1 Frequency Modulation (FM)

1.1 Objective

This experiment deals with the basic performance of Frequency Modulation (FM). Upon completion of the experiment, the student will:

- Understand ANGLE modulation concept.
- Learn how to generate FM signal.
- Learn how to build FM demodulator.
- Become familiar with Bessel Function.

1.2 Prelab Exercise

1. .Find the maximum frequency deviation of the following signal; and verify your results in the laboratory. Carrier sinewave frequency 10 MHz, amplitude 1 Vp-p with frequency deviation constant 10.7 kHz/V ,modulated by sinewave frequency 10 kHz amplitude 1 Vp-p .

2. Explain what is Carson's rule.

3. What is the difference between NBFM and wideband FM refer to the Spectral component of the two signals.

4. Use Matlab to draw an FM signal: $\circ = 15Hz$, carrier amplitude $A = 2.5V$, $A_m = 1V_p$, modulation frequency $f_m = 1Hz$, modulator constant $K_f = 7.5Hz/Volt$, $t = 0 \ to \ 4$ seconds. Show :

   (a) Modulation frequency versus time.

   (b) FM signal.

   (c) Differentiated FM signal.

   (d) Differentiated FM signal followed by a LPF.

5. Write a mathematical expression for Fourier transform of a differentiation, add a graph for the absolute value of the transform, explain the use of a differentiation in a FM detector.

1.3 Background Theory

An angle modulated signal, also referred to as an exponentially modulated signal, has the form

$$S_m(t) = A\cos[\omega t + \theta(t)] = \mathrm{Re}\{A\exp[j\phi(t)]\} \tag{1}$$

were the instantaneous phase $\phi_i(t)$ is defined as

$$\phi_i(t) = \omega t + \theta(t) \tag{2}$$

and the instantaneous frequency of the modulated signal is defined as

$$\omega_i(t) = \frac{d}{dt}[\omega t + \theta(t)] = \omega + \frac{d(\theta(t))}{dt} \tag{3}$$

The functions $\theta(t)$ and $\frac{d(\theta(t))}{dt}$ are referred to as the instantaneous phase and frequency deviations, respectively.

The phase deviation of the carrier $\phi(t)$ is related to the baseband message signal $s(t)$. Depending on the nature of the relationship between $\phi(t)$ and $s(t)$ we have different forms of angle modulation.

$$\frac{d(\theta(t))}{dt} = k_f s(t) \tag{4}$$

$$\phi(t) = k_f \int_{t_0}^{t} s(\lambda)d\lambda + \omega t \tag{5}$$

where k_f is a frequency deviation constant, (expressed in (radian/sec)/volt). It is usually assumed that $t_0 = -\infty$ and $\phi(-\infty) = 0$.

Combining Equations-4 and 5 with Equation-1, we can express the frequency modulated signal as

$$S_m(t) = A\cos[\omega t + k_f \int_{-\infty}^{t} s(\tau)d\tau] \tag{6}$$

Fig. 1 shows a single tone ($s\,(t)$ message signal), frequency modulated a carrier frequency, represented in time domain.

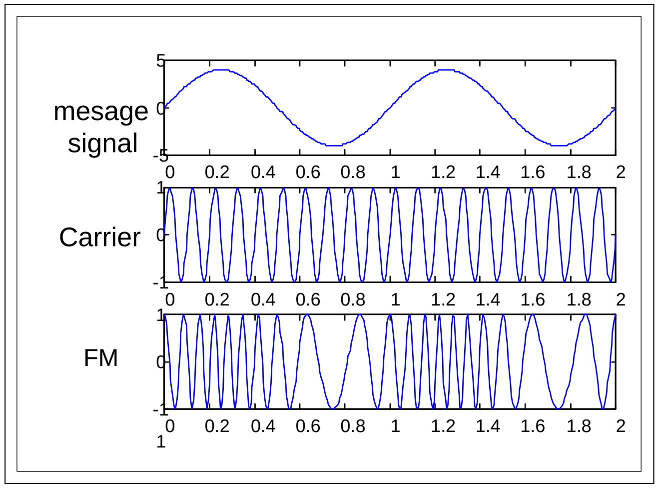

Fig 1: modulation signal (upper), carrier signal (midle), and modulated FM signal(lower)

1.4 Bessel Function

Bessel function of the first kind, is a solution of the differential equation

$$\beta^2 \frac{d^2y}{dx^2} + \beta \frac{dy}{dx} + (\beta^2 - n^2)y(\beta) = 0$$

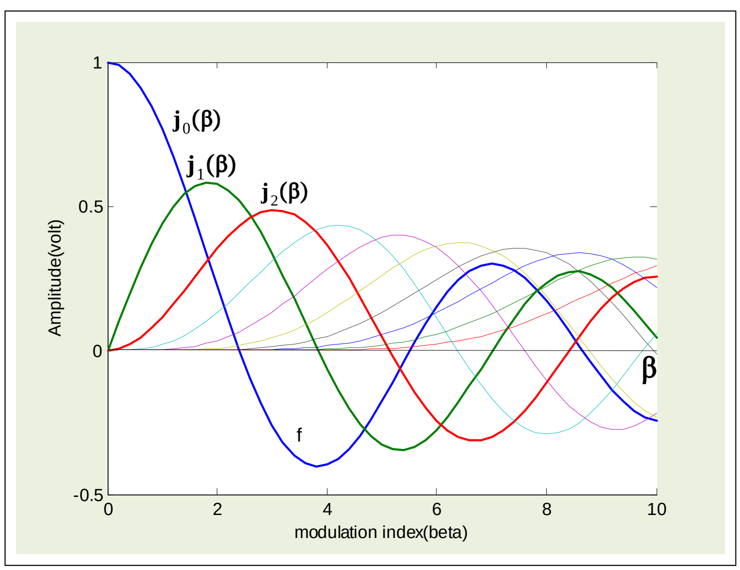

Fig. 2 Bessel function, of kind 1, and order 1 to 10

Bessel function defined for negative and positive real integers. It can be shown that for integer values of n

