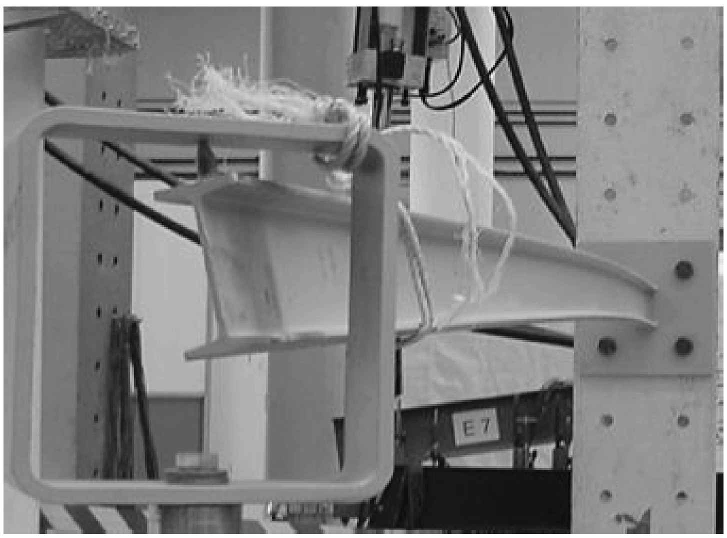
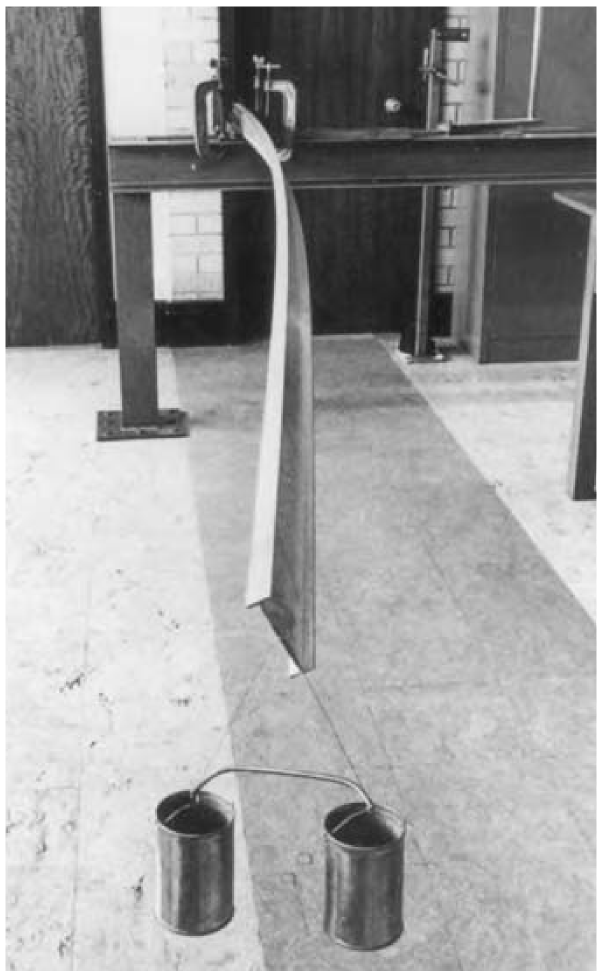
Design of Laterally Unrestrained Beams

In this chapter, the resistance of members against instability phenomena caused by a bending moment will be presented in standard cross sectional shapes, such as I or H bent around the major axis (*y* axis), the typical instability phenomenon is **lateral-torsional buckling**.

