresses did not change much, but the circumferential stresses more than doubled in values, reaching about 22,000 psi at the inner fibers. Such drastic change in the stresses clearly suggests that the wide-flange corrections should be checked as a standard procedure for curved beams with flanges.

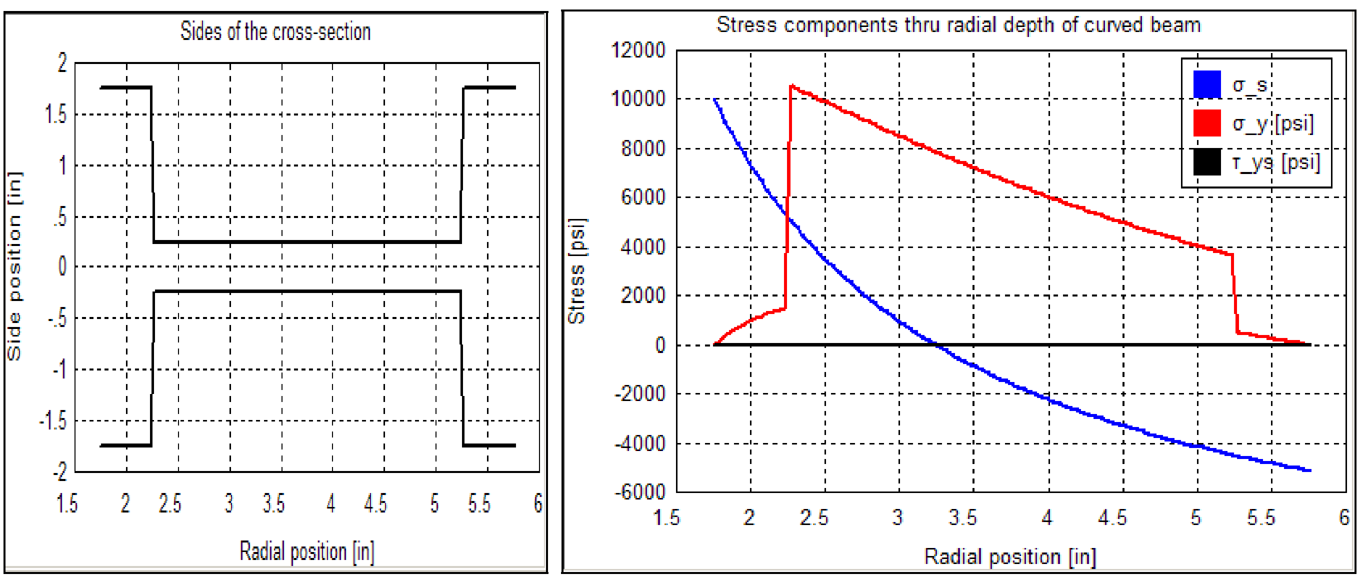

Figure 19 Actual cross-section and predicted stresses

The final predicted final shape, after several sequential corrections, and the predicted corrected stress distribution are graphed in Figure 19.The iterative changes in the flange sizes are listed in Figure 20.

