1-D Kinematics: Horizontal Motion

We discussed in detail the graphical side of kinematics, but now let's focus on the equations. The goal of kinematics is to mathematically describe the trajectory of an object over time. To do that, we use three main equations. However, I will include two more for the sake of convenience. Remember that the acceleration is assumed to be constant!

Deriving the Kinematics Equations:
We use 4 quantities to describe kinematics:

1. Position (x or y)
2. Velocity (m/s)
3. Acceleration (m/s^2)
4. Time (s)

<u>Note:</u> Constant acceleration $\Rightarrow \bar{a} = a$ at all times.

$\bar{a} = a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$ let t_o = 0s $\Rightarrow a = \frac{(v - v_0)}{t} \Rightarrow v = at + v_0$

By mathematical definition: $\bar{v} = \frac{v + v_0}{2}$ ($\rightarrow$ midpoint of a line) and by a physics definition $\bar{v} = \frac{\Delta x}{\Delta t}$
$\Rightarrow \frac{v + v_0}{2} = \frac{1}{2}(v + v_0) = \frac{x - x_0}{t} \Rightarrow x = \frac{1}{2}(v + v_0)t + x_0$
Substitute $v = at + v_0$ into the above result to yield:

$$x = \frac{1}{2}(at + v_0 + v_0)t + x_0 = \frac{1}{2}(at + 2v_0)t + x_0 \Rightarrow x = \frac{1}{2}at^2 + v_0 t + x_0$$

Note that: $v = at + v_0 \Rightarrow t = \frac{v - v_0}{a}$
Substitute the above equation into $x = \frac{1}{2}(v + v_0)t + x_0$ to yield $\Delta x = \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$
$\Rightarrow 2a\Delta x = (v + v_0)(v - v_0) = v^2 + v_0^2$
$\Rightarrow v^2 = 2a\Delta x + v_0^2$
The use of the x direction in these derivations was completely arbitrary. Therefore, these equations apply for the y-direction as well.

The Kinematics Equations Summarized:

Relate the equations to the graphs!

$$\boldsymbol{x = x_0 + v_0 t + \frac{1}{2}at^2}$$
$$\boldsymbol{v = v_0 + at}$$
$$\boldsymbol{v^2 = v_0^2 + 2a(x - x_0)}$$

On the AP Exam Equation Sheet

$$\boldsymbol{x = x_0 + vt - \frac{1}{2}at^2}$$
$$\boldsymbol{\Delta x = \bar{v}t}$$

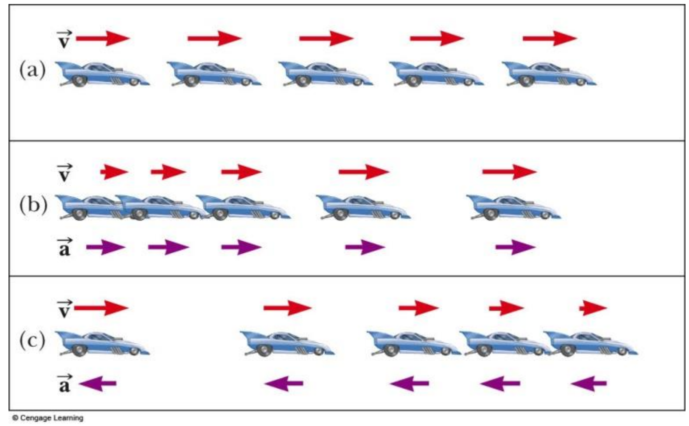
For your convenience on Homework

Don't Forget Your Basic Definitions Either:

$$\boldsymbol{\Delta x = x - x_0}$$
$$\boldsymbol{\bar{v} = \frac{\Delta x}{\Delta t}}$$
$$\boldsymbol{\bar{a} = \frac{\Delta v}{\Delta t}}$$

A Reminder of What the Variables Mean:

x_0	Initial position
x	Final position
Δx	Displacement
v_0	Initial Velocity
v	Final Velocity
$\bar{v}$	Average Velocity
a	Acceleration
t	Time



Tips On How to Use The Equations:

- First make a quick sketch of the situation (optional but strongly recommended).
  - Include your coordinate system!
- Label the quantities you know and identify the quantity (or quantities) you want to find.
- Select the equation(s) that contains the variables you know and the variable you want to find.
  - You may need to use more than one equation to find your desired quantity.
- Solve for the desired quantity and make sure the value seems physically reasonable!