1- Lateral-Torsional Buckling

Consider a member subject to bending about the strong axis of the cross section (y axis). Lateral-torsional buckling is characterized by lateral deformation of the compressed part of the cross section (the compressed flange in the case of I or H sections). This part behaves like a compressed member, but one continuously restrained by the part of the section in tension, which initially does not have any tendency to move laterally. As seen in the below Figures, where this phenomenon is illustrated for a cantilever beam, the resulting deformation of the cross section includes both lateral bending and torsion. This is why this phenomenon is called **lateral-torsional buckling.**

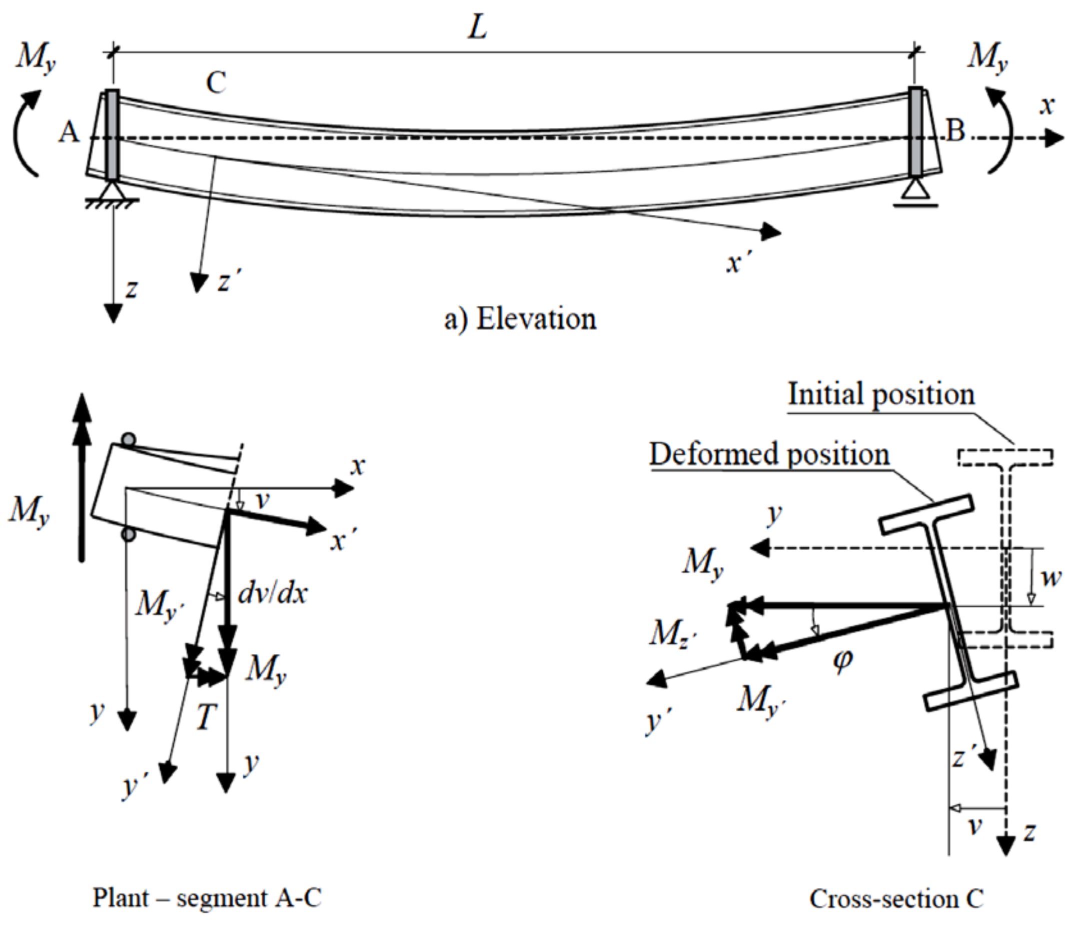




2- Elastic critical moment

To obtain the elastic critical moment, consider the simply supported beam of Figure 3.56, with the supports preventing lateral displacements and twisting but allowing warping and bending rotations around the cross sectional axes (y and z), submitted to a constant bending moment My. Consider the following assumptions:

1- perfect beam, without any type of imperfections (geometrical or material);
2- doubly symmetric cross section;
3- material with linear elastic behaviour;
4- small displacements ($\sin\phi = \phi$; $\cos\phi = \phi = 1$)



b) Internal forces at-section C

Figure 3.56 – Lateral-torsional buckling in a doubly symmetric I section under constant bending moment

The critical value of the moment denoted as $\boldsymbol{M_{cr}^{E}}$ (critical moment of the “standard case”) is obtained:

$$M_{cr}^{E}=\frac{\pi}{L}\sqrt{G\,I_T\,E\,I_z\left(1+\frac{\pi^2\,E\,I_W}{L^2G\,I_T}\right)}, \qquad (3.99)$$

By inspection of equation (3.99), it is observed that the critical moment of a member under bending depends on several factors, such as:

- loading (shape of the bending moment diagram);
- support conditions;
- length of the member between laterally braced cross sections;
- lateral bending stiffness;
- torsion stiffness;
- warping stiffness.