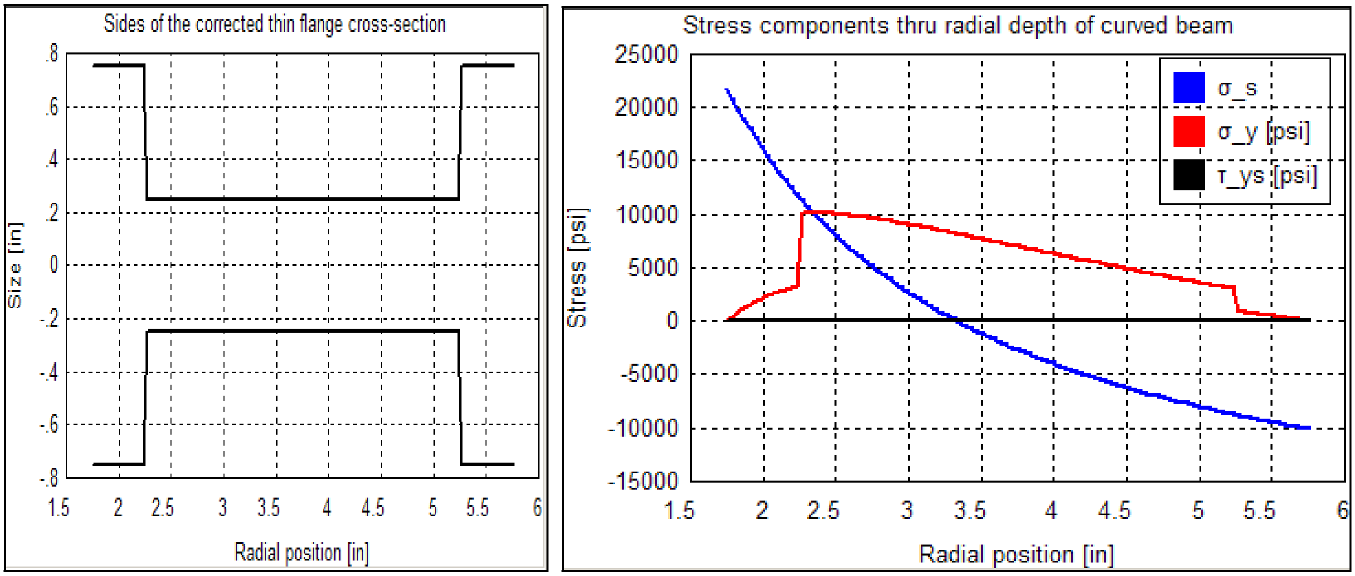

Figure 19 Final estimated effective shape and stresses

1.9358728	width_in		in
	widthin_in	1.76686533	in
.5	width_web		in
2.8077438	width_out		in
	widthin_out	2.56265155	in

1.7668653	width_in		in
	widthin_in	1.66926093	in
.5	width_web		in
2.5626515	width_out		in
	widthin_out	2.41368426	in

1.6692609	width_in		in
	widthin_in	1.60160057	in
.5	width_web		in
2.4136843	width_out		in
	widthin_out	2.30811674	in

1.5	width_in		in
	widthin_in	1.5	in
.5	width_web		in
1.5	width_out		in
	widthin_out	1.5	in

Figure 20 A series of flange width reduction corrections

1.3 Cook-Young Thin Flange Pipe Lift Example

Cook and Young (our text) describes an approximation representing the lifting of a large diameter pipe by reinforcing it with a curved beam around the diameter at the lift section. This example will show that even for large radius curved beams, which almost match the straight beam results, the correction for very wide flanges is still important.

A curved “T” beam (with a top width of 200 mm) was welded to the pipe, at the lifting section, to form a unequal “I” beam where the bottom flange (the pipe) was assumed to be effective over a length of 1501 mm. The initial analysis was carried out for those dimensions, as seen in Figure 21and Figure 22. The effective bottom flange first correction is estimated to be only about 120 mm instead of the assumed 1500 mm.

Input	Name	Output	Unit	Comment
				==> Data for: Cook-Young p177 Ex thin unequal I beam ok
				SECTION GEOMETRY
	A	6790.52237	mm^2	cross-sectional area
	A_m	4.47026262	mm	>0, integral over A of 1 / r dA = A / R_na
3.12	depth_in		mm	unequal I flange depth at r_inner & y_inner
	depth_web	68.75	mm	unequal I center web, >= 0
6.24	depth_out		mm	unequal I flange depth at r_outer & y_outer
	depth	78.11	mm	total depth, depth_in + depth_web + depth_out
	J_y	893846675	mm^4	integral over A of z^2 / (1 - ky) dA
	J_z	5843177.82	mm^4	integral over A of y^2 / (1 - ky) dA
22	r		mm	radius to point y (fill the r list and move to input for plot)
1500	r_inner		mm	inner radius of the section
	r_web	1503.12	mm	radius at beninning of web
	r_web_top	1571.87	mm	top radius of the web
1578.11	r_outer		mm	outer radius of the section
	R	1519.6032	mm	radius to centroid (y = 0 = z)
1501	width_in		mm	unequal I width at r_inner & y_inner, can be width_web
	widthin_in	118.997148	mm	copy to width_in for thin flange correction
12.5	width_web		mm	unequal I width at center web section, >0
200	width_out		mm	unequal I width at r_outer & y_outer, can be width_web
	widthin_out	151.754752	mm	copy to width_out for thin flange correction
	y	1497.6032	mm	distance from centroid, in curved plane
	y_inner	19.6032031	mm	y at inner radius (beam bottom)
	y_outer	-58.506797	mm	y at outer radius (beam top)
0	z		mm	distance from centroid, normal to curved plane
	z_max	750.5	mm	maximum z coordinate on section
	z_min	-750.5	mm	minmum z coordinate on section
	Z	.000372638		int over A of y^2/(1-ky)dA/AR^2 = int over A of y/(1-ky)dA/A

