# Physics P201/P202 Laboratory Manual 

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## Additional Notes (IMPORTANT!):

1) Attendance at ALL laboratory exercises is MANDATORY (see your instructor for further details).
2) A schedule with dates of specific labs will be posted in PS106 and on the web.
3) A pre- and post- tests may be given during the first and the last week in the Fall Semester.
4) One additional lab grade will be given for a set of take home exercises over Appendices A and B. A review of Appendices A and B will be given during the first lab meeting in the Fall Semester.
5) Some labs will be done in advance of when you cover the material in class. This is intentional. Please read ahead in the textbook when necessary.

## Format(s) of Lab Reports

Each week you will turn in a report of the previous week's experiment.

## Every lab report should include the following:

a) Raw data, signed by the instructor before you leave (no Xeroxes please);
b) Lab partners names (no more than three to a group, total!);
c) An analysis of your results following Appendix A if you are reporting graphical data (using LoggerPro);
or
d) An analysis of your results following Appendix B if you are reporting constant data;
e) Answers to all the questions in the lab using complete sentences and proper English, please!
f) A conclusion in your own words including: what you learned during the lab, sources of uncertainty, discussion of which measurements cause the greatest uncertainty, ways to improve the experiment, explanation of whether your results support or do not support what you were trying to prove; if you use other sources for information (e.g. Wikipedia, your textbook, etc.), site your sources.

You will turn in one of two styles of lab reports as specified in each particular experiment:

## For an informal lab report:

a) Turn in one set of results and/or answers for each group. Be sure everyone in your group agrees on the answers you write down. There will be quiz and/or test questions about the laboratory exercises.

## For a formal lab report:

a) Each student turns in their own report. (One report per student.)
b) Include a description of the apparatus and procedure and an indication of the purpose of the experiment in your own words. This should be brief (outline form) but complete enough that someone else (for example another student) could use your report to duplicate the experiment. Diagrams are often helpful. Do not copy the lab manual.

## Policies:

a) Attendance is REQUIRED for ALL students for ALL labs. Special arrangements may be made under exceptional circumstances, but this will usually result in points being taken off. See your instructor for details.
b) You may finish the lab write-ups at home. HOWEVER, any raw data will be examined and signed by the instructor before you leave. The raw data is not to be recopied, Xeroxed or modified but must be turned in as is with the lab report. This policy is intended to get you into the habit of making a neat record of your results as you go along and will help you avoid the problem of scribbling down numbers which you later cannot read or understand.
c) Lab reports must be turned in on time for full credit; points will be deducted for lateness.
d) NOTE: Many labs will be done in advance of when you cover the material in class. This is intentional; please read ahead in the textbook when necessary.
e) Points will be lost for lateness and excessive sloppiness.

## Motion Graphs (with the LabQuest) (formal)

This lab is designed to help you understand the connection between graphs and actual motion along a straight line.

## Part one: Motion Graphs with the LabQuest

The LabQuest is a data collecting device that can be hooked up to several different probes, many of which you will use in other labs. In this lab, the probe is a motion sensor which detects location using an ultrasound signal (much the same way a bat uses echo-location to navigate). The LabQuest collects data from the sensor and presents a graph of the data. Instructions for collecting data are at the end of Part one.


Collect data (directions below) while doing each of the following activities. Be sure to share the LabQuest and probe so that each lab partner gets to make at least one of the graphs.
a) Walk toward the wall at a constant speed.
b) Walk toward the wall at a different constant speed.
c) Walk away from the wall at a constant speed.
d) Walk away from the wall at a constant speed and then suddenly stop and walk towards the wall at the same speed.
e) Walk away from the wall starting out slowly and then speeding up.
f) Walk away from the wall starting out fast and then slow down.

Repeat any cases where you did not get the result you thought you would get. For each of the above cases draw a sketch of the distance vs. time graph and velocity vs. time graph in you lab report (two graphs for each case, a -f ). Draw the velocity vs. time graph right below the position vs. time graph so that their time axes match.

1) Comment on how the slope of the position graph at a certain time is related to the value of velocity at the same time.

For each of the following eight graphs (see next page), create an identical graph on your LabQuest by collecting data while you move in an appropriate manner. You will have to experiment to find out what motion causes each graph each will require different motion.
2) In your lab report, describe in detail (similar to the way it is done in a-f above) the motion you had to perform in order to obtain each graph on the next page. You should have eight descriptions - one for each graph.

NOTE: There are eight separate graphs here - the velocity and position graphs do NOT correspond to the same motion.









Collecting data with the LabQuest (these are nearly the same for all labs using the LabQuest):
A) Turn the LabQuest on (small button at top left of device, above the screen). It will take several seconds to start. Select "LabQuest App" and wait for the program to load.
B) Plug in the probe you want to use.
a) For this lab plug in the motion probe into 'DIG1' port on the side of the LabQuest.
b) The LabQuest will recognize some probes automatically; however it does not recognize the motion detector used in this lab.
c) Use the plastic stylus to tap (gently!) on the 'Sensor' menu. Select 'Sensor Setup'. NOTE: You do not have to tap very hard with the plastic stylus; please try not to scratch the LabQuest screen.
d) Select 'DIG1' and then 'Motion Detector'. Also select ' $m$ ' for meters under 'Units'
e) Tap 'OK'. You should hear the Motion Detector start clicking and a number will appear on the screen. This is the distance in meters from the Motion Detector to the target at which it is pointed.
d) Play around with the Motion Detector to get a feel for its range. Notice that closer than about 30cm it does not give an accurate reading. Pointing at a target too far way also will produce erratic results.
C) Set parameters and make a graph.
a) The LabQuest automatically sets the sample rate at 20 samples per second and length of time for data collection at 5 s which should be adequate for this lab. However you may change these parameters if you wish by tapping on 'Sensor' menu and then 'Data Collection' to get to the screen where they are set.
b) To take data and graph it, first tap on the small graph icon tab at the top of the screen. The clicking will stop and the LabQuest is ready to take 5 s of data and make a graph.
c) To take data, either tap on the forward button on the lower left of the screen or push the button with the same icon on the front of the LabQuest. The LabQuest will take 5 s of data and plot it on the screen.
d) You can change the variables being plotted (position versus time, velocity versus time, acceleration versus time) from the menu choice 'Graph Options' under 'Graph' in the graph screen.
D) Collect more data, save data, plot data, or quit.
a) The small file cabinet icon near the upper right of the screen allows you to temporarily keep data under the names 'Run 1', 'Run 2', etc. You can look at the various runs you have collected by taping on the 'Run 1' bar.
b) Although you can save all your data using 'Save' under the 'File' menu, please do not do that (or delete the files when you are done) so that the memory does not fill up.
c) If the program LoggerPro is installed locally on computers in PS100 you will be able to print out these graphs using a USB cable. The Physics Department is working with IT to install the program. If this option is not available you may have to make hand sketches of your data for your lab report.
e) To shut the LabQuest down when you are done, hold the start button down (upper left button above the screen); when the dialog screen appears tap 'Discard' to discard the data you have collected.

## As always, when in doubt ask the instructor!

## Part two: Motion Graphs with Ticker Tape

In this part you will investigate the linear motion of a cart on a low friction track. The velocity and acceleration of the cart will be measured indirectly by counting dots made on a piece of tape as the cart rolls down the track.


To set up the experiment:
a) Put the timer end of the track onto the stepped block. Use a low step for the motion with constant velocity and the highest step for the motion with constant acceleration.
b) Make sure the timer is turned off and plug it into the wall.
c) Cut a piece of silver tape a little longer than the length of the track. The tape is threaded through the timer and attached to the cart with a clamp so that the shiny side of the tape faces upward after it goes through the timer.
d) Arrange things so the cart will pull the tape smoothly through the timer as it rolls down the track (practice a few times before actually taking data). Be sure the tape is laid out flat on the other side of the timer before you start, so that it does not get tangled and stop the cart.
e) The black sliding switch on the timer should be set to 10 Hz . This means that dots will be made at a rate of 10 per second, so that the time interval between them is 0.1 s .

In order to take data, first push the cart to the high end of the track (next to the timer). Turn the timer on and release the cart. The timer should leave faint dots on the tape as the cart pulls the tape through the timer.

## Motion with constant velocity

To have the cart rolling with approximately constant velocity the track should be set on a low step, so that friction is just compensated for by the tilt in the track. Adjust the leg on the track up or down until the cart moves at constant velocity with the tape attached. You will have to give the cart a slight shove to start it moving (the acceleration is zero after all!).

After you have practiced rolling the cart a few times, take some data as described above. The dots should be evenly spaced (if they are not, adjust the height of the track until they are). Select ten evenly spaced consecutive dots and measure the distance on the tape from the beginning dot to each of the following dots as shown ( $x_{1}, x_{2}, x_{3}$, etc.):


Record the distance traveled to each of these dots ( $x_{1}, x_{2}, x_{3}$, etc.) along with the total time taken (remember that the time between dots is 0.1 s so the time to the third dot is 0.3 s , time to the forth dot is 0.4 s , etc.).

Plot using the computer program LoggerPro a graph of distance (on the y-axis) versus time (on the x -axis). The data points on the graph should follow a straight line, indicating constant velocity. Add the regression line to your graph (remember to enable point protectors and turn off the 'connect points' option).
3) Does your data agree with the regression line? (See Appendix A on how to address this question. The rulers you are using have a precision of about 1 mm , so the uncertainty of each $x_{i}$ should be about $1 \mathrm{~mm}=0.1 \mathrm{~cm}$ in this case.) If your data does not agree with the regression line, discuss possible reasons for the disagreement.
4) Your graph is a distance vs. time graph, so the slope is velocity. From the slope of your graph (from the Linear Fit box in LoggerPro), what is the velocity of the cart?
5) Write the equation of motion which this graph is describing (with $x$ and $t$ as variables).

## Motion with constant acceleration

Repeat the data taking procedure except with the timer end of the track on the highest step. You may wish to use an even steeper slope by putting the starting end of the track on a book to insure you have acceleration. Now the dots should be getting further apart as the cart goes down the track. If they are not, raise the starting end of the track even higher (up to about 10 cm above the table).

Once you have the track properly adjusted, calculate the angle $\theta$ it makes with the horizontal. Do this by measuring the height of the starting end of the track above the table and the length of the track, and using trigonometry. Do not use a protractor because the angle is too small to measure it accurately with a protractor. Record the height, the length, and the angle of the track.

Choose ten consecutive dots separated by ever increasing distances. Make a plot of the distances from a zero dot to the other dots vs. time, just as you did above for the motion with constant velocity. This graph will not be a straight line.
6) Why is this graph not a straight line?

To get a graph which will be a straight line, you can plot the velocity of the cart versus time. For the ten selected dots, measure the distances between the adjacent dots (e.g., $\Delta x_{4,5}$ shown in the figure below is the distance between the $4^{\text {th }}$ and the $5^{\text {th }}$ dots). Divide each such distance by 0.1 s . This gives you the average velocities for each consecutive 0.1 s time intervals. These average velocities are equal to the instantaneous velocities at the times in the middle of each interval (e.g. $\Delta x_{4,5} / 0.1 \mathrm{~s}$ is the velocity at time $t=0.45 \mathrm{~s}$; see the figure). Make a table with the middle times in the first column and the average velocities in the second column.


Plot the velocity (y-axis) versus time (x-axis). This graph should be a straight line graph. Add the regression line to your graph, enable point protectors, and turn off the 'connect points' option.
7) Does your velocity vs. time data agree with the regression line? Explain. (The uncertainty of each $\Delta x_{i}$ should be about $1 \mathrm{~mm}=0.1 \mathrm{~cm}$. This translates into the uncertainty of $1 \mathrm{~mm} / 0.1 \mathrm{~s}=10 \mathrm{~mm} / \mathrm{s}=1 \mathrm{~cm} / \mathrm{s}$ for each velocity value, see Appendix B.) If your data does not agree with the regression line, discuss possible reasons for the disagreement.
8) The slope of velocity vs. time graph is acceleration. Based on the slope of the regression line, what is the acceleration of the cart?
9) From the value of the y-intercept, what is the initial velocity of the cart for the ten-dot time period you have chosen?
10) Write the equation describing how the velocity of the cart changes over time in your experiment.
11) If there were no friction on the track, the acceleration would be given by $a=g \sin \theta$ where $\theta$ is the angle of the track. Calculate the expected value for the acceleration using this formula and the angle of the track you recorded above. Your actual value for the acceleration was reported in question eight above. Do the two values agree? (See Appendix B on how to compare the results of two measurements.) If they do not agree, explain what could have caused the disagreement.
12) Describe in detail how the data and calculations for the cart on the track would change if the switch setting on the timer were set on 50 Hz instead of 10 Hz . Would the values for $x, \Delta x$ and $t$ be different? Why or why not? Would the values for the velocity and acceleration of the cart be different? Why or why not?

Be sure to discuss your findings and provide some comments about the lab in the form of a conclusion.
This is a formal lab: one report per student. Be sure to read the format page in this manual (page 3).

## Vectors (informal)

Up until this course you probably have only used one type of numbers called scalars. Scalars are mathematical objects which have only one piece of information associated with them, their magnitude or size. Examples of scalars in physics are temperature, speed, mass, energy.

This lab investigates mathematical objects called vectors which have two pieces of information associated with them: magnitude and direction. Some examples of physical quantities which must be represented by vectors are velocity, force, momentum, magnetic field. All of these have directions associated with them. We will use a bold face letter to indicate a vector in the following.

Since vectors are not mathematically like scalars, we expect the addition, subtraction, multiplication and division of vectors to be different from the scalar numbers you are accustomed to. Scalars add the following way:
$3 \mathrm{~m}+4 \mathrm{~m}=7 \mathrm{~m}$.
If $\mathrm{A}=5 \mathrm{~N}$ and $\mathrm{B}=6 \mathrm{~N}$, then $\mathrm{A}+\mathrm{B}=11 \mathrm{~N}$.
When doing arithmetic with vectors, however care must be taken to keep track of the additional information which vectors contain. For example:
3 m West plus 4 m East $=1 \mathrm{~m}$ East (not 7 m as it does for scalars).
If $\mathbf{A}=5 \mathrm{~N}$ north and $\mathbf{B}=6 \mathrm{~N}$ East, then $\mathbf{A}+\mathbf{B}=7.8 \mathrm{~N}$ at $40^{\circ}$ Northeast (not 11 N ).
There are three ways to add vectors practiced in this lab: the trigonometric method, the geometric method, and the force table method. Each method is demonstrated below by adding the following two vectors:
$\mathbf{A}=100 \mathrm{~N}$ at $10^{\circ}$ counterclockwise (ccw) from the +x axis
$\mathbf{B}=150 \mathrm{~N}$ at $70^{\circ}$ counterclockwise (ccw) from the +x axis

## Force table

The force table represents vectors by masses hanging from strings. We first choose a scale. For example, we may represent 100 N by 50 g on the force table. Using this scale, $\mathbf{A}$ is represented by $50 \mathrm{~g} \mathrm{at} 10^{\circ}$ and $\mathbf{B}$ is represented by a 75 g mass at $70^{\circ}$. To find the sum $\mathbf{A}+\mathbf{B}$ put the 50 g mass at $10^{\circ}$ (just an empty hanger) and the 75 g mass at $70^{\circ}$ (the hanger plus 25 g ) on the force table. By trial and error add weights to a third string and adjust its position until it balances the other two. The found vector is called equilibrant. It has the same magnitude as the vector $\mathbf{C}=\mathbf{A}+\mathbf{B}$ but it points in the direction opposite to the direction of $\mathbf{C}$. To find the direction of the vector $\mathbf{C}$ itself, take the direction of the equilibrant and add (or subtract) $180^{\circ}$. In our example the equilibrant is at $225^{\circ}$ and is 110 g (including the hanger mass), so we conclude that C is at $45^{\circ}$ and has a magnitude of about 220 N . (Notice this is not the same as adding 100 N and 150 N which would give an incorrect answer of 250 N .)

## Graphical method

The graphical method is done on graph paper. All lengths are drawn to a chosen scale and all angles are measured by protractor in this method. For the above vectors let us choose 5.0 cm on the paper to represent 100 N of force. Vector $\mathbf{A}$ must be 5 cm long and drawn at an angle of $10^{\circ}$ (measured with a protractor) and vector $\mathbf{B}$ will 7.5 cm long at an angle of $70^{\circ}$. To add $\mathbf{A}$ and $\mathbf{B}$ (remember vectors do not add like numbers so the answer is not 12.5 cm ), draw $\mathbf{A}$ first. Then draw $\mathbf{B}$ with its tail starting at the tip of $\mathbf{A}$. The sum $\mathbf{A}+\mathbf{B}$ is the vector $\mathbf{C}$ which stretches from the tail of $\mathbf{A}$ to the tip of $\mathbf{B}$. To get the length of $\mathbf{C}$, measure it in cm then convert back to Newtons using the chosen scale. In the drawing below, $\mathbf{C}$ is 11 cm long so it represents a force of about 220 N . It is measured to be at an angle of about $45^{\circ} \mathrm{ccw}$ from the +x axis, so vector $\mathbf{C}$ is 220 N at $45^{\circ} \mathrm{ccw}$ from the +x axis.


## Trigonometric method

The trigonometric method consists of breaking down the vector into x and y components using trigonometry. To add two vectors, the x components for each are found and added and the y components for each are found and added. The resulting vector has an x component equal to the sum of the two x components and a y component equal to the sum of the $y$ components.


First find the x and y component of $\mathbf{A}$ :
$\mathrm{A}_{\mathrm{x}}=100 \mathrm{~N} \cos 10^{\circ}=98.5 \mathrm{~N}$
$\mathrm{A}_{\mathrm{y}}=100 \mathrm{~N} \sin 10^{\circ}=17.4 \mathrm{~N}$
Then find the x and y components of $\mathbf{B}$ :
$\mathrm{B}_{\mathrm{x}}=150 \mathrm{~N} \cos 70^{\circ}=51.3 \mathrm{~N}$
$B_{y}=150 \mathrm{~N} \sin 70^{\circ}=141 \mathrm{~N}$
$\mathbf{C}=\mathbf{A}+\mathbf{B}$ has components $\mathrm{C}_{\mathrm{x}}=\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}}=150 \mathrm{~N}$ and $\mathrm{C}_{\mathrm{y}}=\mathrm{A}_{\mathrm{y}}+\mathrm{B}_{\mathrm{y}}=158 \mathrm{~N}$. The magnitude of $\mathbf{C}$ is given by $\mathrm{C}=\left[\mathrm{C}_{\mathrm{x}}^{2}+\mathrm{C}_{\mathrm{y}}^{2}\right]^{1 / 2}=218 \mathrm{~N}$. The angle C makes with the +x axis is given by $\tan ^{-1}\left(\mathrm{C}_{\mathrm{y}} / \mathrm{C}_{\mathrm{x}}\right)=46.5^{\circ}$.

## Part one: The Force Table

Find the result of adding the following vectors using the force table.
a) $\quad \mathbf{A}=100 \mathrm{~N}$ at $20^{\circ} \mathrm{ccw}$ from the +x axis and $\mathbf{B}=100 \mathrm{~N}$ at $20^{\circ} \mathrm{cw}$ from the +x axis
b) $\quad \mathbf{A}=100 \mathrm{~N}$ at $45^{\circ} \mathrm{ccw}$ from the +x axis and $\mathbf{B}=100 \mathrm{~N}$ at $45^{\circ} \mathrm{cw}$ from the +x axis
c) $\quad \mathbf{A}=100 \mathrm{~N}$ at $90^{\circ} \mathrm{ccw}$ from the +x axis and $\mathbf{B}=100 \mathrm{~N}$ at $90^{\circ} \mathrm{cw}$ from the +x axis

Let the equilibrant be 100 N at $180^{\circ}$. Find the magnitudes of $\mathbf{A}$ and $\mathbf{B}$ when they are, correspondingly, at
d) $\quad 10^{\circ} \mathrm{ccw}$ and $10^{\circ} \mathrm{cw}$ from the the +x axis
e) $50^{\circ} \mathrm{ccw}$ and $50^{\circ} \mathrm{cw}$ from the the +x axis
f) $80^{\circ} \mathrm{ccw}$ and $80^{\circ} \mathrm{cw}$ from the the +x axis

1) A heavy bird sits on a clothesline. When is the tension in the line greater, when the line sags a lot or when it sags a little? Explain. (Think how this question is related to the above exercises.)