Besides these factors, the point of application of the loading also has a direct influence on the elastic critical moment of a beam. A gravity load applied below the shear centre C (that coincides with the centroid, in case of doubly symmetric I or H sections) has a stabilizing effect ($\boldsymbol{M_{cr,1}>M_{cr}}$), whereas the same load applied above this point has a destabilizing effect ($\boldsymbol{M_{cr,2}<M_{cr}}$), as illustrated in Figure 3.57. The calculation of the critical moment for design of a beam must also incorporate this effect.

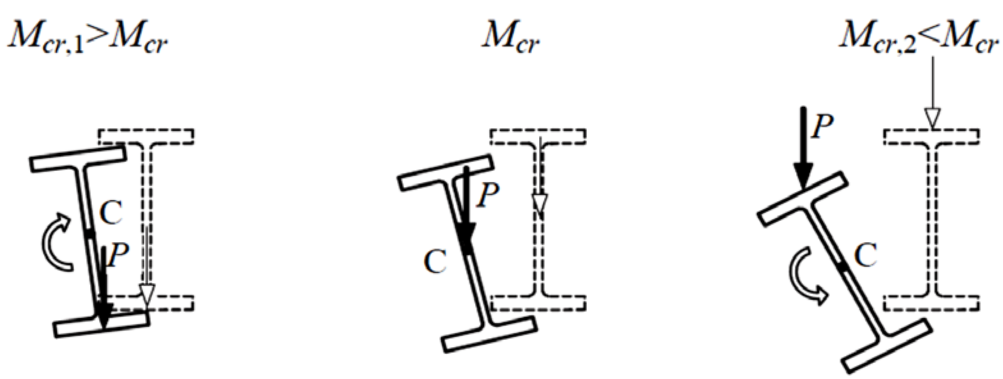


Figure 3.57 – Effect of the point of load’s application

Equation (3.99) is valid for the calculation of the elastic critical moment of a simply supported beam, with a doubly symmetric cross section and subjected to a constant bending moment (the “standard case”). However, in reality, other situations often occur, such as beams with non-symmetrical cross sections, with other support conditions, subject to different loading patterns and, consequently, subject to different bending moment diagrams. The derivation of an exact expression for the critical moment for each case is not practical, as this implies the computation of differential equations of some complexity.

Therefore, in practical applications approximate formulae are used, which are applicable to a wide set of situations.

the elastic critical moment can be estimated using equation (3.107). This is applicable to members subject to bending about the strong axis, with cross sections mono-symmetric about the weak z axis (see Figure 3.59), for several support conditions and types of loading.

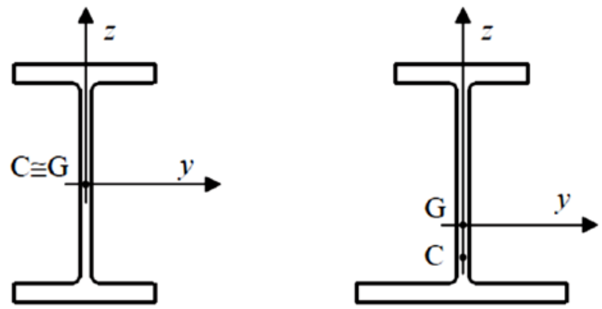


Figure 3.59 – Sections mono-symmetric about the weak axis

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \begin{array}{l} \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} \\ -(C_2 z_g - C_3 z_j) \end{array} \right\}, \tag{3.107}$$