Figure 21 Initial pipe lifting section geometry

Input	Name	Output	Unit	Comment
				SECTION LOADING
	MzOffsetN	6547158.03	N-mm	M_z if due to offset from center by N_z only
-6700000	M_z		N-mm	moment about z-axis, + for tension on +y face
0	M_y		N-mm	moment about y-axis, + in +y-dir
	M_total	6700000	N-mm	resultant moment
	M_angle	180	deg	resultant moment plane c.w. w.r.t. y-axis
4.2	N_s		kN	axial force normal to section
0	offset		mm	optional offset of N_s from curve center
0	p_y		N/mm	load per unit length in + y-direction, along s
0	p_z		N/mm	load per unit length in + z-direction, along s
0	V_y		kN	transverse shear force in the + y-direction
0	V_z		kN	transverse shear force in the + z-direction

Figure 22 Pipe lifting specified loadings

The original assumed web shape and the corresponding stresses are shown at the top of Figure 23, while the final shape and stresses appear at the bottom of the figure. Note that the normal stress has approximately doubled from the original assumptions. A series of the correction steps used along the way to a final geometry are listed in Figures 24-26, while the specific flange widths are shown in Figure 27.

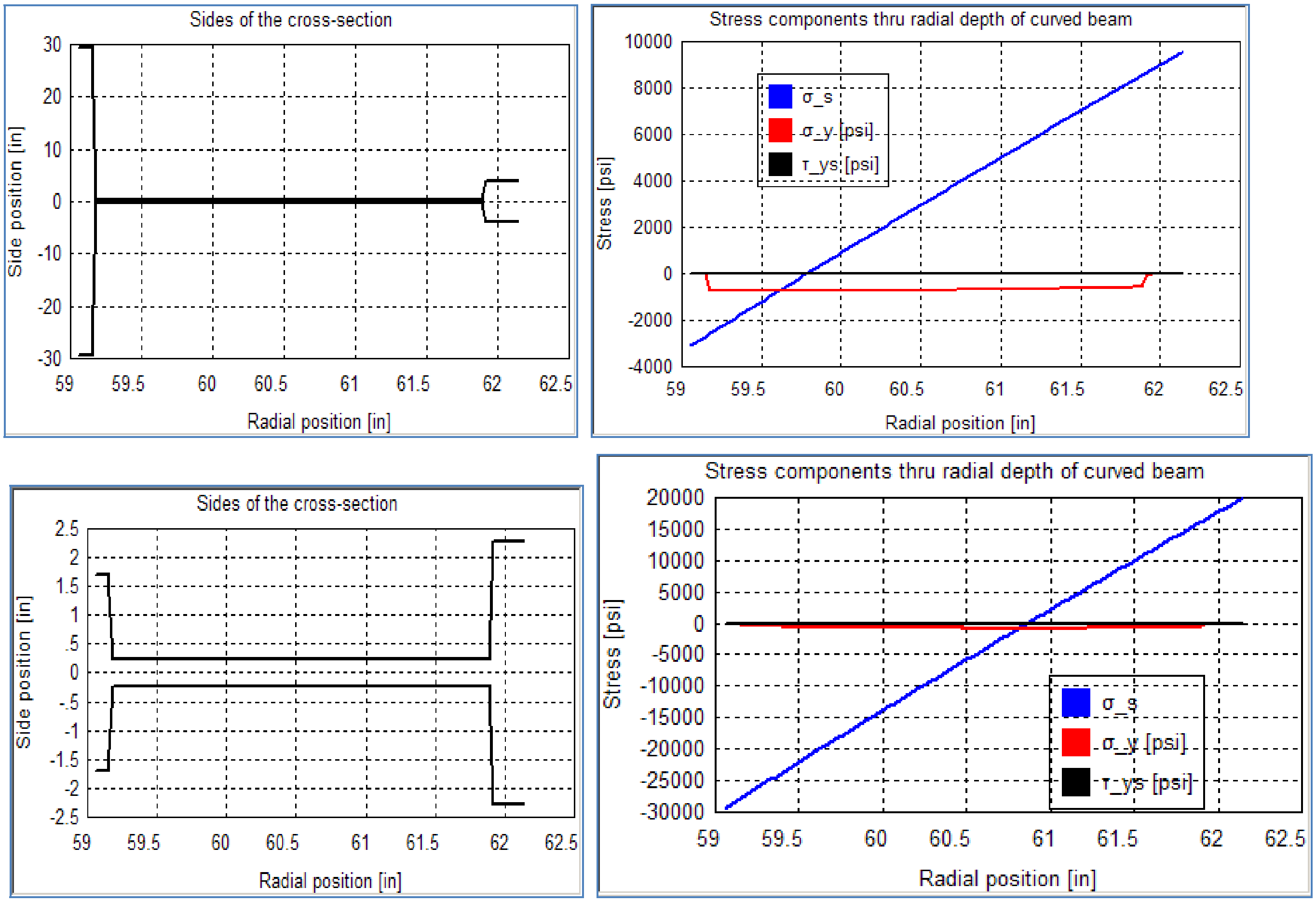

Figure 23 Initial and final flange sizes and beam stresses

Input	Name	Output	Unit	Comment
				==> Data for: Cook-Young p177 Ex thin unequal I beam ok
				SECTION GEOMETRY
	A	2177.60453	mm^2	cross-sectional area
	A_m	1.40753755	mm	>0, integral over A of 1 / r dA = A / R_na