Find the result of adding the following vectors using the force table.
g) $\quad \mathbf{A}=300 \mathrm{~m} / \mathrm{s}$ at $45^{\circ} \mathrm{ccw}$ from the +x axis and $\mathbf{B}=200 \mathrm{~m} / \mathrm{s}$ at $80^{\circ} \mathrm{ccw}$ from the +x axis
h) $\quad \mathbf{A}=300 \mathrm{~T}$ at $45^{\circ} \mathrm{ccw}$ from the +x axis, $\mathbf{B}=200 \mathrm{~T}$ at $80^{\circ} \mathrm{ccw}$ from the +x axis, and $\mathbf{C}=100 \mathrm{~T}$ at $170^{\circ} \mathrm{ccw}$ from the +x axis

## Part two: Graphical Method

Using scale drawings, add the following vectors (all angles measured counterclockwise from the +x axis).
a) $\quad 7 \mathrm{~N}$ at $10^{\circ}$ and 6 N at $75^{\circ}$
b) $\quad 50 \mathrm{~m} / \mathrm{s}$ at $30^{\circ}$ and $65 \mathrm{~m} / \mathrm{s}$ at $210^{\circ}$
c) $\quad 300 \mathrm{~T}$ at $10^{\circ}, 200 \mathrm{~T}$ at $30^{\circ}$ and 100 T at $40^{\circ}$
d) Subtract 6 N at $75^{\circ}$ vector from 7 N at $10^{\circ}$ vector
(Hint: To subtract a vector is the same as to add the opposite of that vector. The 7 N vector is drawn first, then the opposite of the 6 N vector is added.)

## Part three: Trigonometric Method

Verify the results of Part 1g, Part 1h, and Part 2(a-d) by using trigonometry to calculate the answers. Remember that you compare the resultant vector from the force table, not the equilibrant vector.
2) You may have noticed that the answers in Part three are not exactly equal to the results you have found using the force table and the graphical method. Which method is more accurate? How do you know?
3) Which of the above three methods can be used to subtract vectors? Explain.
4) What is the difference between a vector and a scalar (in your own words)?

Be sure to discuss your findings and provide some comments about the lab in the form of a conclusion.
This is an informal lab. You need to turn in one set of answers per group. Be sure everyone in the group understands and agrees on the answers.

## Projectile Motion (formal)

## Follow Appendix B for reporting your results in this lab.

## Part one: Zero launch angle

In the first part you will fire projectiles horizontally from the table using a spring gun to find the launch velocity for two different balls. Do not change the tension in the spring gun during this lab.


The $x$ and $y$ motions occur independently of each other. Only the length of time that the ball spends in the air is common to the two motions. There is no initial component of velocity in the $y$ direction if the ball is fired horizontally. Thus for the motion in y direction we can write $y=1 / 2 g t^{2}$, where g is the acceleration of gravity $\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)$ and $t$ is the time of fall.

1) Measure the initial height of the ball above the floor $(y)$. Calculate the time $t$ the ball spends in the air.

Find the approximate location of the steel ball landing point on the floor and place the large plastic landing surface at that location. Tape a piece of white paper to the floor at that location and lay (do NOT tape) a piece of carbon paper on top. Do ten shots. The landing surface and white paper should not move during the experiment (the carbon paper doesn't matter). Measure the horizontal distance (the $x$-distance) from the gun to the closer edge of the white paper. Record the result. Remove the paper from the landing surface.

Measure and record the distance from the edge of the paper to each of the ten dots. Add the distance between the gun and the edge of the paper to each of the distances you have just measured (you want the total distance from the gun to the landing point). These are the total x -distances traveled by the projectile.
2) Find the mean of your ten $x$ values. Find the standard deviation and the uncertainty of the mean (see Appendix B).

Draw a circle around the cluster of dots containing about one half of the dots (five in this case). The center of the circle should coincide (approximately) with the center of the cluster. Measure the radius of this circle.
3) How does the radius of the circle compare with the standard deviation you found in question 2 ?

The two values should be close to each other. If your values are not close, check your calculation of the standard deviation in 2 . The approximate 'geometric' way of determining the standard deviation (by drawing a circle) will be used in some of the future labs.
4) If you did 100 shots instead of 10 shots, would the standard deviation for $x$-distances be very different or about the same? Would the uncertainty of the mean $x$ be different or about the same? Explain.
5) Geometrically, in this experiment the standard deviation corresponds to the radius of the circle containing one half of your shots. What does the uncertainty of the mean represent geometrically in this experiment?

You can now find the launch velocity of the ball. For the motion in x direction $x=v_{\mathrm{ox}} t$ (since there is no acceleration in the x direction). The initial velocity is therefore given by $v_{\mathrm{ox}}=x / t$.
6) Calculate the mean value for the initial velocity of the steel ball.
7) Calculate the uncertainty of the initial velocity. (Hint: Use the uncertainty of $x$ that you found in question 2 . Estimate the uncertainty of $t$. Is the latter uncertainty important when you evaluate the uncertainty of the initial velocity?)
8) Report the value for the initial velocity of the steel ball (see Appendix B).

Repeat the experiment using aluminum ball (use a different sheet of paper). Answer questions $1-8$ for the aluminum ball.
9) Are the initial velocities of the steel and aluminum balls the same or different? Explain (see Appendix B on how to compare two different measurements).

## Part two: Launch angle different from zero

In the first part the launch angle is zero; the ball leaves the table top horizontally. If a projectile is fired at an angle $\theta$ above the horizontal over level ground with initial velocity $v_{0}$, the x and y components of velocity are given by $v_{\mathrm{o}, \mathrm{y}}=v_{\mathrm{o}} \sin \theta$ and $v_{\mathrm{o}, \mathrm{x}}=v_{\mathrm{o}} \cos \theta$.

In the $y$ direction the projectile goes upward and then downward. For the upward part of the trip the final velocity is zero: $v_{\mathrm{y}}=v_{\mathrm{o}, \mathrm{y}}-g t=0$. So the time of the upward motion is $t=v_{\mathrm{o}, \mathrm{y}} / g=v_{\mathrm{o}} \sin \theta / g$. Twice this time is the length of time for the entire trip (if the projectile lands at the same level from which it was launched).

In the x direction we have no acceleration, so $x=v_{\mathrm{o}, \mathrm{x}} t=v_{\mathrm{o}, \mathrm{x}}\left(2 v_{\mathrm{o}} \sin \theta / g\right)$. In the figure below $x$ is the horizontal range $R$ of the gun. Substituting $v_{\mathrm{o}, \mathrm{x}}=v_{\mathrm{o}} \cos \theta$, we obtain

$$
R=2 v_{0}^{2} \cos \theta \sin \theta / g=v_{\mathrm{o}}^{2} \sin 2 \theta / g
$$

Solving the last equation for $v_{0}$, we get

$$
v_{\mathrm{o}}=(g R / \sin 2 \theta)^{1 / 2}
$$

The obtained equation relates the launch velocity of the ball to the horizontal range $R$ and the launch angle $\theta$.


Use the steel ball in this part. Clamp the apparatus to the very end of the table in such a way that the ball lands on the table (if the range is such that the ball misses the table, check with the instructor). Do not change the tension in the spring gun. The ball should land at the same level as the launcher.

Launch the steel ball at five different launch angles $\theta$, starting at $25^{\circ}$ and increasing by $10^{\circ}$ each time. Do five trials for each angle. For each angle, draw a circle around the five dots (like you did in part one) and measure the range $R$ from the gun to the center of the circle. (You are averaging over the five trials for each launch angle thus reducing the uncertainty of $R$ for each $\theta$ !)

For each launch angle, calculate the initial velocity $v_{\mathrm{o}}$ of the steel ball using $v_{\mathrm{o}}=(g R / \sin 2 \theta)^{1 / 2}$. Organize you results into a table (launch angle $-1^{\text {st }}$ column, $R-2^{\text {nd }}$ column, $v_{0}-3^{\text {rd }}$ column).
10) Find the mean launch velocity for the five angles. (The launch velocity is supposed to be the same for different launch angles, so it does make sense to find the mean of the five values of $v_{0}$ that you found above!)
11) Find the standard deviation and the uncertainty of the mean launch velocity.
12) Report your value for the launch velocity of the steel ball.
13) Is your result for the launch velocity of the steel ball reported in question 12 more precise or less precise than the result you reported in Part one for the steel ball? How do you know?
14) Assuming you have not changed the tension in the spring, the values for the initial velocity of the steel ball in Part one and in Part two should be the same. Are the initial velocities found in parts one and two close enough to each other so that you could say they are the same? If they differ significantly, calculate relative discrepancy between the two values.
15) Based on the equation $R=v_{0}{ }^{2} \sin 2 \theta / g$, what launch angle should give the greatest range?
16) Does your data in Part two support your prediction in 15 ?

Be sure to discuss your findings and provide some comments about the lab in the form of a conclusion.
This is a formal lab: one report per student. Be sure to read the format page at the beginning of this manual.

## Friction Forces (with the LabQuest) (informal)

Friction is a force (measured in Newtons) that resists the motion of an object. The coefficient of friction $\mu$ is a unitless constant which depends on the mechanical properties of the two surfaces in contact with one another and, to some extent, on the relative velocity of the two surfaces. It is defined to be the ratio of the friction force $f$ (parallel to the surfaces) to the normal force $F_{N}$ pushing the two surfaces together (perpendicular to the surfaces): $\mu=f / F_{N}$.

In this lab you will investigate the friction force between a wooden block and a wooden surface as the block slides along the surface. The LabQuest will be attached to a force probe in this lab to measure the tension in the string that pulls the block along the surface. The force of friction can be related to the tension in the string as described below in parts two and three.

## How to collect data:

NOTE: The metal lever on the probe should always be perpendicular to the applied force (to the string).
A) The force probe should be plugged into the CH1 slot on the top of the LabQuest.
B) From the 'Sensor Setup' screen select ' $\mathrm{CH1}$ ', scroll down to 'Force', and choose 'Student Force'. Under 'Units', leave the selection as N (Newtons). Click OK to go back to the main screen.
C) Zero and calibrate the probe:
a) Take all weight off the probe. Under the 'Sensor' menu select 'Zero' and 'CH1: Student Force'. The probe should now read zero if there is no force on it.
b) You probably will not have to calibrate the probe. However you can do this in the following way. Hang a known mass from the probe. The metal bar on the probe should be horizontal for this measurement so that the weight is perpendicular to it.
c) Choose 'Calibrate' and 'CH1: Student Force' from the 'Sensor' menu. Tap on 'Calibrate Now' and enter the known weight (in Newtons!) of your hanging mass. To get the weight in Newtons, multiply the mass in kilograms by $9.80 \mathrm{~m} / \mathrm{s}^{2}$. Tap 'OK'. You can use a second mass to get a better calibration.
NOTE: If the force that you get in part two or three is negative then you probably did not zero or calibrate correctly.
D) Set up the trigger level. Tap on the 'Sensor' menu and then on the 'Data Collection'. Enable triggering and set up the trigger level to about 0.3 N . You can also change the number of samples per second, the length of sampling, etc., but the default values should be good enough for this lab.
E) You are now ready to take data. Go to the graphing screen (tap the tiny graph icon tab). The start button at the lower left of screen, or on the LabQuest below the screen, starts the data collection.
a) Slowly pull the probe and the block. Be sure that the metal lever on the probe is perpendicular to the string. When the applied force (the tension in the string) exceeds the trigger level, the LabQuest will start taking data.
b) The graph on the LabQuest screen shows the pulling force in Newtons on the $y$-axis vs. the time on the x -axis (in seconds). Using the plastic stylus you can highlight a particular region of the graph and zoom in. To find the average pulling force for that particular region, choose 'Statistics' from the 'Analyze' menu and tap on 'Force'. The mean, max, min, standard deviation, and the number of samples for the selected region will appear at the right of the screen. Zoom out to go back to your original graph. You can turn off the 'Statistics' by unselecting 'Force' under the 'Analyze' menu.

## Part one: Newton's third law



Find another group of students. Hook two probes together so that the force is perpendicular to the lever for a push or a pull for both probes. Be sure that the LabQuest settings (trigger levels, etc.) are the same for both groups. While the LabQuest is taking data, push (gently!) or pull (gently!) on both probes.

NOTE: The range that the force probe can measure is zero to 19.6 N (equivalent of a 2.00 kg weight). Do not apply a larger force to the probe.

1) How do the graphs on the two LabQuests compare? Sketch both graphs, one below the other, in your lab report.
2) Does your data support the Newton's third law?

## Part two: Kinetic and static friction on a horizontal surface

Place the wooden block on the horizontal surface with a string attached to the force probe. The string should stay parallel to the surface and perpendicular to the metal lever on the probe (as shown in the figure below).


In this part the forces acting on the block parallel to the surface are friction $f$ and the tension in the string, $T$. When acceleration is zero (the block is at rest or moves at constant velocity), we have in the $x$ direction $T-f=0$. Thus the friction force is equal to the tension $T$, which is read off the LabQuest display. The normal force in this part is the weight of the block, $m g$, (plus any weights on top of it).
3) Draw a free body diagram for the wooden block. What will happen if the string is not horizontal (not parallel to the surface)?
4) Why is it important to move the block at constant velocity while taking data?

Start taking data and pull the force probe, gradually increasing the pulling force. Once the pulling force exceeds the trigger level, the LabQuest will start recording data. Keep increasing the pulling force until the block breaks loose and starts moving. When this happens, pull the probe and the block at a slow constant velocity. The graph on the LabQuest should have a peak on the left of the screen and a flat region on the right. You may have to practice a few times before you get a nice looking graph.

The maximum of the peak is reached when the pulling force equals the maximum force of static friction for this block. For the maximum force of static friction we have $f_{\mathrm{s}, \max }=\mu_{\mathrm{s}} F_{N}$, where $\mu_{\mathrm{s}}$ is the coefficient of static friction and $F_{N}$ is the normal force. Record the values of $F_{N}$ and $f_{\mathrm{s}, \max }$ into a table ( $F_{N}$ into the $1^{\text {st }}$ column, $f_{\mathrm{s}, \max }$ into the $2^{\text {nd }}$ column).

The flat region on the LabQuest screen corresponds to the motion of the block at constant velocity. In this case the pulling force is equal to the force of kinetic friction acting on the block. The force of kinetic friction is related to the normal force as $f_{\mathrm{k}}=\mu_{\mathrm{k}} F_{N}$, where $\mu_{\mathrm{k}}$ is the coefficient of kinetic friction. Highlight a section of data that is relatively flat (on average) and zoom in. To find the average pulling force for this region, choose 'Statistics' from the 'Analyze' menu and tap on 'Force'. The mean, max, min, standard deviation, and the number of samples for the selected data will appear at the right of the screen. Record the mean (equal to the average $f_{\mathrm{k}}$ ) into the $3^{\text {rd }}$ column of the table.

Place a $200-\mathrm{g}$ mass on top of the block. Repeat the experiment. When the block is moving, be sure to maintain about the same velocity as in the previous trials. Record the new values of $F_{N}, f_{\mathrm{s}, \text { max }}$, and $f_{\mathrm{k}}$, into the table. Repeat the experiment five times, each time increasing the additional mass by 200 g .

Using LoggerPro, make graphs of $f_{\mathrm{s}, \text { max }}$ vs. $F_{N}$ and $f_{\mathrm{k}}$ vs. $F_{N}$ (with the friction forces on $y$-axes and $F_{N}$ on $x$-axes). Add the regression line to each graph (delete the connecting line and enable point protectors).
5) Does your data support the laws of kinetic and static friction? Base your conclusion on the values of Correlation and Y-Intercept for each graph. (See Appendix A. The Y-Intercepts should be consistent with zero if your data supports the laws of kinetic and static friction.)
6) Based on the slopes of your graphs, what are the values of the static and kinetic friction coefficients? Report their values, including their uncertainties and units.
7) How do the values of the static and kinetic friction coefficients compare? Are they the same or different given their uncertainties?

Repeat the experiment with the wooden block on its edge (without any additional masses). Be sure to pull the block at about the same constant velocity as previously. Record the values of $F_{N}, f_{\mathrm{s}, \max }$, and $f_{\mathrm{k}}$.
8) Do the values of $f_{\mathrm{s}, \max }$ and $f_{\mathrm{k}}$ depend noticeably on the contact area? (Are the differences between the values of $f_{\mathrm{s}, \text { max }}$ and $f_{\mathrm{k}}$ you have just obtained and the corresponding regression lines much bigger than the RMSE or about the same as the RMSE? See Appendix A.)

## Part three: Kinetic and static friction on a ramp

In part two the surface is horizontal and the normal force is equal to $m g$. What happens when the surface is not horizontal and the normal force is no longer $m g$ ? Is the law of friction, $f=\mu F_{N}$, still valid? In this part you will repeat the measurements with the block on an incline at five different angles.

The calculation of friction and normal forces is more complicated in this case (the normal force is no longer equal to $m g$ and the friction force is no longer equal to the tension in the string). The normal force is the component of the weight perpendicular to the surface, $F_{N}=m g \cos \theta$. The friction force is $f=T-m g \sin \theta$, where $T$ is the tension in the string. The latter expression is valid when the block is pulled at constant velocity (zero acceleration) up the ramp.
9) Draw a free body diagram for the block on the ramp. Derive the expressions for $F_{N}$ and $f$ given above.


Set up the experiment as shown in the above picture. Do not add any extra weights to the block in this part. For better results, set up the trigger level to $m g \sin \theta+0.3 \mathrm{~N}$, with $m$ being the mass of the block and $\theta$ the angle of the ramp. (Can you justify such a choice for the trigger level?)

Note that the force probe is fairly sensitive. Since the metal lever of the probe is no longer vertical, the zero of the probe drifts (try it!). The amount of drift depends on the angle. Be sure to zero the probe before taking data for each new angle of the ramp.

Perform the experiment for $\theta=5^{\circ}, 20^{\circ}, 30^{\circ}, 35^{\circ}$, and $40^{\circ}$. Be sure to set up the proper trigger level and zero the probe for each new angle. Start taking data and pull the force probe, gradually increasing the pulling force. Like in part two, keep increasing the pulling force until the block breaks loose and starts moving. When this happens, pull the probe and the block at a slow constant speed. Be sure to pull the block at about the same speed each time.

Like in part two, record the maximum tension (related to the static friction) and the average tension where your data is flat (related to the kinetic friction). Organize your results into a table (the angle of the ramp $-1^{\text {st }}$ column, $\max T-$ $2^{\text {nd }}$ column, average $T-3^{\text {rd }}$ column). Calculate the normal force, the force of static friction, and the force of kinetic friction ( $4^{\text {th }}, 5^{\text {th }}$, and $6^{\text {th }}$ columns).

Using LoggerPro, make graphs of $f_{\mathrm{s} \text {, max }}$ vs. $F_{N}$ and $f_{\mathrm{k}}$ vs. $F_{N}$ (with the friction forces on y-axes and $F_{N}$ on x-axes) for the five angles. Add the regression line to each graph (delete the connecting line and enable point protectors).
10) Does your data in this part support the laws of kinetic and static friction? Base your conclusion on the values of Correlation and Y-Intercept for each graph (see Appendix A).
11) Based on the slopes of your graphs, what are the values of the static and kinetic friction coefficients in this part? Report their values, including their uncertainties and units.
12) How do the values of the static and kinetic friction coefficients found in this part compare to their values found in part two? Are the static coefficients found in parts two and three the same, given their uncertainties? Are the kinetic coefficients the same, given their uncertainties? Comment on any possible disagreement between the values found in parts two and three.
13) Explain the difference between the coefficient of friction and the friction force.
14) What happens to the friction force when the angle of the ramp $\theta=90^{\circ}$ ?
15) What happens to the coefficient of friction when $\theta=90^{\circ}$ ? (Careful, think about what the coefficient represents.)

Be sure to discuss your findings with your lab partners (this material will eventually appear on a test) and provide some comments about the lab in the form of a conclusion on your report.

## Ballistic Pendulum (with the LabQuest) (informal)

In this experiment you will find the initial velocity of a ball fired from a spring gun two ways. One method employs the ballistic pendulum. The other method uses the LabQuest to record the time of flight of the ball through a photogate. For the ballistic pendulum the principles of conservation of energy and conservation of momentum are used to determine the initial velocity of the ball. Please read ahead in your book about these two principles if you have not covered them in class yet.

## Part one: The ballistic pendulum

This method uses conservation of energy and conservation of momentum to determine the velocity of the ball. The apparatus is shown in the figure below. When the ball is fired it is caught by the pendulum, causing the pendulum to swing upward to a vertical height $h$.


