## UNIT 1-- LEXICAL ANALYSIS

Structure of compiler - Functions and Roles of lexical phase - Input buffering - Representation of tokens using regular expression - Properties of regular expression - Finite Automata - Regular Expression to Finite Automata - NFA to Minimized DFA.

## 1.STRUCTURE OF COMPILER:

Compiler is a translator program that reads a program written in one language -the source language- and translates it into an equivalent program in another language-the target language. As an important part of this translation process, the compiler reports to its user the presence of errors in the source program.


Fig. 1.1. A Compiler

### 1.1 A LANGUAGE-PROCESSING SYSTEM:



Fig. 1.2. A language-processing system
The input to a compiler may be produced by one or more preprocessor, and further processing of the compiler's output may be needed before running machine code is obtained.

## Preprocessors:

Preprocessors produce input to compilers. They may perform the following functions:

1. Macro processing: A preprocessor may allow a user to define macros that are shorthand for longer constructs.
2. File inclusion: A preprocessor may include header files into the program text. For example, the C
3. preprocessor causes the contents of the file 〈stdio.h> to replace the statement \#include<stdio. $h>$ when it processes a file containing this statement.
4. "Rational" preprocessors. These processors augment older languages with more modern flow-of-control and data-structuring facilities. For example, such a preprocessor might provide the user with built-in macros for constructs like while-statements or if-statements, where none exist in the programming language itself.
5. Language extensions. These processors attempt to add capabilities to the language by what amounts to built-in-macros.

## Assemblers:

Some compilers produce assembly code, which is passed to an assembler for producing a relocatable machine code that is passed directly to the loader/linker editor. The assembly code is the mnemonic version of machine code. A typical sequence of assembly code is:

MOV a, R1
ADD \#2, R1
MOV R1,b

## Loaders and Linker-Editors:

- A loader program performs two functions namely, loading and link-editing. The process of loading consists of taking relocatable machine code , altering the relocatable addresses and placing the alters instructions and data in memory at the proper locations.
- The link-editor allows to make a single program from several files of relocatable machine code.


### 1.2 THE PHASES OF A COMPILER

## Analysis-Synthesis Model of Compilation:

There are two parts to compilation: analysis and synthesis. The analysis part breaks up the source program into constituent pieces and creates an intermediate representation of the source program. The synthesis
part constructs the desired target program from the intermediate representation. The analysis consists of
three phases:

1. Linear analysis, in which the stream of characters making up the source program is read from left to right and grouped into tokens that are sequences of character having a collective meaning.
2. Hierarchical analysis, in which characters or tokens are grouped hierarchically into nested collections with collective meaning.
3. Semantic analysis, in which certain checks are performed to ensure that the components of a program fit together meaningfully.
A compiler operates in phases, each of which transforms the source program from one representation to another. The structure of compiler is shown in Fig.1.3.The first three phases form the analysis portion of the compiler and rest of the phases form the synthesis phase.


Fig. 1.3. Phases of a compiler

## Lexical Analysis:

In a compiler, linear analysis is called lexical analysis or scanning. For example, in lexical analysis the characters in the assignment statement,

$$
\text { position }=\text { initial }+ \text { rate } * 60
$$

would be grouped into the following tokens:

1. The identifier position.
2. The assignment symbol $=$.
3. The identifier initial.
4. The plus sign.
5. The identifier rate.
6. The multiplication sign.
7. The number 60 .

The blanks separating the characters of these tokens would be eliminated during this phase.

## Syntax Analysis:

Hierarchical analysis is called parsing or syntax analysis. It involves grouping the tokens of the source program into grammatical phrases that are used by the compiler to synthesize the output. The source program is represented by a parse tree as one shown in Fig. 1.5.


Fig. 1.5. Parse tree for position $=$ initial + rate $* \mathbf{6 0}$
The hierarchical structure of a program is expressed by recursive rules .i.e by context-free grammars.The following rules define an expression:

1. Any identifier is an expression. i.e. $\mathrm{E} \rightarrow \mathrm{id}$
2. Any number is an expression. i.e. $\mathrm{E} \rightarrow$ num
3. If expression ${ }_{1}\left(\mathrm{E}^{1}\right)$ and expression ${ }_{2}\left(\mathrm{E}^{2}\right)$ are expressions , then,

$$
\mathrm{E} \rightarrow \mathrm{E}+\mathrm{E}|\mathrm{E} * \mathrm{E}|(\mathrm{E})
$$

Rules (1) and (2) are basic rules which are non-recursive, while rule(3) define expression in terms of operators applied to other expressions.