3.12	depth_in		mm	unequal I flange depth at r_inner & y_inner
	depth_web	68.75	mm	unequal I center web, >= 0
6.24	depth_out		mm	unequal I flange depth at r_outer & y_outer
	depth	78.11	mm	total depth, depth_in + depth_web + depth_out
	J_y	2248628.06	mm^4	integral over A of z^2 / (1 - ky) dA
	J_z	1946315.49	mm^4	integral over A of y^2 / (1 - ky) dA
1500	r		mm	radius to point y (fill the r list and move to input for plot)
1500	r_inner		mm	inner radius of the section
	r_web	1503.12	mm	radius at beninning of web
	r_web_top	1571.87	mm	top radius of the web
1578.11	r_outer		mm	outer radius of the section
	R	1547.67331	mm	radius to centroid (y = 0 = z)
118.99715	width_in		mm	unequal I width at r_inner & y_inner, can be width_web
	widthin_in	102.863535	mm	copy to width_in for thin flange correction
12.5	width_web		mm	unequal I width at center web section, >0
151.75475	width_out		mm	unequal I width at r_outer & y_outer, can be width_web
	widthin_out	136.500823	mm	copy to width_out for thin flange correction
	y	47.6733125	mm	distance from centroid, in curved plane
	y_inner	47.6733125	mm	y at inner radius (beam bottom)
	y_outer	-30.436687	mm	y at outer radius (beam top)
0	z		mm	distance from centroid, normal to curved plane
	z_max	75.8773758	mm	maximum z coordinate on section
	z_min	-75.877376	mm	minmum z coordinate on section
	Z	.000373145		int over A of y^2/(1-ky)dA/AR^2 = int over A of y/(1-ky)dA/A

