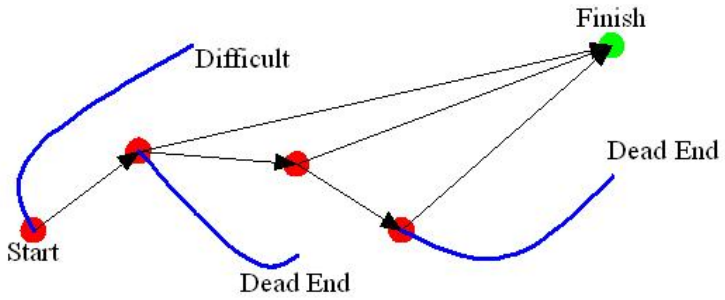
This document contains information on problem solving in mathematics, lecture preparation, the three step study session, exam preparation, and grading.

1 Mathematical Mountaineering: Problem Solving in Mathematics

Here is a diagram that represents conceptually the problem solving process in mathematics:



For different problems you will get different diagrams, with different possible paths. A solution to the problem would be represented by the path you followed to get from the Start to the Finish.

Notice that:

- there is more than one path from Start to Finish,
- some paths (thick blue lines) do not lead to the Finish, (the mathematics used is correct, but either they lead to mathematical dead ends or become too difficult to follow),
- there are choices (red circles) along the way.

You might think of the diagram as a mountaineering problem. You begin at the starting line and end at the finish line, and two people might choose different routes to get there. Experienced mountaineers read maps, survey the landscape around them, and rely on past experience to help them choose paths wisely. Even so, sometimes they choose poorly, and the path they are following becomes too difficult to traverse, so they have to backtrack to a point where they previously made a choice and choose differently. Some paths to the finish line are shorter than others, but just because a path is shorter than another doesn't mean it is necessarily better–they both got you to the finish line, which is the most important thing!

In the past, you may have practiced problem solving by having a guide who led you along the path from the beginning to the end, focussing solely on the path, possibly not even noticing that there were choices along the way. You learned this path through having the guide lead you along it many times, until you had memorized the path. In the end, you did memorize the path, and could follow it precisely every time. In terms of our mountaineering metaphor, you had learned how to get from one specific destination to another specific destination, following one specific path.

In university this is not how problem solving is approached. Certainly we want you to be able to solve problems, but we don't want you to do it by memorizing the path for each problem you study. We want you to be able to understand the choices that are made during the problem solving process, and make wise choices based on your understanding of where it is you are headed (what am I ultimately trying to do?), the mathematical landscape around you (are there square roots in the problem? Denominators? Equations to be solved?) and your past mathematical experience.

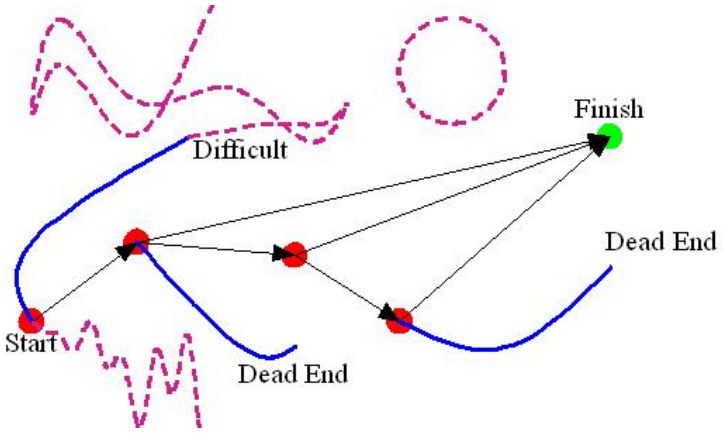
We want you to develop these skills since they can be applied to new problems you encounter, and you can then find the path to the solution without the help of a guide. Also, sometimes studying the different paths to a solution can help you better understand the solution and what it means.