The process can be divided into two steps for analysis: ball capture and pendulum swing. First notice that conservation of kinetic energy does not occur in the ball capture step since the collision is inelastic. However, we can apply conservation of momentum to the capture of the ball because momentum is conserved in any collisions. Before the collision, the total momentum is $p_{\text {before }}=m v_{\mathrm{o}}+0$ where $m$ is the ball's mass and $v_{\mathrm{o}}$ is the ball's initial velocity. The pendulum is not moving so its contribution to the total momentum is zero. After the collision the total momentum is $p_{\text {after }}=(m+M) V$, where $m$ is the ball's mass, $M$ is the pendulum mass, and $V$ is the velocity of the pendulum with the ball.


Using conservation of momentum we have: $p_{\text {before }}=p_{\text {after }}$, so $m v_{\mathrm{o}}=(m+M) V$.

1) The friction force between the ball and the pendulum during the ball's capture is quite large. Why doesn't it affect the conservation of momentum in this experiment?

So far you still have two unknowns, $V$ and $v_{0}$. In order to find the initial velocity $v_{0}$ you need to know the velocity $V$ of the pendulum and ball after collision. This can be found by using conservation of energy after the ball is captured, since mechanical energy is conserved when the pendulum with the trapped ball swings upward.


Right after the collision the ball and pendulum have zero potential energy while their kinetic energy is $1 / 2(M+m) V^{2}$. When they reach the highest point of the swing, the kinetic energy is zero and potential energy is $(M+m) g h$. Kinetic energy has been converted into potential energy. Conservation of energy gives $1 / 2(M+m) V^{2}=$ $(M+m) g h$, so $V=(2 g h)^{1 / 2}$. Replacing $V$ in the momentum equation yields an expression for $v_{0}$ :

$$
v_{\mathrm{o}}=[(M+m) / m](2 g h)^{1 / 2} .
$$

2) Why can't you use conservation of momentum when the pendulum-ball system swings upward?

The mass of the ball, $m$, is measured on a balance. The pendulum mass, $M$, is measured by hanging the heavy end of the pendulum from a spring scale so that the pendulum is horizontal and still attached to the apparatus. Do not remove the pendulum from the apparatus.

Your pendulum has an angular scale. Fire the ball several times into the pendulum to determine the deflection angle. The pointer on the apparatus has a fair amount of friction. With each new trial the deflecting pendulum pushes the pointer a little bit higher, until the true deflection angle is found after several trials. Record the angle of deflection after about ten shots. The value of $h$ is $h=r(1-\cos \theta)$, where $r$ is the distance from the pivot to the center of the pendulum bob and $\theta$ is the angle.
3) Prove the formula $h=r(1-\cos \theta)$ with a drawing and trigonometry in your lab report.
4) Calculate $v_{o}$ using the equation given above.
5) Estimate the uncertainties of $m, M, r$ and $\theta$. Evaluate the uncertainty of $v_{\mathrm{o}}$ (see Appendix B). Which measurement introduces the greatest uncertainty into the value of $v_{\mathrm{o}}$ in this experiment? Explain.
6) Report your value of $v_{o}$ as described in Appendix B.

## Part two: The photogate (with the LabQuest)

Pull the pendulum up out of the way of the ball's path for this part of the experiment. Set up the photogate so that when the ball is fired, it breaks the laser beam of the photogate (see the figure on the next page). The red light on the photogate comes on when the beam is blocked. Be sure to set up the photogate in such a way that the diameter of the ball crosses the beam.

Measure the diameter $d$ of the ball (use a caliper). Since the ball travels a distance equal to its diameter in the time $\Delta t$ recorded by the LabQuest, $d=v_{0} \Delta t$. Knowing $d$ and $\Delta t$, one can calculate the ball's velocity: $v_{\mathrm{o}}=d / \Delta t$.


Photogate

## To run the LabQuest with the photogate:

A) Start the LabQuest and plug the photogate into DIG 1.
B) Test your photogate with your hand to see if it is working (the LabQuest should say 'Blocked' when your hand breaks the beam and the red light on the photogate should come on).
C) Tell the LabQuest how you want to use the photogate:
a) Tap on the 'Sensors' menu and choose 'Data Collection';
b) Choose 'Photogate Timing' from the Mode selection;
c) Choose 'Gate' from the Photogate Mode selection;
d) Enter the diameter of your ball in meters;
e) Tap 'OK' to finish.
D) Align the photogate so that the diameter of the ball crosses the beam. Tap the forward button to start data collection. Fire the ball through the photogate five times. The gate time (the time it takes the ball to cross the beam) and the velocity of the ball calculated by the LabQuest should not change very much from trial to trial. Tap the stop button after you have collected five data points.
7) Find the average velocity of the ball for the five trials. Find the standard deviation and uncertainty of the average velocity (see Appendix B).
8) Report the value of $v_{o}$ found using the photogate.
9) Do the values of $v_{0}$ found in parts one and two agree (see Appendix B)?
10) Which method is more precise (part one or part two)? How do you know?
11) Which method is more accurate? Why? (Discuss possible sources of systematic uncertainty in each case.)
12) Discuss how you could improve the precision of each method.
13) Discuss how you could improve the accuracy of each method.
14) What is the difference between precision and accuracy (see Appendix B)?

Be sure to discuss your findings and provide some comments about the lab in the form of a conclusion.

## Conservation of Momentum (informal)

In this experiment you will investigate conservation of momentum as it applies to two colliding balls. You will need to review the section in your book on collisions in two dimensions.

Clamp the aluminum ramp to a lab bench top with the c-clamp provided. The end of the base plate should extend past the edge of the table so that the lower end of the aluminum rail is even with the edge of the table.

1) Record the distance from the bottom of the rail to the floor. Using $h=1 / 2 \mathrm{gt}{ }^{2}$, calculate the time $t$ it takes a ball to reach the ground. Record the time.
2) Does this time depend on the mass of the ball? Explain.

Tie the string provided to the eye of the screw eye to form a plumb line. Tie the other end to the bottom of the set screw. The screw eye should almost reach the floor. Place the plastic landing surface so that the ball will land on it. Tape paper to the surface starting where the plumb line is hanging and go out far enough so that the balls will both land on the paper when the roll off the track. Make sure the paper is secure and will not slide around during the experiment. Mark the location of the plumb line on the paper (the point directly below the hanging screw eye).

Lay (do not use tape) carbon paper on top of the white paper taped to the floor, carbon side down so that as a ball strikes it will leave a mark on the white paper. Roll a steel ball down the ramp (do a trial run before putting the carbon paper down). Do a few (about ten) trials with a single ball (no collision) with the carbon paper in place. Be sure you start the ball from the same place on the ramp each time.
3) Remove the carbon paper and draw a circle around the group of points containing about a half of the points like you did in the 'Projectile Motion' lab. The radius of this circle gives you an estimate for the standard deviation of the horizontal displacement of the ball. Record this radius (in centimeters).
4) Draw a line (using a meter stick) from the center of the circle to the point directly under where the plumb line hangs. The length of this line is the average horizontal displacement of the ball. Record the horizontal displacement of the ball (in centimeters) together with its uncertainty (radius of the circle divided by the square root of the number of trials).
5) The horizontal displacement of the ball $d$ is related to the velocity of the ball at the bottom of the ramp $v$ as $v=d / t$, where $t$ is the time of flight. Calculate and record $v$ together with its uncertainty. (Hint: The time of flight $t$ is known fairly accurately, so the uncertainty of $v$ should be mainly determined by the uncertainty of $d$.)
6) Measure and record the mass of the ball (in grams). The momentum of the ball $p$ at the bottom of the ramp is $p=m v$. Calculate and record $p$ together with its uncertainty.

The momentum recorded above is the initial momentum, or momentum before a collision. You will use the initial momentum direction as your y-axis. Make a coordinate system on the paper by drawing a line perpendicular to your $y$-axis which passes through the point directly under the plumb line.
7) What is the initial $x$-momentum?

Before replacing the carbon paper, place one of the steel balls in the depression at the top of the set screw (which is just past the lower end of the rail). Hold the second steel ball on the lower end of the ramp and adjust the set screw up or down so that the ball on the screw is level with the ball on the ramp.

Notice that the set screw will move from side to side making collisions at different angles possible. Do a few practice runs at one collision angle. Glancing collisions will give the best results, nearly head-on collisions do not.

When you are ready to collect data, lay carbon paper on top of the white paper taped to the floor, carbon side down so that as the balls strike they will leave a mark on the white paper.

For the same collision angle do ten collisions. Be sure to start the ball at the same place on the ramp each time and be sure the angle to the set screw does not change in between trials. When you have ten points for that collision angle, draw a circle around each of the two clusters of points. Each circle should contain about a half of the points in each cluster. Label the two circles of data points so you will be able to tell them apart from data taken later.

Repeat the experiment for a different collision angle using the same balls. Be sure to keep the data for each angle separate.

Repeat the experiment using the glass ball as a target (keep using the same steel ball to roll down the track). Measure and record the mass of the glass ball.

For each of the three data sets perform the following analysis (questions $8-12$ ).
8) Measure and record the $x$ and $y$-coordinates of each of the two data clusters (the $x$-coordinate is the distance from the $y$-axis to the center of a cluster while the $y$-coordinate is the distance from the $x$-axis to the center of a cluster).
Estimate the uncertainties of $x$ and $y$-coordinates for each cluster (radius of each circle divided by the square root of ten).
9) Calculate x and y -velocities of the balls after the collision ( $v_{x}=x / t$ and $v_{y}=y / t$, where $t$ is the time of fall). Calculate the uncertainties of x and y -velocities for each ball.
10) Calculate the x and y-components of momentum of each ball after the collision with their uncertainties $\left(p_{x}=m v_{x}\right.$ and $p_{y}=m v_{y}$, where $m$ is the mass of the ball).

Record your results (questions $8-10$ ) in the table (a separate table for each of the three data sets):

|  | $x \pm \delta x$ | $v_{x} \pm \delta v_{x}$ | $p_{x} \pm \delta p_{x}$ | $y \pm \delta y$ | $v_{y} \pm \delta v_{y}$ | $p_{y} \pm \delta p_{y}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ball 1 |  |  |  |  |  |  |
| Ball 2 |  |  |  |  |  |  |

11) Does the sum of the two y-components of momentum after the collision agree with the total y-momentum before the collision (question 6), given the uncertainties of each quantity? If they do not agree, how big is the relative discrepancy?
12) Since the $x$-momentum before the collision is zero, the two $x$-components of momentum after the collision should add to zero. Note that since the two components are in opposite direction, one of them has to be negative. Does the sum of the two $x$-components of momentum after the collision agree with zero given their uncertainties?
13) For the glass ball - steel ball data, calculate the total kinetic energy of the balls before and after the collision. Calculate the uncertainty of $\mathrm{KE}_{\text {before }}$. Assume that $\delta \mathrm{KE}_{\text {after }} \approx \delta \mathrm{KE}_{\text {before }}$. Was the collision elastic? (Is $\mathrm{KE}_{\text {before }}=\mathrm{KE}_{\text {after }}$ given their uncertainties?)
14) Is momentum conserved in your experiments? Explain why you expect momentum to be conserved in these experiments while kinetic energy may or may not be conserved.

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Uniform Circular Motion (informal)

According to Newton's first law, objects in motion tend to move along a straight line unless acted on by a force. In order to change the magnitude or direction of velocity, a force must be applied. The force needed to keep an object moving in a circle is called the centripetal force. It acts towards the center of the circle. A centripetal force gives rise to a centripetal acceleration. The centripetal acceleration $a_{c}$ is related to the speed of an object moving in a circle as $a_{c}=v^{2} / r$, where $v$ is the speed and $r$ the radius of the circle.

In this lab a mass hanging from a string supplies a constant centripetal force needed to keep the smaller mass (the rubber stopper) going in a circle at a constant tangent speed $v$. Newton's $2^{\text {nd }}$ law for the components of forces and acceleration along the string is $M g=m v^{2} / r$, where $M g$ is the weight at the lower end of the string, $m$ is the mass of the stopper, and $v^{2} / r$ is the centripetal acceleration of the stopper.


1) Draw a free body diagram for the system (the stopper, the string, and the counter weight). Derive the relation between $M g$ and $v^{2} / r$ given above (neglect the fact that the string keeping the stopper on the circular path is not exactly horizontal).
2) For an object on a circular path, which direction does the centripetal force vector point?
3) For an object on a circular path, which direction does the centripetal acceleration vector point?

Before taking data, practice swinging the stopper at approximately constant speed and radius. You may want to put a paper clip on the string to mark the radius. It may help you keep the radius constant while you swing but be sure the paper clip does not touch the tube while you swing, as this will introduce additional forces.

In order to measure the speed $v$, find the distance around the circle $(2 \pi r)$ and measure the time needed for 20 revolutions with a stopwatch. Since $d=v t$, the speed $v$ is given by $v=20(2 \pi r) / t$. Record the values of $t$ and $v$ for five different values of $r$. Begin with $r=40 \mathrm{~cm}$ and increase the radius by about 20 cm each time.
4) Notice that as you increase the radius of the circle, the time to complete 20 revolutions also increases. Explain why this should be the case in this experiment.

Since the magnitude of centripetal force is constant in this experiment, the magnitude of centripetal acceleration of the stopper should be constant as well. Because $a_{c}=v^{2} / r$, the graph of $v^{2}$ vs. $r$ should be a straight line with the slope equal to $a_{c}$.

Plot $v^{2}$ vs. $r$ using LoggerPro ( $v^{2}$ on the y-axis and $r$ on the x - axis). Add the regression line to your graph, enable point protectors, delete the connecting line.
5) Does your data agree with the straight line? Base your conclusion on the values of Correlation and Y-Intercept (see Appendix A). Note that the relation $a_{c}=v^{2} / r$ is not an empirical law. It is an exact mathematical relation. A disagreement with the straight line indicates the presence of some systematic effect (systematic uncertainty). If your data does not agree with the line, comment on what may have caused the disagreement.
6) Based on your graph, what is the centripetal acceleration of the stopper in this experiment? Report the value of $a_{c}$ with its uncertainty and units (see Appendix B).
7) Now calculate $M g / m$ from your recorded values of $M$ and $m$. Estimate its uncertainty. Report the value of $M g / m$. Which quantity introduces the largest uncertainty into the result? Explain.
8) Compare the results found in questions 6 and 7. Do the values agree given their uncertainties? If they do not agree, discuss possible reasons for the disagreement.

Double the hanging counter weight and repeat the experiment. Repeat the same analysis as you did above. Answer questions $5-8$ for your new data.
9) What conclusions can you draw from repeating the experiment with a different counter weight? Do you get a larger value for $a_{c}$ ? Is this what you would expect?

Now try to swing the stopper without any counter weight (Careful!).
10) Can you do it? Explain why not.

Set the stopper into motion with some counter weight and with the radius of rotation of about 1 m . Pull down on the counter weight with your other hand.
11) What happens to the speed of the stopper? Explain your observation. (Hint: Think about the angular momentum of the stopper with respect to the axis passing through the plastic tube. Is it conserved?)
12) What happens to the kinetic energy of the stopper? Is the work done by you on the system positive, negative, or zero?

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Temperature (informal)

What is temperature? It is commonly associated with how hot or cold an object feels when we touch it. In physics a more objective definition of temperature is needed. It is known that most substances expand when heated. This simple fact was used in the past to define temperature. In this lab you will follow this route. You will make your own thermometer and come up with your own temperature scale. You will then relate your temperature scale to the Celsius scale.

Despite its simplicity, the definition of temperature based on volume expansion has obvious shortcomings. Different materials expand differently when heated, so thermometers made with different thermometric substances may not always agree with each other. In the $2^{\text {nd }}$ half of $19^{\text {th }}$ century, when first thermodynamics and then statistical mechanics were developed, the concept of temperature was put on a solid physical foundation. It was shown that temperature is related to the average energy of internal motion of atoms and molecules. Nevertheless, the old way of measuring temperature, based on volume expansion, can still be used in most practical situations.

In this lab you have a thermometer which has no marks on it. Since the physical temperatures of boiling water and melting ice are the same everywhere in the world (assuming the air pressure is the same and the water is a pure sample), one can use these two reference temperature points to make a thermometer.

Put the thermometer into the provided steam bath (be careful not to burn yourself). When the thermometer reaches equilibrium with the bath, mark where the red liquid is with an indelible pen. (Note: You have to do this carefully since the level will start to change as soon as you remove the thermometer from the steam bath. Even though you are using an indelible marker the marks can still be rubbed off so you will have to be careful not to disturb the marks during the rest of the lab.)

Once you have the high point marked let the thermometer cool to room temperature. When it has cooled, put the thermometer into the ice bath and let it come to equilibrium. Mark this second level with the indelible pen, again being careful to mark where the low point is while the thermometer is in the bath.

Lay your thermometer next to a ruler and divide the distance between the high and the low points into equal portions and mark these with the pen. You can choose whatever number of marks you want, ten or more but less than 100 is a good choice. Do not use the same number of divisions as the group next to you. Be sure the marks are evenly spaced.

Now choose what numbers you want to associate with the marks you have made on your thermometer. If you choose the low point to be zero and the high point to be 100 and put exactly 100 marks between the high and the low point, you would have a thermometer which measures in Celsius (or centigrade). If you choose the low point to be 32 and the high point to be 212 and mark exactly 180 points between, you would have a Fahrenheit thermometer. Since you cannot get 180 or even 100 marks on the thermometer you are making, you will have to come up with your own temperature scale.

1) What number do you want to associate with the low end of your scale?
2) How many marks did you put between the low and the high end? How much of an increase do you want to associate with each mark?
3) From the choice of the low number and the choice of increments associated with the divisions the upper number is fixed. What is the number associated with the high end of your temperature scale?
4) What are you going to call these degrees on your thermometer? (The Celsius and Fahrenheit scales are named after people so you could name your scale after someone in the group.)
5) Can you think of any reason you could not start with a big number for the low reading and count downwards to the temperature at the boiling point? Explain how this would work. (There is a temperature scale which does this.)
6) Compare your thermometer with a Celsius thermometer. Are the locations of the boiling and freezing marks the same or different on the two thermometers?
7) Lay a Celsius thermometer next you your thermometer and wait until they are in equilibrium with the room. What temperature does the Celsius thermometer read? What temperature does your thermometer read (in your scale)?
8) Find another group which has a different number of divisions than your thermometer and compare room temperature readings between the two.
9) Find a formula to convert from your temperature scale to the Celsius scale. (Hint: To find a conversion formula from Fahrenheit to Celsius you do two things. First you find out how much one degree in the Fahrenheit scale compares to one degree in the Celsius. Since there are 180 marks on the Fahrenheit scale between melting and boiling but 100 marks on the Celsius thermometer, the ratio $100 / 180$ converts a change of one degree Fahrenheit to a change of one degree Celsius. The second part of the conversion has to do with the starting points of the two different scales. The Celsius scale starts at 0 but the Fahrenheit scale starts at 32 so the Fahrenheit scale has to be slid back to zero by subtracting 32 in order to compare the two scales. So the following formula will change from Fahrenheit to Celsius: $\left(\mathrm{T}_{\mathrm{F}}-32\right)^{*} 100 / 180=\mathrm{T}_{\mathrm{C}}$.) What is the conversion formula from your temperature scale to Celsius?
10) Check your conversion formula. Measure and record the temperature of a glass of tap water using your thermometer. Do the same with Celsius thermometer. Convert your temperature to Celsius scale. Do the two results agree? Comment on any possible discrepancy.
11) Why is it important to have evenly spaced markings on the thermometer in order for your conversion formula to work?
12) Does it matter that the high and low points' locations on a particular thermometer do not exactly correspond to the high and low locations on a different thermometer? Why or why not?
13) How would you go about extending the range of your thermometer to account for temperatures higher than your highest mark or lower than your lowest mark?
14) Can you think of any temperature measurement which could not be made with your thermometer but could be made with the Celsius thermometer?
15) Are there any real advantages or disadvantages of the scale you have chosen?
16) What one thing would have made your thermometer more precise?
17) The Celsius thermometer you are using has increments of $1^{\circ} \mathrm{C}$. This means you can "eyeball" the temperature to about half of a degree Celsius. Write the "eyeball" precision of your thermometer in your units and then change this precision to Celsius. Which thermometer is more precise?
18) Can you think of any errors which could occur in the procedure used by you to make your thermometer which would make it less accurate than the Celsius thermometer?
19) What is the purpose of picking the boiling and freezing points of water in the above procedure? Could you use other temperatures? Explain.
20) The boiling temperature of water changes a lot if you go to a higher altitude, whereas the melting temperature of ice does not change very much. Explain what would happen if a group of students followed the same procedure as you did but at a different altitude. Could you find a way to make your thermometers agree? Why or why not?