Figure 24 Second flange correction estimate

			NEUTRAL AXIS DATA
e	.577290833	mm	eccentricity for pure bending
h_in	47.0960217	mm	distance from na to inner fiber
h_out	31.0139783	mm	distance from na to outer fiber
na_in_top	0		1 if NA is in top (outer) section, else 0
na_in_web	1		1 if NA is in web (middle) section, else 0
na_in_bot	0		1 if NA is in bottom (inner) section, else 0
NA_angle	-180	deg	neutral axis angle c.w. w.r.t. z-axis
NA_b	1.09271604	mm	neutral axis y intercept
NA_m	0		neutral axis slope
R_na	1547.09602	mm	radius to neutral axis at z = 0
			THIN FLANGES CORRECTION via Bleich
α_in	.848506618		inner flange length reduction factor, <1
α_out	.890460264		outer flange length reduction factor, <1
β_in	1.35372903		inner flange stress increase factor, > 1, < 1.732
β_out	1.18859932		outer flange stress increase factor, > 1, < 1.732
v	.3		Poisson's ratio, default = 0.3

Figure 25 Second Bleich correction coefficients

Input	Name	Output	Unit	Comment
118.99715	width_in		mm	unequal I width at r_inner & y_inner, can be width_web
	widthin_in	102.863535	mm	copy to width_in for thin flange correction
12.5	width_web		mm	unequal I width at center web section, >0
151.75475	width_out		mm	unequal I width at r_outer & y_outer, can be width_web
	widthin_out	136.500823	mm	copy to width_out for thin flange correction

Input	Name	Output	Unit
102.86353	width_in		mm
	widthin_in	94.830435	mm
12.5	width_web		mm
136.50082	width_out		mm
	widthin_out	127.366262	mm

Input	Name	Output	Unit
94.830435	width_in		mm
	widthin_in	89.5535073	mm
12.5	width_web		mm
127.36626	width_out		mm
	widthin_out	120.935596	mm

Input	Name	Output	Unit
89.553507	width_in		mm
	widthin_in	85.67156	mm
12.5	width_web		mm
120.9356	width_out		mm
	widthin_out	116.02371	mm

Input	Name	Output	Unit
85.67156	width_in		mm
	widthin_in	82.6278704	mm
12.5	width_web		mm
116.02371	width_out		mm
	widthin_out	112.07951	mm

Figure 26 Sequences of Bleich correction flange widths

1.4 References

1. Cook, R.D. and W.C. Young, Advanced Mechanics of Materials. 1999: Prentice Hall.
2. Oden, J.T. and E.A. Ripperger, Mechanics of Elastic Structures. 2-nd ed. 1981: McGraw-Hill.
3. Seely, F.B. and J.O. Smith, Advanced Mechanics of Materials. 2-nd ed. 1952: John Wiley.
4. Timoshenko, S., Strength of Materials, Part 2. 1954: Van Nostrand.
5. UTS, TK Solver User Guide. 2006, Universal Technical Systems Inc.

Appendix: The Rules Sheet and Summary Results

Rule
; Normal stress in unsymmetric curved or straight beams, J.E. Akin 2007
; In Honor of Prof. J.T. Oden's 70-th Year. Ref: Mech. of Elastic Structures, 1981
; GENERAL SECTION GEOMETRY
depth = r_outer - r_inner ; radial depth
depth_web = depth - depth_in - depth_out
A = width_in*depth_in + width_web*depth_web + width_out* depth_out
2 * z_max = max (width_in, width_web,width_out)
z_min = - z_max
R1 = (width_in - width_web) * depth_in
R2 = (width_out - width_web) * depth_out
R3 = width_web * depth
R_top = depth * R3 / 2 + R1 * depth_in / 2 + R2 * (depth - depth_out / 2)
R_bot = R1 + R2 + R3
R = r_inner + R_top / R_bot ;Hall
Rn1 = width_in * ln ((r_inner + depth_in) / r_inner)
Rn2 = width_web * ln((r_outer - depth_out) / (r_inner + depth_in))
Rn3 = width_out * ln(r_outer / (r_outer - depth_out))
R_na = (R1 + R2 + R3) / (Rn1 + Rn2 + Rn3) ; Hall
R_na = A / A_m
r_inner = R - y_inner
r_outer = R - y_outer
r_web = r_inner + depth_in
r_web_top = r_outer - depth_out
; set logic flags
if r <= r_web then r_in_bot = 1 else r_in_bot = 0
if and(r > r_web, r <= r_web_top) then r_in_web = 1 else r_in_web = 0
if r > r_web_top then r_in_top = 1 else r_in_top = 0
if r <= R then r_le_R = 1 else r_le_R = 0
if r > R then r_gt_R = 1 else r_gt_R = 0
if R_na <= r_web then na_in_bot = 1 else na_in_bot = 0
if and(R_na <= r_web_top, R_na > r_web) then na_in_web = 1 else na_in_web = 0
if R_na > r_web_top then na_in_top = 1 else na_in_top= 0