$$j_{-n}(\beta) = (-1)^n j_n(\beta) \tag{7}$$

$$j_{n-1}(\beta) + j_{n+1}(\beta) = \frac{2n}{\beta} j_n(\beta) \tag{8}$$

$$\sum_{n=-\infty}^{\infty} j_n^2(\beta) = 1 \tag{9}$$

A short listing of Bessel function of first kind of order n and discrete value of argument β, is shown in Table-1, and graph of the function, is shown in Fig. 2

Note that for very small β,value $j_0(\beta)$ approaches unity, while $j_1(\beta)$ to $j_n(\beta)$ approach zero.

$n \backslash \beta$	0	0.2	0.5	1	2	5	8	10
0	1.00	0.99	0.938	0.765	0.224	-0.178	0.172	-0.246
1	0	0.1	0.242	0.440	0.577	-0.328	0.235	0.043
2		0.005	0.031	0.115	0.353	0.047	-0.113	0.255
3				0.02	0.129	0.365	-0.291	0.058
4				0.002	0.034	0.391	-0.105	-0.22
5					0.007	0.261	0.186	-0.234
6						0.131	0.338	-0.14
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9							0.126	0.292
10							0.061	0.208
11							0.026	0.123

Table-1 Bessel function $j_n(\beta)$

1.5 Properties of Bessel function

1. Eq. -1.9 indicates that the phase relationship between the sideband components is such that the odd-order lower sidebands are reversed in phase .

2. The number of significant spectral components is a function of argument β (see Table-1). When $\beta \ll 1$, only J_0, and J_1, are significant so

that the spectrum will consist of carrier plus two sideband components, just like an AM spectrum with the exception of the phase reversal of the lower sideband component.

3. A large value of β implies a large bandwidth since there will be many significant sideband components.

4. Transmission bandwidth of 98% of power always occur after $n = \beta + 1$, we note it in table-1 with underline.

5. Carrier and sidebands null many times at special values of β see table-2 .

Order	0	1	2	3	4	5	6
β for 1st zero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β for 2nd zero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β for 3rd zero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β for 4th zero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β for 5th zero	14.93	16.47	17.96	19.41	20.83	22.21	23.59
β for 6th zero	18.07	19.61	21.12	22.58	24.02	25.43	26.82

Table-2 Zeroes of Bessel function: Values for β when $j_n(\beta) = 0$

1.6 Spectrum of Frequency Modulated Signal

Since frequency modulation is a nonlinear process, an exact description of the spectrum of an frequency-modulated signal for an arbitrary message signal is more complicated than linear process. However if $s(t)$ is sinusoidal, then the instantaneous frequency deviation of the angle-modulated signal is sinusoidal and the spectrum can be relatively easy to obtained. If we assume $s(t)$ to be sinusoidal then

$$s(t) = A_m \cos \omega_m t \tag{10}$$

then the instantaneous phase deviation of the modulated signal is

$$\phi(t) = \frac{k_f A_m}{\omega_m} \sin \omega_m t \tag{11}$$

The modulated signal, for the (FM signal) , is given by

$$S_m(t) = A \cos(\omega t + \beta \sin \omega t) \tag{12}$$

where the parameter β is called the modulation index defined as

$$\beta = \frac{k_f A_m}{\omega_m} \quad \text{For FM}$$

The parameter β is defined only for sinewave modulation and it represents the maximum phase deviation produced by the modulating signal. If we want to compute the spectrum of $S_m(t)$ given in Equation 11, we can express $S_m(t)$ as

$$S_m(t) = \mathrm{Re}\{A \exp(j\omega t) \exp(j\beta \sin \omega_m t)\} \tag{13}$$

In the preceding expression, $\exp(j\beta \sin \omega_m t)$ is periodic with a period $T_m = \frac{2\pi}{\omega_m}$. Thus, we can represent it in a Fourier series of the form

$$\exp(j\beta \sin \omega_m t) = \sum_{-\infty}^{\infty} C_x(nf_m) \exp(j2\pi nf) \tag{14}$$

Where

$$\begin{aligned} C_x(nf_m) &= \frac{\omega_m}{2\pi} \int_{-\frac{\pi}{\omega_M}}^{\frac{\pi}{\omega_M}} \exp(j\beta \sin \omega_m t) \exp(-j\omega_m t) dt \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin\theta - n\theta)] d\theta = j_n(\beta) \end{aligned} \tag{15}$$

Where $j_n(\beta)$ known as Bessel functions. Combining Equations 1.13, 1.14, and 1.15, we can obtain the following expression for the FM signal with tone modulation:

$$S_m(t) = A \sum_{-\infty}^{\infty} j_n(\beta) \cos[(\omega + n\omega_m)t] \tag{16}$$

The spectrum of $S_m(t)$ is a spectrum of a sinusoidal signal multiplied by a constant (Bessel function), such a spectrum consist of infinite number of Dirac delta function. The number of significant (energy contained) spectral lines, is limited , so we can use the FM nodulation, with finite bandwidth. An example of narrow band and wideband FM spectrum is shown in Figure-3.