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Coefficient of Linear Expansion (informal)

Most common substances expand when they are heated and contract when cooled (an important exception being water between four and zero degrees Celsius). If we have a metal rod, we expect that the change in length $\Delta L$ will be proportional to the increase in temperature $\Delta T$ and to the original length $L_{0}: \Delta L=\alpha L_{0} \Delta T$.

The proportionality constant $\alpha$ is called the coefficient of thermal expansion and has units of $\left(C^{0}\right)^{-1}$. Usually $\alpha$ is positive, indicating that the substance will expand when heated, but it can be negative, too (in the case of substances which contract when heated). You will find the coefficient of thermal expansion for one unknown rod. Based on the value of the coefficient you will determine the composition of the rod.

1) Measure and record the room temperature and the original length of the rod. Measure and record the diameter of the rod (use a caliper to measure the diameter). Calculate and record the cross-sectional area of the rod.

The apparatus is shown in the drawing. The micrometer measures the change in length of the rod. Each division on the main dial represents one hundredth of a millimeter $(0.01 \mathrm{~mm})$. The needle may make more than one revolution around the main dial as the rod expands. The number of revolutions is counted by the second, smaller, dial. The micrometer can be zeroed by rotating its outer ring.


Place the rod into the metal tube. Make sure the rod is not bent inside the tube. Both ends of the rod must slightly protrude past the ends of the tube. Place the tube with the rod into the base. Attach the micrometer. Secure the micrometer with the nut and make sure that the micrometer shaft is touching the end of the rod. The other end of the rod should be touching the screw at the other end of the base. Assemble the rest of the setup as shown in the picture. Zero the micrometer.

## IMPORTANT!

a) To ensure proper drainage, the steam should enter at the top but drain at the lower end of the metal tube.
b) Insert the thermometer LAST to reduce the chance of breaking it when assembling the rest of the setup.

Once the steam is on, the thermometer will reach a maximum temperature near $100^{\circ} \mathrm{C}$. After the rod has remained at the maximum temperature for several minutes record that temperature. Record the change in length of the rod from the micrometer.
2) Use your values of $\Delta L, L_{0}$, and $\Delta T$ to calculate the expansion coefficient $\alpha$.
3) Estimate and record the uncertainties of $\Delta L, L_{0}$, and $\Delta T$.
4) Calculate the uncertainty introduced into the value of $\alpha$ by the uncertainties of $\Delta L, L_{0}$, and $\Delta T$ (cf. Appendix B).
5) Which quantity, $\Delta L, L_{0}$, or $\Delta T$, introduces the largest uncertainty into the value of $\alpha$ ? Explain.
6) In this lab $\Delta L$ is measured quite precisely (up to 0.01 mm ) while the precision of $L_{0}$, is not as high ( 1 mm or so). Why should $\Delta L$ be measured more precisely than $L_{\mathrm{o}}$ in this lab?
7) Report your value of $\alpha$ (cf. Appendix B).
8) Compare the value of $\alpha$ which you have found to the values of linear thermal expansion coefficients in your book. According to your measured expansion coefficient, what substance is your rod made of? What is the relative discrepancy between your value and the accepted value of $\alpha$ for that substance?

Under mechanical stress a metal rod expands or contracts according to the following equation: $F / A=Y \Delta L / L_{0}$. In this equation $F$ is the force applied to the rod, $A$ is the cross-section area of the rod, $L_{\mathrm{o}}$ is the initial length, and $\Delta L$ is the change in length. The coefficient $Y$ is called Young's modulus. It depends only on the material the rod is made of (it is independent of size or shape of the rod). The ratio $F / A$ is called stress (or tensile stress). It has the same units as pressure $\left(\mathrm{Pa}\right.$, or $\left.\mathrm{N} / \mathrm{m}^{2}\right)$.
9) In your book find the value of Young's modulus for the material your rod is made of. Using the values of $\Delta L$ and $L_{\mathrm{o}}$ which you measured before, calculate the stress that would build up in the rod if it were not allowed to expand freely when heated in this experiment. Compare this stress to the atmospheric pressure $\left(\mathrm{P}_{\mathrm{atm}}=101.3 \mathrm{kPa}\right)$.
10) Using the cross-sectional area of the rod which you previously found, calculate the force that would be needed to keep the rod from expanding in this experiment. Express your answer in $\mathrm{lb}(1 \mathrm{lb}=4.45 \mathrm{~N})$.
11) Based on your results in questions 9 and 10 discuss why it is necessary to put thermal expansion joints on bridges and insert $U$-shaped sections into gas or oil carrying pipelines. What other examples of practical implications of thermal expansion you can think of?
12) How is the behavior of water different near $0^{\circ} \mathrm{C}$ than that of the rods you are using in this lab? (Consult your book if necessary.) Describe the behavior of a pond as it freezes if water did not behave the way it does.

Be sure to discuss your findings and provide some comments about the lab in the form of a conclusion.

## Mechanical and Electrical Equivalents of Heat (with the LabQuest) (informal)

James Prescott Joule was one of the first scientists to realize that a temperature increase can occur by mechanisms other than direct heat transfer from a flame or some other hot object. Mechanical, electrical, and some other forms of energy can be transformed into thermal energy, resulting in a temperature increase of an object. In this lab you will first do an experiment which compares the loss of mechanical energy to the thermal energy gain of a sample of lead. In the second part you will find the electrical equivalent of heat. Note that conservation of energy applies in both cases.

## Part one: Mechanical equivalent of heat

Insert the temperature probe into CH1 slot of the LabQuest. Turn the LabQuest on. It should recognize the probe and display the current room temperature on the screen.

1) Measure and record the length through which the shot will fall inside the provided cardboard tube.
2) Measure and record the mass of lead shot you will be using (use about 200 g ) and its initial temperature.

Place the lead in the tube and replace the stopper. One student should now grip the tube on both ends (so there is no chance of the stoppers coming out) and hold the tube straight up and down. While the other lab partners count, rotate the tube by $180^{\circ}$ and wait till the lead hits the bottom. Do this exactly 50 times, making sure the lead has time to reach the bottom after each rotation. When you are done, open the tube, pour the shot into a container, and measure its new temperature.
3) What is the new temperature of the lead shot? What is the temperature change?
4) The mechanical energy available is 50 mgh , where $m$ is the mass of the shot, $g$ is the acceleration due to gravity, and $h$ is the length the shot falls during one rotation. How much mechanical energy in Joules is available in this experiment?
5) The thermal energy gain $Q$ of the shot can be measured by its temperature increase: $Q=C_{\text {lead }} m \Delta T$. Look up the specific heat of lead in your book. How much thermal energy in Joules is added to the shot?

You will notice that the two numbers are not equal. The available mechanical energy will be larger than the thermal energy gain.
6) What is the relative discrepancy between the mechanical energy available initially and the thermal energy gain?
7) Where does the missing mechanical energy go? What other sources of systematic uncertainty are present in this experiment?
8) The probe measures temperature up to $0.1^{\circ} \mathrm{C}$ while the thermometer used in the previous lab can measure temperature only up to $1^{\circ} \mathrm{C}$. Why do you have to measure temperature more precisely in this experiment?
9) Do you expect $\Delta T$ to depend on $m$ (the mass of the lead shot)? If you used twice as much of lead shot, would $\Delta T$ be different or about the same?

## Part two: Electrical equivalent of heat

In this part you will compare the energy lost by a current carrying resistor to the thermal energy gain of a sample of water in a calorimeter cup. You should get a much better agreement between the two quantities in this part.
10) Measure and record the mass of the calorimeter cup (just the inside part).

Using a mixture of tap water and fountain water, fill the calorimeter cup about $3 / 4$ full with water that is about $5^{\circ} \mathrm{C}$ below room temperature. There should be enough water to completely cover the resistor coils when they are placed in the cup.
11) Measure and record the mass of the water and cup. Subtract the mass of the empty cup to find the mass of your water sample. Record the mass of your water sample.

When electrical current flows through a resistor, energy is dissipated in the form of heat (which is why stereos, TV, and computers have to be ventilated). The rate at which energy is dissipated (the dissipated power) is given by $P=V \cdot I$, where $I$ is the current flowing through the resistor and $V$ is the voltage across the resistor. Since power is work done (or energy dissipated) per unit time, the energy given off depends on how long the current flows. The energy given off by the resistor in Joules is $W=V \cdot I \cdot t$, where $t$ is the duration of time the current flows.

Hook up the following circuit and have the instructor check it. When you are ready to begin, turn on and adjust the power supply so that a current of about five amperes flows through the circuit, and start the stopwatch at the same time.
12) Record the current flow $I$ and the voltage $V$ (current from the analog ammeter and voltage from the digital voltmeter).


The current and voltage should remain about constant during the experiment. Adjust the power knob on the power supply if they change. Continue mixing the water and keep monitoring its temperature until it reaches 4 or $5^{\circ} \mathrm{C}$ above room temperature ( 9 or $10{ }^{\circ} \mathrm{C}$ above the starting temperature of water in the cup). When this happens, turn off the current and record the stopwatch reading. Wait a minute or so for the water to reach its highest temperature (keep mixing it) and record that temperature.
13) What is the time and what is the temperature change of the water and the cup?
14) Calculate the energy given off by the resistor in Joules using $W=V \cdot I \cdot t$.
15) Estimate the uncertainties of $V, I$, and $t$. Calculate the uncertainty of $W$. Which quantity, $V, I$, or $t$, introduces the biggest uncertainty into the value of $W$ ?
16) Report your value of $W$ (see Appendix B).

The thermal energy gained by the water sample and the calorimeter cup is given by $Q=C_{\text {cup }} \cdot m_{\text {cup }} \cdot \Delta T+C_{\mathrm{w}} \cdot m_{\mathrm{w}} \cdot \Delta T$, where $C_{\text {cup }}$ is the specific heat of aluminum (the cup is made of aluminum) and $C_{\mathrm{w}}$ is the specific heat of water (look these values up in your book). We neglect the small amount of heat absorbed by the coil itself.
17) Calculate $Q$, the thermal energy gain in Joules, using the above equation.
18) Evaluate the uncertainty of $Q$. Which measurement introduces the largest uncertainty into the value of $Q$ ?
19) Report your value of $Q$ (see Appendix B).
20) Do the values of $W$ and $Q$ found by you agree (see Appendix B)?
21) If they do not agree, what is the relative discrepancy? Discuss possible reasons for the observed disagreement. 22) Why do you start below and end above room temperature in this experiment? What other precautions were taken in this part to minimize the energy transfer to the environment?

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Simple Harmonic Motion (with the LabQuest) (informal)

The simple case of a single mass on a spring is the basic model from which a deeper understanding of the behavior of atoms in a solid is built. Melting, thermal expansion, heat capacity, propagation of sound waves are examples of physical processes that can be explained using models which start by assuming that the atoms of the solid are held together by tiny spring-like forces.

## Part one: Mass on a spring

Hooke's law says that the restoring force of a spring is proportional to the amount by which it is stretched: $F=-k y$, where $k$ is the spring constant (a measure of the stiffness of the spring) and $y$ is the distance the spring is stretched from its equilibrium point. A system with a Hooke's law force will undergo harmonic vibrations when set into motion (if the mass on the spring is given a nudge, it will bob up and down). By measuring the period of these vibrations you can find $k$, the spring constant.


Read the directions in the first lab on motion and set up the motion detector using those instructions. You may want to change the length of time for data collection for this lab. You may also want to change the views in the graph window to examine the velocity vs. time and acceleration vs. time graphs for your data.

Set up the apparatus as shown in the above picture. Attach a paper plate to the mass in order to have a bigger target for the sound waves emitted by the motion detector. Also remember that the detector cannot accurately detect anything closer than about 30 cm .

Start the mass bobbing up and down and take the data. Try to prevent side to side motion and do not let the mass bounce up so high that it compresses the spring all the way. Be sure the mass is bobbing for the entire time the LabQuest is taking data. After the LabQuest has finished taking data explore the graphs on the screen.

1) What is the velocity when the displacement is at its maximum? What is the velocity when the displacement is at its minimum? What is the value of the velocity when the displacement is zero?
2) Describe in detail the relationship between the position, velocity, and acceleration vs. time graphs. (As the position decreases the velocity does what? As the position increases the velocity is doing what?)
3) Using plastic stylus, tap on the position graph to find the vertical distance (in cm ) from the highest to the lowest point on the graph. Divide this distance by two. This value (one half of the vertical distance from maximum to minimum) is the amplitude of oscillations, $A$. Record the amplitude $A$.
4) Record the horizontal distance on the screen (in seconds) from one peak to the next (you may want to get this value for several pairs of peaks and take the average to get a better result). This is the period $T$ of the oscillations.

Set the mass in motion with a different amplitude and collect the data. Explore the graphs on the screen. 5) Record the new $A$ and $T$. Is the period $T$ different or about the same?

Repeat the experiment and measure the $A$ and $T$ for a different mass on the spring (double the mass).
6) Is the new period $T$ different or about the same as in question 5 ?
7) Apply the following equation to find the spring constant $k$ (using your last data set): $T=2 \pi \sqrt{m / k}$.
8) Estimate the uncertainty of $k$.
9) Report your value of $k$ (see Appendix B).

## Part two: Modeling

In this part you will use the LabQuest to find the equation which matches the data. The function which describes simple harmonic motion is a sine or cosine wave. In the following the cosine is used but the sine function could be used as well. The equation of motion for un-damped oscillations can be written as $y=A \cos (\omega t+\phi)$, where $A$ is the amplitude (maximum displacement from equilibrium), $\omega=2 \pi / T$ is the angular frequency, and $T$ is the period. The phase angle $\phi$ is determined by the oscillator's starting position at $t=0$.
a) With a good set of data on the screen, select 'Model' from the 'Analyze' menu and select 'Position'.
b) From the 'Select Equation' menu scroll down and select either $A \cos (B x+C)+D$ or $A \sin (B x+C)+D$.
c) By trial and error, select different values of the parameters $A, B, C$, and $D$ until the function you have matches the data you took. Note that with $\mathrm{D}=0$ the function may not appear on the graph with the data if the graph scale does not go down to $y$ equal zero.
10) What does changing $A$ do to the equation graph?
11) What does changing $B$ do to the equation graph?
12) If the spring was stiffer what would happen to the value of B? Explain.
13) What does changing $C$ do to the equation graph?
14) How far from equilibrium does the mass start its motion if $\mathrm{C}=0$ ? What if $\mathrm{C}=\pi / 2$ ?
15) What does changing $D$ do to the equation graph?
16) What does $D$ represent physically?
17) What input values of $A, B, C$ and $D$ gave a graph that most closely matched your data?
18) Write the equation for the motion of the mass using the values of $A, B, C$ and $D$ you have determined.
19) Do the values of A and B you have determined agree with the corresponding quantities you got for this set of data in part one? (Compare using relative discrepancy.)
20) You may have noticed that for cases where data was taken for a long time period the amplitude of oscillations is not the same at the end of the period as it was at the beginning. Why does this happen?

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Resistance (formal)

Moving charge is called electric current. It is measured in amperes (A). A current of one ampere means one coulomb of charge passes through the cross-sectional area of a wire each second.

Electric potential, measured in volts $(\mathrm{V})$, is the electrical potential energy stored by a unit charge at a certain point in electric field. Positive charges tend to move from higher electric potential to lower electric potential (just like a mass rolls downhill, from higher gravitational potential to lower gravitational potential).
Negative charges tend to move from lower electric potential to higher electric potential (negative charges roll up an "electric potential hill"). Voltage is the difference between electric potentials at two different points in an electric field.

When a conductor is present and a voltage is applied across it, charges will move (current will flow) along the conductor. The property of a conductor that makes it a better or worse conductor is called resistance which is measured in ohms $(\Omega)$. The three quantities are related by the Ohm's law: $V=I R$.

Electric current $I$ is measured by the ammeter. The ammeter is always connected in series with the circuit element(s) in which the current is measured. An ideal ammeter has zero resistance. Voltage $V$ is measured by the voltmeter. The voltmeter is always connected in parallel with the circuit element(s) across which the voltage is measured. An ideal voltmeter has infinite resistance.

An analog ammeter is used to measure currents in this lab. Voltages are measured using a digital multimeter set up for DC (direct current) voltage measurements. The power supply should be set to DC for the entire lab.

## Part one: Ohm's law

Hook up the circuit shown and have your instructor check it. The resistor in this lab is a piece of high resistance wire. Connect the voltmeter last, after the rest of the circuit has been assembled.


Turn the power on. If the ammeter needle goes backwards, the ammeter is hooked up backwards immediately turn the power off and change the leads so the current goes through the meter in the proper direction.

Measure and record the voltage across the wire for six different values of current, increasing the current by about 0.1 A each time starting from zero. Note that the power supply knob does not tell you what the voltage is. The voltage must be measured with the voltmeter (the digital multimeter set up for DC voltage).

Plot $V$ vs. $I$ (voltage across the wire in Volts vs. current in the wire in Amps). Add the regression line to
your graph, remove the connecting line, and enable point protectors (see Appendix A).

1) Does your data support the Ohm's law? (Is Correlation $\approx 0.99$ ? Is the Y-Intercept consistent with zero?)
2) Based on the graph, what is the resistance of your wire? (cf. Appendix B on how to report your value for the resistance.)

In the picture on the previous page the ammeter is on the ' $B$ ' side of the power supply. Keeping the setting of the power supply unchanged, move the ammeter to the ' A ' side. (In your case, if the ammeter has been between the negative terminal of the power supply and the high resistance wire, move it to a point between the positive terminal and the wire, or vice versa.) Record the current at the new location.
3) The current at A and B should be the same: no current is "lost" in the wire. This means that every electron which entered the wire from the left, exited on the right. So, what is "lost" or "used up" as the current goes through the wire if it is not current? (Hint: When the current is large, the wire gets hot.)

Now move the ammeter to a point corresponding to point ' C ' in the figure. Record the current at point ' C '.
4) The current at point 'C' should be zero (or very close to zero). Explain why.

## Part two: Resistance and resistivity

The resistance of a wire depends on its length and cross-sectional area. It is proportional to the length $L$ and inversely proportional to the cross-sectional area $A: R=\rho L / A$. The coefficient of proportionality $\rho$ is called resistivity. It depends only on the material the wire is made of.

For a constant current in the wire of about 0.3 A , measure and record the potential drop from the beginning of the wire out to five different points along the wire. Also measure and record the distance $L$ from the beginning to each point. Calculate $R$ using $R=V / I$ for each $L$.

Plot $R$ vs. $L$. Add the regression line to your graph, remove the connecting line, and enable point protectors.
5) Does your data support the $R=\rho L / A$ equation?
6) What does the slope of your graph physically represent?

Measure the diameter of the wire with a micrometer at several different points. Find the average diameter. Calculate the average cross-sectional area of the wire. Calculate the resistivity of the material your wire is made of.
7) What value do you get for the resistivity?
8) Estimate the uncertainty of the resistivity value you have found.
9) Report your value of resistivity (cf. Appendix B on how to report).
10) Compare your resistivity value with the accepted values for different materials to determine the substance your wire is made of. According to your measurements, what material is your wire made of? What is the relative discrepancy between your value and accepted value?
11) What is the difference between resistance and resistivity? Which wire has a lower resistance, one with a diameter of 0.01 mm or one with a diameter of 0.1 mm ?
12) Why should the current in this experiment be kept relatively low? (Hint: What would happen to the temperature and the resistivity of the wire if the current were very large?)

NOTE: This is a formal lab (one report per student). Be sure to read the format page in this manual. Discuss your findings and include some comments about the lab in a conclusion.

## Resistors in Series and Parallel (informal)

An analog ammeter is used to measure currents in this lab. Voltages are measured using a digital multimeter set up for DC voltage measurements. The power supply should be set to DC for the entire lab.

## Series Circuit:

Hook up the series circuit shown in the figure below and have your instructor check it.

1) Start with the power supply knob turned all the way counterclockwise and gradually increase the power until the voltage across the power supply reads 5.0 V . Monitor the current through the circuit as you increase the power. How does the brightness of the bulbs relate to the current?
2) With the voltage across the power supply equal 5.0 V , measure and record the voltages across individual bulbs. Also, record the current measured by the ammeter. Note the intensities of the bulbs, so you can compare them to the case with four bulbs (question 7). How does the sum of the voltages across the three bulbs compare with the voltage across the power supply?