Where

- C_1, C_2 and C_3 are coefficients depending on the shape of the bending moment diagram and on support conditions, given in Tables 3.6 and 3.7 for some usual situations.
- k_z and k_w are effective length factors that depend on the support conditions at the end sections. Factor k_z is related to rotations at the end sections about the weak axis z, and k_w refers to warping restriction in the same cross sections. These factors vary between 0.5 (restrained deformations) and 1.0 (free deformations), and are equal to 0.7 in the case of free deformations at one end and restrained at the other. Since in most practical situations restraint is only partial, conservatively a value of $k_z = k_w = 1.0$ may be adopted;

- $z_g = (z_a - z_s)$, where z_a and z_s are the coordinates of the point of application of the load and of the shear centre, relative to the centroid of the cross section; these quantities are positive if located in the compressed part and negative if located in the tension part;

- $z_j = z_s - \left(0.5 \int_A \left(y^2 + z^2 \right) \left(z / I_y \right) dA \right)$ is a parameter that reflects the degree of asymmetry of the cross section in relation to the y axis. **It is zero for beams with doubly symmetric cross section (such as I or H cross sections with equal flanges)** and takes positive values when the flange with the largest second moment of area about z is the compressed flange, at the cross section with maximum bending moment;

For more details about details and application of equation 3.107, review the textbook.

3- Effect of imperfections and plasticity

In the previous sub-section the elastic critical moment was obtained for an ideal member with constant bending moment (the "standard case"), and formulae were also presented, some exact and some approximate, for the calculation of the elastic critical moment in members with other support and/or loading conditions.

In the verification of the lateral-torsional buckling resistance, the effect of the following geometrical imperfections should be considered:

- the initial lateral displacements;
- the initial torsional rotations;
- the eccentricity of the transverse loads relative to the shear centre of the cross sections;
- residual stresses.

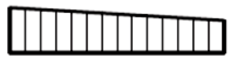
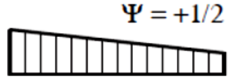
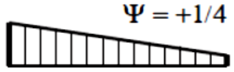
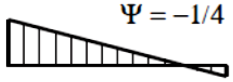
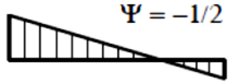
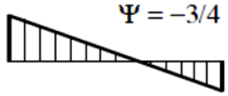
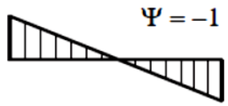
Due to the presence of geometrical imperfections, the real behaviour of a member diverges from the theoretical behaviour and the elastic critical moment is never reached.

The effect of geometrical imperfections may be introduced into the design procedure of a member under major axis bending in a similar way to that for design of a member under pure compression.

As for compressed members, residual stresses and other geometrical imperfections affect the lateral-torsional resistance of beams. In a simplified way, all these imperfections are taken into account through the equivalent imperfection concept.

Based on extensive numerical, experimental and parametric simulations (Boissonnade et al, 2006) it was concluded that the design of the majority of steel members (including members composed by rolled and welded I or H sections) could be done according to the European buckling curves, previously obtained for the design of members under pure axial compression.