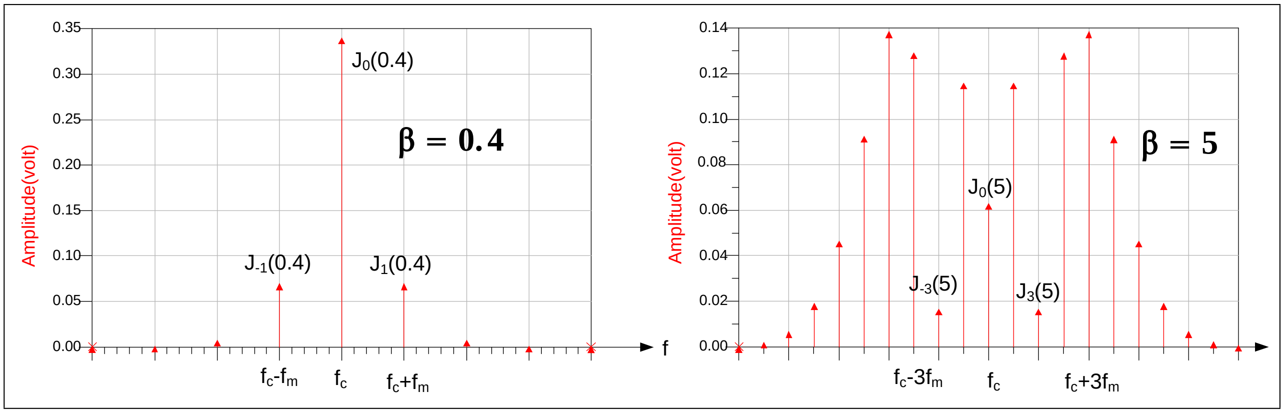

Fig 3 : Narrow, and wideband FM spectrum.

The spectrum of an FM signal has several important properties:

1. The FM spectrum consists of a carrier component plus an infinite number of sideband components at frequencies $f \pm nf_m$ $(n = 1, 2, 3...)$. But the number of significant sidebands depend primarily on the value of β. In comparison, the spectrum of an AM signal with tone modulation has only three spectral components (at frequencies f, $f + f_m$, and $f - f_m$).

2. The relative amplitude of the spectral components of an FM signal depend on the values of $j_n(\beta)$. The relative amplitude of the carrier depends on $j_0(\beta)$ and its value depends on the modulating signal (unlike AM modulation where the amplitude of the carrier does not depend on the value of the modulating signal).

1.7 Power and Bandwidth of FM Signals

In the previous section we saw that a single tone modulated FM signal has an infinite number of sideband components and hence the FM spectrum seems to have infinite spectrum. Fortunately, it turns out that for any β a large portion of the power is contained in finite bandwidth. . Hence the determination of FM transmission bandwidth depends to the question of how many significant sidebands need to be included for transmission, if the distortion is to be within certain limits.

To determine FM transmission bandwidth, let us analyze the power ratio S_n, which is the fraction of the power contained in the carrier plus n

sidebands, to the total power of FM signal. We search a value of number of sidebands n, for power ratio $S_n \geq 0.98$.

$$S_n = \frac{\frac{1}{2}A \sum_{k=-n}^{n} j_k^2(\beta)}{\frac{1}{2}A \sum_{k=-\infty}^{\infty} j_k^2(\beta)} \tag{17}$$

$$S_n \geq 0.98 \tag{18}$$

Using the value properties of Bessel function,, and Table 1, we can show that the bandwidth of FM signal B_T, depends on the number of sidebands n, and FM modulation index β. which can be expressed as

$$B_T \approx 2(\beta + 1) f_m \tag{19}$$

1.8 Narrow Band FM

Narrowband FM is in many ways similar to DSB or AM signals. By way of illustration let us consider the $NBFM$ signal

$$\begin{aligned} S_m(t) &= A\cos[\omega t + \phi(t)] = A\cos\omega t\cos\phi(t) - A\sin\omega t\sin\phi(t) \\ &\approx A\cos\omega t - A\phi(t)\sin\omega t \end{aligned} \tag{20}$$

Using the approximations $cos\phi = 1$ and $sin\phi \approx \phi$, when ϕ is very small. Equation-26 shows that a $NBFM$ signal contains a carrier component and a quadrature carrier linearly modulated by (a function of) the baseband signal. Since $s(t)$ is assumed to be bandlimited to f_m therefore $\phi(t)$ is also bandlimited to f_m,. Hence, the bandwidth of $NBFM$ is $2f_m$, and the $NBFM$ signal has the same bandwidth as an AM signal.

1.9 Narrow Band FM Modulator

According to Equation-1.20, it is possible to generate $NBFM$ signal using a system such as the one shown in Fig-4 . The signal is integrated prior to modulation and a DSB modulator is used to generate the quadrature component of the $NBFM$ signal. The carrier is added to the quadrature component to generate an approximation to a true $NBFM$ signal.