Re-configure the circuit so that the ammeter measures the current between bulbs one and two.
CAUTION: Watch the ammeter carefully when you turn the power on. If the needle goes all the way across the scale and pegs on the right side, turn the circuit off immediately and consult with the instructor.
3) With the same power supply setting as in question 2, record the current flowing from bulb one into bulb two. How does the current in this case compare with the current recorded in question 2? Is any current "used up" in the first bulb and, as a consequence, bulb two receives less current?
4) What happens if you unscrew one bulb?
5) With one bulb unscrewed, measure and record the voltage across the power supply. Explain why the power supply voltage is not zero but the current in the circuit is zero.


Now add one more bulb so that it is in series with the first three. Keep the voltage across the power supply equal to 5.0 V (you may have to adjust slightly the power knob setting).
6) How does the total current change if a bulb is added with the same voltage ( 5.0 V ) across the power supply? Explain your observation.
7) How does the intensity of each bulb in this case compare to the intensity of each bulb in the three bulb case? Explain.

## Parallel Circuit:

Turn the power supply back to zero and then off. Hook up the parallel circuit shown in the figure below and have your instructor check it. Turn on the power supply (watch the ammeter again!) and adjust the knob until the voltage across the power supply reads 5.0 V again.
CAUTION: More than 5.0 V may blow the bulbs in this part!

8) How does the intensity of the bulbs compare with the intensity for the series circuit with the same power supply setting?
9) Record the total current flowing through the power supply. How does the total current in the parallel circuit compare with the total current for the series circuit with the same power supply setting?
10) Measure and record the voltage across each bulb. How do these values compare with the value of the voltage across the power supply?
11) What happens if you unscrew one bulb?

Re-configure the wiring, but do not change the power supply setting, so that the ammeter measures only the current through the first bulb.
12) Record the current. How does the current through the first bulb compare with the total current?

Re-configure the wiring so that the ammeter measures only the current through the second bulb.
13) Record the current. How does the current through the second bulb compare with the total current?

Re-configure the wiring so that the ammeter measures the current through the third bulb.
14) Record the current. How does the current through the third bulb compare with the total current?
15) How does the sum of currents through bulbs 1,2 , and 3 compare with the total current?
16) Why are the bulbs brighter in the parallel circuit than in the series circuit (with the same voltage across the power supply)?

Re-configure the wiring to measure the total current. Add one more bulb so that it is in parallel with the first three. Keep the voltage across the power supply equal to 5.0 V (you may have to adjust slightly the power knob setting).
17) How does the total current change when you add a bulb in parallel? Explain what you observe.
18) How does the intensity of the bulbs compare to the case with three bulbs? Explain your observation.
19) What is the difference between voltage and current?
20) Why should the ammeter be in series with the circuit element where the current is measured?
21) Why should the voltmeter be in parallel with the circuit element across which the voltage is measured?
22) Why should an ammeter have low resistance while a voltmeter high resistance?
23) List some examples of the use of series circuits in your home. Why is it important to have the series connection in these examples?
24) List some examples of the use of parallel circuits in your home. Why is it important to have the parallel connection in these examples?

Be sure to discuss you findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Batteries and Kirchhoff's Rules (informal)

In the previous labs you observed that the electric potential changes across a resistor when the current flows through the resistor. The potential at B is less than the potential at A in the figure, so there is a potential drop from A to B as current flows from A to B.


A battery or power supply introduces a potential rise equal to the $\operatorname{emf} \varepsilon$ of the battery or power supply. In the following figure there is a potential rise in going from D to C .


In addition to the emf, a real battery has an internal resistance $r$ associated with it:


The terminal voltage $V_{\mathrm{T}}$, or voltage across the external terminals of the battery, is emf $\varepsilon$ minus the potential drop across the internal resistance $r: V_{\mathrm{T}}=\varepsilon-I r$. Notice that the terminal voltage depends on the current that flows through the battery.

## Part one: Emf, internal resistance, and terminal voltage

The terminal voltage is equal to the emf when the current is zero. Thus you can find the emf of a battery by measuring the voltage across its terminals while the battery is not in a circuit. Since almost no current flows through a voltmeter when it is connected to the battery, we have $I=0$ and $V_{\mathrm{T}}=\boldsymbol{\varepsilon}$.

1) Measure and record the emfs for two batteries (choose a 1.5 V battery and a 9 V battery).

When a current is flowing through the battery, the terminal voltage will be smaller than the emf because of the potential drop across the internal resistance of the battery.

Set up the circuit shown at the top of next page. Adjust the rheostat (the variable resistor R ) to get six different values of current. Record the current values along with the terminal voltage $V_{\mathrm{T}}$ across the battery for each setting. Let the current flow only as long as necessary to take the meter readings, as you do not want the battery to go dead during the experiment. Do this for both batteries.


You can now determine the internal resistance of your batteries using $V_{\mathrm{T}}=\boldsymbol{\varepsilon}-I \cdot r$, where $r$ is the internal resistance. Plot $V_{\mathrm{T}}$ vs. I for each battery. Add the regression line to each graph (enable point protectors and remove the connecting line, cf. Appendix A).
2) Does your data support the equation $V_{\mathrm{T}}=\varepsilon-I r$ ? Note that this equation is only an approximation to the real, and more complicated, behavior. You may (and probably will) see deviations between your data and the regression line. Describe how your results deviate from the ideal case.
3) Based on your graphs, what is the internal resistance $r$ for each battery?
4) What does Y-intercept of each graph represent? How does it compare to what you found in question 1?

## Part two: Kirchhoff's rules

Notice that for the circuit shown below it is not possible to use the rules for adding series and parallel resistors, because the batteries are in the way. Instead you can use the rules formulated by Kirchhoff.

Kirchhoff's first rule (the junction rule) says that the sum of currents flowing into any junction in a circuit is equal to the sum of currents flowing out of that junction. Kirchhoff's second rule (the loop rule) says that the sum of all potential rises and all potential drops in any closed loop in a circuit should be zero.


Hook up the circuit shown above. The resistor color code is given at the end of this lab. With the voltmeter, measure the potential rises and drops across each element in the loop a-b-c-f-g-a (measure the voltage from $\mathbf{a}$ to $\mathbf{b}$, then from $\mathbf{b}$ to $\mathbf{c}$, etc.; some of the values may be zero or negative). Be sure to keep the voltmeter leads oriented in the same direction when you go around the loop. For example, red at a black at $\mathbf{b}$, then red at $\mathbf{b}$ black at $\mathbf{c}$, etc.
5) Do these voltages ( $\mathbf{a}$ to $\mathbf{b}$ plus $\mathbf{b}$ to $\mathbf{c}$ plus ...) add up to be zero? They should if you found them correctly! 6) Now measure and record each voltage around the loop a-d-h-e-f-g-a. Is Kirchhoff's loop rule valid for this loop (do these voltages add up to zero)?
7) Now verify Kirchhoff's loop rule for the loop b-d-h-e-c-b. Do these voltages add up to zero?
8) What value of voltage do you get between $\mathbf{f}$ and $\mathbf{c}$ in this circuit? Why?
9) Does it matter which way you choose to go around a loop in applying Kirchhoff's rules (say a-g-f-c-b-a instead of a-b-c-f-g-a)? What changes when you reverse the direction around the loop?
10) Are you measuring terminal voltages or emfs across the batteries in this part?
11) Note that the internal resistances of the batteries are not shown. Why are the voltages across the internal resistances of the batteries ignored when Kirchhoff's loop rule is verified? Are they ignored?

Use the Ohm's law $(V=I R)$ to find the current in each of the three resistors. The currents are very small in this part so you cannot measure them accurately with an ammeter. However, by dividing the voltage across a resistor by the resistance, you can find the current through the resistor. Also, determine the direction of current in each resistor. (Hint: Current in a resistor always flows from higher to lower potential.) Record the currents indicating their directions (e.g., from $\mathbf{g}$ to $\mathbf{f}$, etc.).
12) Using the currents you have found, does the junction rule work at the junction between $\mathbf{a}, \mathbf{b}$, and $\mathbf{d}$ ? How about at the junction between $\mathbf{c}, \mathbf{e}$, and $\mathbf{f}$ ? What is the relative discrepancy between the sum of currents flowing into each junction and the sum of currents flowing out of that junction?
13) Why do we say voltage across the resistor but talk about current through the resister?
14) For the following circuit, calculate the current using Kirchhoff's rules. Do not set up the circuit.


Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Resistor color code

The bands on the resistor tell what the resistance is supposed to be with the following code;

| Bands one \& two | Band three | Band four |
| :---: | :---: | :---: |
| Black $=0$ | Silver $=$ X . 01 | Silver $=+$ or - 10\% |
| Brown $=1$ | Gold = X . 1 | Gold $=+$ or - 5\% |
| Red $=2$ | Black $=$ X 1 | nothing $=+$ or $-15 \%$ |
| Orange $=3$ | Brown $=$ X 10 |  |
| Yellow $=4$ | Red $=$ X 100 |  |
| Green $=5$ | Orange $=$ X 1000 |  |
| Blue $=6$ | Yellow $=$ X 10000 |  |
| Violet $=7$ | Green $=$ X 10 ${ }^{5}$ |  |
| Gray $=8$ | Blue $=$ X 106 |  |
| White $=9$ | Violet $=$ X 10 ${ }^{7}$ |  |
|  | Gray $=$ X 108 ${ }^{8}$ |  |
|  | White $=$ X $10{ }^{9}$ |  |

For example, a resistor with bands Brown, Red, Orange, and Silver is a resistor of $12000 \Omega \pm 10 \%$.

## Breadboards

A common method of temporary construction for the hobbyist is the solderless breadboard, a device made of plastic with hundreds of spring-loaded connection sockets joining the inserted ends of components and/or 22-gauge solid wire pieces. A photograph of a real breadboard is shown here, followed by an illustration (next page) showing a simple series circuit constructed on one.


Underneath each hole in the breadboard face is a metal spring clip, designed to grasp any inserted wire or component lead. These metal spring clips are joined underneath the breadboard face, making connections between inserted leads. The connection pattern joins every five holes along a vertical column (as shown with the long axis of the breadboard situated horizontally):

## Lines show common connections

 underneath board between holes

Thus, when a wire or component lead is inserted into a hole on the breadboard, there are four more holes in that column providing potential connection points to other wires and/or component leads. The result is an extremely flexible platform for constructing temporary circuits.

As an example, the three-resistor circuit shown below can also be built on a breadboard:

## Schematic

diagram


NOTE: Do not force wires, resistors or leads to other electronic devices into the breadboard holes if they are too thick. This stretches the holes so that they will not make contact with normal sized components.

## Diodes and Transistors (informal)

In this lab you will investigate the behavior of a diode and of a transistor. These devices are the basis behind signal amplifiers, computer circuitry, and many other electronic devices. A typical computer chip has thousands of diodes and transistors on it.

## Part one: The diode

A diode is a device that allows current to flow in one direction but not in the other.


Set up the circuit shown. Use a $100-\mathrm{k} \Omega$ resistor, $R=100 \mathrm{k} \Omega$. Start at zero volts across the power supply terminals. Measure the applied voltage (voltage directly across the power supply), the voltage across the diode, and the voltage across the resistor. Adjust the power to get 10 readings of applied voltage at 0.5 V intervals form zero to +5 V and record all three voltages for each of these 10 settings. Organize your data into a table.

1) How does the sum of the voltages across the resistor and the diode compare to the voltage across the power supply? Explain what you observe.

Since $V_{\mathrm{R}}=I R$, the current in the circuit can be found from the voltage across the resistor: $I=V_{\mathrm{R}} / R$. Calculate and record the current in the circuit for each of the resistor voltage readings.

Now reverse the power supply leads and repeat the three voltage measurements across the diode, the resistor and the power supply for 10 power supply readings between zero and 5 V (which are now really negative voltages out to -5 V , since you have reversed the power supply leads). The resistor voltage should be zero (or very close). Organize your data into a table recording the voltages as negative numbers. Using $I=V_{\mathrm{R}} / R$, calculate and record the current in the circuit for each of the 10 power supply settings

Plot the current values on the $y$-axis vs. the corresponding power supply voltages on the $x$-axis (you should have 10 negative voltages and the corresponding currents and 10 positive voltages and currents). Highlight the positive- $x$ side (the right hand side) of the graph and add the regression line to it.
2) What is the slope of the line? What does the slope physically represent in this case?
3) Why are the current values all zero (or near zero) for the reversed (negative) voltage? (Hint: what do diodes do in a circuit?)
4) What would happen if a sinusoidal voltage (an AC voltage), which varies from -5 V to +5 V , were applied to this circuit? Draw a sketch showing an AC voltage and the corresponding current in this circuit, one below the other. (This is often the first step in changing an alternating current into a direct current.)

## Part two: The transistor

A transistor uses a small signal current ( $I_{\mathrm{b}}$ in the next page figure) to control or change a much larger current ( $I_{\mathrm{c}}$ in the figure). In this way a weak signal gets amplified. For example, $I_{\mathrm{b}}$ might be the weak signal from a radio station and $I_{\mathrm{c}}$ could be the larger current in your speaker wire. You want small changes in the signal to cause large changes in the output. The transistor does this for you.

A transistor has three terminals: the Base, the Collector, and the Emitter. To determine which lead is which, hold the transistor with the leads facing you and the small tab on the side pointing to the left. The three terminals will be oriented as shown in the figure.


Set up the circuit shown. $V_{\mathrm{bb}}$ is the variable power supply. It represents the input signal (an incoming radio signal, for example). The 12 V potential and the ground are supplied on the breadboard and represent the amplifier's power source (supplied by house current for your radio receiver). $R_{1}$ is a $100 \mathrm{k} \Omega$ resistor. $R_{2}$ is a $3.3 \mathrm{k} \Omega$ resistor (this is where the output signal appears, your radio speaker, for example).


Start with $V_{\mathrm{bb}}$ set to zero. Measure the voltages across $R_{1}, R_{2}$, and $V_{\mathrm{ce}}$ shown in the figure. From these measurements and the Ohm's law find the currents $I_{\mathrm{b}}$ and $I_{\mathrm{c}}$. Now, change the setting on $V_{\mathrm{bb}}$ so that the voltage across $R_{1}$ reads about 0.2 V and measure the voltages across $R_{2}$ and $V_{\text {ce }}$. Calculate the currents $I_{\mathrm{b}}$ and $I_{\mathrm{c}}$. Repeat the measurements for six different settings of $V_{\mathrm{bb}}$, increasing $V_{\mathrm{R} 1}$ by about 0.2 V each time. Do not exceed 1.0 V for $V_{\mathrm{R} 1}$. Calculate and record the currents $I_{\mathrm{b}}$ and $I_{\mathrm{c}}$ for each setting.
5) Look at your values for the voltages across $R_{2}$ and $V_{\mathrm{ce}}$. How do they change relative to each other? How does their sum change? Explain what you observe.

Make a plot of $I_{\mathrm{c}}$ vs. $I_{\mathrm{b}}$. This should be a straight line (approximately - you may have to discard the zero voltage measurement). If the data begins to curve at the top, you have saturated the circuit - use lower applied voltages and make sure that the $V_{\mathrm{R} 1}$ values do not exceed 1.0 V . Add the regression line to your plot. The slope of the line is called the gain of the circuit.
6) According to the slope of your $I_{\mathrm{c}}$ vs. $I_{\mathrm{b}}$ graph, what is the gain for this circuit?

Notice that small changes in $I_{\mathrm{b}}$ cause much bigger changes in $I_{\mathrm{c}}$ (the slope is large), so a small change in the input signal will be amplified in the output. Another thing you may notice about the way this circuit is set up is that an increase in $V_{\mathrm{bb}}$ causes a decrease in $V_{\mathrm{ce}}$. This circuit is said to be an inverting circuit.
7) Draw a sketch of an AC voltage (representing the input signal) and, directly below it, draw a sketch of the output $V_{\text {ce }}$ voltage for your amplifying circuit. (Hint: The output signal will be inverted and amplified.)

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Magnetic Field (with the LabQuest) (informal)

In this lab you will map the magnetic field of a magnet and also the magnetic field of a current carrying coil of wire. First you will map the direction of the magnetic fields using a compass. Then you will use the LabQuest with a magnetic field probe to find the magnitude of the field along the axes of the magnet and the coil.

IMPORTANT: Keep the magnet far away from your calculator, your cell phone, credit cards, USB memory drives and the LabQuest! These are VERY strong magnets! Do NOT let two magnets get close together (they are very hard to separate once they are attached to each other).

1) The operation of the magnetic field probe is based on the Hall effect. In your report for this lab explain in a paragraph how the Hall effect enables the probe to detect the magnetic field. (You may have to consult sources other than your textbook for an explanation.)

## Part one: Magnetic field of a bar magnet

The magnets you are using are in the shape of a coin with north on one side and south on the other. Place the magnet on edge on a sheet of paper. Trace the location of the magnet, indicating the orientation of its sides. Use the compass to find the direction of the magnetic field at various points around the magnet. At each point draw an arrow on the paper indicating the direction in which the north pole of the compass points. This is the direction of the magnetic field at that point. Make sure your individual arrows form continuous field lines. Magnetic field lines outside of a magnet start at the magnet's north pole and end at the magnet's south pole.
2) How does the map of the magnetic field of a bar magnet compare to that of the Earth? (Find a picture of the Earth's magnetic field in your book.) Include your drawing of the field lines in your lab report.

The magnetic field (in Tesla) as a function of distance from the magnet's center along the axis of the magnet is given by $B=M / r^{3}$, where $r$ is the distance from the magnet's center and $M$ is a constant. You will verify this equation by recording the magnetic field at different distances from the magnet.


Set up the apparatus as shown in the figure. The magnetic probe should be plugged into CH 1 on the LabQuest. Follow the instructions in the end of the lab on how to collect the data. Analyze your data as described in the Analysis section in the end of the lab. Answer the questions in the end of the Analysis section.

## Part two: Magnetic field of a current-carrying coil

A magnetic field will be present anytime there is moving charge (a current flow). A coil of wire carrying a direct current (DC) has a magnetic field which maps out to have the same shape as the field of a bar magnet.


Set up the apparatus as shown, with about 1 A of current flowing to produce a magnetic field in and around the coil. If the coil starts getting hot, turn off the circuit and let the coil cool before proceeding further.

Put the coil onto a sheet of graph paper so that the axis of the coil is parallel to the paper. For better results, you may want to cut a hole in the paper around the coil so that the axis of the coil lies exactly in the plane of the paper. Map the magnetic field of the coil with the compass as you did for the magnet in Part one.
3) How does the map of the field of a coil compare to that of a bar magnet? Describe similarities and differences.

Measure the magnetic field on the axis of the coil with the LabQuest the same way you did for the magnet. Be sure the white dot on the probe faces the center of the coil (not down on the table below the center).

NOTE: The magnetic field of the coil is a bit weaker than the field of the magnet so you may have to start a little closer to the coil and change the distance by less (by 0.5 cm ) each time.

Analyze your data as described in the Analysis section. Answer the questions in the end of the Analysis section.
4) How do the strengths of the magnetic fields of the magnet and the coil compare in this lab?
5) The electric field of a point charge decreases as $1 / r^{2}$, with $r$ being the distance from the charge. The magnetic field of a magnet decreases as $1 / r^{3}$ far away from the magnet. Why is the dependence on the distance $r$ different in these two cases?

Discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## To collect data:

REPEAT: Do not lay the magnet near your calculator or the LabQuest unit, or near a disk drive or your credit cards or another magnet!
a) Start the LabQuest using the button on the upper left, above the screen. (You may want to review the general directions for using LabQuest which can be found in the first lab of this manual.)
b) From the 'Sensor' menu choose 'Sensor Setup'. For 'CH1 choose 'Magnetic Field High'. The switch on the probe control box should also be set on high. Click 'OK' to exit.
c) Now you want to zero the probe. Hold it on the line along the magnet's axis well away from the magnet (or the coil) and any other possible magnetic or electrical devices. The white dot on the probe must face the magnet. Under the 'Sensor' menu choose 'Zero' and select 'CH1: Magnetic Field High'.
d) Slowly move the probe back and forth in front of the magnet to get an idea of the range of the magnetic field of the magnet. Be sure the white dot on the probe faces the magnet. If the magnetic field is too strong, the probe will saturate (give the same reading at different points).
e) Choose 'Data Collection' from the 'Sensor' menu. For 'Mode' select 'Events with Entry'. Tap in the 'Name' box and use the stylus to enter 'distance' with units of 'cm'. Click 'OK' to exit.
f) Place the probe on the magnet's axis close to the magnet. Be sure the probe is not saturated and the white dot faces the magnet. Tap the forward button in the lower left of the screen, or the button on the LabQuest with the same icon below the screen, to start taking data.
g) Tap the 'KEEP' button on the screen to record the first data point. Enter the distance in cm from the center of the magnet (or the coil).
h) Move the probe one centimeter away from the magnet and tap the 'KEEP' button on the screen. Record the new distance in cm from the center of the magnet.
i) Repeat the previous step, moving the probe one centimeter each time and tapping 'KEEP' to record the data points. When you have reached a distance such that the magnetic field has dropped to zero, tap to stop data collection. You should be able to get about 8 to 10 data points.
j) The graph on the LabQuest screen should curve downward. If the initial part of the graph is flat but then the graph curves down, you may want to retake the data starting a little further away from the magnet (so that the probe is not saturated at the beginning).
k) Analyze the data as explained below in the Analysis section.