Problems

1. An object with an initial velocity of $4\frac{m}{s}$ moves in a straight line under a constant acceleration. Three seconds later, its velocity is $14\frac{m}{s}$. (a) How far did the object travel during this time? (b) What was the acceleration of the object?

a) $X = \frac{1}{2}(V + V_0)t = \frac{1}{2}(14\frac{m}{s} + 4\frac{m}{s})(3\,s) = 27\,m$

b) $a = \frac{\Delta V}{\Delta t} = \frac{14\frac{m}{s} - 4\frac{m}{s}}{3.0\,s} = 3.3\,\frac{m}{s^2}$

2. An object starts from rest and uniformly accelerates at a rate of 2 m/s^2 for 5.0 seconds. **(a)** What is the object's displacement during this 5-second time period? **(b)** What is the object's final velocity? **(c)** How many seconds does it take the object to have a displacement of 15 meters? **(d)** Suppose the object had an initial velocity of $-3\ m/s$, how would the object's displacement compare the displacement found in part (a)?

$$(a) x = \frac{1}{2}at^2 + v_o t + x_0 = \frac{1}{2}\left(2\frac{m}{s^2}\right)(5.0\ s)^2 + \left(0\frac{m}{s}\right)(5\ s) + 0m = 25\ m$$

$$(b) v = at + v_0 = \left(2\frac{m}{s^2}\right)(5.0\ s) + 0\frac{m}{s} = 10\frac{m}{s}$$

$$(c) x = \frac{1}{2}at^2 + v_0 t + x_0 \rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{(2)(15m)}{2\frac{m}{s^2}}} = 3.873\ s$$

(d) The displacement would be a smaller value than that found in part a because the object would have an initial negative displacement until it reached a positive velocity. In part a the object's displacement was always in the positive direction.

3. A train is traveling down a straight track at $20\frac{m}{s}$ when the engineer applies the brakes, resulting in an acceleration of $-1.0\frac{m}{s^2}$ as long as the train is in motion. How far does the train move during a 40-s time interval starting at the instant the brakes are applied?

Using the uniformly accelerated motion equation $\Delta x = v_0 t + \frac{1}{2}at^2$ for the full 40 s interval yields $\Delta x = (20\ \text{m/s})(40\ \text{s}) + \frac{1}{2}(-1.0\ \text{m/s}^2)(40\ \text{s})^2 = 0$, which is obviously wrong.
The source of the error is found by computing the time required for the train to come to rest. This time is

$$t = \frac{v - v_0}{a} = \frac{0 - 20\ \text{m/s}}{-1.0\ \text{m/s}^2} = 20\ \text{s}$$

Thus, the train is slowing down for the first 20 s and is at rest for the last 20 s of the 40 s interval.

The acceleration is not constant during the full 40 s. It is, however, constant during the first 20 s as the train slows to rest. Application of $\Delta x = v_0 t + \frac{1}{2}at^2$ to this interval gives the stopping distance as

$$\Delta x = (20\ \text{m/s})(20\ \text{s}) + \tfrac{1}{2}(-1.0\ \text{m/s}^2)(20\ \text{s})^2 = \boxed{200\ \text{m}}$$

4. A bullet 2.00 cm long is fired at $420\frac{m}{s}$ and passes straight through a 10.0 cm thick board, exiting at $280\frac{m}{s}$. (a) What is the average acceleration of the bullet through the board? (b) What is the total time that the bullet is in contact with the board? (c) What minimum thickness could the board have if it was supposed to bring the bullet to a stop?

We assume that the bullet begins to slow just as the front end touches the first surface of the board, and that it reaches its exit velocity just as the front end emerges from the opposite face of the board.

(a) The acceleration is

$$a = \frac{v_{\text{exit}}^2 - v_0^2}{2(\Delta x)} = \frac{(280\ \text{m/s})^2 - (420\ \text{m/s})^2}{2(0.100\ \text{m})} = \boxed{-4.90 \times 10^5\ \text{m/s}^2}$$

(b) The average velocity as the front of the bullet passes through the board is

$$\bar{v} = \frac{v_{\text{exit}} + v_0}{2} = \frac{280\ \text{m/s} + 420\ \text{m/s}}{2} = 350\ \text{m/s}$$

and the total time of contact with the board is the time for the front of the bullet to pass through plus the additional time for the trailing end to emerge (at speed v_{exit}),

$$t = \frac{(\Delta x)_{\text{board}}}{\bar{v}} + \frac{L_{\text{bullet}}}{v_{\text{exit}}} = \frac{0.100\ \text{m}}{350\ \text{m/s}} + \frac{0.0200\ \text{m}}{280\ \text{m/s}} = \boxed{3.57 \times 10^{-4}\ \text{s}}$$

(c) From $v^2 = v_0^2 + 2a(\Delta x)$, with $v = 0$, gives the required thickness is

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (420\ \text{m/s})^2}{2(-4.90 \times 10^5\ \text{m/s}^2)} = 0.180\ \text{m} = \boxed{18.0\ \text{cm}}$$

5. An inattentive driver is traveling $18.0\frac{m}{s}$ when he notices a red light ahead. His car is capable of a braking acceleration of $3.65\frac{m}{s^2}$. If it takes him 0.200 s to get the brakes on and he is 45.0 m from the intersection when he sees the light, will he be able to stop in time?