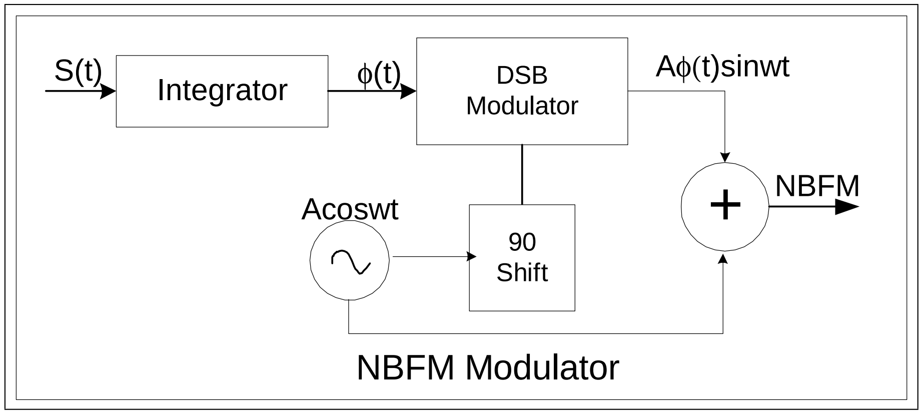

Fig 4 NBFM Modulator

1.10 Wide Band FM Modulator

There are two basic methods for generating FM signals known as direct and indirect methods. The direct method makes use of a device called voltage controlled oscillator (VCO) whose oscillation frequency depends linearly on the modulation voltage.

A system that can be used for generating an FM signal is shown in Figure-5.

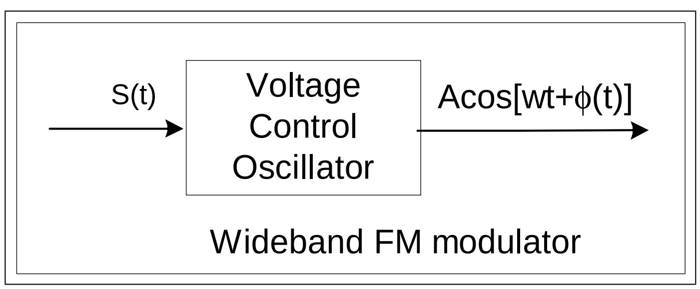

Fig. 5 VCO as wideband FM modulator

The combination of message differentiation that drive a VCO produces a PM signal. The physical device that generates the FM signal is the VCO whose output frequency depends directly on the applied control voltage of the message signal. $VCO's$ are easily implemented up to microwave frequencies.

1.11 Demodulation of FM Signals

An FM demodulator is required to produce an output voltage that is linearly proportional to the input frequency variation. One way to realize the requirement, is to use discriminators- devices which distinguish one frequency from another, by converting frequency variations into amplitude variations. The resulting amplitude changes are detected by an envelope detector, just as done by AM detector.

$$S_m(t) = A\cos[\omega t + k_f \int_{-\infty}^{t} s(\tau)d\tau]$$

the discriminator output will be

$$y_d(t) = k_d k_f s(t)$$

where k_d is the discriminator constant. The characteristics of an ideal discriminator are shown in Fig. 6. Discriminator can be realized by using a filter in the stopband region, in a linear range, assuming that the filter is differentiation in frequency domain.

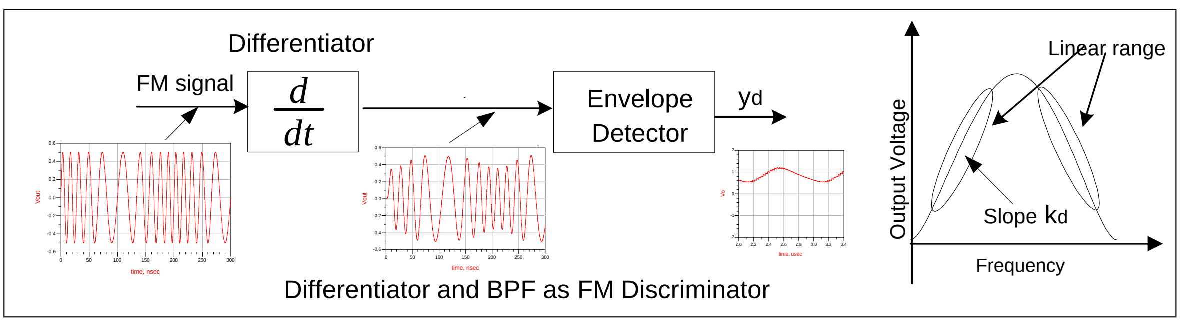

Fig 6 Ideal and real frequency demodulator

An approximation to the ideal discriminator characteristics can be obtained by the use of a differentiation followed by an envelope detector (see Figure-6) . If the input to the differentiator is $S_m(t)$, then the output of the differentiator is

$$S_m'(t) = -A[\omega + k_f s(t)]\sin[\omega t + \phi(t)] \quad (21)$$

With the exception of the phase deviation $\phi(t)$, The output of the differentiator is both amplitude and frequency modulated. Hence envelope detection can be used to recover the message signal. The baseband signal is recovered without any distortion if $\text{Max}\{k_f s(t)\} = 2\pi\Delta f < \omega$, which is easily achieved in most practical systems.