## Analysis

Use the LabQuest to find the equation that matches the data. Theory says the magnetic field far away from a bar magnet or a current-carrying coil should vary with distance as $B=M / r^{3}$, where $M$ is a constant.
a) With a good set of data on the screen, select 'Model' from the 'Analyze' menu and select 'Position'. b) From the 'Select Equation' menu scroll down and select $y=\mathrm{A} / x^{\mathrm{B}}$. Note that the B coefficient in $y=\mathrm{A} / x^{\mathrm{B}}$ is not the magnetic field, while $B$ in $B=M / r^{3}$ is. ( $B$, the field, is represented by $y$ in the model equation.) c) By trial and error, select different values for the parameters A and B until the function matches the data you took.

1-A) What values for the $A$ and $B$ parameters give the best fitting function for your data?
2-A) How well does your data verify the theoretical equation $B=M / r^{3}$ ? (Is the B parameter value you have found close to 3 ?) Comment on any observed discrepancy.
3-A) What value have you found for the $M$ constant? What does $M$ physically represent?

## Faraday's Law (with the LabQuest) (formal)

Faraday's law is $V=N\left|\Delta \Phi_{\mathrm{B}} / \Delta t\right|$, where $N$ is the number of turns in the coil, $\Phi_{\mathrm{B}}$ is the magnetic flux through each turn of the coil, and $V$ is the induced emf (in volts). Notice that Faraday's law says there must be a change in the flux in order for a voltage (an emf) to appear. The changing flux for this lab is provided by a bar magnet which changes its location relative to a wire coil. In this lab the LabQuest measures the emf in the coil exactly like a voltmeter would, with the exception that it records a series of voltages rather than just one. Follow the instructions in the end of the lab on how to collect the data.


## Part one: Same magnet speed, different coils

In this part you will verify that the induced emf $V$ is proportional to the number of turns in the coil, $N$. You will have to use five different coils for this part (you may have to swap the coils with other groups in the lab).

For five coils, each with different number of turns $N$, follow the directions in the end on how to collect data. Drop the magnet from the same height each time and with the same end pointing down. The graph on the LabQuest screen should be mostly flat except for a peak above and a peak below zero as the magnet first falls into the coil (first peak) and then out of the coil (second peak).

By tracing through the graph with the arrow keys, find and record the maximum and minimum voltages (the $y$-values of the + and - peaks) on the graph for each trial.

1) Do the maximum and minimum voltages have the same absolute values? Explain why not.
2) What happens to the graph on the screen if you turn the magnet over and drop it with the other pole facing downward? Explain your observation.


In the picture above the curves represent the real voltages and the tick marks represent two different data collection rates (a voltage measurement of the actual voltage is made at each tick mark). Notice that in the second case no voltage measurement is made when the second peak occurs.
3) Which set of readings (top or bottom) will give more accurate peak voltage readings and why?
4) Step through the last graph on the LabQuest using the trace button. Judging from the spacing of the data points, what is the uncertainty of your recorded value of the maximum voltage?
5) If there were only three or four tick marks within the peaks in your graph (like in the bottom graph in the above picture), what would you want to change on the LabQuest to get better results?

Find the average of the absolute values of the maximum and minimum voltages for each trial. Graph these average values $V_{\text {av }}$ vs. $N$ (the number of turns in the coil). The graph should be approximately a straight line. Add the regression line to your graph, remove the connecting line, and enable point protectors.
6) Does your data agree with the straight line (see Appendix A)?
7) What does the slope of this graph physically represent?
8) Report the value of the slope of your graph (see Appendix B).
9) If you were dropping the magnet from a higher point, would the value of the slope be larger, smaller, or the same?

## Part two: Same coil, different magnet speeds

The rate of change of the flux, $\left|\Delta \Phi_{\mathrm{B}} / \Delta t\right|$, is proportional to the speed at which the magnet falls through the coil. In order to find the speed of the magnet as it passes through the coil you will need two measurements: the time the magnet takes to pass the center of the coil and the length of the magnet.

Using the arrow keys on the LabQuest, determine the time span between the peaks. The first peak occurs when the front tip of the magnet is near the center of the coil. The second peak happens when the back tip of the magnet passes near the center. The time between the peaks, to a good approximation, gives you the time the magnet takes to cross the center of the coil. The length of the magnet divided by this time is the average speed of the magnet while passing the coil.

Also from the LabQuest screen record the + and - peak voltages (the y-values) and find $V_{\text {av }}$ as you did in Part one. Find and record $V_{\mathrm{av}}$ and the time between the voltage peaks for 5 different trials, changing the distance the magnet falls by at least 20 cm each time to have different magnet speeds in different trials. Record the distance fallen for each trial. Use the same coil in each of these trials.

Calculate the speed of the magnet for each trial. Graph $V_{\mathrm{av}}$ vs. the speed of the magnet. The graph should be approximately a straight line. Add the regression line to your graph, remove the connecting line, and enable point protectors.
10) Does your data agree with the straight line (see Appendix A)?
11) What does the slope of this graph physically represent?
12) Report the value of the slope of your graph (see Appendix B).
13) What voltage would you get if the magnet were not moving in this experiment?
14) For each trial, calculate the speed based on the acceleration of $9.80 \mathrm{~m} / \mathrm{s}^{2}$ and the distance fallen by the magnet. Do they agree with the speeds you measured above (relative discrepancy, trial by trial)? If you see a discrepancy, comment on its possible causes.

## NOTE: This is a formal lab (one report per student). Follow the format page of this manual. Discuss your findings and provide some comments about the lab in the form of a conclusion.

## To collect data:

NOTE: Do not lay the magnet near your calculator or the LabQuest unit or near a disk drive or diskette or your credit cards!
NOTE: The magnet should not hit the floor (this can destroy the magnetism of the magnet) - always be sure some foam rubber is underneath the tube.
a) Turn on the LabQuest
b) Press the Home button (you can do this by clicking on the Home button on the screen or the buttons below the screen).
c) Select "LabQuest App". Make sure the data cables or voltage probes are connected into CH1 of the channels on the top of the LabQuest.
d) Your Screen should automatically start reading CH 1 :Potential and show a potential reading in Volts.
e) We now will want to change our how the LabQuest chooses its data points and time intervals; under 'Sensors' choose Select Data Collection...
f) A new window will open that has three menu choices that you can edit (Timing, Triggering, and Advanced).
g) Under Timing click your stylus on the Rate box. A keypad will now open up and you can edit the number of samples per second you want to take. A good number is 1000 .
h) For this lab you will want to change the length of time your LabQuest takes data. In the same manner as the Rate box you will want to change this number to around $0.15-0.20$ seconds.
i) Next select 'Triggering'. The triggering menu will now drop down and you will want to select the box next to "Enable Triggering". Scroll down and change the trigger voltage about 0.01 V . The LabQuest will trigger when the voltage reaches this value.
j) You may also want to change the "collect points before trigger" to around 10 or 20 in order to see the full graph.
k) Now select 'OK'. You are now ready to start taking data.

1) Press the forward button either on the LabQuest screen or the LabQuest. The LabQuest will wait until it is triggered by the falling magnet to take data, which will then appear on the screen in a graph.

## Capacitors and Inductors (informal)

In this lab you will investigate the behavior of RC and LR circuits. Review the properties of capacitors and inductors in your book before you begin. You will also learn to operate an oscilloscope.

## Part one: The oscilloscope

The oscilloscope is really just a fancy voltmeter. The main advantage of the oscilloscope over a voltmeter is that it can be used to follow rapid changes of voltage in an AC circuit. A normal voltmeter cannot react fast enough to follow rapid voltage changes.

The heart of the oscilloscope is the cathode ray tube. In this vacuum tube, free electrons are created at the back of the tube and accelerated toward the front. At the front, the electrons strike a screen coated with phosphorous which then gives off light. The beam of electrons is swept across the screen from left to right by a changing electric field inside the oscilloscope. The rate at which the beam is swept across the screen is controlled by the TIME/DIV knob. Try changing the setting of this knob to see what it does to the beam. When the time VARIABLE knob is turned all the way clockwise, the settings on the TIME/DIV knob tell you how long it takes for the beam to sweep across one block on the screen.

Vertical motion of the electron beam is controlled by the voltage applied to the leads of the oscilloscope. If a constant (DC) voltage is applied to the leads, the beam will shift up by an amount proportional to the applied voltage. If an alternating sine wave voltage is applied to the oscilloscope leads, the beam will sweep up and down as the sine wave voltage alternates between its maximum positive value and its maximum negative value. Since the beam is also sweeping left to right at the same time, a sine wave shaped curve will appear on the screen. The height of the curve will be proportional to the input voltage. The VOLTS/DIV knob sets the vertical scale on the screen (the voltage VARIABLE knob should be all the way clockwise). As an example, if the center to peak distance of a signal on the screen is two blocks and the VOLTS/DIV knob is set on 3 V , then the amplitude of the signal is 6 volts.

NOTE: Some probes (the leads of the oscilloscope) divide the input signal by a factor of 10 . In this case, you need to multiply the voltage you read off the screen by 10 to get the actual input voltage.

Attach the leads of the oscilloscope to the output of the sine wave generator and play with changing the VOLT/DIV and TIME/DIV knobs to see how they interact. Set the sine wave generator to a frequency between 1 kHz and 100 kHz . Adjust the oscilloscope until you have a nice picture of a single sine wave on the screen.

1) What frequency did you pick on the sine wave generator?
2) Use the oscilloscope to measure the frequency. To measure the frequency, count the number of horizontal blocks on the screen within one period of the sine wave. Multiply the number of blocks by the TIME/DIV setting you have chosen. This number is the time per oscillation, or the period. The inverse is the frequency. Does your measured frequency match the frequency setting on the generator?
3) Use the oscilloscope to measure the amplitude voltage of this wave (count the number of vertical blocks and multiply by the VOLTS/DIV setting on the oscilloscope). What is the amplitude voltage?

Connect the probe to the back of the function generator to see a square wave.
4) What are the peak to peak voltage and frequency of this wave (according to the readings on the oscilloscope)? Does the frequency match the generator setting?

## Part two: The Capacitor ( $C=0.1 \mu \mathrm{~F}$ )

When a DC voltage is applied to a capacitor current begins to flow and charge builds up on the capacitor at an exponential rate. Since $Q=C V$, at some point the applied voltage cannot cause any more charge to be added to the capacitor. At this point the charge on the capacitor and the voltage across the capacitor reach their maximum values and the current in the circuit drops to zero.

For most capacitors a single charging occurs too quickly to observe. So in this lab we will let the capacitor charge and discharge many times a second by applying a square wave voltage, which turns on to a fixed positive voltage and then off to zero voltage over and over again. The oscilloscope plots each charge/discharge cycle on top of the previous cycle, giving the illusion of a single charge and discharge frozen in time.

Determine the maximum voltage output of your square wave generator by hooking the leads of the oscilloscope directly to the square wave output (choose a frequency between 200 and 1000 Hz ). NOTE: The square wave signal comes from the back side of the generator used in this lab.

Set up the circuit shown and put the leads of the oscilloscope across the capacitor. Make sure that the ground lead of the probe (the one with the crocodile clip) is connected to the ground of the square wave output.

square wave
Pick a frequency and adjust the oscilloscope settings so that you get a shark fin shaped picture on the oscilloscope. This should look like a combination of an inverted exponential increase followed by an exponential decrease like the ones shown in your book for a charging and discharging capacitor. Every time the square wave goes positive, the capacitor begins to charge up (an inverted exponential increase). When the square wave goes to zero, the capacitor discharges (an exponential decrease). Draw a picture of the voltage on the capacitor as a function of time as seen on the oscilloscope.

The equation for the voltage across a discharging capacitor is $V=V_{0} \mathrm{e}^{-t / R C}$, where $V_{\mathrm{o}}$ is the maximum voltage on the capacitor. The time constant $\tau$ is defined as $\tau=R C$, where $R$ is the resistance of the resistor in ohms and $C$ is the capacitance of the capacitor in Farads. Notice that when $\mathrm{t}=\tau=R C, V=V_{\mathrm{o}} \mathrm{e}^{-1}$ (or $V \approx 0.368 V_{\mathrm{o}}$ ).
5) What is the theoretical time constant for this circuit?
6) The time constant can be measured from the oscilloscope. Find the time (the horizontal distance on the screen times the Time/Div setting) between the moment when the voltage just starts to decrease from its maximum value and the moment when the voltage is 0.368 of its maximum value. This time is your time constant. What value does the oscilloscope give for the time constant?
7) How do the theoretical time constant and the oscilloscope reading compare (relative discrepancy)?
8) What is the maximum voltage on the capacitor? How does this voltage compare to the maximum voltage of the square wave determined previously (relative discrepancy)?
9) Why are the two voltages about equal (they should be!)?

## Part three: The Inductor $(L=\mathbf{2 5 m H})$

When voltage is applied across an inductor, a reverse voltage (called a back emf) is generated in the coil because of the Faraday's law. In the case of a square wave applied voltage, the voltage across the inductor is different from zero for only short periods of time, right after the square pulse changes direction (positive to zero or zero to positive). The oscilloscope is needed because this change in voltage occurs too quickly to see with a voltmeter.

Set up the circuit below and measure and make a sketch of the voltage across the resistor (which is proportional to the current in the circuit). Again, make sure that the ground lead of the probe is connected to the ground of the square wave output. NOTE: You will have to adjust the frequency of the square wave in order to see the fin shark shaped picture on the screen.


## square wave

The decreasing voltage across the resistor (proportional to the decreasing current in the circuit) is given by $V$ $=V_{0} \mathrm{e}^{-t /(L / R)}$, where $L$ is the inductance of the coil, $R$ is the resistance, and $V_{\mathrm{o}}$ is the voltage across the resistor when the current is near its maximum value. The time constant for this circuit is $L / R$.
10) What is the theoretical value for the time constant of this circuit?
11) Measure the time constant from the oscilloscope as you did previously for the capacitor circuit. What value does the oscilloscope give for the time constant?
12) How do the theoretical time constant and the oscilloscope reading compare (relative discrepancy)?
13) What value do you get for $V_{o}$ ? How does it compare to the maximum voltage of the square wave?
14) Why are the two voltages different in this case?

Now measure the voltage across the inductor. Again, the ground lead of the probe should be connected to the ground of the square wave output (you will have to rewire the circuit!). Make a sketch of the picture on the oscilloscope screen.
15) Why is the picture on the screen different from the resistor voltage case? Discuss why the graph on the screen looks the way it does.
16) Describe what would happen to the $R C$ and $L R$ graphs if the time constants ( $R C$ or $L / R$ ) were increased. Draw a sketch which shows the original graph compared to a graph with a larger time constant. What would happen to the graphs if the time constants were decreased?
17) Using your book or other resources, describe what an inductor does and how it is used.
18) Using your book or other resources, describe what a capacitor does and how it is used.

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## LRC Resonance Circuit (informal)

We know from mechanics that there is an optimal driving frequency, the resonance frequency, which will give the largest swings of a driven, damped harmonic oscillator (such as a mass on a spring). When the driving frequency equals the resonance frequency, we get the maximum amplitude of the swing and this is called resonance. Here is the equation for a damped, driven, harmonic oscillator:

$$
F \cos \left(\omega_{\mathrm{D}} t\right)-k x-b \mathrm{~d} x / \mathrm{d} t=m \mathrm{~d}^{2} x / \mathrm{d} t^{2}
$$

where $k$ is the spring constant, $\omega_{\mathrm{D}}$ is the driving frequency, $F$ is the amplitude of the driving force, $b$ is like the friction coefficient, and $m$ is the mass hanging on the spring.

A similar effect, also called resonance, is used in a stereo tuner to tune in your favorite radio station. The following is the equation for a series LRC circuit, derived using Kirchhoff's loop rule. Notice the similarity of this equation to the damped, driven, harmonic oscillator above (the form of the equations is the same, only the names of the variables are different):

$$
V_{\mathrm{o}} \cos \left(\omega_{\mathrm{D}} t\right)-q / C-R \mathrm{~d} q / \mathrm{d} t=L \mathrm{~d}^{2} q / \mathrm{d} t^{2}
$$

where $R$ is the resistance, $C$ is the capacitance, $L$ is the inductance, $\omega_{\mathrm{D}}$ is the driving angular frequency, $V_{\mathrm{o}}$ is the amplitude of the driving voltage, and $q$ is the charge on the capacitor (notice that the current $I=\mathrm{d} q / \mathrm{d} t$ appears in the equation). The LRC equation is the same as the damped, driven, harmonic oscillator equation but with different quantities. Because they are described by the same equation an LRC will have a behavior similar to a harmonic oscillator. For example, we know a mechanical system, for example a mass on a spring, can show resonance when it is driven at a particular frequency called the resonance frequency. The LRC circuit will show the same effect; for one particular frequency the circuit has very large amplitude oscillations.

NOTE: The above are calculus based differential equations but it is not necessary to understand calculus to do this lab. Your instructor will explain these equation if you have not seen them previously.

1) Pair the terms of the two equations (for example, inductance $L$ plays the role of mass $m$, so $L$ in the second equation replaces $m$ in the first equation). What do $R, 1 / C, q$, and $V_{\mathrm{o}}$ replace?

Set up the circuit shown using the capacitance box for $C$. Although the box has numbers that say MF next to them, the numbers are actually in micro $\left(10^{-6}\right)$ Farads $(\mu \mathrm{F})$. Start with a value of $0.001 \mu \mathrm{~F}$ (top and bottom knobs set to zero, middle knob set to 0.001 MF ) for the capacitor. Be sure the resistor is the first element from the ground side of the signal generator.


Connect the oscilloscope leads across the resistor, turn the generator on with a frequency of $10,000 \mathrm{~Hz}$, and make adjustments of the oscilloscope controls until you have a clear picture of the signal. Be sure the ground side of the oscilloscope goes to the ground side of the resistor (which is the ground side of the generator). The vertical axis on the screen is voltage and the horizontal is time. Make sure the oscilloscope remains in the calibrated mode during the whole experiment!
a) Adjust the frequency of the generator until the voltage across the resistor is at its maximum. This is the resonance frequency for this circuit. Record the amplitude voltage at the resonance and the resonance frequency. Use the oscilloscope screen to find the frequency and the amplitude voltage as you did in the previous lab. (The frequency readings on the function generator are not sufficiently precise.)
b) Reduce the frequency by about $10 \%$ from its resonance value and record the frequency and voltage amplitude from the oscilloscope. Do this for five measurements, decreasing the frequency by about $10 \%$ each time toward the lower frequencies and recording the voltage amplitude and the frequency on the oscilloscope.
c) Go back to the resonance frequency and increase the frequency by about $10 \%$ from its resonance value, recording the frequency and amplitude from the oscilloscope. Do ten $10 \%$ steps toward the higher frequencies, recording the amplitude voltage and the frequency each time.

Change the capacitance box to $0.002 \mu \mathrm{~F}$ and repeat all the measurements (steps $\mathrm{a}-\mathrm{c}$ ). Do this again for $0.0005 \mu \mathrm{~F}($ not $0.005 \mu \mathrm{~F}$ ) ).

Plot the three sets of voltages versus frequency on a single graph using LoggerPro (voltages on the vertical axis, frequency on the horizontal axis). Enable point protectors, keep the connecting lines. Print your graph. Now change the horizontal axis scale on the graph to logarithmic (click on the logarithmic checkbox under 'Graph Options'; 'Axis Options' for the $x$-axis). Print the new graph.
2) Which graph is a better representation of the data? Explain.
3) The resonance frequency of a damped driven harmonic oscillator is given by $f_{0}=1 /(2 \pi)(k / m)^{1 / 2}$. Since the equation for an LRC circuit is identical in form, what would you expect the formula for the resonance frequency to be for the LRC circuit? (Hint: What do $m$ and $k$ correspond to in the LRC equation?)
4) Use the equation you just wrote to calculate the theoretical resonance frequencies for the three values of capacitance in the above measurements. Compare, using relative discrepancy, these theoretical values with the three experimental values you found.