## Semantic Analysis:

- The semantic analysis phase checks the source program for semantic errors and gathers type information for the subsequent code-generation phase.
- An important component of semantic analysis is type checking. i.e .whether the operands are type compatible.
- For example, a real number used to index an array.


Fig. 1.6. Semantic analysis inserts a conversion from integer to real

## Intermediate Code Generation:

After semantic analysis, some compilers generate an explicit intermediate representation of the source program. This representation should be easy to produce and easy to translate into the target program. There are variety of forms.

- Three address code
- Postfix notation
- Syntax Tree

The commonly used representation is three address format. The format consist of a sequence of instructions, each of which has at most three operands. The IR code for the given input is as follows:

$$
\begin{aligned}
& \text { temp1 }=\text { inttoreal }(60) \\
& \text { temp2 }=\text { id3 } * \text { temp1 } \\
& \text { temp3 }=\operatorname{id} 2+\text { temp2 } \\
& \operatorname{id1}=\text { temp3 }
\end{aligned}
$$

## Code Optimization:

- This phase attempts to improve the intermediate code, so that faster running machine code will result.
- There is a better way to perform the same calculation for the above three address code ,which is given as follows:

$$
\text { temp } 1=\mathrm{id} 3 * 60.0
$$

$\mathrm{id} 1=\mathrm{id} 2+$ temp 1

- There are various techniques used by most of the optimizing compilers, such as:

1. Common sub-expression elimination
2. Dead Code elimination
3. Constant folding
4. Copy propagation
5. Induction variable elimination
6. Code motion
7. Reduction in strength.....etc..

## Code Generation:

- The final phase of the compiler is the generation of target code, consisting of relocatable machine code or assembly code.
- The intermediate instructions are each translated into sequence of machine instructions that perform the same task. A crucial aspect is the assignment of variables to registers.
- Using registers R1 and R2, the translation of the given example is:

$$
\begin{aligned}
& \text { MOV id3, R2 } \\
& \text { MUL \#60.0, R2 } \\
& \text { MOV id2, R1 } \\
& \text { ADD R2, R1 } \\
& \text { MOV R1, id1 }
\end{aligned}
$$

## Symbol-Table Management:

- An essential function of a compiler is to record the identifiers used in the source program and collect its information.
- A symbol table is a data structure containing a record for each identifier with fields for attributes.(such as, its type, its scope, if procedure names then the number and type of arguments etc.,)
- The data structure allows to find the record for each identifier and store or retrieve data from that record quickly.


## Error Handling and Reporting:

- Each phase can encounter errors. After detecting an error, a phase must deal that error, so that compilation can proceed ,allowing further errors to be detected.
- Lexical phase can detect error where the characters remaining in the input do not form any token.
- The syntax and semantic phases handle large fraction of errors. The stream of tokens violates the syntax rules are determined by syntax phase.
- During semantic, the compiler tries to detect constructs that have the right syntactic structure but no meaning to the operation involved. e.g. if we try to add two identifiers ,one is an array name and the other a procedure name.


Fig. 1.4. Translation of statement

## 2. FUNCTIONS AND ROLES OF LEXICAL PHASE :

- It is the first phase of a compiler.
- Its main task is to read input characters and produce tokens.
- "get next token "is a command sent from the parser to the lexical analyzer(LA).
- On receipt of the command ,the LA scans the input until it determines the next token and returns it.
- It skips white spaces and comments while creating these tokens.
- If any error is present the LA will correlate that error with source file and the line number.


Fig. 1.5. Interaction of lexical analyzer with parser
Issues in Lexical Analysis:
There are several reasons for separating the analysis phase of compiling into lexical and parsing:

1. Simpler design is the most important consideration.
2. Compiler efficiency is improved. A large amount of time is spent reading the source program and partitioning into tokens. Buffering techniques are used for reading input characters and processing tokens that speed up the performance of the compiler.
3. Compiler portability is enhanced.