2 Experiment Procedure

2.1 Required Equipment

1. Spectrum Analyzer (SA) $HP - 8590L$. or equivalent
2. Oscilloscope $HP - 54600A$.
3. Signal Generator (SG) $HP/Agilent - 8647A$.
4. Function Generator $HP - 33120A$.
5. Double Balanced Mixer Mini-Circuit $ZAD - 2$.
6. Phase Shifter Mini-Circuit ZSCQ-2-90.
7. 10.7 MHz band pass filter. Mini Circuit BBP-10.7

2.2 VCO as Frequency Modulator

2.2.1 Simulation

In this simulation we drive the VCO (FM modulator) by DC and AC signal, and record the output signal.

1. Simulate a VCO as FM modulator as indicated in Fig. 7

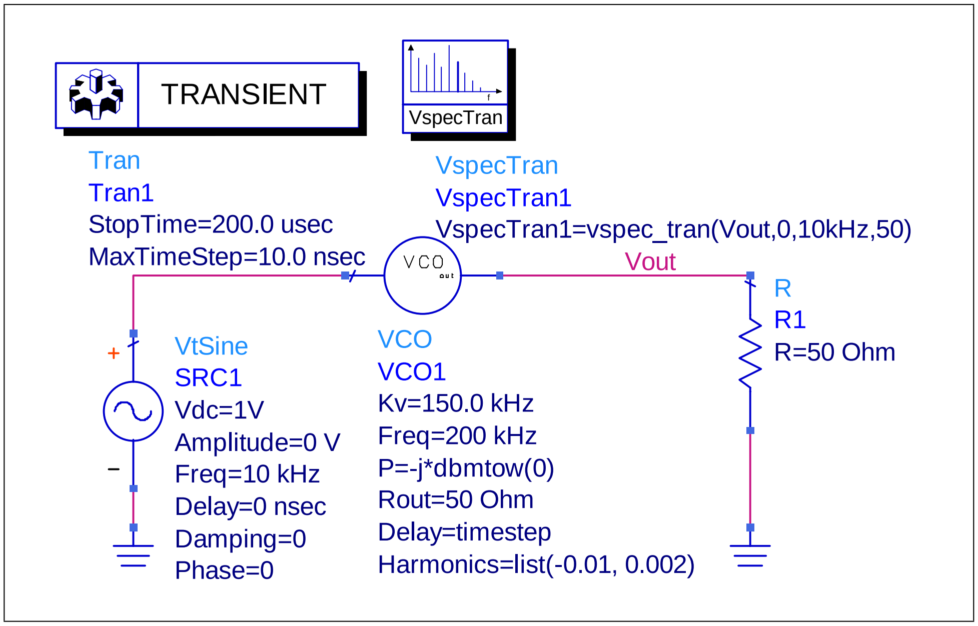

Fig. 7 VCO as FM modulator

2. What is the expected frequency of $Vout$ signal? Draw a graph of the signal in time domain and frequency domain and prove your answer..

3. Set the voltage of the modulating frequency to $Vdc = 0$ and Amplitude to 1V.

4. Draw a graph of FM signal in frequency and time domain.

5. Calculate modulation index β of the modulated signal,and use MATLAB command $J = besselj(0 : 16, 15)$ to find the amplitude of sidebabands components. compare the results to the simulation.

2.2.2 VCO as Frequency Modulator- Measurement

During this experiment you learn how to measure the FM modulation characteristics and Bessel function in frequency domain using spectrum analyzer..

1. Connect the system as indicated in Fig.. 8.

2. Set the Signal-generator to: frequency 10 MHz, amplitude 0 dBm. External DC FM modulation, frequency deviation 20 kHz.

3. Set the function generator to DC volt, amplitude 530 mV.

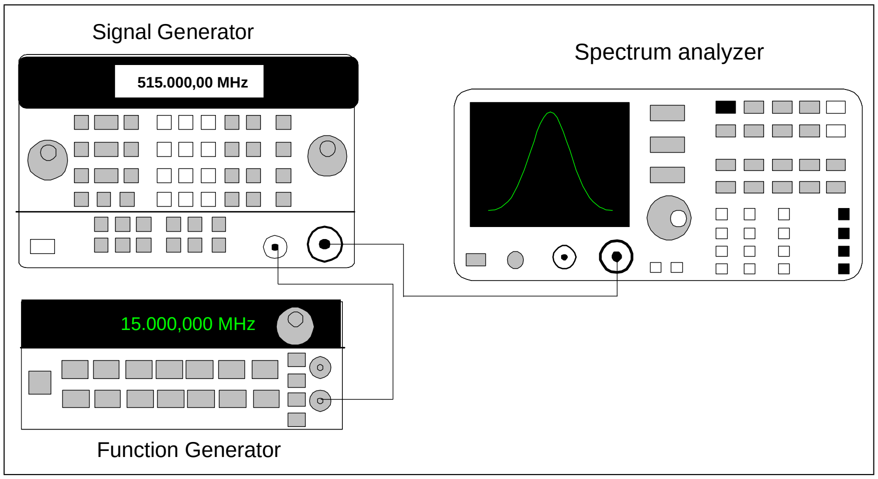

Fig 8 VCO as FM modulator

4. Adjust the amplitude (if necessary) of the function generator in order to get full frequency deviation.

5. Measure and record the frequency of the signal generator with spectrum analyzer.

6. Change the DC voltage, and watch the output signal, Switch off the modulation, measure and record the frequency of the signal generator.