Figure 27 Rule set 1

Rule
if known('R) then k = 1 / R ; for curved beam, give k=0 for straight beam
y = R - r
denom = 1 - y * k ; work term
R - R_na = e
R_s = R* ln (r_outer / r_inner) ; Akin
Ae = A * (R - R_na) ; debug aid
A_m = First_s * k ; integral over A of 1 / r dA
First_s = A + J_z * k^2 ; integral over A of 1 / (1-y / R) dA
First_y = J_z * k ; integral over A of y / (1-y / R) dA
R_s1 = R* ln (r_web / r_inner) ; Akin
R_s2 = R* ln (r_web_top / r_web) ; Akin
R_s3 = R* ln (r_outer / r_web_top) ; Akin
J_y = (width_in^3 * R_s1 + width_web^3 * R_s2 + width_out^3 * R_s3) / 12
J_z * k^3 = A_m - k * A ; Brickford
J_z * k^2 = Z * A ; ; Seely-Smith
Z = First_s / A - 1 ; Seely-Smith
Z1 = width_out * ln(r_outer) ; work term
Z2 = (width_web - width_out) * ln (r_web_top) ; work term
Z3 = (width_in - width_web) * ln (r_web) ; work term
Z4 = width_in * ln(r_inner) ; work term
Z = R * (Z1 + Z2 + Z3 - Z4) / A - 1 ; Seely-Smith
; NORMAL STRESS
MzOffsetN = (offset + R) * N_s
s_term = (N_s - M_z * k) / A ; work term
y_term = M_z / J_z ; work term
z_term = M_y / J_y ; work term
σ_axial = N_s/A ; σ_s due to axial load
σ_sMz = M_z * (A - r * A_m) / (A * r * (R*A_m -A)) ; σ_s due to M_z only; Pilkey
σ_s = s_term + (y_term * y + z_term *z) / denom ; stress at y,z
σ_s_inner = s_term + (y_term * y_inner + z_term *z) / (1 - y_inner*k) ; at y_inner,z
σ_s_outer = s_term + (y_term * y_outer + z_term *z) / (1 - y_outer*k) ; at y_outer,z
σ_s_ratio = abs (σ_s_inner / σ_s_outer) ; ratio of inner and outer stress
σ_z = 0 ; place holder

Figure 28 Rule set 2

Rule
; RADIAL STRESS
; Geometric options
Aq_bot = width_in * depth_in
Aq_web = Aq_bot + width_web * depth_web
Aq_bot_na = width_in * (R_na - r_inner)
Aq_web_na = Aq_bot + width_web * (R_na - r_web)
Aq_top_na = Aq_web + width_out*(R_na - r_web_top)
Qz_bot = -R * width_in * (r_web - r_inner - R * ln (r_web / r_inner)) ; Oden
Qz_web = Qz_bot - R*width_web*(r_web_top - r_web - R * ln (r_web_top / r_web))
Qz_bot_na = -R * width_in * (R_na - r_inner - R * ln (R_na / r_inner))
Qz_web_na = Qz_bot - R*width_web*(R_na - r_web - R * ln (R_na / r_web))
Qz_top_na=Qz_web-R*width_out*(R_na - r_web_top - R*ln (R_na/r_web_top)) ; Oden
; neutral axis logic
if na_in_bot then Aq_na = Aq_bot_na
if na_in_web then Aq_na = Aq_web_na
if na_in_top then Aq_na = Aq_top_na
if na_in_bot then b_na = width_in
if na_in_web then b_na = width_web
if na_in_top then b_na = width_out
if na_in_bot then Qz_na = Qz_bot_na
if na_in_web then Qz_na = Qz_web_na
if na_in_top then Qz_na = Qz_top_na
; position logic
if r_in_bot then A_q = width_in * (r - r_inner)
if r_in_web then A_q = Aq_bot + width_web * (r - r_web)
if r_in_top then A_q = Aq_web + width_out * (r - r_web_top)
if r_in_bot then b = width_in
if r_in_web then b = width_web
if r_in_top then b = width_out
if r_in_bot then Q_z = - R*width_in*(r - r_inner - R*ln (r / r_inner))
if r_in_web then Q_z = Qz_bot - R*width_web*(r - r_web - R*ln (r / r_web))
if r_in_top then Q_z = Qz_web - R*width_out*(r - r_web_top - R*ln (r / r_web_top))
F = s_term * A_q + y_term*Q_z
dVy_ds = -(p_y + k * N_s) * A_q / A
σ_y = (k*F + dVy_ds) / (b * (1 - k * y))
F_na = s_term * Aq_na + y_term*Qz_na
dVds_na = -(p_y + k * N_s) * Aq_na / A
σ_y_na = (k*F_na + dVds_na) / (b_na * (1 - k * e))