Notice that the resonance frequency depends on the capacitance. Set the frequency on the generator to about 25 kHz . Adjust the capacitance until you find resonance.
5) What value of capacitance gives you the resonance? How does it compare to the theoretical expectation (relative discrepancy)?
6) Explain how to tune an LRC circuit to a specific resonance frequency (this is, in fact, how a radio or TV tuner works).
7) What other value in your LRC circuit can you change in order to tune it to a specific frequency? (Old crystal radio sets used this method.)
8) Give some other examples of resonance, identifying the oscillating system and the periodic driving force. Explain how you know that this is an example of resonance.

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Ray Tracing (informal)

This lab investigates the reflection and refraction of light as it interacts with various objects. For each part of the experiment place the ray box light source and the object (lens, prism, etc.) on top of a sheet of paper and trace the arrangement onto the paper. Trace the objects and the rays of light onto the paper and label the directions of the rays. Notice that the ray box can be set to produce one or more rays by changing the cardboard divider. The following arrangements are to be set up.

1) Flat mirror with one ray at an angle of incidence other than $90^{\circ}$. Draw a perpendicular to the surface of the mirror with a protractor and measure the incident and reflected rays. Is the law of reflection obeyed?
2) Concave mirror with several parallel rays along the axis of the mirror. Find the focal length (the distance from the center of the mirror to where the rays come together). Would the focal length be longer or shorter for a mirror which is more curved? Explain.
3) Convex mirror with several parallel rays along the axis. Remove the mirror after drawing the rays and extend the rays backward until they cross to find the focal point. Measure the focal length. Would the focal length be longer or shorter for a mirror which is more curved? Explain.
4) Look up the definition of spherical aberration in your book. Does either of these mirrors exhibit spherical aberration?
5) Glass block, with parallel sides, with one ray at an angle other than $90^{\circ}$. Lay the block down flat on the paper. Draw a perpendicular to the surface at the point that the ray enters the block. What is the angle of incidence? By removing the glass and connecting the points where the ray entered and left the glass, find the refracted ray inside the glass block. What is the angle of refraction as the ray enters the glass block?
6) Calculate the index of refraction of the glass block using Snell's law (consult in your book if necessary). Be sure you have measured the incident and refracted angles for Snell's law from a perpendicular drawn to the glass surface.
7) What causes refraction?
8) The index of refraction is greater than one for most materials. What does this tell you about the speed of light in these materials?
9) Glass triangle with one ray. Lay the glass triangle flat on the paper. Orient the triangle in such a way that the light ray inside the glass undergoes total internal reflection (the ray is reflected back into the glass and does not come out into the air). Find and draw the ray which undergoes total internal reflection in the glass triangle. Be sure to indicate where the ray would have come out had it not totally reflected.
10) Prism with one ray (or use the entire light source with no slits). Find such an orientation for the prism that a spectrum of colors appears. Which color is bent more and which less? Why does this separation of colors occur?
11) Circular glass plate with the maximum number of rays entering. Does this arrangement demonstrate spherical aberration for the rays furthest from the center? Explain.
12) Convex glass (at least one side convex) with several rays entering. What is the focal length (measured from the center of the lens to the focus)?
13) Concave glass (at least one side concave). Remove the glass after drawing the rays and extend the rays backward to find the focal point. What is the focal length?
14) Combined concave and convex lenses with several rays. Does it make a difference which comes first? Try this with the lenses touching first and then with them separated by about 3 or 4 cm .

Be sure to discuss your findings with your partners and provide some comments about the lab in the form of a conclusion in your report.

## Optical Bench (informal)

## Part one: Lenses

For a converging lens, set up the light source and the lens, and find the image on the screen. Measure $q$ (the distance to the image from the lens), $p$ (the distance to the object from the lens - the arrow on the light source is your object), the object height $h_{0}$ (the size of the arrow - not its height from the table), and the image height $h_{\mathrm{i}}$ for five different positions of the object. Organize your data into a table.

1) For each of the five trials, calculate the focal length $f$ using the thin lens formula: $1 / f=1 / p+1 / q$. Record the focal length for each trial in your table. Calculate and record the average focal length.
2) For each trial, calculate the absolute value of magnification $m$, first using $m=q / p$ and then $m=h_{\mathrm{i}} / h_{\mathrm{o}}$. Record both values of $m$ for each trial in your table (two more columns). Trial by trial, compare the two values of $m$ using relative discrepancy. Are they close to each other?

3) Pick one of object distances from your table and, using the average focal length calculated, make a scale drawing of the arrangement. Choose an arbitrary (not too small and not too large!) height for the object on your drawing.

To make a scale drawing: Choose a scale (so that a certain length on the paper represents a certain distance in the experiment). Draw the object distance and the focal length to scale. For example, if 1 mm on the paper represents 1 cm in the experiment, then an object 50 cm in front of a 30 cm focal length lens will be drawn at a distance of 50 mm from the lens on the paper, while the focal point will be 30 mm from the lens. Draw the principal rays to determine where the image is on the drawing. Measure the distance to the image on the drawing and, using the scale you have chosen, convert it back to the actual size. Ask your instructor if you have questions.
4) How does the value of $q$ you have found from the scale drawing compare with the value you have found experimentally with the optical bench? What is the relative discrepancy between the two values?

A second way of finding the focal length of a converging lens is to focus the image of a distant object onto a screen. In this case, the object distance $p$ is very large and $1 / p$ goes to 0 , so that $q=f$. Pick a distant object (you may want to go into the hallway for this part - the window at the end of the hallway can be your distant object) and focus its image onto the screen. Measure and record the distance between the lens and the screen. This is your $q$, which is equal to $f$ in this case.
5) How does the focal length you have just measured compare with the average focal length found with the optical bench? What is the relative discrepancy between the two values?

## Part two: Mirrors

For a concave (converging) mirror find the image on the screen. The screen and the source have to be on the same side, with the mirror at the other end of the optical bench.
6) Measure the distance to the image, the distance to the object, and the object and image heights for five different positions of the object. Organize your data into a table.
7) Repeat steps $1-4$ from Part one.

## Part three: Lenses in combination

A diverging lens will not produce an image on the screen (try it!). So, how can you find its focal length? This can be done by using two lenses in contact with each other, the diverging lens and a stronger converging lens. When two thin lenses are placed in contact with each other, the combined focal length is given by $1 / \mathrm{f}=1 / f_{1}+1 / f_{2}$, where $f$ is the total focal length, $f_{1}$ is the focal length of the first lens and $f_{2}$ is the focal length of the second lens.

Put the converging lens from part one in contact with a diverging lens of weaker power (use scotch tape). Note: If the lenses are not of the same diameter, be sure to block the portion of the larger lens sticking out past the smaller one. The image in this part will be farther away from the lenses than in part one. If you cannot find the image at all, you may have a converging lens that is weaker than the diverging lens. In this case, swap lenses until you have a converging - diverging pair that does produce a real image. (You will have to find the focal length of the new converging lens!)
8) Find the image of a distant object formed by the combination of lenses and record the distance between the lenses and the image. This distance is equal to the focal length $f$ of the combination.
9) Use the double-lens formula given above, the focal length $f$ of the combination, and the known focal length $f_{2}$ of the converging lens to calculate the focal length $f_{1}$ of the diverging lens.
10) Make a scale drawing of a $15-\mathrm{cm}$ high object placed 25 cm in front of a diverging lens with the focal length of 10 cm . In this drawing, draw the object height to scale.

## Part four: A Telescope

Make a telescope by using two converging lenses (not in contact with each other, just hold the lenses up to your eye, use a distant object to focus on, and make a few general observations).
11) About what lens separation gives the sharpest picture? How does this separation compare with the sum of focal lengths of the lenses you have chosen? Is the picture erect or inverted?
12) Look in your book to see the different kinds of telescopes. What kind do you have?

Discuss your findings and provide some comments about the lab in the form of a conclusion.

## Wave Properties of Electromagnetic Radiation (informal)

Radio waves, microwaves, IR radiation, visible light, UV radiation, X-rays, and $\gamma$-rays are different parts of electromagnetic spectrum. They are all electromagnetic waves and differ only by their wavelength (and, of course, frequency since the speed of light $c$ is constant and $c=\lambda f$ ). In this lab you will study the wave behavior of visible light. You will also experiment with microwaves.

When the wavelength is much smaller than the typical size of objects involved in the experiment (the usual situation in the case of visible light), waves propagate in straight lines as if they were geometric rays. You explored the behavior of light under this condition in the two previous labs. When the wavelength is close to the typical size of the objects involved, the wave nature of electromagnetic radiation becomes obvious: electromagnetic waves exhibit diffraction and interference patterns, as waves of any other kind do.

## Part one: Wave properties of the visible light

A green laser $(\lambda=532 \mathrm{~nm})$ and a red laser $(\lambda=650 \mathrm{~nm})$ provide the light sources for this part.

## Do not shine the lasers in anyone's eyes!

## Single slit diffraction pattern

When light passes through a narrow slit, it bends into the regions where you would expect to have shadow (in geometric optics). This phenomenon is called diffraction. The pattern formed on a remote screen behind the slit is a result of diffraction and interference of light waves coming from different parts of the slit.

Set up the green laser and the single slit (mounted in a slide holder) on one side of the room and a large sheet of white paper on the other side of the room. Use the slit labeled A. You should arrange things so as not to be in other people's way. Adjust the alignment very carefully and turn off the lights to get good results.


Trace the pattern on the paper (turn this in with your report). Measure and record the distance between the slit and the paper. Replace the green laser with the red laser. Trace the red laser pattern on the same sheet of paper.

1) Which laser gives you a broader diffraction pattern? NOTE: The red laser is fainter than the green one. This may create a false impression that its pattern is narrower.

The single slit diffraction formula relates the angles toward diffraction minima to the wavelength of light used in the experiment: $a \cdot \sin \theta_{\mathrm{m}}=m \lambda$. In this formula, $a$ is the slit width, $m$ is the order of diffraction minimum, $\theta_{\mathrm{m}}$ is the angle from the slit toward $m$-th minimum, and $\lambda$ is the wavelength.

For the green laser diffraction pattern, measure and record distances from the center of the pattern (the middle of the central bright spot) to the centers of $m= \pm 1, \pm 2$, and $\pm 3$ minima (dark bands between the spots). Calculate the angles from the slit toward each minimum (e.g., in the figure $\theta_{1}=\tan ^{-1}\left(x_{1} / L\right)$ is the angle toward $m=1$ minimum).

Now use the formula $a \cdot \sin \theta_{\mathrm{m}}=\mathrm{m} \lambda$ and the known value of the slit width, $a=0.12 \mathrm{~mm}$, to determine the wavelength $\lambda$. You should have six values of $\lambda$, corresponding to the six ( $m= \pm 1, \pm 2$, and $\pm 3$ ) distance measurements. Calculate the average $\lambda$.
2) Estimate the uncertainty of your average $\lambda$ value (see Appendix B).
3) Report your value of $\lambda$ for the green laser.
4) Does your value of $\lambda$ agree with the known value of 532 nm ? What is the relative discrepancy?
5) Why does the value of $L$ not have to be measured as precisely in this experiment as the distances between the diffraction minima and the center of the pattern on the paper?
6) Would it help to use a caliper instead of a ruler to measure the distances on the paper? Explain.
7) Why does the red light produce a broader diffraction pattern than the green light?
8) Suppose instead of the monochromatic laser light you used sunlight in this part (used a system of mirrors to direct sun rays onto the slit). How would the single slit diffraction pattern look like in this case?
9) When you point the laser beam through the doorway of the classroom, it does not form a diffraction pattern. Yet when it goes through the slit on the slide, there is a pattern. Why do you get a diffraction pattern in one case and not in the other?

## Double slit interference pattern

When waves from two (or more) different sources arrive in phase to a certain point, they enhance each other (at that point). The waves interfere constructively in this case. When waves from two sources arrive $180^{\circ}$ out of phase to a certain point, they weaken each other at that point. The waves interfere destructively in this case. The two sources of light waves in this experiment are the two narrow slits separated by a small distance.

Set up the green laser and the double slit (mounted in a slide holder) on one side of the room and a sheet of white paper on the other side of the room. Use the double slit labeled A. Trace the double slit interference pattern on the paper. Replace the green laser with the red laser. Trace the red laser pattern on the same sheet of paper.

The pattern on the paper in this case is the result of double slit interference pattern superimposed on the single slit diffraction pattern. Note that each bright diffraction spot is now subdivided into smaller spots (each wide spot is now a cluster of smaller spots).
10) Compare the double slit patterns produced by the green and red lasers. How are they similar? How are they different?
11) Compare the double and single slit patterns produced by the green laser. How are they similar? How are they different?
12) The small spots inside the central cluster in the double slit pattern look brighter than the wide spot in the single slit pattern. Do you think they are actually brighter? Discuss what happens to the total energy radiated by the slit(s) toward the central wide spot in the double and single slit cases.

## Part two: Microwaves

In this part you will investigate another type of electromagnetic radiation - microwaves. The laws of reflection, refraction, and interference hold for microwaves as they do for waves of any kind. Compared to visible light, we expect microwaves to interact with matter differently, because the wavelength of microwaves is longer. In fact, microwaves have just the right wavelength and frequency to be absorbed by water molecules. This is why the microwaves in a microwave oven heat so efficiently. The microwaves you will be working with are of very low power $(5 \mathrm{~mW})$. They are therefore harmless.
13) The frequency of microwaves used in this lab is $9,500 \mathrm{MHz}$. What is their wavelength?

The unit with the electrical plug is the sender and is to be attached to the numbered track which has a disk with angular readings on it. The receiver unit has an ammeter on its side which measures the intensity of the arriving microwaves. Connect the other numbered arm to the circle and place the receiver on it. Microwaves in this lab may reflect off of the table, chairs, the sink, and your hands. There may also be some interference from the other microwave units in the room. From what you know about reflection, set up the apparatus to minimize these effects.

With the sender and receiver facing each other, place different wire screens between them. Experiment with changing the type of the screen and the screen orientation in the vertical plane (perpendicular to the beam). Be sure to keep your hands from interfering with your readings.
14) Which screen, with larger or smaller holes, transmits better?
15) Why is one screen better than the other at letting the microwaves through? (Hint: Does the size of the wavelength have anything to do with it?)

Now place the sender and receiver on the same side of one screen, pointing at the screen as shown. Leave the sender and screen in place and vary the angle to the receiver by moving the receiver (but keeping the distance to the screen the same). Find the location of the receiver which picks up the strongest reflection. Locate a line perpendicular to the screen (you can slide a piece of paper under the apparatus and draw the perpendicular if this will help). Measure the angle from the perpendicular to the sender beam (the incident angle) and the angle between the perpendicular and the beam found by the receiver (the reflected angle).

16) Are the incident and reflected angles the same (approximately)? Is the law of reflection obeyed?
17) Repeat the experiment with the other screen. Do the microwaves reflect better off the screen with smaller holes or the one with larger holes? Explain what you observe.

The beam produced by the sender is polarized. With the sender and receiver facing each other, use the plastic board with metal strips on it to determine which direction the beam is polarized (horizontal or vertical). Remember that microwave polarizer works so that an unpolarized beam of microwaves becomes polarized perpendicular to the metal strips. If the metal strips block the beam when they are horizontal, then the beam is polarized horizontally. NOTE: Do this part without a holder for the plastic plate to be sure that the beam does not get past the plate by going through the holder.
18) What does it mean to say a wave is polarized? Is your microwave beam polarized vertically or horizontally?
19) Can sound waves in air be polarized? Why or why not?

Now hang the metal sheet with a double opening on the sender unit. Keep the distance between sending unit and receiver constant but change the angle from the sender to the receiver. You will need to measure the angle between the normal direction to the double slit and the direction toward the receiver. (Hint: To simplify your task you can place the sender unit so that the double slit screen hangs right above the center of the disk with degree marks. This way, the angles can simply be read off the disk. Move the receiver as far from the disk and the sender unit as the track allows.)

Measure and record the intensity of the beam at different angles, from -90 to $+90^{\circ}$. You may want to slowly sweep through the angles first, to see where the maxima and minima are located. Measure and record the intensity changing the angle by about $5^{\circ}$ each time.
20) Plot the intensity measured by the receiver vs. the angle between the normal direction to the double slit and the direction toward the receiver (intensity on the $y$-axis vs. angle on the $x$-axis).

The angle from the double slit to the interference maxima is related to the slit separation and the wavelength as $d \cdot \sin \theta_{\mathrm{m}}=m \lambda$, where $d$ is the distance between the slits, $\lambda$ is the wavelength, and $\theta_{\mathrm{m}}$ is the angle toward the $m$-th order maximum. On your graph you should see two broad peaks corresponding to $m= \pm 1$ double slit interference maxima.
21) Measure the distance $d$ between the centers of the openings on the metal sheet. Based on you graph, calculate the wavelength of microwaves used in this lab. Does it matter for your calculation what units are used to express the intensity of microwaves in this lab? Explain.
22) Compare the wavelength which you have just found to the value you calculated in question 13 . What is the relative discrepancy between the two values?
23) What are the differences and similarities between the double slit interference patterns for visible light and microwaves?

Be sure to discuss your findings with your lab partners and provide some comments about the lab in the form of a conclusion in your report.

## Appendix A: Graphing experimental data

## Should I make a graph of my data?

A graph is used when the data is expected to change for each measurement. Suppose you measure the height of the same tree every year in a biology experiment. You do not expect to get the same answer each year, so it would not make any sense to find the average of the numbers you collect nor does it make sense to find a standard deviation or do a Q test (see Appendix B), because each value is supposed to be different. You are interested in how the height of the tree changes over time.

## All graphs should be done with a computer program for graphing. No hand drawn graphs will be accepted for any lab (except when you are asked to sketch a graph from the LabQuest or oscilloscope).

Every computer on campus has access to the program called LoggerPro. Go to any of the computing labs on campus and log onto your account. In the 'Student Apps' window select 'Shortcuts'. You will see an icon for LoggerPro. Double click on it to get started. If you cannot find the program, ask one of the student assistants. If they cannot find it, have them call professor Forinash at extension 2039. You may use other graphing programs as long as they perform a linear regression fit and display the parameters of the fit with their uncertainties.

Follow the examples below to answer the following questions about the data you collect when the data is supposed to be in a graph:
a) Does the data support the physical model I am exploring?
b) What value(s) do I report?
c) What if my data does not make a straight line?

Suppose you have a small moving cart. You measure the velocity of the cart as it rolls down a ramp at intervals of half a second. After you are done you have the following data for the motion of the cart:

| $\mathrm{t}(\mathrm{s})$ | 1.50 | 2.00 | 2.50 | 3.00 | 3.50 | 4.00 | 4.50 | 5.00 | 5.50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{v}(\mathrm{m} / \mathrm{s})$ | 8.6 | 10.1 | 12.2 | 13.7 | 15.4 | 17.0 | 19.1 | 20.5 | 22.3 |

Practice by putting this data into LoggerPro with $t$ values in the $x$ column and $v$ values in the $y$ column. In the program you will want to select 'Graph Options' under 'Options' to add titles, select point protectors, turn off 'connect points', and choose 'autoscale' for both $x$ and $y$ axis to get the graph shown on the next page. You can also click on the column headings to change the names.

NOTE: To enter scientific notated numbers in LoggerPro you must use E notation. For example, to enter $2.3 \times 10^{-3}$ type 2.3E-3 in LoggerPro.

Plot the best fitting straight line to this data by clicking on the 'Linear Fit' button (second from the right at the top of the screen) or drag across the data with the mouse and choose 'Linear Fit' in the 'Analyze' menu. This will also cause the program to calculate the formula for this line (called the regression line) which will appear in a box on the graph. A graph of this data will look like the one shown on the next page (modify your graph until it does). Note that the 'connect points' option is turned off. It should be turned off when your graph shows the regression line (to make the graph easier to read). You must enable point protectors to show your actual data points.