7. Calculate the VCO Frequency tuning sensitivity (Hz/Volt) K_V.

2.2.3 Frequency Modulation and Bessel Function

1. Connect the function generator directly to the spectrum analyzer as indicated in Fig. 9

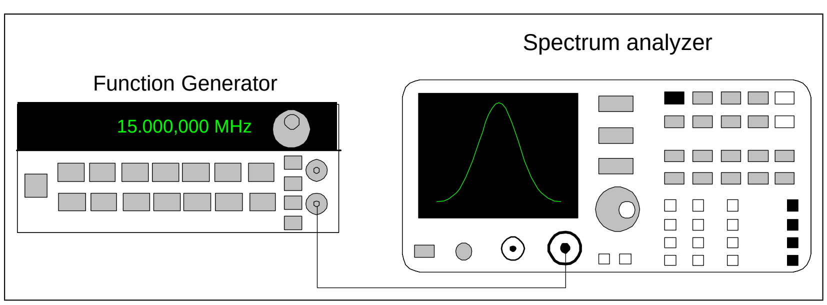

Fig. 9 Frequency modulation and Bessel function.

2. Set the function generator to sinewave frequency 10MHz, amplitude $1V_{RMS}$, FM, modulating frequency 5 kHz, frequency deviation 25kHz.

3. Calculate the modulation index β, switch off the modulation, and verify by the spectrum analyzer that the amplitude of the carrier only is exactly $1V_{RMS}$.

4. Switch on the FM modulation, and measure the amplitude, and frequency of the sidebands is according to Bessel function, see table-1. **Save the image on magnetic media.**

5. Adjust the function generator to proper frequency deviations to get first null of the carrier $j_0 = 0$, and first null of the sideband $j_1 = 0$, **Save the image on magnetic media.**

2.3 Power and Bandwidth of FM Signal-Carson's Rule

2.3.1 Simulation

1. Simulate a frequency modulated signal, as indicated in Fig. 10

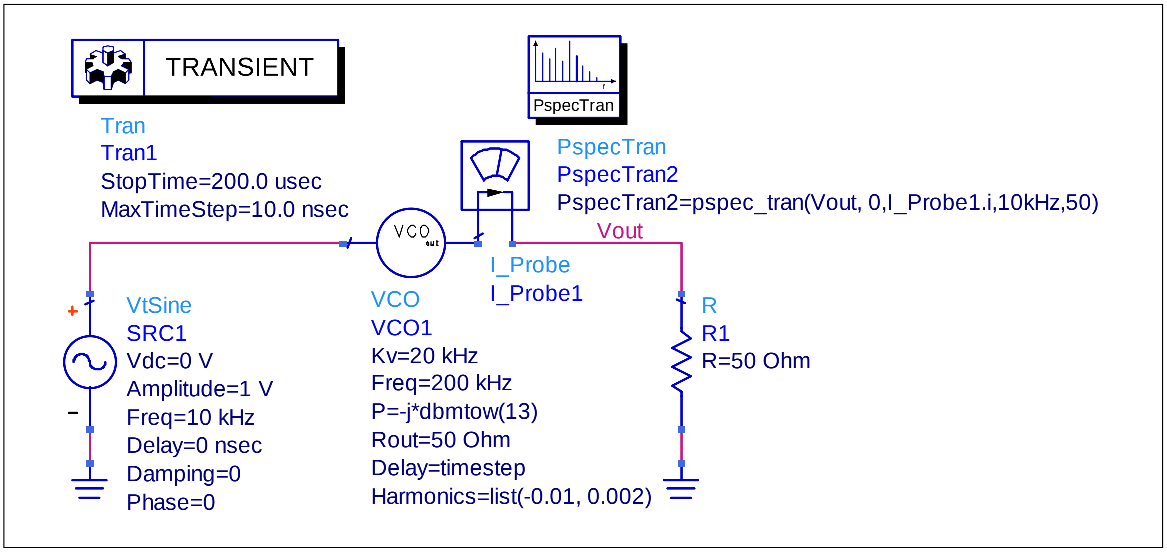

Fig. 10 Simulation of power and bandwidth of FM signal

2. According to Carson's rule find the bandwidth and power of the FM signal,
3. Switch off the FM modulator (set the amplitude of VtSine to 0V), measure the total power of the signal and show that, at least 98% of power contained in Carson's bandwidth.

2.3.2 Measurement of Power and Bandwidth of FM Signal

1. Connect the function generator to the spectrum analyzer as indicated in Fig. 8.
2. Set the function generator to sinewave frequency 10MHz, amplitude $1V_{RMS}$, FM, modulating frequency 5 kHz, frequency deviation 10kHz.
3. Find the bandwidth of the signal according to Carson's rule, and show such a bandwidth contains at least 98% of total power. (total power is the power of the signal without modulation).

2.4 Narrow Band FM Modulator

In this part of the experiment, you implement narrow FM modulator (see Fig. 4), without the first stage- integrator, since we assume that our sinewave modulating signal, is a phase shifted of other sinewave signal.