Δx Before Brakes: $\Delta X_1 = (18.0 \frac{m}{s})(.200 s) = 3.6 m$

Δx while Stopping: $V^2 = 2a\Delta X_2 + V_0^2$

"at rest"

$0 = 2a\Delta X_2 + V_0^2$

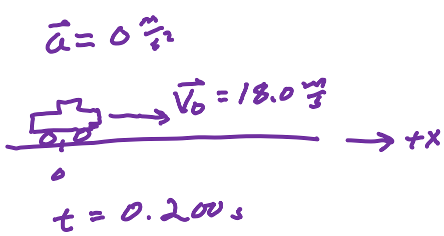
$$\Delta X_2 = \frac{-V_0^2}{2a} = \frac{-(18.0\frac{m}{s})^2}{2(-3.65\frac{m}{s^2})} = 44.4 m$$

Total Δx: $\Delta X = \Delta X_1 + \Delta X_2 = 3.6 m + 44.4 m = 48.0 m$

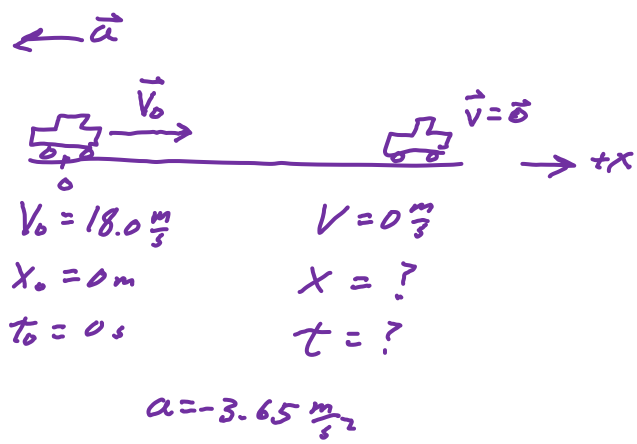
No, he will not stop in time. He will cross into the intersection

Initial Sketches:

1 - Before



2 - During Braking



6. Suppose your car has a maximum straight-line braking acceleration of $5.0\,\frac{m}{s^2}$. Calculate the stopping distance for the following initial speeds: (a) $24.6\,\frac{m}{s}$ [$\sim 55\ mph$], (b) $31.3\,\frac{m}{s}$ [$\sim 70\ mph$], and (c) $38\,\frac{m}{s}$ [$\sim 85\ mph$]. (d) How is stopping distance related to your initial speed? (e) What effect does doubling your speed have on stopping distance? (f) Suppose you increase your speed by 15%, what is the percent increase of your stopping distance?

a] $\overset{0}{\cancel{v}^2} = 2a\Delta x + v_0^2$

$$\Delta x = \frac{-v_0^2}{2a} = \frac{-\left(24.6\,\frac{m}{s}\right)^2}{2\left(-5.0\,\frac{m}{s^2}\right)} = 60.5\ m$$

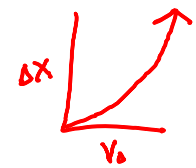
$(a < 0)$

~200 ft

Safety:

$\Delta x \propto v_0^2$

Double v_0, quadruple Δx!



Similarly ...

b] 98.0 m

~320 ft

c] 144.4 m

~475 ft

(f) $\mathbf{1.15^2 = 1.3225 \rightarrow 132.25\%}$

7. Two bank robbers are in their get-away car travelling at its top speed of $30\,\frac{m}{s}$. They drive by a parked police officer who was on the lookout for them. If the police officer takes 2.0 s to react and engage in pursuit, how long will it take him to catch the robbers if his car has a maximum acceleration of $4.5\,\frac{m}{s^2}$. You may assume the police officer's car has a greater top speed than the robbers' car.

Let forward be the positive direction

In the 2.0 reaction time, the robbers gain a $(2.0s)(30\,\frac{m}{s}) = 60m$ head start.

Let $X_{oc} = 0$ m. Then $X_{oR} = 60m$

We want to know when $X_c = X_R$.

We also know that $t_c = t_R \equiv t$ since we already accounted for the head start.

$$X_R = V_R t + X_{oR} \qquad X_c = \frac{1}{2}a_c t^2 + \cancelto{0}{V_{oc} t} + \cancelto{0}{X_{oc}}$$

$$V_R t + X_{oR} = \frac{1}{2}a_c t^2 \Rightarrow \frac{1}{2}a_c t^2 - V_R t - X_{oR} = 0$$

$$(2.25\,\tfrac{m}{s^2})t^2 - (30\,\tfrac{m}{s})t - 60m = 0$$

for pursuit $\rightarrow$ $$t = \frac{30\frac{m}{s} \pm \sqrt{(30\frac{m}{s})^2 - 4(2.25\frac{m}{s^2})(-60m)}}{2(2.25\,m/s^2)} = 15.1s,\ \cancel{-1.8s}$$

$$t_{total} = 2.0s + 15.1s = \boxed{17.1s}$$

Note: $(30\,\frac{m}{s})(17.1s) = 513\,m$ or $\sim\frac{1}{3}$ mile down the road!