As we solve problems, focus on the wise choices more than memorizing the path that we follow. Understand the choices we make, why we choose the way we do, and never ever settle for the answer "because it works". You must strive to understand why it works, and why we choose the paths we do. In this way you will learn the mathematical skills necessary to solve many problems, and problems which are different from the ones studied in class. In essence, you will become an experienced mathematical mountaineer, able to effectively survey the landscape around you and find your way through unfamiliar territory using your mathematical mountaineering skills effectively.

I will help you by being a guide, a guide who shows you one or more paths that lead to the solution but more importantly a guide who explains why we followed the path we did, why we had the choices we had along the way, and why we made the choices we did. At first, this may seem like extra work, since the path to the solution is short and easy to follow. You may be tempted to memorize the path, and although this may work for awhile, it is not what you should be striving to do. There are simply too many paths to memorize, and you can expect to see problems in your life which you have not seen before which will require you to survey the mathematical landscape and make wise choices in order to find a solution.

Mathematics builds on itself, so your past mathematical experience is very important to help you solve problems, and this is especially important in calculus. You cannot understand integrals without first understanding derivatives, and you cannot understand derivatives without first understanding limits. Also, calculus is performed on functions, so you must understand functional notation, and the behaviour of functions like cosine, sine, logarithm, exponential, polynomials, and rational functions. Calculus uses algebra, so you must have strong algebra skills (solving equations, finding common denominators, rationalizing square roots, etc.).

Below is a diagram that represents what happens when someone without a good understanding of these prerequisite skills tries to solve problems. The paths they end up on are the dashed red paths. A good mathematical mountaineer would never find themselves on these paths, because you only get on these paths by using mathematics incorrectly.



Without the skills necessary to follow the mathematically correct paths, it is easy to stray into unproductive alleys, sometimes changing the problem into some different problem through algebra or other errors. It is hard to focus on the choices you have to make when all your energy is spent trying to stay on the path! This would be like a mountaineer with heatstroke, whose entire energy is spent simply trying to walk instead of surveying the landscape and making wise choices–the odds are this mountaineer will not make it to the finish line.

I have drawn the lines that are using correct mathematics, whether they lead to a solution or not, as gently curving lines, since these lines are the ones an experienced mathematical mountaineer would follow. Always following only mathematically correct paths means the experienced mathematical mountaineer has a good chance of success in finding solutions to a wide variety of problems.

2 How to Succeed in My Courses

The basic rule of thumb for success in my classes is that you should be striving to understand concepts and never simply trying to memorize techniques or formulas. Memorization may have served you well in the past, but it will not be sufficient for success in university. You will need to understand concepts, and apply these concepts to problems which are different from the ones studied in class. See the Three Step Study Session below for a studying technique that does not rely on memorization.

To succeed in my 4cr courses you will need to be willing to spend, per week, nine hours outside of class reading the textbook and working problems (UMM policy is that one credit is defined as three hours of learning effort per week for an average student to earn an average grade in the course: 4 credits times 3 hours/week/credit - 3 hours/week in lecture = 9 hours/week outside class). For Calculus I and II, which are 5cr courses, you will need to spend about 10 hours per week outside of class. You might plan on blocking out a schedule for the week that ensures you spend the appropriate amount of time on each of the courses you are taking.

The majority of your learning will take place outside of lectures, as you work problems and read the text. You will not learn everything you need to learn in a university course simply by coming to lectures, nor if you miss lectures. You must come to lectures and put in the time outside of class to master the material.