2.4.1 Simulation

1. Simulate a narrow band modulator (see Fig. 4) use the elements of Fig. 11.

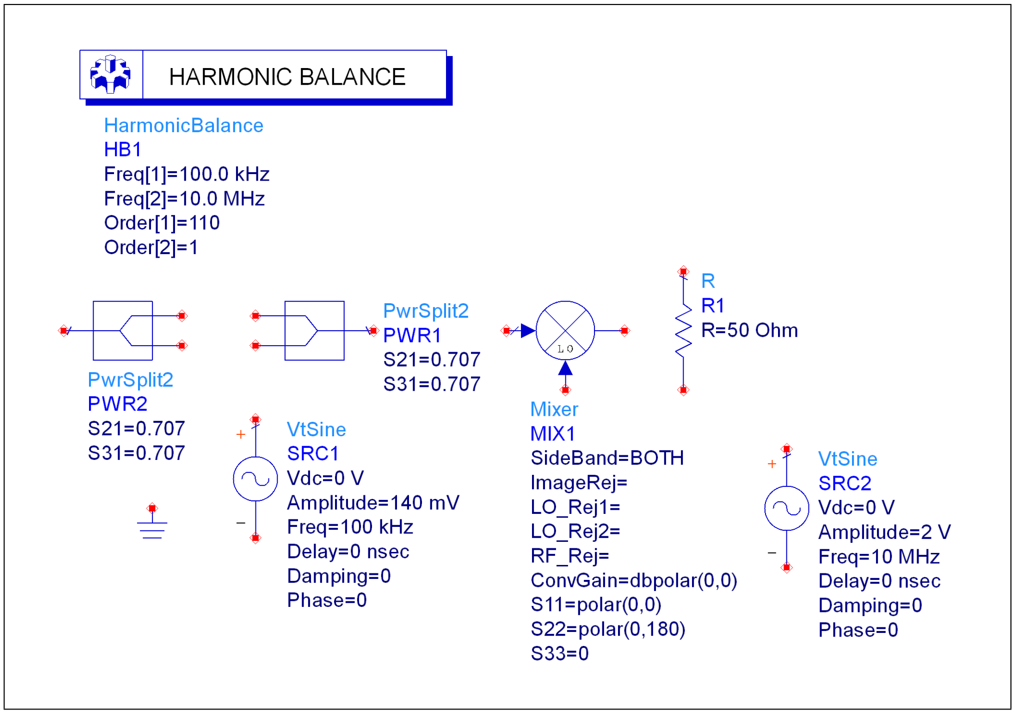

Fig.-11 Narrow band FM signal

(a) Carrier-sinewave frequency 10MHz amplitude $2V_p$.

(b) modulating frequency- sinewave frequency 100kHz, amplitude 140mV.

2. According to Carson's rule find the bandwidth of the FM signal. Draw a frequency domain graph signal Magnitude (voltage) versus frequency, and find the voltage of the carrier and sidebands? Use Table-1 to find the modulation index β.

2.4.2 Narrow Band FM-Measurement

1. Connect the system according to Fig.-12.
2. Adjust the equipment as follow:
function generator-LO - Sinewave frequency 10 MHz amplitude 7dbm.
function generator- RF- Sinewave frequency 10 kHz amplitude -10dbm.(integral of the cosine input wave).

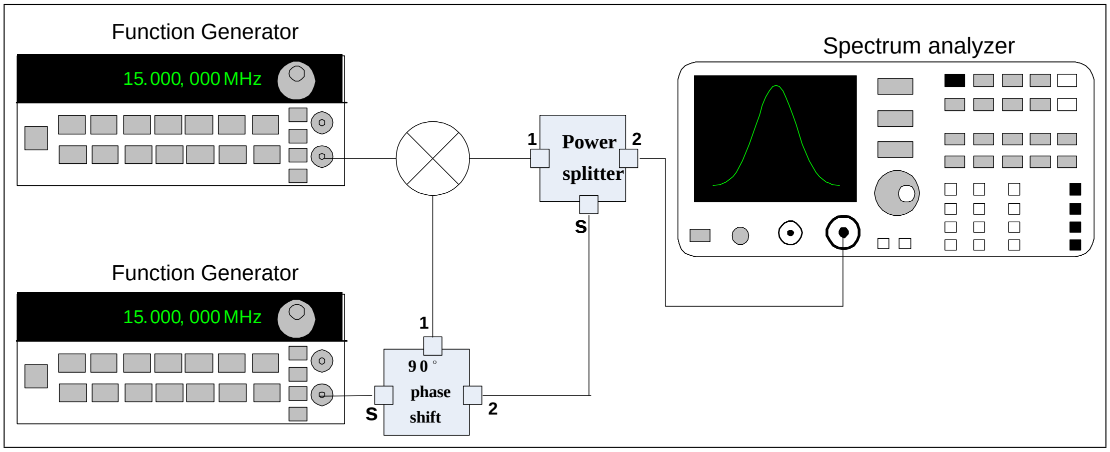

Fig.-12 Narrow band FM modulator.

3. Set the spectrum analyzer to 10 MHz span 50kHz , watch the FM signal at spectrum analyzer, change the amplitude and frequency of the modulating frequency generator, which component of the FM signal changed?