## Tokens ,Patterns, Lexemes:

- Token is a sequence of characters in the input that form a meaningful word. In most languages, the tokens fall into these categories: Keywords , Operators, Identifiers , Constants, Literal strings and Punctuation.
- There is a set of strings in the input for which a token is produced as output. This set is described a rule called pattern.
- A lexeme is a sequence of characters in the source program that is matched by the pattern for a token.

| Token | Sample Lexemes | Informal Description of Pattern |
| :---: | :---: | :---: |
| const | const | const |
| If | if | if |
| relation | <, < $=$, \%, <>, >, > $=$ | $\leqslant$ or <n or $\pm$ or <> or $>=$ or > |
| id | pi, count, D2 | letter followed by letters and digits |
| nam | 3. 1416, 0, 6.02E23 | any numcric copstant |
| Hiteral | "core dumped" | any characters between " and " execpl " |

Fig. 1.6. Examples of tokens

## Attributes of tokens:

- The lexical analyzer collects information about tokens into their associated attributes. The tokens influence parsing decisions and attributes influence the translation of tokens.
- Usually a token has a single attribute i.e. pointer to the symbol table entry in which the information about the token is kept.
Example: The tokens and associated attribute values for the statement given,

```
    \(E=M * C * 2\)
are written below as a sequence of pairs:
    <id, pointer to symbol-table entry for E>
    <assign_op, >
    <id, pointer to symbol-table entry for \(M\) >
    <mult_op,>
    <id, pointer to symbol-table entry for \(\mathrm{C}>\)
    <exp-op, >
    <num, integer value 2>
```


## Lexical errors:

Few errors are discernible at the lexical level alone, because a LA has a very localized view of the source program. A LA may be unable to proceed because none of the patterns for tokens matches a prefix of the remaining input.

## Error-recovery actions are:

i. Deleting an extraneous character.
ii. Inserting a missing character.
iii. Replacing an incorrect character by a correct character.
iv. Transposing two adjacent characters.

## 3. INPUT BUFFERING

A two-buffer input scheme that is useful when look ahead on the input is necessary to identify tokens is discussed. Later, other techniques for speeding up the LA ,such as the use of "sentinels" to mark the buffer end is also discussed.

## Buffer Pairs:

A large amount of time is consumed in scanning characters, specialised buffering techniques are developed to reduce the amount of overhead required to process an input character. A buffer divided into two N -character halves is shown in Fig. 1.7.Typically, N is the number of characters on one disk block, e.g., 1024 or 4096.


Fig. 1.7. An input buffer in two halves.

- $\mathbf{N}$,input characters are read into each half of the buffer with one system read command, instead of invoking a read command for each input character.
- If fewer than $\mathbf{N}$ characters remain in the input, then a special character eof is read into buffer after the input characters.(eof---end of file)
- Two pointers, forward and lexeme_beginning are maintained. The string of characters between the two pointers is the current lexeme.
- If the forward pointer has moved halfway mark, then the right half is filled with $\mathbf{N}$ new input characters. If the forward pointer is about to move the right end of the buffer, then the left half is filled with $\mathbf{N}$ new input characters and the wraps to the beginning of the buffer. Code for advancing forward pointer is shown in Fig. 1.8.

```
if forward at end of first half then begin
    reload second half:
    forward := forward + 1
end
else if formurd at end of sccond hatf then begin
    rcload first hatf;
    move forward to beginning of first half
end
else fonvard:= forward + 1;
```

Fig. 1.8. Code to advance forward pointer

## Sentinels:

- With the previous algorithm, we need to check each time we move the forward pointer that we have not moved off one half of the buffer. If so, then we must reload the other half.
- This can be reduced ,if we extend each buffer half to hold a sentinel character at the end.
- The new arrangement and code is shown in Fig. 1.9and 1.10.This code performs only one test to see whether forward points to an eof.


Fig. 1.9. Sentinels at end of each buffer half.

```
forvard:= forword + 1;
if fonward: = eof then begin
    if forward at end of first half then begin
        reload second balf;
        fonward := forward + I
    end
    ulse if forward at end of second half then begin
        reload first half;
        move fonward to begioning of first half
    end
    else /* eof within a buffer signifying end of inpat */
        terminate lexical analysis
end
```

Fig. 1.10. Lookahead code with sentinels.

## 4. REPRESENTATION OF TOKENS USING REGULAR EXPRESSION

Regular expression are an important notation for specifying patterns. The term alphabet or character class denotes any finite set of symbols. Symbols are letters and characters. The set $\{0,1\}$ is the binary alphabet. ASCII and EBCDIC are two examples of computer alphabets.

- A string over some alphabet is a finite sequence of symbols drawn from that alphabet. The length of a string $s$ is written as $|\mathbf{s}|$, is the number of occurrences of symbols in $s$.
- The empty string, denoted by $\varepsilon$, is a special string of length zero.
- The term language denotes any set of strings over some fixed alphabet.
- The empty set $\{\varepsilon\}$, the set containing only the empty string.
- If $x$ and $y$ are strings , then the concatenation of $x$ and $y$, written as $x y$, is the string formed by appending y to x .
Some common terms associated with the parts of the string are shown in Fig. 1.11.

| Term | Definition |
| :---: | :---: |
| prefix of $s$ | A string obtained by removing zero or more trailing symbols of string $s ; e, g$., ban is a prefix of banana. |
| suffix of 3 | A string formed by deleting zeso or more of the leading symbols of $5 ;$ e.g., nama is a suffix of banana. |
| subsiring of $s$ | A string obtained by deleting a prefix and a suffix from s: e.g., nan is a substring of banana. Every prefix and every suffix of $s$ is a substring of $s$, but not every substring of $s$ is a prefix or a suffix of $s$. For every string $s$, both $s$ and $\epsilon$ are prefixes, suffixes, and substrings of $s$. |
| proper prefix, suffix, or substring of $s$ | Any nonempty string $x$ that is, respectively, a prefix, suffix, or substring of $s$ such that $s \neq x$. |
| subsequence of s | Any string formed by deleting zero or more not necessarily contiguous symbols from s; e.g., Daaa is a subsequence of banana. |

Fig. 1.11. Terms for parts of a string

## Operations on Languages

There are several important operations that can be applied to languages. The various operation and their definition are shown in Fig.1.12.

| Operation | Defmition |
| :---: | :---: |
| union of $L$ and $M$ written LUM | $L \cup M=\{s \mid s$ is in $L$ or $s$ is in $M\}$ |
| concalenation of $L$ and $M$ written LM | $L M=\{s t \mid s$ is in $L$ and $t$ is in $M\}$ |
| Kleene closure of $L$ written $L^{*}$ | $L^{*}=\bigcup_{i=0} L^{i}$ <br> $L^{*}$ denotes "zero or more concatenations of" $L$. |
| positive closure of $L$ written $L^{*}$ | $L^{*}=\bigcup_{i=1}^{\infty} L^{i}$ <br> $L^{\prime}$ denotes "one or more concatenations of" $\boldsymbol{L}$. |

Fig. 1.12. Definitions of operations on languages.

Let L be the set $\{\mathrm{A}, \mathrm{B}, \ldots . . ., \mathrm{Z}, \mathrm{a}, \mathrm{b}, \ldots . . ., \mathrm{Z}\}$ consisting upper and lowercase alphabets, and D the set $\{0,1,2, \ldots . ., 9\}$ consisting the set of ten decimal digits. Here are some examples of new languages created from $L$ and $D$.

1. $L \cup D$ is the set of letters and digits.
2. $L D$ is the set of strings consisting of a letter followed by a digit.
3. $L^{4}$ is the set of all four-letter strings.
4. $L^{*}$ is the set of all strings of letters, including $€$, the empty string.
5. $L(L \cup D)^{*}$ is the set of afl strings of letters and digits beginning with a letter.
6. $D^{+}$is the set of all strings of one or more digits.

## Regular Language

- A regular language over an alphabet $\Sigma$ is the one that can be obtained from the basic languages using the operations Union, Concatenation and Kleene *.
- A language is said to be a regular language if there exists a Deterministic Finite Automata (DFA) for that language.
- The language accepted by DFA is a regular language.
- A regular language can be converted into a regular expression by leaving out $\}$ or by replacing \{\} with () and by replacing U by + .


## Regular Expressions

Regular expression is a formula that describes a possible set of string.
Component of regular expression..

| X | the character x |
| :---: | :---: |
|  | any character, usually accept a new line |
| [ xyz ] | any of the characters $x, y, z, \ldots .$. |
| R? | a R or nothing (=optionally as $\mathbf{R}$ ) |
| R* | zero or more occurrences..... |
| R+ | one or more occurrences ...... |
| $\mathbf{R}_{1} \mathbf{R}_{\mathbf{2}}$ | an $\mathbf{R}_{1}$ followed by an $\mathbf{R 2}$ |
| $\mathbf{R 2}_{2} \mathbf{R}_{1}$ | either an $\mathbf{R}_{1}$ or an $\mathbf{R}_{2}$. |

A token is either a single string or one of a collection of strings of a certain type. If we view the set of strings in each token class as an language, we can use the regular-expression notation to describe tokens.

Consider an identifier, which is defined to be a letter followed by zero or more letters or digits. In regular expression notation we would write.
Identifier = letter (letter | digit)*

Here are the rules that define the regular expression over alphabet .

Regular expression is used to describe the structure of tokens. Formal definition is,

1. $\boldsymbol{E}$ is said to be a regular expression.
2. $\boldsymbol{\Phi}$ is said to be a regular expression.
3.a in $\sum$ is said to be regular expression.
4.Suppose $\mathbf{r}_{1}$ and $\mathbf{r}_{2}$ are regular expressions denoting the languages $\mathbf{L}\left(\mathbf{r}_{1}\right)$ and $\mathbf{L}\left(\mathbf{r}_{2}\right)$. Then,
a) $\left(\mathbf{r}_{1}\right) \mid\left(\mathbf{r}_{2}\right)$ is a regular expression denoting $\mathbf{L}\left(\mathbf{r}_{1}\right) \mathbf{U}^{\prime} \mathbf{L}\left(\mathbf{r}_{2}\right)$.
b) ( $\mathbf{r}_{1}$ ). ( $\mathbf{r}_{2}$ ) is a regular expression denoting $\mathbf{L}\left(\mathbf{r}_{1}\right) . \mathbf{L}\left(\mathbf{r}_{2}\right)$.
c) $\left(\mathbf{r}_{1}\right)^{*}$ is a regular expression denoting $\mathbf{L}\left(\mathbf{r}_{1}\right)^{*}$.

Unnecessary parentheses can be avoided in regular expressions if we adopt the conventions that:

1. The unary operator * has the highest precedence and is left associative.
2. concatenation has the second highest precedence and is left associative.
3. |, the alternate operator has the lowest precedence and is left associative.

## 5. PROPERTIES OF REGULAR EXPRESSION

There are a number of algebraic laws obeyed by regular expressions and these can be used to manipulate regular expressions into equivalent forms. Algebraic properties are shown in Fig. 1.13.

| Axıм | Description |
| :---: | :---: |
| $r\|s=s\| r$ | \| is commutative |
| $\underline{r(s)} \mid s)=(r \mid s) \mid t$ | \| is associasive |
| $(r s) t=r(s t)$ | concatenation is associative |
| $\begin{aligned} & r(s \mid t)=r s \mid r t \\ & (s \mid t) r=s r \mid t r \end{aligned}$ | concatcnation distributes over [ |
| $\begin{aligned} & \epsilon r=r \\ & r \epsilon=r \end{aligned}$ | $\boldsymbol{\epsilon}$ is the identity element for concatenation |
| $r^{*}=(r \mid \epsilon)^{*}$ | relation between * and $\epsilon$ |
| $r^{* *}=r^{*}$ | * is idempotent |

Fig. 1.12. Algebraic laws of regular expression

## Regular Definitions

For notational convenience, we may wish to give names to regular expressions and to define regular expressions using these names as if they were symbols.

Identifiers are the set or string of letters and digits beginning with a letter. The following regular definition provides a precise specification for this class of string.

## Example-1,

$a b * \mid c d ?$ Is equivalent to $\left(a\left(b^{*}\right)\right) \mid(c(d ?))$
Pascal identifier
Letter - A $|\mathrm{B}| \ldots . . .|\mathrm{Z}| \mathrm{a}|\mathrm{b}| \ldots . . .|\mathrm{z}|$
Digits - $0|1| 2|\ldots|$.
Id - letter (letter / digit)*
Transition Diagrams for Relational operators and identifier is shown below.


## 6. FINITE AUTOMATA

A recognizer for a language is a program that takes as input a string x and answer "yes" if x is a sentence of the language and "no" otherwise. The regular expression is compiled into a recognizer by constructing a generalized transition diagram called a finite automaton. A finite automaton can be deterministic or nondeterministic, where "nondeterministic" means more than one transition out of a state may be possible on the same input symbol.

## Deterministic Finite Automata (NFA)

A Finite Automata (FA) or Finite State Machine (FSM) is a 5-tuple (Q, $\sum, \mathrm{q}_{0}, \mathrm{~A}, \delta$ ) where,

- Q is the set of finite states
- $\quad \sum$ is the set of input symbols (Finite alphabet)
- $\mathrm{q}_{0}$ is the initial state
- A is the set of all accepting states or final states.
- $\delta$ is the transition function, $\mathrm{Q} \times \sum \rightarrow \mathrm{Q}$

For any element q of Q and any symbol $\mathrm{a} \in \sum$, we interpret $\delta(\mathrm{q}, \mathrm{a})$ as the state to which the Finite Automata moves, if it is in state $q$ and receives the input ' $a$ '.

## How to draw a DFA?

1. Start scanning the string from its left end.
2. Go over the string one symbol at a time.
3. To be ready with the answer as soon as the string is entirely scanned.

## Non-Deterministic Finite Automata (NFA)

## Definition:

A NFA is defined as a 5 -tuple, $\mathrm{M}=\left(\mathrm{Q}, \sum, \mathrm{q}_{0}, \mathrm{~A}, \delta\right)$ where,

- Q is the set of finite states
- $\quad \sum$ is the set of input symbols (Finite alphabet)
- $\mathrm{q}_{0}$ is the initial state
- A is the set of all accepting states or final states.
- $\delta$ is the transition function, $\mathrm{Q} \times \sum \rightarrow 2^{\mathrm{Q}}$

Regular expression to NFA:

## Thompson's construction method

1. $R=\wedge$

2. $R=r_{1}$

3. $R=r_{1} r_{2}$

4. $R=r_{1} / r_{2}$ (or) $r_{1}+r_{2}$

5. $R=r^{*}$


## 7. REGULAR EXPRESSION TO FINITE AUTOMATA - NFA TO MINIMIZED DFA

## Converting NFA to DFA: The Subset Construction Algorithm

The algorithm for constructing a DFA from a given NFA such that it recognizes the same language is called subset construction. The reason is that each state of the DFA machine corresponds to a set of
states of the NFA. The DFA keeps in a particular state all possible states to which the NFA makes a transition on the given input symbol. In other words, after processing a sequence of input symbols the DFA is in a state that actually corresponds to a set of states from the NFA reachable from the starting symbol on the same inputs.

There are three operations that can be applied on NFA states:

| Operation | Definition |
| :---: | :--- |
| $\in$-closure $(s)$ | set of NFA states reachable from state $s$ on $\in$-transition |
| $\in$-closure $(T)$ | set of NFA states reachable from some $s$ in $T$ on $\in$-transition |
| move $(T, a)$ | set of NFA states to which there is transition on input $a$ <br> from some state $s$ in the set $T$ |

The staring state of the automaton is assumed to be $s_{0}$. The $\in$-closure $(s)$ operation computes exactly all the states reachable from a particular state on seeing an input symbol. When such operations are defined the states to which our automaton can make a transition from set $T$ on input $a$ can be simply specified as:

$$
\in \operatorname{closure}(\operatorname{move}(T, a))
$$

## Subset Construction Algorithm

Initialize: Let $\in$-closure ( $s_{0}$ ) be the only state in Dstates ( of the DFA )
Repeat: while there are unmarked states $T$ in Dstates do
mark $T$
for each symbol $a$ do
$U=\quad \in-\operatorname{closure}(\operatorname{move}(T, a))$
if $U$ is not in Dstates then
add $U$ as unmarked state in Dstates
$\operatorname{Dtran}[T, a]=U$
end
end

## Algorithm for Computation of $\in$-closure

Initialize: Let all states from $T$ are on the Stack, and $\in$-closure $(T)=T$

Repeat: while Stack is not empty do
pop $t$ from the top of the Stack
for each state $u$ with edge from $t$ to $u$ labeled $\in$ do
if $u$ is not in $\in$-closure ( $T$ ) then
add $u$ to $\in$-closure ( $T$ )
push $u$ onto the Stack
end
end

## Example: Convert the NFA for the expression: $(a \mid b) * a b b$ into a DFA using the subset construction algorithm.

Step 1: Convert the above expression in to NFA using Thompson rule constructions.


Step 2: Convert the above NFA in to DFA.
The derivation of the states and transitions of the DFA, including the computation of the $\in$-closure and move functions, can be demonstrated as follows:

```
\(\in\)-closure \((\{0\})=\{1,2,4,7\}=A\)
\(\in-\operatorname{closure}(\operatorname{move}(\mathrm{A}, a))=\{3,8\}\)
\(\in\)-closure ( \(\{3,8\}\) ) \(=\{1,2,3,4,6,7,8\}=B\)
\(\in\)-closure(move (A,b)) \(=\{5\}\)
\(\in\)-closure \((\{5\})=\{1,2,4,5,6,7\}=C\)
\(\in-\operatorname{closure}(\) move \((\mathrm{B}, a))=\in-\operatorname{closure}(\{3,8\})=B\)
\(\in-\) closure(move (B,b) \(=\in\)-closure \((\{5,9\})=\{1,2,4,5,6,7,9\}=D\)
\(\in-\operatorname{closure}(\operatorname{move}(\mathrm{C}, a))=\in-\operatorname{closure}(\{3,8\})=B\)
\(\in-\operatorname{closure}(\) move \((\mathrm{C}, b))=\in-\operatorname{closure}(\{5\})=C\)
\(\in-\operatorname{closure}(\operatorname{move}(\mathrm{D}, a))=\in-\operatorname{closure}(\{3,8\})=B\)
\(\in-\operatorname{closure}(\operatorname{move}(\mathrm{D}, b))=\in-\operatorname{closure}(\{5,10\})=\{1,2,4,5,6,7,10\}=E\)
\(\in-\operatorname{closure}(\operatorname{move}(\mathrm{E}, a))=\in-\operatorname{closure}(\{3,8\})=B\)
\(\in\)-closure(move \((\mathrm{E}, b))=\in\)-closure \((\{5\})=C\)
```

The above procedure is repeated until no new state can be found.

## The transition table for DFA :

| STATE | $\boldsymbol{a}$ | $\boldsymbol{b}$ |
| :---: | :---: | :---: |
| $A$ | $B$ | $C$ |
| $B$ | $B$ | $D$ |
| $C$ | $B$ | $C$ |
| $D$ | $B$ | $E$ |
| $E$ | $B$ | $C$ |

The DFA is:


## Minimized DFA.

Step 3: Convert the above DFA in to minimized DFA by applying the following algorithm.
Minimized DFA algorithm:
Input: DFA with 's' no of states
Output: Minimized DFA with reduced no of states.
Steps:

1. Partition the set of states in to two groups. They are set of accepting states and non accepting states.
2. For each group G of $\pi$ do the following steps until $\pi=\pi$ new .
begin
divide $G$ in to as many groups as possible, such that two states $s$ and $t$ are in the same group only when for all states $s$ and $t$ have transitions for all input symbols ' $s$ ' are in the same group itself. Place newly formed group in $\pi$ new.
end
3. Choose representative state for each group.
4. Remove any dead state from the group. After applying minimized DFA algorithm for the regular expression (a|b)*abb , the transition table for the minimized DFA becomes

## Transition table for Minimized state DFA :

| STATES | Input Symbol |  |
| :---: | :---: | :---: |
|  | $\mathbf{a}$ | $\mathbf{b}$ |
| A | B | A |
| B | B | D |
| D | B | E |
| E | B | A |

## Minimized DFA:



## Exercises:

Convert the following regular expression in to minimized state DFA,

1. (a|b)*
2. (b|a)*abb(b|a)*
3. $\left((\mathrm{a} \mid \mathrm{c})^{*}\right) \mathrm{ac}(\mathrm{ba})^{*}$