2.1 Lecture Preparation

To get the most out of the lectures you should:

- Come to class.
  It's true–coming to lectures is the most important thing you can do to help you succeed.
  If you do miss a lecture, you absolutely must find out what you missed. The answer to the question "Did I miss anything important?" is always *Yes*. Talk with a friend in the class to get caught up on any notes you missed. Missing class puts you behind, and you will have to work harder than usual to catch up.
- Work on homework for the course every day (even Tuesday and Thursday).
  I can not stress enough how important it is that you work problems! The homework identifies the types of problems from the text that should be mastered.
  Any homework solutions I provide serve as models for what constitutes complete solutions. Write out your solution first, then compare your solution to mine and make note of anything that you don't understand. Make sure to get answers to any questions you have as soon as possible!
- Read the section before the lecture and do not fall behind.
  Make sure when you are reading that you are reading for comprehension. This means you are thinking about what you are reading, rereading paragraphs when necessary, and pausing to work through examples to ensure you understand them completely. Make notes about the material, especially anything you don't fully understand. Then come see me, your study group, or a tutor to discuss these concepts so that you do understand them. Reading for comprehension takes practice, but it is an essential skill to develop to help you succeed at university. As you read, try to focus on understanding rather than memorization.
- Debrief after lectures.
  After a lecture, you should spend a few minutes (maybe fifteen) going over your lecture notes and making sure you understood the topics that were discussed that day. Compare your lecture notes to the textbook, since the topic may be explained slightly differently in the text.
- Discuss any difficulties with me during office hours, or by appointment.
  Please make the most of my Office Hours! The content of the course can be difficult at times and I expect to see you all in my office at some time or other. When you come to office hours, come prepared: for conceptual questions bring your notes on the topic and any problems you have done relating to that topic; for homework questions, bring the work you have done on that problem.
- Visit the Math Room in Briggs Library.
- Form a study group.

2.2 The Three Step Study Session

Thanks to Kathryn Gonier-Klopfleisch from Academic Assistance in Briggs Library who informed me of this format for a study session.

If you are looking for some structure for your study sessions, the following might work for you. It can work in any course, but it is tailored here specifically for mathematics where you are trying to learn a specific skill or concept. It is similar to the warm-up/workout/cool-down that athletes use to train, so a particular study session is broken up into three parts, with the goal being able to solve problems without the aid of a text. The time you spend on each section should be tailored to whatever you feel comfortable with.

- Warm-up
  Review the lecture notes, textbook, or other resources that pertains to the material you are studying. This will warm up your mental muscles, and prepare you to solve problems. The warm-up might take 15 to 25 minutes.
- Drill
  Work problems from the homework. Use resources at hand (textbook, notes, solutions which are provided, study partners) to help you solve the problems. The drill might take an hour or longer.
- Cool-down
  In this period your goal is to make the knowledge you have acquired during the study session "portable", that is, to make sure you can solve the problems you have completed without reference to the textbook, your notes, or the help of others. This is extremely important if you are preparing for a no-aids test! The cool-down might take 45 minutes.
  During the cool-down, you can do the following:
  - Review the method of solution for the problems you studied. Write the method of solution down in words, or say it aloud to your study partner or to yourself.
    The goal is to, as much as possible, be able to talk about the solution without using mathematical notation.
  - Go back and work through the problems again–this time without reference to resources.
    If you need to use a text or some notes to help you complete the solution, and you won't be able to use these resources on the test, you know you haven't mastered the concepts and should spend more time on them.
  - Make a list of any unanswered questions that arose during your study session.
    These questions are the ones you will ask your tutor, your study group, or your professor during office hours.

2.3 Exam Preparation

Here are some suggestions to guide your preparation for tests. You probably won't use all of these techniques, but you should try to find techniques which work for you. If you have a technique which works for you and isn't listed here, please let me know so I can pass it on to your peers!

- A good way to prepare for an exam is to follow the lecture preparation techniques above faithfully. This will give you a good foundation to begin focussed exam preparation several days before the exam.
- Review assignments and homework solutions posted on the course webpage.
  Attempt the questions before reading my solution; if you get stuck, read only as much of the solution as you need to get unstuck, and then try to finish the problem. Refer to the text as necessary.
  This will provide you with an overview of the material you need to be studying.
- Review the concepts described in the concept reviews for each test on the course webpage.
  Can you talk about the concepts? Do you know the basic results from the concept review?
- Make notes on the topics you are studying as you review.
  Write short sentences to describe how to solve problems. Describe verbally to a friend how you would solve a particular type of problem. This verbal description will help you remember the process of solving particular problems during the test, and helps you become independent from the text.
  Include example problems if a certain type of problem appears frequently.