Figure 29 Rule set 3

Rule
; SHEAR STRESS
Qy_term = Q_z / J_z
τ_ys = V_y * (Qy_term - k * A_q / A) / (b * (1 - y*k))
τ_na = V_y * (Qy_term - k * Aq_na / A) / (b_na * (1 - e*k))
;τ_ys = (Qy_term * V_y - V_y * A_q / A) / (b * (1 - y*k)) ; Oden
;τ_na = (Qy_term * V_y - V_y * Aq_na / A) / (b_na * (1 - e*k))
; NEUTRAL AXIS ITEMS equation and angle w.r.t. z : y = NA_m *z + NA_b
M_angle = atan2d (M_y, M_z) ; Resultant moment plane w.r.t. y-axis
M_total = sqrt (M_z^2 + M_y^2) ; Resultant moment value
NA_angle = atan2d (-z_term, (y_term - s_term)) ;atand (NA_m) ; NA slope angle
NA_b = -s_term / (y_term - s_term) ; NA y-intercept
NA_m = -z_term / (y_term - s_term) ; NA slope
;if M_y = 0 then NA_angle = 0
;if M_y = 0 then NA_m = 0
; GENERAL SECTION GEOMETRY
h_in = R_na - r_inner ; debug aid
h_out= r_outer - R_na ; debug aid
if k=0 then J_z = I_z
if k=0 then J_y = I_y
if k=0 then J_yz = I_yz
; STRAIGHT BEAM COMPARISONS
A_in = width_in * depth_in
A_web = width_web * depth_web
A_out = width_out * depth_out
I_y = (A_in * width_in^2 + A_web * width_web^2 + A_out * width_out^2) / 12
Iz1 = width_in * depth_in^3 / 12 + A_in * (R - (r_inner + r_web) / 2)^2
Iz2 =width_web * depth_web^3 / 12 + A_web * (R - (r_web + r_web_top) /2)^2
Iz3 = width_out * depth_out^3 / 12 + A_out * (R - (r_web_top + r_outer) / 2)^2
I_z = Iz1 + Iz2 + Iz3
Q_p = A_in * (R - (r_inner + r_web)/2) + width_web * (R - r_web)^2 / 2
;add two unlikely logics below
rw = r_web ; shorthand
rt = r_web_top ; shorthand
ww = width_web ; shorthand
wo = width_out ; shorthand

Figure 30 Rule set 4

Rule
if and(r_le_R,r_in_bot)   then Q = width_in*(r-r_inner)*(R-(r+r_inner)/2)
if and(r_le_R,r_in_web) then Q=A_in*(R-(rw+r_inner)/2)+ww*(r-rw)*(R-(r+rw)/2)
if and(r_gt_R,r_in_web) then Q = Q_p - ww * (r - R)^2 / 2
if and(r_gt_R,r_in_top) then Q = Q_p - ww*(rt - R)^2 / 2 - wo*(r-rt)*(rt-2*R+r)/2
term_y = M_z / I_z
term_z = M_y / I_y
st_beam = N_s / A + term_y * y   + term_z * z   ; straight beam normal stress
st_τ   = V_y * Q / (b * I_z)   ; straight beam shear stress
st_in   = N_s / A + term_y * y_inner + term_z * z   ; normal stress at inner radius
st_out   = N_s / A + term_y * y_outer + term_z * z   ; normal stress at outer radius
if r_in_bot   then Aq =   width_in   * (r - r_inner)
if r_in_web then Aq = A_in   + width_web * (R - r_web)
if r_in_top   then Aq = A_in + A_web + width_out * (r - r_web_top)
if r_in_bot   then Q =   width_in * (r - r_inner)*(R - r_inner - (r - r_inner) / 2)
if r_in_web then Q = Q_bot   + width_web * (r - r_web)*(R - r_web - abs(r - R))
if r_in_top   then Q = Q_web + width_out * (r_outer - r)*(R - r_outer + (r_outer - r)/2)
;if M_y = 0 then st_beam = N_s / A + M_z * y / I_z
;if M_y = 0 then st_in   = N_s / A + M_z * y_inner / I_z
;if M_y = 0 then st_out   = N_s / A + M_z * y_outer / I_z
;   THIN FLANGE CORRECTION Via Bleich
L_f_in = (width_in - width_web) /2
L_f_out = (width_out - width_web) /2
r_f_in = r_inner + depth_in / 2
r_f_out = r_outer - depth_out / 2
if not(given('v')) then v = 0.3
if depth_in   > 0 then λ_in^4   = 3 * (1 - v^2) / (r_f_in   * depth_in)^2   else λ_in   = 9
if depth_out > 0 then λ_out^4 = 3 * (1 - v^2) / (r_f_out * depth_out)^2 else λ_out = 9
Λin   = λ_in * L_f_in
Λout = λ_out * L_f_out
in_α   = (sinh(2*Λin)   + sin(2*Λin)) / ((2 + cosh(2*Λin)   + cos(2*Λin)) * Λin)
out_α = (sinh(2*Λout) + sin(2*Λout)) / ((2 + cosh(2*Λout) + cos(2*Λout)) * Λout)
in_β   = sqrt(3) * (cosh(2*Λin)   - cos(2*Λin)) / (2 + cosh(2*Λin)   + cos(2*Λin))
out_β = sqrt(3) * (cosh(2*Λout) - cos(2*Λout)) / (2 + cosh(2*Λout) + cos(2*Λout))
if L_f_in <= 0 then α_in = 1 else α_in = in_α
if L_f_in <= 0 then β_in = 1 else β_in = in_β

Figure 31 Rule set 5

Rule
if L_f_out <= 0 then α_out = 1 else α_out = out_α
if L_f_out <= 0 then β_out = 1 else β_out = out_β
L_B_in = L_f_in * α_in
L_B_out = L_f_out * α_out
;if L_B_in < width_web then B_f_in = width_web else B_f_in = L_B_in
;if L_B_out < width_web then B_f_out = width_web else B_f_out = L_B_out
;if L_B_in < width_web then σs_B_in = σ_s_inner else σs_B_in = β_in*σ_s_inner
;if L_B_out < width_web then σs_B_out = σ_s_outer else σs_B_out = β_out*σ_s_outer
B_f_in = L_B_in
B_f_out = L_B_out
σs_B_in = β_in*σ_s_inner
σs_B_out = β_out*σ_s_outer
widthin_in = 2 * B_f_in + width_web
widthin_out = 2 * B_f_out + width_web
; SHAPE SIDE PLOTS
if r_in_bot then Side_R = width_in / 2
if r_in_web then Side_R = width_web / 2
if r_in_top then Side_R = width_out / 2
Side_L = - Side_R
if r_in_bot then Thin_R = B_f_in + width_web / 2
if r_in_web then Thin_R = width_web / 2
if r_in_top then Thin_R = B_f_out + width_web / 2
Thin_L = - Thin_R
; EMPIRICAL PROPERTIES
Ey = I_y * R_s / depth ;approximate J_y
Ez = I_z * R_s / depth ;approximate J_z

Figure 32 Rule set 6