$n\backslash\beta$	0.1	0.15
0	0.00(Ref.)	0.00(Ref)
1	-26.0(dB)	-22.5(dB)

Table-3 NBFM, sidebands amplitude

4. Change the amplitude and frequency of the local oscillator , which component of the FM signal changed?

5. According to table-3 set the system to $\beta = 0.1, 0.15$, calculate the frequency deviation for each β, **save image on magnetic media.**

2.5 FM Demodulator- discrimination Method.

2.5.1 Simulation

In this part you will demodulate an FM signal using a discriminator. A discriminator may be realized by a LPF (differentor in frequency domain) followed by an envelope detector.

1. Simulate an FM demodulator based on discriminator as indicated in Fig. 13

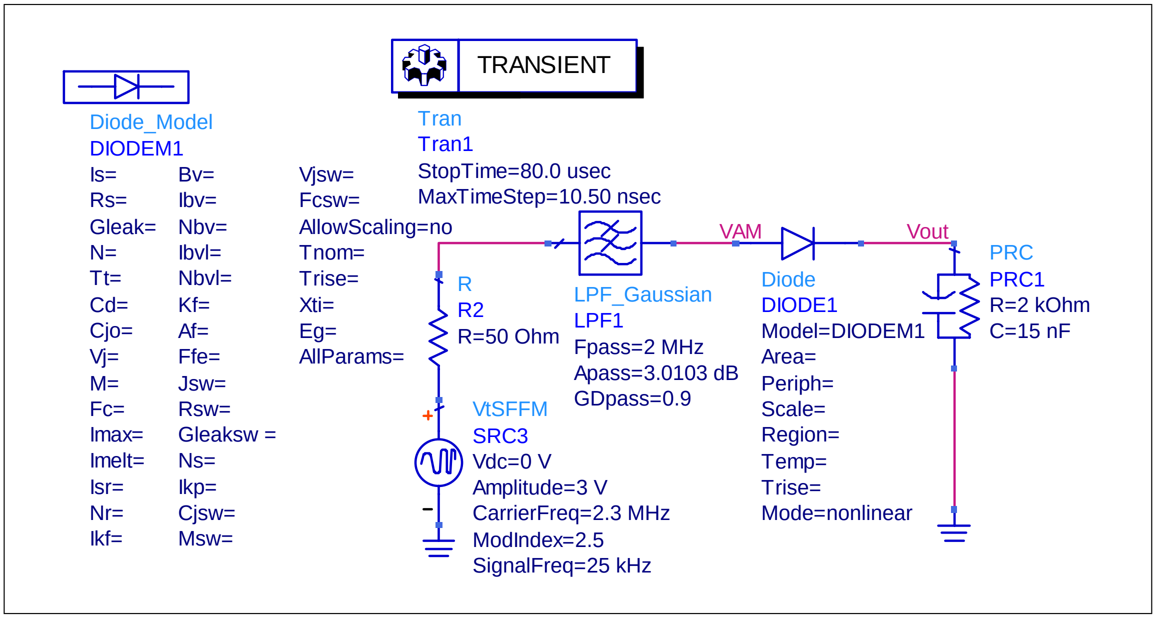

Fig. 13 FM detector- discrimination method

2. Draw a graph of VAM , and Vout, explain the idea of the circuit.

2.6 FM Demodulator

2.6.1 Measurement

We start in the firs part with a low pass filter as discriminator, in the second part we use the IF filter of the spectrum as a discriminator, and peak detector as demodulator.

1. Connect the filter and function generator to the oscilloscope,as indicated in Fig.- 13.

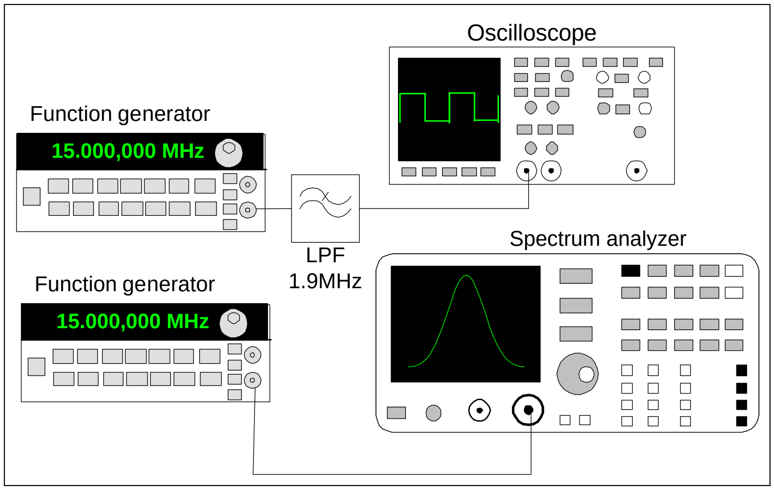

Fig. 14 FM Discriminator

2. Set the function generator to FM modulation, carrier frequency 2.4 MHz, Amplitude 0 dBm, modulating frequency 1k Hz. deviation frequency 50 kHz.

3. The oscilloscope display an FM signal after differentiation, which signal you identify, **save image on magnetic media**

4. Disconnect the filter and connect the function generator to the spectrum analyzer.

5. Set the spectrun analyzer to center frequency 2.4MHZ, span 300kHz, IF bandwidth to 10kHz. Center the signal on display if necessary.

6. Set the span to 0kHz, now the spectrum display a signal like an oscilloscope, change the center frequency slightly to recover the modulating frequency (1kHz.).