## a) Does the data support the physical model I am exploring?

Notice that the individual data points in the figure do not lie perfectly on the line. Some of them deviate more from the line and some deviate less. This is normal and should be the case in any real experiment. The reason is that each data point has its own uncertainty (cf. Appendix B). In our example the velocity is measured to about $0.1 \mathrm{~m} / \mathrm{s}$ (as it is suggested by the precision of the entries in the $2^{\text {nd }}$ row of the data table). Thus it should be expected that the data points will deviate from the line by $0.1 \mathrm{~m} / \mathrm{s}$ on average, even when the acceleration of the cart is constant.

In the 'Linear Fit' box in the figure above, the second to last line has Correlation $=0.9995$. This number tells you how well the data is correlated with the straight line. A correlation of 1.00 is perfect. A correlation greater than 0.99 is good, it means the data follows the regression line reasonably well. In our example Correlation is $>0.99$, so it is good. If the data points were scattered all over the place or followed a curve, the correlation would be much less than 1.

The last line in the 'Linear Fit' box, the RMSE or Root Mean Square Error, shows how much on average individual data points differ from the regression line. The value of RMSE should be numerically close to the uncertainty of your individual data points (in this example, the uncertainty of the individual measurements of velocity, $0.1 \mathrm{~m} / \mathrm{s}$ ). If this is the case, you can conclude that the data agrees with the regression line. In our example, the RMSE $(0.15 \mathrm{~m} / \mathrm{s})$ is not very different from the uncertainty $(0.1 \mathrm{~m} / \mathrm{s})$, so the data agrees with the regression line.

In the opposite case, when the uncertainty of individual measurements is much less than the RMSE, you would conclude that the data does not agree with the regression line. In our example, if the uncertainty of the velocity measurements were much less than $0.15 \mathrm{~m} / \mathrm{s}$ (say it were $0.01 \mathrm{~m} / \mathrm{s}$ ), you would conclude that the data does not agree with the regression line. (This could mean that the ramp is bumpy and the acceleration of the cart is not constant.)

NOTE: If you do not know the uncertainty of your individual measurements (do not have an independent estimate for it ), the RMSE can be used as an estimate of the uncertainty provided that the Correlation is greater than 0.99 . In our example, based on the RMSE you would conclude that the individual measurements of the velocity of the cart have an uncertainty of $0.15 \mathrm{~m} / \mathrm{s}$.

To summarize, your data agrees with the regression line if the Correlation is greater than 0.99 and the RMSE agrees with the uncertainty of your individual measurements. Assuming that the physical model you are exploring predicts a linear dependence (in this example, a linear dependence of velocity of the cart on time), your conclusion about whether your data supports or does not support the model will be based upon the agreement or disagreement of the data with the regression line.

## b) What value(s) do I report?

The 'Linear Fit' box shows two other numbers. The slope ( $m$ in the 'Linear Fit' box; do not confuse this with mass or meters) tells you how steep the graph is. The units of the slope are found by dividing the units of the vertical axis ( $\mathrm{m} / \mathrm{s}$ in this example) by the units of the horizontal axis ( s in this example). Here the graph is a velocity versus time graph, so the slope will be the acceleration of the cart in $(\mathrm{m} / \mathrm{s}) / \mathrm{s}$ or $\mathrm{m} / \mathrm{s}^{2}$. From the graph you conclude that the acceleration of the cart is $3.467 \mathrm{~m} / \mathrm{s}^{2}$.

The y-intercept ( $b$ in the 'Linear Fit' box) tells you where the line crosses the vertical axis. In this example the $y$ intercept is the velocity value at $t=0$. From the graph you conclude that the initial velocity of the cart (when you started taking data) was $3.405 \mathrm{~m} / \mathrm{s}$.

You may also want to display the uncertainties of the slope and y-intercept (not shown in the above picture) in order to report your results as discussed in Appendix B. That is you can report $m \pm \delta m$ and $b \pm \delta b$, where $\delta m$ and $\delta b$ are the uncertainties of the slope and y-intercept, respectively. Right click on the Linear Fit box, select 'Linear Fit Options', and then select 'Show Uncertainty' to see the values of $\delta m$ and $\delta b$.

NOTE: The values for $m$ and $b$ from the Linear Fit box can only be trusted when the data agrees with the regression line (when the Correlation $>0.99$ and the RMSE is close to the uncertainty of the individual data points if you know that uncertainty).

The equation for a straight line has the form $y=m x+b$. Now it is possible to write an equation describing the motion of the cart in this example. The velocity is on the vertical $(y)$ axis and time is on the horizontal $(x)$ axis, so the equation of motion of the cart is $v=3.467 \mathrm{~m} / \mathrm{s}^{2} t+3.405 \mathrm{~m} / \mathrm{s}$, where $t$ is in seconds and $v$ is in $\mathrm{m} / \mathrm{s}$. This corresponds to the equation $v=a t+v_{0}$ from your book.

Notice that knowing the equation of a straight line graph gives you more information than you started with. By using the equation above you can determine what velocity the cart has at some time $t$ for which you did not take a data point. For points in between the points you collected this is called interpolation. It is also possible to predict what velocity values would occur at times less than the first measured $t$ or greater than the last measured $t$ value which is called extrapolation.

For some labs you want to compare your value of the slope to another value (perhaps determined by a second method, found in a table, or given in the lab manual). If this is the case, follow Appendix B on how to compare two values for the same quantity obtained by two different methods.

## c) What if my data does not make a straight line graph?

Not all data will make a straight line graph. For a cart starting from rest and accelerating down the ramp, the traveled distance $d$ at time $t$ is given by $d=1 / 2 a t^{2}+d_{0}$. If you make a $d$ vs. $t$ graph, the data points will not follow a straight line. You cannot do a linear regression fit in this case (if you do, the Correlation will be $\ll 0.99$ ). However, if you make a separate column of the values of $t^{2}$ and plot $d$ versus $t^{2}$ (distance versus time squared), the graph will become a straight line and you will be able to do the linear fit. Thus, by a clever choice of what is plotted, you can sometimes transform your data into a straight line graph.

## Appendix B: Interpreting experimental results

In science experiments you often measure the same quantity many times, sometimes in two or more different ways. All your measurements are supposed to give the same value for the measured quantity but the actual numbers are not exactly the same. For example, if different students in the class measure the length of the room they should get the same answer but likely they will not get exactly the same number. In such a situation, what value should the students as a group report for the length of the room?

More generally, every scientific measurement has finite precision. Often the final result depends on several independently measured quantities, and each one of them has its own precision and its own uncertainty. How can one estimate the precision and the uncertainty of the final result? Follow the examples below to analyze and interpret the data you collect. These examples should help you answer the following questions about your data.
a) Can I throw out this point?
b) What are the mean and standard deviation for my set of data?
c) What is the uncertainty of my result?
d) How do I report my result?
e) Is my result any good?
f) If my final answer depends on several different quantities, how do I combine their uncertainties to find the uncertainty of the result?
g) If I have a second answer (from a second set of data or from a table in a book), is it close enough to my first value to be able to say they are equivalent?

NOTE: You may want to practice with the formulas to be sure you can reproduce the answers in these examples.
Suppose a group of students makes several measurements of the length of a hallway. The length has a true value known to be 6.35 m . Here is the set of data obtained by the students (the values are supposed to all be the same but of course, like in any real experiment, they are not exactly equal).

| $L(\mathrm{~m})$ | 6.31 | 6.27 | 6.59 | 6.28 | 6.36 | 6.11 |
| :--- | :--- | :--- | ---: | ---: | ---: | ---: |

## a) Can I throw out this point?

The Q-test is a procedure which allows you to decide whether you can discard an outlier (a data point that is far away from the rest of the data). It is done by calculating the so called Q value for a suspicious data point.
Formula to Q-test a point: $\quad \mathrm{Q}=\frac{\left|x_{\text {tested }}-x_{\text {near }}\right|}{\left|x_{\text {max }}-x_{\text {min }}\right|}$
In this formula $x_{\text {tested }}$ is the data point you are testing, $\boldsymbol{x}_{\text {near }}$ is the nearest data point, $x_{\text {max }}$ is the largest data point, and $x_{\min }$ is the smallest data point. Once you have calculated Q for a suspicious data point, compare it to the critical Q value for the total number of data points $N$ you have. The critical Q values for different $N$ are given in the table.

| N | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Q | 0.94 | 0.76 | 0.64 | 0.56 | 0.51 | 0.47 | 0.44 | 0.41 |

If the value of Q you have calculated is greater than the value in the table, you can throw out that data point. In our example, we see that 6.59 m looks rather different from the other values. Can we throw it out? The Q value for the 6.59 m measurement is $|6.59-6.36||6.59-6.11|=0.48$. This is less than 0.51 (the critical Q value given for seven
data points in the above table), so the point should be kept.
NOTE: A Java Applet which does the Q-test calculation can be found at http://homepages.ius.edu/kforinas/physlets/graphing/graph.html (Click on the link 'Statistics').

## b) What are the mean and standard deviation for my set of data?

When you do repeated measurements of the same quantity, your numbers will not be exactly the same. In this situation, what would be the best estimate for the quantity you have measured? Usually, it is the mean, or average, of your results.
Formula for the mean: $\langle x\rangle=\frac{x_{1}+x_{2}+\cdots+x_{N}}{N}$

In this formula $x_{1}, x_{2}, \ldots . x_{N}$ are the data points and $N$ is the total number of data points. In our example $x$ is $L$, the length of the hallway. Substituting the values from the above table into this formula, we get $<L>=6.32 \mathrm{~m}$.

How broadly is your data distributed around the mean? The quantity which describes this is called standard deviation.
Formula for the standard deviation: $\quad$ S.D. $=\sqrt{\frac{\left(x_{1}-\langle x\rangle\right)^{2}+\left(x_{2}-\langle x\rangle\right)^{2}+\cdots+\left(x_{N}-\langle x\rangle\right)^{2}}{N-1}}$

In this formula $x_{1}, x_{2}, \ldots x_{N}$ are the data points, $\langle x\rangle$ is the mean, and $N$ is the number of data points. In our example, $x_{1}, x_{2}, \ldots x_{N}$ are the individual measured lengths and $\langle x\rangle$ is $\langle L\rangle$, the mean length. Substituting the values from the table into the formula for the standard deviation, we get S.D. $=0.14 \mathrm{~m}$ (check to make sure you can get this result).

Standard deviation is often called 'the uncertainty of a single measurement' (because it shows by how much on average the individual measurements in a data set deviate from the mean). We will use the term 'standard deviation' in this course. The use of a more precise measuring device and/or more precise measuring procedure results in a smaller standard deviation for a set of data.

## c) What is the uncertainty of my result?

The uncertainty, or the uncertainty of the mean, tells you how close your mean may be to the true value of the quantity you are measuring. Alternatively, it tells you how close the true value may be to the mean of your data sample.

NOTE: In science literature the term 'error' is often used as an equivalent for the uncertainty. In this course we will use the term 'uncertainty'.
Formula for the uncertainty of the mean, $\boldsymbol{\delta}: \quad \delta=\frac{\text { S.D. }}{\sqrt{N}}$
In this formula S.D. is the standard deviation for a sample of data and $N$ is the number of data points in the sample. In our example the uncertainty is $0.05 \mathrm{~m}(0.14 \mathrm{~m}$ divided by the square root of seven).

For the mean of several independent measurements of the same quantity there is a $68 \%$ probability that the true value lies within $\pm \delta$ of the mean and a $95 \%$ probability that it lies within $\pm 2 \delta$ of the mean.

What is the difference between the standard deviation and the uncertainty of the mean? The standard deviation shows how broadly the individual data points are distributed around the mean. The uncertainty of the mean shows how far the true value of the measured quantity may lie from the mean. If you are using the same measuring device
and procedure, no matter how many data points you take the standard deviation will be about the same. Notice, however, that since $\delta$ is the standard deviation divided by the square root of $N$, as $N$ gets bigger $\delta$ gets smaller. Thus as you take more and more data points, the mean of your data sample gives you more and more precise estimate of the true value of the quantity you are measuring.

If you perform a single measurement of some quantity, e.g. measure the width of a table only once using a meter stick, you can use the precision of your measuring device as an estimate for the uncertainty of your measurement. In the case of a meter stick, the uncertainty would be 1 mm (or, perhaps, 0.5 mm ). The $1 \delta$ and $2 \delta$ probability figures cited above ( $68 \%$ and $95 \%$ ) are no longer valid in the case of a single measurement.

## d) How do I report my result?

$\underline{\text { Scientific results are reported including their uncertainties (and units!). }}$

## Reported value: (mean value $\pm$ uncertainty) units.

The reported value must include the mean and the uncertainty with the proper number of significant figures retained. The uncertainty should be rounded to one significant figure. The reported mean must have the same number of decimal places as the uncertainty. In our example, when we divide 0.14 m by $\sqrt{7}$ to find the uncertainty, we get $0.0529 \ldots \mathrm{~m}$. We should round it to 0.05 m . The reported value for the length of the hallway in our example should be $(6.32 \pm 0.05) \mathrm{m}$ (note that the m for meters must be included).

NOTE: When dealing with large ( $N \gg 10$ ) data samples, two significant figures may be retained in the uncertainty.

## e) Is my result any good?

To be considered good your result should be sufficiently precise. This means that the relative uncertainty of your result should be small.

Formula for the relative uncertainty: Relative Uncertainty $=\delta_{x} /\langle x\rangle$.
The relative uncertainty should be less than $10 \%$ in most of the labs in this course. In our example, $\delta_{L} /<L>=0.05 / 6.32 \approx 0.008=0.8 \%$, so this result is sufficiently precise.

In addition, if the true (or accepted) value for the quantity you have measured is available, this true value should lie within $\pm 2 \delta$ of the mean (the $95 \%$ confidence interval). If this is the case, your result provides an accurate estimate for the true value of the quantity you have measured. In our example, the true value ( 6.35 m ) does lie within $\pm 2 \delta$ of the mean.

Another way to quantify how close your result is to the true value is relative discrepancy.
Formula for the relative discrepancy: $\quad$ Relative Discrepancy $=\left|\frac{\text { your result }- \text { true value }}{\text { true value }}\right|$.
Smaller relative discrepancy means more accurate result, the result which is closer to the true value. In our example the relative discrepancy is $|(6.32-6.35) / 6.35| \approx 0.005=0.5 \%$.

Notice the difference between precision and accuracy. Precision, characterized by the relative uncertainty, tells you how good the result is compared to itself. Accuracy, characterized by the relative discrepancy, tells you whether the result is close to the true value. If your measuring device has some systematic defect (e.g. you are using a bent meter stick), your result may be precise but it will not be accurate!

## f) If my final answer depends on several different quantities, how do I combine their uncertainties to find the uncertainty of the result?

Suppose the students in our example now want to measure the speed of a toy car as it runs along the hallway. The car is battery operated so its speed does not change from trial to trial. The students use a stopwatch to measure the time it takes the car to get from one end of the hallway to the other. In several trials they find $t=(18.4 \pm 0.3) \mathrm{s}$ (where 18.4 s is the mean time and 0.3 s is the uncertainty). They can calculate the speed of the car using $v=L / t$, where $\mathrm{L}=(6.32 \pm 0.05) \mathrm{m}$. What value should they report for $v$ ?

The mean speed is given by $\langle v\rangle=\langle L\rangle /\langle t\rangle=6.32 \mathrm{~m} / 18.4 \mathrm{~s}=0.3435 \mathrm{~m} / \mathrm{s}$. What is the uncertainty of $\langle v\rangle$ ? Here are some rules for combining (or propagating) uncertainties.

## Rules for combining uncertainties:

Sums and differences: If a quantity $x$ is a sum or a difference of quantities $y$ and $z$, i.e. $x=y+z$ or $x=y-z$, then the uncertainty of $x$ is

$$
\delta_{x}=\sqrt{\delta_{y}^{2}+\delta_{z}^{2}}
$$

Note that if $\delta_{y} \gg \delta_{z}$, then $\delta_{x} \approx \delta_{y}$. Thus in the case of a sum or a difference of two (or more) quantities, the quantity with the largest uncertainty introduces the dominant contribution to the uncertainty of the result.

Products and quotients: If a quantity $x$ is a product or a quotient of quantities $y$ and $z$, i.e. $x=y z$ or $x=y / z$, then for the uncertainty of $x$ we have

$$
\frac{\delta_{x}}{x}=\sqrt{\left(\frac{\delta_{y}}{y}\right)^{2}+\left(\frac{\delta_{z}}{z}\right)^{2}}
$$

Note that if $\delta_{y} / y \gg \delta_{z} / z$, then $\delta_{x} / x \approx \delta_{y} / y$. Thus in the case of a product or a quotient of two (or more) quantities, the quantity with the largest relative uncertainty introduces the dominant contribution to the uncertainty of the result.

Powers and roots: If $x=y^{n}$, then $\frac{\delta_{x}}{x}=n \frac{\delta_{y}}{y}$. If $x=\sqrt[n]{y}$, then $\frac{\delta_{x}}{x}=\frac{1}{n} \frac{\delta_{y}}{y}$. For example, if $x=y^{2}$, then $\frac{\delta_{x}}{x}=2 \frac{\delta_{y}}{y}$.

Trigonometric functions: If $x=\sin \theta$, then $\delta_{x}=|\cos \theta| \delta_{\theta}$
If $x=\cos \theta$, then $\delta_{x}=|\sin \theta| \delta_{\theta}$
If $x=\tan \theta$, then $\delta_{x}=\left(1 / \cos ^{2} \theta\right) \delta_{\theta}$
NOTE: In the above trigonometric formulas $\theta$ must be measured in radians.

In our example, $v$ is a quotient of $L$ and $t$. Hence, $\delta_{v} / v=\sqrt{\left(\delta_{L} / L\right)^{2}+\left(\delta_{t} / t\right)^{2}}=0.018$ and $\delta_{v}=0.018 \cdot v \approx$ $0.006 \mathrm{~m} / \mathrm{s}$. Notice that, as discussed above, only one significant figure is kept for $\delta_{v}$. The reported value for the speed in this example should be $v=(0.344 \pm 0.006) \mathrm{m} / \mathrm{s}$, or $(34.4 \pm 0.6) \mathrm{cm} / \mathrm{s}$.

Notice that in our example $\delta_{t} / t=0.016$ while $\delta_{L} / L=0.008$. So in this example the uncertainty of $t$ introduces a greater uncertainty in $v$ than the uncertainty of $L$ does. To improve the precision of $v$, the students must first try to improve the precision of their time measurement.

## You should always determine and report which measurement is more critical in your experiment and why.

## g) What if I have a second set of data to compare with the first?

Here is the second set of measurements of the same hallway done by another group of students. The length of the hallway still has the same true value of 6.35 m .

| $L(\mathrm{~m})$ | 6.34 | 6.29 | 6.39 | 6.28 | 6.22 | 6.21 | 6.30 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :--- |

Using the formulas above (Try the formulas on your own to see if you get the same numbers!) we get the following results in this case.

The Q-test shows that all the data points should be kept.
The mean $L=6.29 \mathrm{~m}$.
The standard deviation for this set of values $=0.06 \mathrm{~m}$.
The uncertainty of the mean for this data set is $\delta_{L}=0.02 \mathrm{~m}$.
The reported value for this set is $L=(6.29 \pm 0.02) \mathrm{m}$.
This second set of data is more precise than the first set. We know this because the standard deviation and the uncertainty are smaller for the second data set ( $\delta_{L}=0.02 \mathrm{~m}$ for the second set compared to $\delta_{L}=0.05 \mathrm{~m}$ for the first set). However, the second data set is less accurate than the first data set: the discrepancy between the mean and the true value of $L$ is larger for the second data set.

Is the reported value $L_{1}$ close enough to the reported value $L_{2}$ so that one could say that the two results agree? The difference between the two mean values is 0.03 m . The uncertainty of this difference is $0.054 \mathrm{~m} \approx 0.05 \mathrm{~m}$ (see the rules for combining uncertainties). Thus we have $L_{1}-L_{2}=(0.03 \pm 0.05) \mathrm{m}$. The difference is consistent with zero and, hence, the two results do agree when their uncertainties are taken into account.

The relative discrepancy between the mean values of two data sets is calculated as follows:

$$
\text { Relative Discrepancy between the mean values of data sets } 1 \text { and } 2=\left|\frac{\text { difference of the two means }}{\text { average of the two means }}\right| \text {. }
$$

In our example it is equal to $|(6.32-6.29) / 6.305| \approx 0.005$, or $0.5 \%$.

If you want to know more, the following books may be helpful.

Data Reduction and Error Analysis for the Physical Sciences by P.R. Bevinton and D.K. Robinson (McGraw Hill, 1992)

A Practical Guide to Data Analysis for Physics Science Students
by L. Lyons (Cambridge University Press, 1991)