Table 3.6 – Coefficients C_1 and C_3 for beams with end moments

<table>
<tr><th rowspan="2">Loading and support conditions</th><th rowspan="2">Diagram of moments</th><th rowspan="2">k_z</th><th rowspan="2">C_1</th><th colspan="2">C_3</th></tr>
<tr><th>$\psi_f \leq 0$</th><th>$\psi_f > 0$</th></tr>
<tr><td rowspan="18"></td><td rowspan="2">$\Psi = +1$
</td><td>1.0</td><td>1.00</td><td colspan="2">1.000</td></tr>
<tr><td>0.5</td><td>1.05</td><td colspan="2">1.019</td></tr>
<tr><td rowspan="2">$\Psi = +3/4$
</td><td>1.0</td><td>1.14</td><td colspan="2">1.000</td></tr>
<tr><td>0.5</td><td>1.19</td><td colspan="2">1.017</td></tr>
<tr><td rowspan="2">$\Psi = +1/2$
</td><td>1.0</td><td>1.31</td><td colspan="2">1.000</td></tr>
<tr><td>0.5</td><td>1.37</td><td colspan="2">1.000</td></tr>
<tr><td rowspan="2">$\Psi = +1/4$
</td><td>1.0</td><td>1.52</td><td colspan="2">1.000</td></tr>
<tr><td>0.5</td><td>1.60</td><td colspan="2">1.000</td></tr>
<tr><td rowspan="2">$\Psi = 0$
</td><td>1.0</td><td>1.77</td><td colspan="2">1.000</td></tr>
<tr><td>0.5</td><td>1.86</td><td colspan="2">1.000</td></tr>
<tr><td rowspan="2">$\Psi = -1/4$
</td><td>1.0</td><td>2.06</td><td>1.000</td><td>0.850</td></tr>
<tr><td>0.5</td><td>2.15</td><td>1.000</td><td>0.650</td></tr>
<tr><td rowspan="2">$\Psi = -1/2$
</td><td>1.0</td><td>2.35</td><td>1.000</td><td>$1.3 - 1.2\psi_f$</td></tr>
<tr><td>0.5</td><td>2.42</td><td>0.950</td><td>$0.77 - \psi_f$</td></tr>
<tr><td rowspan="2">$\Psi = -3/4$
</td><td>1.0</td><td>2.60</td><td>1.000</td><td>$0.55 - \psi_f$</td></tr>
<tr><td>0.5</td><td>2.45</td><td>0.850</td><td>$0.35 - \psi_f$</td></tr>
<tr><td rowspan="2">$\Psi = -1$
</td><td>1.0</td><td>2.60</td><td>$-\psi_f$</td><td>$-\psi_f$</td></tr>
<tr><td>0.5</td><td>2.45</td><td>$-0.125 - 0.7\psi_f$</td><td>$-0.125 - 0.7\psi_f$</td></tr>
</table>

- In beams subject to end moments, by definition $C_2 z_g = 0$.
- $\psi_f = \dfrac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$, where I_{fc} and I_{ft} are the second moments of area of the compression and tension flanges respectively, relative to the weak axis of the section (z axis);
- C_1 must be divided by 1.05 when $\dfrac{\pi}{k_w L}\sqrt{\dfrac{EI_W}{GI_T}} \leq 1.0$, but $C_1 \geq 1.0$.

Table 3.7 – Coefficients C_1, C_2 and C_3 for beams with transverse loads

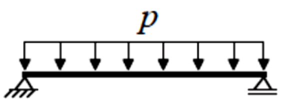
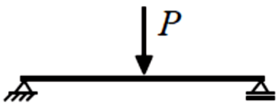
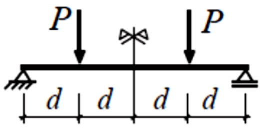
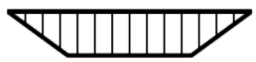
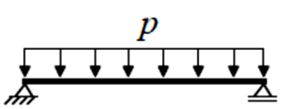
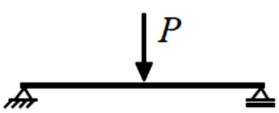
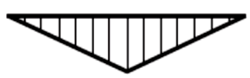
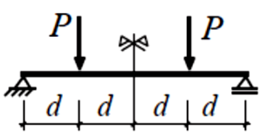
Loading and support conditions	Diagram of moments	k_z	C_1	C_2	C_3
		1.0 0.5	1.12 0.97	0.45 0.36	0.525 0.478
		1.0 0.5	1.35 1.05	0.59 0.48	0.411 0.338
		1.0 0.5	1.04 0.95	0.42 0.31	0.562 0.539

Table 3.7 – Coefficients C_1, C_2 and C_3 for beams with transverse loads

Loading and support conditions	Diagram of moments	k_z	C_1	C_2	C_3
 p		1.0 0.5	1.12 0.97	0.45 0.36	0.525 0.478
 P		1.0 0.5	1.35 1.05	0.59 0.48	0.411 0.338
 P P d d d d		1.0 0.5	1.04 0.95	0.42 0.31	0.562 0.539