- Do problems from the text for which you have solutions, that are similar to the problems you have seen so far in your test preparation.
- Branch out and do other types of problems that appeared less frequently throughout the section.
- Studying in many short sessions is more effective than one or two marathon studying sessions.
  Consider making a time schedule which maps out when and what you will study.
  You might choose a long term time frame (Friday Morning: History, Friday Afternoon: Precalculus, etc), and a short term time frame for each day that lists what exactly you will focus on. The short term time frame can be created every day and be more flexible.
  Create goals which you can reasonably be expected to meet.
- Get as much sleep as possible while you study for tests. Come to your exams well rested, and mentally sharp.
- Study in an environment that mimics the environment the test will take place in. It should be quiet and clear of clutter.
- For a given chapter (or section), create practice "tests" for yourself, maybe three or four questions which you have the solution to, and then answer them without reference to the text. Correct your test yourself, or work with a friend and have them correct your test and you correct theirs. Do not move on to other questions until you have mastered these ones. You might consider imposing a time limit on these mini-tests.
- If you do study in groups, also study alone so you can focus on the types of questions you need to work on.
- Come and talk with me (email me to set an appointment if necessary) if there are questions you have.

Other Resources

Talk with, work with and get to know your fellow students. University is an incredibly exciting and fun time, and you would be wise to share the experience with those around you. Work together on assignments, discuss study habits...but also get together with your peers and listen to music, talk about things you know nothing about (that is how you will learn!), and help others. OK, this advice goes beyond math!

Make use of all the resources that UMM has to help you succeed. We want you to succeed! There are many services available in the following offices; you may wish to surf their web sites to see what they have to offer. If you are looking for something specific, I suggest you start at the Directories Page (http://www.morris.umn.edu/directories/) for UMM.

- The Academic Assistance Center 362 Briggs (http://www.morris.umn.edu/services/dsoaac/aac/)
  Services:
  - Peer Tutoring
  - Drop in Services (includes Math Room)
  - Study Tables
  - Academic Counselling
  - and lots more
- Student Counselling 235 Behmler (http://www.morris.umn.edu/services/counseling/)
- Disability Services 362 Briggs (http://www.morris.umn.edu/services/dsoaac/dso)
- Multi-Ethnic Student Program (MSP) 110 Multi-Ethnic Resource Center (http://www.morris.umn.edu/services/msp/)
- The Writing Room 327 Briggs (http://www.morris.umn.edu/academic/writing_room/)

3 Grading

Some of you may be used to seeing math graded based on the final answer, and either correct or incorrect with little attention paid to the steps that led you to the answer. I am grading you based on the process you describe to obtain your solution, as well as your final answer.

- I read each solution as a self contained document, and look for internal consistency in the solution you provide. Questions I have in my head are:
  - Are the steps clearly explained?
  - Is the mathematics you used correct?
  - Is the mathematical notation used correctly?
  - Is the mathematics you used the correct mathematics needed to solve the problem?
- I do not compare your solution to my solution. Solutions which are different from mine can be 100% correct, as long as they are explained well and use correct mathematics.
- All errors are not equal, some are more serious than others. For example, forgetting to distribute a minus sign in a well presented solution is not as serious as a solution that is hard to follow and which contains errors in mathematical notation. The grade for the second will be lower, even if the final answer is correct.
- I will comment on things you have done well, and also on things you could do differently to improve your solution.
- If you do not understand a comment, ask me to explain it in more detail.

Grading Notations:

- $\surd$: A check mark means "this is correct", usually in terms of the step you used in the process of solution.
- X or $\otimes$: these symbols mean "there is an error here". I will usually explain what the error is, or the type of error, such as
  - arithmetic error,
  - algebra error,
  - functional notation error,
  - incorrect derivative, etc.
- I will often use an arrow to indicate that an error has occurred between two steps.

Example: