To Relay or Not to Relay? Optimizing Multiple Relay Transmissions by Listening in Slow Fading Cooperative Diversity Communication

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Abstract

Recent study has shown that appropriate selection of a single *"opportunistic"* relay that maximizes a function of the end-to-end, instantaneous channel conditions, achieves the same diversity-multiplexing gain tradeoff with schemes that require multiple and simultaneous relay transmissions (possibly at the same frequency band) and employ distributed space-time coding. In this work, we present low SNR analysis in slow fading environments that shows equivalence of opportunistic relaying to optimal decode-and-forward, under an aggregate power constraint, outperforming schemes based on distributed space-time coding. Amplify-and-forward schemes under an aggregate power constraint are also examined, demonstrating improved performance when a single opportunistic relay is used. The findings suggest that cooperative diversity benefits, under an aggregate power constraint, are maximized when cooperative relays choose not to transmit, but rather choose to cooperatively *listen*, giving priority to the transmission of a single opportunistic relay. In other words, cooperation benefits are maximized when relays act as sensors of the wireless channel and not necessarily as active retransmitters. Since no simultaneous transmissions are utilized, opportunistic relaying can be implemented in existing radio front ends and a radio implementation is briefly discussed.

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I. Introduction

Utilization of terminals distributed in space can provide dramatic performance gains in wireless communication. For example, when channel state information (CSI) is available at a pair of neighboring nodes, then they could appropriately adjust the *phases* of the transmitted signals and therefore, cooperatively *beamform* towards the final destination, increasing total capacity [17]. When CSI is not available or when radio hardware cannot support *beamforming*, then cooperation between the source and a single relay provides for improved resistance to wireless fading [11]. There has been a tremendous interest in the research community around the theme of cooperation and basic results of single-relay cooperation analysis are summarized in [10].

Scaling cooperation to more than one relays is still an open area of research. Distributed space-time coding could be used among the participating nodes, to achieve the optimal diversity-multiplexing gain tradeoff [12]. In practice, such code design becomes difficult given the *distributed* and *ad-hoc* nature of cooperative links, as opposed to multiple-input multiple-output (MIMO) links, where antennas belong to common terminals and for which, space-time coding was originally invented. For example, it is difficult in practice for each relay to acquire information about the channel state of other relays, as needed in the scheme proposed in [13]. It is also difficult for the receiver to acquire information about the channel state between source and *all* relays, because the receiver has no means to estimate such information and therefore, those channel states need to be communicated to the destination. Such space-time coding scheme that requires global CSI at the receiver, including information about the paths between source and relays, was proposed in [9] and was based on linear dispersion codes in analog, amplify-and-forward relays.

An additional difficulty in applying MIMO space-time coding into the cooperative relay channel is the fact that the number of *useful* antennas (relays) for cooperation is, in general, unknown and varying. Therefore, additional steps of coordination are needed among the cooperating nodes, before a specific space-time coding scheme, invented for a fixed number of transmitting antennas, is utilized.

Previous studies implicitly assumes perfect *carrier phase* recovery at the receiver, even when multiple cooperative relays are simultaneously transmitting, allowing *coherent* reception at the receiver, with gains that scale with the number of transmitting elements [7], [8]. Carrier phase

recovery in MIMO links involves estimation and tracking of carrier phase differences among two participating oscillators, one at the transmitter and one at the receiver, in the presence of additive thermal noise (due to thermodynamics in the receiver) and multiplicative noise (due to *multipath*). However, carrier phase recovery in multiple cooperative relay links involves estimation and tracking of carrier phase differences among *several* transmit-receive pairs, proportionally to the number of participating relays. This increases implementation complexity and therefore, the cost of the receiver.

In short, there is important overhead associated with any type of cooperation, ranging from CSI estimation (and its communication to the appropriate nodes) in different parts of the network, to reception energy at each participating node, for transmitting information destined for another node. Such overhead also scales with the number of cooperating nodes, and therefore cannot be ignored, especially when radio implementations of cooperative protocols are sought.

In an effort to minimize the required cooperation overhead and at the same time realize the benefits of cooperation between multiple relays in slow fading wireless environments, a simple, distributed, single-relay selection algorithm was proposed [4]. Each intermediate relay overhears pilot signals from source or destination and evaluates its own end-to-end quality, for efficient information relaying. The relay that maximizes a function of its own channel conditions towards source and destination is selected to forward information to the final destination in a distributed manner.

Specifically, a medium access protocol based on timing was introduced to allow the “best” relay to access the channel first, according to the quality of its wireless path between source and destination. The main idea is that once a race condition among all relays is introduced, individual relays did not need to acquire CSI information about other relays, towards source or destination but instead, their own CSI towards source and destination would suffice. In slow fading environments, selection need not be repeated within every symbol period, but instead, relay selection should be repeated at least twice, within the channel *coherence time*. Analysis of opportunistic relaying for any kind of wireless fading quantified its associated overhead and revealed a flexible tradeoff between how well the algorithm selects the relay that maximizes a function of the *instantaneous* path channel conditions, and how *fast* the algorithm is executed, in a distributed way. For more details on the overhead of opportunistic relaying, the interested reader could refer to [4], [5].

Interestingly, diversity-multiplexing gain analysis [5], as a characterization of reliability versus throughput speed at the high SNR-high spectral efficiency regime, revealed no performance loss, compared to schemes that rely on distributed space-time coding, under the same main assumptions:

- no CSI at the source;
- no *beamforming*[1] radio capabilities;
- slow quasi-static fading, with channel coherence time longer than the transmitted symbols' block;
- no CSI knowledge at each relay, regarding CSI of other relays;
- no feedback (Automatic Repeat Request) from the destination.

A. Contribution

In this work, we maintain the above basic assumptions and perform low-SNR analysis that provides exact closed-form expressions for *outage probability* under *opportunistic relaying*. We compare it with regenerative relaying based on space-time coding and amplify-and-forward relaying, as well as with other relay selection choices found in the literature, under an aggregate relay power constraint:

$$P_{\text{source}} = m\ P,\ P_{\text{relays}} = \sum_{i=1}^{M} P_{i\text{d}} = (1-m)\ P \tag{1}$$

where $\mathrm{P}_{\text{source}}$ is the transmission power of a single source, $\mathrm{P}_{i\text{d}}$ is the transmission power of relay i towards destination, and M is the number of relays. Moreover, $m \in (0,1]$ denotes the fraction of total end-to-end power P allocated to the source transmission and $1-m$ denotes the fraction of total power to the transmissions of all relays.

The motivation behind imposing an aggregate power constraint is threefold: 1) transmission power is a network resource that affects both network life time in battery-operated terminals, as well as network scalability, since a transmission from one node can be considered as *interference* to another. Therefore, transmission power should be always constrained; 2) regulatory agencies always impose a total transmission power limit; 3) we want to show that benefits of cooperation

[1]also known as distributed phased arrays.

can arise even when relays *do not* transmit (and therefore, do not add transmission energy into the system).

The observed gains of opportunistic single-relaying presented in this work, compared to optimal space-time coding (that possibly requires simultaneous, same-frequency transmission among several relays) suggest that relays are useful, even when they do not actively transmit, provided that they adhere to the "opportunistic" cooperation rule and give priority to the "best" available relay. The simplicity of the technique allows immediate implementation in custom radio hardware and a demonstration is briefly discussed.

In section II we present the basic protocols examined in this work and in sections III and IV we analyze decode-and-forward (regeneration) and amplify-and-forward processing at the relays, respectively. Section V briefly describes the implemented radio demonstration and section VI concludes this work.

II. Protocols Examined

We consider slow Rayleigh fading, where the source of information has a poor link towards the final destination. We also assume no CSI at the source. Under the assumption of slow quasi-static fading, there is no throughput rate that could guarantee reliable communication between source and destination and therefore, the *Shannon* capacity is zero. In Fig. 1, we depict this *highly inconvenient* communication scenario: source and destination are blocked by an intermediate wall, while relays are located at the periphery of the obstacle, *around-the-corner*. The relays are able to communicate with both endpoints (source and destination).

We further consider the simplest, two step, *reactive* transmission scheme for half-duplex radios [12]: during the first phase, the source transmits a given number ($N/2$) of symbols and the relays listen, while during the second step, the relays forward a version of the received signal using the same number of symbols[2] (Fig. 2). The channel conditions remain constant during the two phases, following a Rayleigh distribution and corresponding to *channel coherence time* at least equal to the transmission symbols' block of N symbols.

[2]Notice that if the source is allowed to transmit a different symbol, while the relays are forwarding the previous, improved performance could be observed, compared to the above *half-duplex* scheme [2], [16], since one channel degree of freedom would not be wasted. However, in this work we are interested in finding out the optimal strategy for relay transmissions and hence, simplify their operation.

Notice that cooperation is coordinated at the transmission symbols' block level, minimizing overhead and simplifying protocol implementation. An alternative approach could have the duration of the two phases variable, requiring relay selection at the symbol level among participating nodes. Even though such assumption might be theoretically appealing, it is hard to implement in practice, since relay selection at the symbol level would require coordination overhead proportional to the number of transmitted symbols, increasing overall complexity.

We consider two kinds of block-level cooperation: 1) reactive cooperation among decode-and-forward relays that have managed to successfully decode the message during the first stage and 2) proactive cooperation among decode-and-forward or amplify-and-forward relays, where specific relay paths are selected before information is transmitted (see Fig. 2).

The relay transmission strategies that we are going to explore include simultaneous transmissions (possibly at the same frequency band) using distributed space-time coding [12] and we refer to that scheme as "All-relays" scheme. We further include single relay schemes that select the "best" relay according to *average* signal strength ("Single relay") [14] or *instantaneous* signal strength ("Opportunistic"). The former advocate relay selection according to which relay has the *smallest distance* [19] towards destination[3] while the later advocate relay selection based on which relay has the *strongest end-to-end signal* towards destination [4].

Note that the optimal power allocation across source and relays depends on the CSI conditions and might be different than ($\mathrm{P}_{\text{source}} = \mathrm{P}_{\text{relays}}$) [1]. However, optimal power allocation across source and relays is meaningful when a) there is CSI information at the source regarding the whole network (including channel conditions between relays and destination) and b) there is a good direct link between source and destination. None of the above apply in our study. We discuss power allocation, even though the main focus in this work is not just optimal power allocation but the more general question about what the relays should optimally do: re-transmit or not?

III. Decode and Forward Analysis

In this section we assume that the received signal between any two points (s-d) is $y_{\text{d}} = a_{\text{sd}}\, x_{\text{s}} + n_{\text{d}}$, where a_{sd} is complex, circularly symmetric Gaussian random variable with $\mathbb{E}\left\{|a_{\text{sd}}|^2\right\} =$

[3]under an isotropic propagation model that does not include shadowing.

$\mathbb{E}\{\gamma_{\text{sd}}\} = \overline{\gamma}_{\text{sd}}$ and n_{d} is a complex, zero mean, circularly symmetric additive white gaussian noise (AWGN) with $\mathbb{E}\{|n_{\text{d}}^2|\} = N_0$. We denote as R the end-to-end (source-relay-destination) target spectral efficiency in bps/Hz and $\mathsf{SNR} = P/N_0$, the end-to-end transmitted SNR. Note that under the above assumptions, $\gamma_{\text{sd}} \stackrel{\triangle}{=} |a_{\text{sd}}|^2$ has the exponential distribution with parameter $1/\overline{\gamma}_{\text{sd}}$ and corresponds to Rayleigh fading.

A. Proactive Decode and Forward

In Opportunistic relaying [4], [5], the "best" relay b is chosen among a collection of M possible candidates, in a distributed fashion that requires each relay to know its own instantaneous signal strength (but not phase), towards source and destination. The relay selection completes within a fraction of the channel *coherence time* and then, that single relay is used for information relaying. A method of distributed timers is used to allow the "best" relay to be selected, even though each relay has no CSI information regarding the links of other relays. The "best" relay b is chosen to maximize the following function of the channel conditions towards source and destination[4]:

$$\min\{\gamma_{\text{sb}}, \gamma_{\text{bd}}\} \geq \min\{\gamma_{\text{s}i}, \gamma_{i\text{d}}\}, \forall\ i \in \{1, .., M\} \Leftrightarrow b = \arg \max_{i \in \{1,..,M\}} \{\min\{\gamma_{\text{s}i}, \gamma_{i\text{d}}\}\} \quad (2)$$

Communication through the "best" opportunistic relay fails due to outage when the following event happens:

$$\mathcal{E}_{\text{proact}} \equiv \left\{ \left(\frac{1}{2} \log_2 \left(1 + \gamma_{\text{sb}}\ \frac{P_{\text{source}}}{N_0} \right) < R \right)\ \bigcup\ \left(\frac{1}{2} \log_2 \left(1 + \gamma_{\text{bd}}\ \frac{P_{\text{relays}}}{N_0} \right) < R \right) \right\} \quad (3)$$

or equivalently,

$$\mathcal{E}_{\text{proact}} = \left\{ (\gamma_{\text{sb}} < \Theta_1)\ \bigcup\ (\gamma_{\text{bd}} < \Theta_2) \right\} \quad (4)$$

where Θ_1 and Θ_2 are given respectively by:

$$\Theta_1 = \frac{(2^{2R} - 1)}{m\ \mathsf{SNR}} \text{ and } \Theta_2 = \frac{(2^{2R} - 1)}{(1 - m)\ \mathsf{SNR}} \quad (5)$$

Since communication happens in two steps using half-duplex with same frequency radios, the required spectral efficiency per hop is now $2R$, so that the end-to-end spectral efficiency is R and therefore comparable to direct non-cooperative communication (Fig. 2). Note that (4) simply

[4]Apart from the $\min$ function, the harmonic mean of the two path strengths has been also considered in [4], [5].

states that opportunistic relaying fails if either of the two hops (from source to best relay or from best relay to destination) fails.

For the case of equal power allocation to the source and the best relay (i.e. $m = 0.5 \Rightarrow \Theta = \frac{2^{2R}-1}{\mathsf{SNR}}$), the outage probability follows:

$$\begin{aligned}
&\mathbb{P}\left\{\mathcal{E}_{\text{proact}}\right\} = \mathbb{P}\left\{(\gamma_{\text{sb}} < \Theta) \bigcup (\gamma_{\text{bd}} < \Theta)\right\} \\
&\equiv \mathbb{P}\left\{\min\{\gamma_{\text{sb}}, \gamma_{\text{bd}}\} < \Theta\right\} = \mathbb{P}\left\{\max_{i\in\{1,..,M\}}\{\min\{\gamma_{\text{s}i}, \gamma_{i\text{d}}\}\} < \Theta\right\} \qquad (6) \\
&\overset{(a)}{=} \mathbb{P}\left\{\max_{i\in\{1,..,M\}}\{\gamma_{\text{s}i\text{d}}\} < \Theta\right\} = \prod_{i=1}^{M}\mathbb{P}\left\{\gamma_{\text{s}i\text{d}} < \Theta\right\} = \prod_{i=1}^{M}\left(1 - e^{-\Theta(\frac{1}{\overline{\gamma}_{\text{s}i}} + \frac{1}{\overline{\gamma}_{i\text{d}}})}\right) \qquad (7)
\end{aligned}$$

where (a) follows from the fact that the minimum of two independent exponentials is again an exponential random variable with parameter equal to the sum of the two parameters. Therefore, $\gamma_{\text{s}i\text{d}}$ has exponential distribution with parameter $1/\overline{\gamma}_{\text{s}i\text{d}}$:

$$\frac{1}{\overline{\gamma}_{\text{s}i\text{d}}} = \frac{1}{\overline{\gamma}_{\text{s}i}} + \frac{1}{\overline{\gamma}_{i\text{d}}} \qquad (8)$$

For example, for the case of $M = 2$ "see-around-corner" opportunistic relays, the outage probability becomes:

$$\mathbb{P}\left\{\mathcal{E}_{\text{proact}}\right\} = (1 - e^{-\Theta(\frac{1}{\overline{\gamma}_{\text{s}1}} + \frac{1}{\overline{\gamma}_{1\text{d}}})})(1 - e^{-\Theta(\frac{1}{\overline{\gamma}_{\text{s}2}} + \frac{1}{\overline{\gamma}_{2\text{d}}})}) \qquad (9)$$

For the case of unequal power allocation among the source and relays, we slightly modify the relay selection rule (2):

$$\min\{m\gamma_{\text{sb}}, (1-m)\gamma_{\text{bd}}\} \geq \min\{m\gamma_{\text{s}i}, (1-m)\gamma_{i\text{d}}\}, \forall\ i \in \{1,..,M\} \qquad (10)$$

Following the same derivation as in (7), the outage probability becomes:

$$\mathbb{P}\left\{\mathcal{E}_{\text{proact}}\right\} = \prod_{i=1}^{M}(1 - e^{-\frac{1}{\overline{\gamma}_{\text{s}i}}\ \Theta_1}\ e^{-\frac{1}{\overline{\gamma}_{i\text{d}}}\ \Theta_2}) \qquad (11)$$

B. Reactive Decode and Forward

In a reactive decode-and-forward scheme, the relays that successfully decode the message regenerate and transmit it, possibly through a distributed space-time code, as originally proposed in [12][5]. In other words, the multiple relay transmission during the second stage is performed by

[5]Decoding at each relay was assumed to be error-free if outage does not take place.

a subset $\mathrm{D}(k)$ of the relays that includes k relays that successfully decoded the message during the first stage:

$$\frac{1}{2}\log_2\left(1+\gamma_{\mathrm{s}(i)}\ \frac{P_{\text{source}}}{N_0}\right) > R,\ i\in \mathrm{D}(k) \quad \Leftrightarrow \tag{12}$$

$$\gamma_{\mathrm{s}i} > \Theta_1,\ i\in\ \mathrm{D}(k) \tag{13}$$

There is no guarantee of successful decoding from (12). We assume however that error-free decoding happens when the outage does not take place, inline with [11], [12]. Using appropriate distributed space-time coding that allows simultaneous transmissions (possibly at the same frequency bands), the outage probability is given by:

$$\mathbb{P}\left\{\mathcal{E}_{\text{react}}\right\} = \sum_{k=0}^{M}\sum_{\mathrm{D}(k)}\ \mathbb{P}\left\{\mathcal{E}_{\text{react}}\mid \mathrm{D}(k)\right\}\ \mathbb{P}\left\{\mathrm{D}(k)\right\} \tag{14}$$

where the conditional outage probability is

$$\mathbb{P}\left\{\mathcal{E}_{\text{react}}\ \middle|\ \mathrm{D(k)}\right\} = \mathbb{P}\left\{\frac{1}{2}\log_2\left(1+\sum_{\mathrm{D}(k)}\gamma_{i\mathrm{d}}\ \frac{P_{i\mathrm{d}}}{N_0}\right) < R\right\} \tag{15}$$

$$\text{with}\ \sum_{i\in\mathrm{D}(k)} P_{i\mathrm{d}} = P_{\text{relays}}$$

Notice that there are 2^M possible decoding sets for M relays, including $\mathrm{D}(0)$ i.e., the set that has no relays, at the event that no relay successfully decoded the message during the first stage of the protocol. Note also that there are $\binom{M}{k}$ different decoding sets that include exactly k relays that have successfully decoded the message.

It is interesting to see that a careful selection of a single relay minimizes outage probability under the aggregate power constraint in the aforementioned reactive scheme. This minimization is performed by using a single relay in the decoding set $\mathrm{D}(k)$: the relay that belongs to $\mathrm{D}(k)$ and also has the maximum *instantaneous* channel $\gamma_{b\mathrm{d}}$ towards destination. That is due to the following inequality:

$$\sum_{i\in\mathrm{D}(k)}\gamma_{i\mathrm{d}}\ \frac{P_{i\mathrm{d}}}{N_0} \le \sum_{i\in\mathrm{D}(k)}\gamma_{b\mathrm{d}}\ \frac{P_{i\mathrm{d}}}{N_0} = \gamma_{b\mathrm{d}}\ \frac{P_{\text{relays}}}{N_0} \tag{16}$$

yielding

$$\mathbb{P}\left\{\frac{1}{2}\log_2\left(1+\gamma_{b\mathrm{d}}\ \frac{P_{\text{relays}}}{N_0}\right) < R\ \middle|\ b\in\mathrm{D}(k)\right\} \le \mathbb{P}\left\{\frac{1}{2}\log_2\left(1+\sum_{i\in\mathrm{D}(k)}\gamma_{(i)\mathrm{d}}\ \frac{P_{i\mathrm{d}}}{N_0}\right) < R\ \middle|\ \mathrm{D}(k)\right\}$$

Therefore, selection of the relay b that belongs to $\mathrm{D}(k)$ with $\gamma_{bd} \geq \gamma_{i\mathrm{d}}, \forall\, i \ \in \mathrm{D}(k)$ minimizes the outage probability and optimizes performance. For slow fading environments, a simple method can be devised to select the relay with the strongest path strengths towards the destination in a fast and distributed manner, alongside the work in [4]. Notice that the minimization holds irrespectively to the power allocation m.

Given a specific decoding set $\mathrm{D}(k)$, the conditional outage probability under the optimal scheme described above becomes

$$\mathbb{P}\left\{\mathcal{E}_{\text{react}} \middle| \mathrm{D}(k)\right\} = \prod_{i \in \mathrm{D}(k)} \mathbb{P}\left\{\gamma_{i\mathrm{d}} \leq \Theta_2\right\} \tag{17}$$

which simply states that if the "best" relay fails, then all relays should fail given that the best relay has the strongest path γ_{bd} towards destination. The probability for a given decoding set is given by:

$$\mathbb{P}\left\{\mathrm{D}(k)\right\} = \prod_{i \in \mathrm{D}(k)} \mathbb{P}\left\{\gamma_{\mathrm{s}i} \geq \Theta_1\right\} \prod_{j \notin \mathrm{D}(k)} \mathbb{P}\left\{\gamma_{\mathrm{s}j} \leq \Theta_1\right\} \tag{18}$$

It is interesting to see that the outage probability for the reactive scheme is *exactly* the same with that achieved by opportunistic proactive relaying, described in (7), (11). For example, eq. (14) with $M = 2$ can be analytically expressed as:

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}_{\text{react}}\right\} = & \underbrace{(1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}1}})(1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}2}})}_{\mathbb{P}\{\mathrm{D}(0)\}} + (1 - e^{-\Theta_2/\overline{\gamma}_{1\mathrm{d}}})\ (1 - e^{-\Theta_2/\overline{\gamma}_{2\mathrm{d}}})\ \underbrace{e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}1}}\ e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}2}}}_{\mathbb{P}\{\mathrm{D}(2)\}} \\
& + (1 - e^{-\Theta_2/\overline{\gamma}_{1\mathrm{d}}})\ \underbrace{e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}1}}(1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}2}})}_{\mathbb{P}\{\text{relay } 1 \in \mathrm{D}(1)\}} + (1 - e^{-\Theta_2/\overline{\gamma}_{2\mathrm{d}}})\ \underbrace{e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}2}}(1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}1}})}_{\mathbb{P}\{\text{relay } 2 \in \mathrm{D}(1)\}} \\
& = (1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}1}}\ e^{-\Theta_2/\overline{\gamma}_{1\mathrm{d}}})\ (1 - e^{-\Theta_1/\overline{\gamma}_{\mathrm{s}2}}\ e^{-\Theta_2/\overline{\gamma}_{2\mathrm{d}}})
\end{aligned} \tag{19}$$

For $m = 0.5 \Rightarrow \Theta_1 = \Theta_2 = \Theta = \frac{(2^{2R}-1)}{\mathsf{SNR}}$, (19) is exactly the same, as the outage probability for proactive opportunistic relaying in (9). For $m \neq 0.5$, (19) is exactly the same, as in (11). The same result holds for larger numbers of M. From (14), (17), (18), we have:

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}_{\text{react}}\right\} &= \mathbb{P}\left\{\mathrm{D}(0)\right\} + \sum_{k=1}^{M} \sum_{\mathrm{D}(k)} \mathbb{P}\left\{\mathcal{E}_{\text{react}} \mid \mathrm{D(k)}\right\}\ \mathbb{P}\left\{\mathrm{D(k)}\right\} \\
&= \mathbb{P}\left\{\mathrm{D}(0)\right\} + \sum_{k=1}^{M} \sum_{\mathrm{D}(k)} \left\{ \prod_{i \in \mathrm{D}(k)} \mathbb{P}\left\{\gamma_{i\mathrm{d}} \leq \Theta_2\right\} \mathbb{P}\left\{\gamma_{\mathrm{s}i} \geq \Theta_1\right\} \prod_{i \notin \mathrm{D}(k)} \mathbb{P}\left\{\gamma_{\mathrm{s}i} \leq \Theta_1\right\} \right\}
\end{aligned} \tag{20}$$

$$= \prod_{i=1}^{M} (1 - e^{-\frac{1}{\overline{\gamma}_{\mathrm{s}i}} \Theta_1}) + \sum_{k=1}^{M} \sum_{\mathrm{D}(k)} \left\{ \prod_{i \in \mathrm{D}(k)} (1 - e^{-\frac{1}{\overline{\gamma}_{i\mathrm{d}}} \Theta_2}) \; e^{-\frac{1}{\overline{\gamma}_{\mathrm{s}i}} \Theta_1} \prod_{i \notin \mathrm{D}(k)} (1 - e^{-\frac{1}{\overline{\gamma}_{\mathrm{s}n}} \Theta_1}) \right\} \quad (21)$$

$$= \prod_{i=1}^{M} (1 - e^{-\frac{1}{\overline{\gamma}_{\mathrm{s}i}} \Theta_1} \; e^{-\frac{1}{\overline{\gamma}_{i\mathrm{d}}} \Theta_2}) \quad (22)$$

where the last equality follows from Theorem 1 in Appendix.

Note that (22) implies that the outage occurs only when all the potential relays are in outage. Hence, (22) shows that the optimal strategy in reactive decode-and-forward, i.e. the relay strategy that minimizes end-to-end outage probability, under the basic assumptions of this work (no *beamforming*, aggregate power constraint) is to select a single relay that maximizes the instantaneous forward channel path, instead of employing a distributed space-time code. This strategy is equivalent to proactive (opportunistic) decode-and-forward for the case of selection strategy (2) and power allocation fraction $m = 0.5$ or selection strategy (10) and any m, where the "best" path is selected before information is transmitted.

This finding suggests that the choice of the *min* function as a quality measure for a 2-hop link (as proposed in [4], [5]) in a proactive relay selection scheme is indeed appropriate i a sense that it minimizes the outage probability under an aggregate relay power constraint.

Proactive relay selection requires smaller energy for information reception since relays that are not selected can avoid reception during the first stage of the protocol. In contrast, reactive schemes need all relays to receive information during the first stage and therefore scale the reception energy proportionally to the network size. That might be inappropriate when heavy forward error correction (FEC) is used that requires energy-expensive reception routines, especially in battery operated wireless networks.

1) A Note on Optimal Power Allocation: Note that optimal power allocation is feasible, when the source has knowledge of the network topology, in terms of the average channel gains $\overline{\gamma}_{\mathrm{s}i}, \overline{\gamma}_{i\mathrm{d}}$, for all participating relays. That might be difficult in practice, given that the number of relays is unknown and varying and therefore, estimation of the above parameters might require considerable overhead. In this work, we have assumed no CSI at the source. Therefore the interesting question is how much performance loss is observed, when source and best relay use suboptimal power allocation $m = 0.5$, simply because they have incomplete knowledge of network channel conditions. Monte Carlo simulations show that $m = 0.5$ achieves performance

very close to that of optimal m^*, with a corresponding loss in terms of SNR, on the order of 0.7 dB at the worst cases. This is due to the fact that adaptive schemes, like opportunistic relaying, select the forwarding path according to instantaneous channel conditions instead of average, and therefore become less variant to the topology of the network that affects the optimal power allocation m^*[6].

2) A Note on Diversity-Multiplexing Gain Tradeoff: Given that $e^{-x} \to 1-x$ for $x \to 0$, the limit of outage probability at high SNR can be easily computed. Setting $m = 0.5$, $R = r \log_2 \mathsf{SNR}$ and $\mathsf{SNR} \to \infty$ in (22), we have:

$$\mathbb{P}\left\{\mathcal{E}_{\text{proact}}\right\} = \mathbb{P}\left\{\mathcal{E}_{\text{react}}\right\} = \prod_{i=1}^{M}(1 - e^{-(\frac{1}{\overline{\gamma}_{\mathrm{s}i}}+\frac{1}{\overline{\gamma}_{i\mathrm{d}}})\ \Theta}\ ) \longrightarrow \frac{1}{\mathsf{SNR}^{M(1-2r)}} \prod_{i=1}^{M}(\frac{1}{\overline{\gamma}_{\mathrm{s}i}} + \frac{1}{\overline{\gamma}_{i\mathrm{d}}}) \qquad (23)$$

Therefore, the diversity-multiplexing gain tradeoff $d(r)$ is $d(r) = M(1-2r)$. This is the result reported in [5] for the case of proactive (opportunistic) relay selection scheme, where a different derivation methodology was used, based on exponential order results for Rayleigh fading[7]. Here, the derivation is based on the exact calculation of outage probability and covers the reactive case as well.

C. Numerical Results

We compute the outage probability as a function of m SNR with power allocation $m = 0.5$ and the symmetric case of $M = 6$ relays ($\overline{\gamma}_{\mathrm{s}i} = \overline{\gamma}_{i\mathrm{d}} = 1,\ i = 1, 2..., M$). Notice that the derivations of the previous section are not limited to the symmetric case, but apply for any possible network configuration. Proactive decode-and-forward ("Opportunistic") is evaluated using (7), where the opportunistic relay transmits with full power $\mathrm{P}_{\text{relays}} = \mathrm{P}_{\text{source}},\ m = 0.5$. Reactive Space-time coding where all relays that have decoded the message, transmit during the second stage, is depicted as ("All relays") and its performance can be easily evaluated: all successful relays have the same mean channel gains towards destination and total power $\mathrm{P}_{\text{relays}}$ is evenly distributed among them. Then $\mathbb{P}\left\{\mathcal{E} \mid \mathrm{D}\,(k)\right\}$ amounts to estimating the probability distribution function of a chi-square random variable with $2k$ degrees of freedom and therefore, overall performance

[6]It is straightforward to compute the optimal power allocation, using eq. (22).

[7]The diversity order in [5] is $M+1$ and not just M, since direct communication between source and destination was also exploited.

can be easily obtained by (14). Finally, selecting a single successful relay according to average channel conditions is depicted as ('Single') and for the symmetric case, it amounts to selecting just one successful relay randomly (since all relays have the same mean channel gain to the destination) that transmits with full power $\mathrm{P}_{\text{source}}$.

Fig. 3 presents the analytical results, verified by Monte Carlo simulations (not depicted). Fig. 3 shows that Opportunistic relaying in slow fading environments outperforms the two other schemes. In fact, it can be seen that opportunistic relaying, not only is simpler than approaches based on space-time coding, but also it is more efficient, corresponding to a gain in SNR on the order of 2 dB. This is because proactive relay selection based on instantaneous channel gains (via the *min* function) and decode-and-forward is equivalent to optimal reactive decode-and-forward. The "All relays" and the "Single relay" are special cases of reactive decode-and-forward.

This finding suggests that cooperative diversity gains do not necessarily arise from simultaneous transmissions but instead, resilience to fading arises from the availability of several potential paths towards the destination. It is therefore optimal, to select the best one. The main difficulty here is to have the network as a whole entity cooperate in order to discover that path, with minimal overhead and *fast*, within a fraction of the channel coherence time. Ideas on how such selection can be performed in a distributed manner, were demonstrated in [4], [5] for slow fading environments.

Notice that a single relay selection based on average channel gains ("Single") is suboptimal, with a substantial penalty loss. This is due to the fact, that selecting a relay based on average channel gains, removes potential selection diversity benefits as the above experiment clearly demonstrates. Similarly, an alternative scheme would be to select a subset of the decoding set (instead of selecting just one), based on average channel gains and distribute the relay power $\mathrm{P}_{\text{relays}}$ appropriately. That is a scheme analyzed in [15] and can be viewed as a special case of reactive decode-and-forward, for which the optimal strategy is to select a single relay based on *instantaneous* channel conditions (and not average). Selection based on average channel conditions is more appropriate in *ergodic* (fast fading) environments, where it may be practically difficult to discover the "best" relay with small overhead, long before the channel changes again.

IV. Amplify and Forward Analysis

In this section, we follow the notation used in single-relay analysis (one source, one relay and one destination) presented in [16]. Here, we generalize the analysis to the case of multiple relays. The received signal between any two points (s-d) is $y_{\text{sd}} = \sqrt{P_{\text{sd}}}\ a_{\text{sd}}\ x + n_{\text{d}}$ where P_{sd} is the average normalized received power between source s and destination d and depends on the transmitted power, as well as other propagation phenomena, like shadowing. Note that a_{sd} is a unit-power, complex, circularly symmetric, Gaussian random variable corresponding to Rayleigh fading and n_{d} is the AWGN noise term, as defined in section III.

We analyze the general case of amplify-and-forward when the source sends unit power message x_1 during the first stage and unit power message x_2 during the second stage. Later at the analysis, we dismiss the terms due to x_2, according to the scenario of this paper. The system equations for the first stage follow:

1st Stage:

$$y_{\text{d},1} = \sqrt{P_{\text{sd}}}\ a_{\text{sd}}\ x_1 + n_{d,1} \tag{24}$$

$$y_i = \sqrt{P_{\text{s}i}}\ a_{\text{s}i}\ x_1 + n_i,\ \forall\ i\ \in\ \{1, 2, .., M\} \tag{25}$$

Notice that the expected power of each symbol y_i received at each relay i can be easily calculated as: $\mathbb{E}\left\{|y_i|^2\right\} = P_{\text{s}i} + N_0$. Each relay normalizes its received signal with its average power and transmits $\frac{y_i}{\sqrt{\mathbb{E}\{|y_i|^2\}}}$ [16], [20]. Then, the destination receives:

2nd Stage:

$$y_{\text{d},2} = \sqrt{P_{\text{sd}}}\ a_{\text{sd}}\ x_2 + \sum_{i=1}^{M} \sqrt{P_{\text{s}i}}\ a_{\text{s}i}\ \frac{y_i}{\sqrt{\mathbb{E}\left\{|y_i|^2\right\}}} + n_{\text{d},2} \tag{26}$$

$$= \sqrt{P_{\text{sd}}}\ a_{\text{sd}}\ x_2 + \sum_{i=1}^{M} \frac{\sqrt{P_{\text{s}i}}\ \sqrt{P_{i\text{d}}}}{\sqrt{P_{\text{s}i} + N_0}}\ a_{\text{s}i}\ a_{i\text{d}}\ x_1 + \widetilde{n}_{\text{d},2} \tag{27}$$

$$\text{where } \widetilde{n}_{\text{d},2} = n_{\text{d},2} + \sum_{i=1}^{M} \frac{\sqrt{P_{i\text{d}}}}{\sqrt{P_{\text{s}i} + N_0}}\ a_{i\text{d}}\ n_i \tag{28}$$

From (28), we can see that the received signal at the destination can be written as the sum of two terms, corresponding to the two transmitted information symbols plus one noise term. Assuming that the destination has knowledge of the wireless channel conditions $H_{\text{R}\rightarrow\text{D}}$ between the relays and itself (for example, the receiver can estimate the channel using preamble

information), the noise term in (28) becomes complex Gaussian with variance given by[8]:

$$\mathbb{E}\left\{\widetilde{n}_{\mathrm{d},2}\ \widetilde{n}^{*}_{\mathrm{d},2}\mid H_{\mathrm{R}\to\mathrm{D}}\right\}=\underbrace{(1+\sum_{i=1}^{M}\frac{P_{i\mathrm{d}}\ |a_{i\mathrm{d}}|^{2}}{P_{\mathrm{s}i}+N_{0}})}_{\omega^{2}}\ N_{0}=\omega^{2}\ N_{0} \tag{29}$$

In matrix notation, the input-output relation can be written as:

$$\underbrace{\begin{bmatrix} y_{\mathrm{d},1} \\ \\ \frac{y_{\mathrm{d},2}}{\omega} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\left[\begin{array}{c|c} \sqrt{P_{\mathrm{sd}}}\ a_{\mathrm{sd}} & 0 \\ \hline \frac{1}{\omega}\ \sum_{i=1}^{M}\frac{\sqrt{P_{\mathrm{s}i}}\sqrt{P_{i\mathrm{d}}}}{\sqrt{P_{\mathrm{s}i}+N_{0}}}\ a_{\mathrm{s}i}a_{i\mathrm{d}} & \frac{1}{\omega}\sqrt{P_{\mathrm{sd}}}a_{\mathrm{sd}} \end{array}\right]}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{\mathrm{d},1} \\ \\ \frac{\widetilde{n}_{\mathrm{d},2}}{\omega} \end{bmatrix}}_{\mathbf{n}}$$

where

$$H_{21}=\frac{1}{\omega}\ \sum_{i=1}^{M}\frac{\sqrt{P_{\mathrm{s}i}}\sqrt{P_{i\mathrm{d}}}}{\sqrt{P_{\mathrm{s}i}+N_{0}}}\ a_{\mathrm{s}i}a_{i\mathrm{d}} \tag{30}$$

The noise term has covariance matrix given by

$$\mathbb{E}\left\{\mathbf{n}\ \mathbf{n^{T}}\ |H_{\mathrm{R}\to\mathrm{D}}\right\}=N_{0}\ \mathbf{I_{2}} \tag{31}$$

where $\mathbf{I_2}$, is the 2x2 identity matrix and the superscript $\mathbf{T}$ denotes a complex conjugate-transpose.

According to the scenario described in the previous sections, we do not allow the source to transmit a new symbol x_2 during the second stage, when the half-duplex relays forward their information. In that way, the second column of matrix $\mathbf{H}$ is zero and $\mathbf{H}$ becomes a column vector (the first column of $\mathbf{H}$ above).

The mutual information is given by [18]:

$$I_{\mathrm{AF}}=\frac{1}{2}\ \log_{2}\left(1+\frac{P_{\mathrm{sd}}}{N_{0}}\ |a_{\mathrm{sd}}|^{2}+\frac{|H_{21}|^{2}}{N_{0}}\right) \tag{32}$$

Alongside the assumption of having a very poor connection (or no connection) between initial source and final destination, the mutual information becomes:

$$I_{\mathrm{AF}}=\frac{1}{2}\ \log_{2}\left(1+\frac{|H_{21}|^{2}}{N_{0}}\right) \tag{33}$$

[8]notice that knowledge of the wireless channels conditions between source and relays, is not needed at the receiver, for the above assumption to hold.

A. Numerical Results

We present results for the symmetric case of M relays ($\overline{\gamma}_{Si} = \overline{\gamma}_{iD} = 1, \forall\ i\ \ \in \{1, 2, .., M\}$. Denoting $\mathrm{P}_{\text{source}}$ the transmitted power from the source, (33) becomes:

$$I_{\text{AF}} = \frac{1}{2}\ \log_2\left(1 + \frac{P_{\text{source}}}{N_0}\ |\widetilde{H}_{21}|^2\right) \tag{34}$$

where $|\widetilde{H}_{21}|^2$ depends on the relaying strategy: a) all power $\mathrm{P}_{\text{relays}}$ is used at one random relay, b) power is distributed at all relays $\mathrm{P}_{i\text{d}} = \mathrm{P}_{\text{relays}}/M$ and c) all power $\mathrm{P}_{\text{relays}}$ is used at the best opportunistic relay. The exact representation of $|\widetilde{H}_{21}|^2$ follows:

$$|\widetilde{H}_{21}|^2_{\text{one}} = \frac{1}{\frac{P_{\text{source}}+N_0}{P_{\text{relays}}} + |a_{i\text{d}}|^2}\ |a_{s\text{i}}\ a_{i\text{d}}|^2 \tag{35}$$

$$|\widetilde{H}_{21}|^2_{\text{all}} = \frac{1}{\frac{P_S+N_0}{P_{\text{relays}}/M} + \sum_{i=1}^{M}|a_{i\text{d}}|^2}\ |\sum_{i=1}^{M} a_{s\text{i}}\ a_{i\text{d}}|^2 \tag{36}$$

$$|\widetilde{H}_{21}|^2_{\text{opp}} = \frac{1}{\frac{P_S+N_0}{P_{\text{relays}}} + |a_{b\text{d}}|^2}\ |a_{s\text{b}}\ a_{b\text{d}}|^2,\ \text{with} \tag{37}$$

$$\min\{|a_{sb}|^2,\ |a_{b\text{d}}|^2\} \geq \min\{|a_{s\text{i}}|^2,\ |a_{i\text{d}}|^2\}, \forall\ i\ \ \in \{1, 2, .., M\}$$

The first weighting term in (35), (37) is greater than that in (36). The second term in (36) corresponds to the magnitude of the sum of complex numbers with random phases. Therefore, the addition of an increasing number of those terms does not necessarily results in a proportional increase of the magnitude: that would be possible, only under equal phases (beamforming). The cumulative distribution function $\text{CDF}(x) = \mathbb{P}\left\{I_{\text{AF}} \leq x\right\}$ is depicted in Fig. 4 for the three cases above. Selecting the opportunistic relay outperforms the case of having all relays transmit. It is also shown, that choosing a random relay is a suboptimal technique, compared to the "all relays" case, since the probability of transmitting a low SNR signal increases.

This figure shows again that the advantages of multiple nodes in a relay network do not arise because of complex transmission/reception techniques, as the "all relays transmit" approach requires, but rather emerge because of the fact that multiple possible paths exist between source, the participating relays and the destination. Opportunistic relaying simply exploits the best available path.

V. Cooperating Relays as Wireless Channel Sensors: A Demo

In an effort to realize wireless networks that adapt to the wireless channel conditions and facilitate cooperation, we built a small-scale, cooperative diversity demonstration. The simplicity of *opportunistic* relaying allowed the use of simple low-cost radios. We interfaced a low-cost 8051 micro-controller to the baseband output of a 916.5 MHz Industrial Scientific Medical (ISM) transceiver module, in a custom Printed Circuit Board (PCB). Then we developed all the necessary software functions for transmission, opportunistic relaying and reception [3].

The goal of the demo setup was to demonstrate in human-perceived scales, the fact that the network as a whole, chose a different relay-path, depending on the wireless channel conditions, especially when people were moving inside the room. In Fig. 5, three colored relays are depicted ("red", "yellow" and "green") which are willing to cooperatively assist a source-destination pair (not depicted in Fig. 5). The source is connected to a weather report service over the internet (through a Personal Digital Assistant) and the destination is connected to a large, *store* display, that displays the received information, without any type of error correction.

As people moved inside the room (a.k.a. changing indoor wireless channel conditions), the best relay path changed and a different relay assisted the communication, as shown in Fig. 6: blocking the "red" relay resulted in information forwarding from the "yellow" relay, depicting the received message at the store display with yellow color. Blocking the yellow relay resulted in selecting the "red" relay-path. More information regarding the demo implementation can be found in [3]. The relay selection requires only partial CSI at each relay (but no CSI regarding the other cooperating relays) and a detailed description and analysis can be found in [5].

The purpose of the above description is to emphasize that the simplicity of the scheme allowed implementation using *existing* radio hardware. Simultaneous transmission at the same frequency band are not needed, since a "smart" relay selection at the medium access layer (layer 2) eliminates the need for space-time coding (and simultaneous transmissions) at the physical layer.

VI. Conclusion

Under the assumptions followed in this work, we showed that the cooperative diversity benefits are increased when cooperative relays choose not to transmit, giving priority to the transmission of a single *opportunistic* relay. We also demonstrated the equivalence of opportunistic relaying

(under the *min* function rule) with the optimal reactive and regenerative (decode-and-forward) multiple relays scheme, when no CSI is exploited at the source.

Therefore, cooperation should be viewed not only as a transmission problem (using distributed space-time codes) but also as a distributed relay selection task. For the cases studied in this work, there is no performance loss compared to distributed space-time coding, in fact there is improved performance, under an aggregate power constraint. Additionally, the proactive nature of the opportunistic scheme reduces the required energy, needed for reception at the relays, which is significant in modern error-correcting radios. Moreover, it was shown that benefits of cooperation arise and improve under opportunistic relaying, even when *dumb* processing is conducted at each relay (the case of amplify-and-forward). The scheme requires no same-frequency, simultaneous transmissions and it is simple enough to be implemented in existing RF front ends. An implementation example in low cost radio was briefly discussed.

Since the optimal strategy for relays is to elect a single relay/retransmitter, the power allocation problem can be simplified, and now it remains to be seen what is the optimal power distribution across only two transmitting nodes: the source and the "best" relay. Future work could include analysis and implementation extensions in fast fading environments, where average channel conditions might be more practical for relay selection. Additionally, extensions can be explored in the interference-limited regime.

Hopefully, this work will spark interest in the exploration of schemes that view cooperative nodes not only as active re-transmitters, but also as distributed sensors of the wireless channel. This work demonstrated that cooperative relays can be useful even when they do not transmit, provided that they cooperatively *listen*. In that way, improved performance is realized and implementation becomes feasible.

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APPENDIX

Theorem 1: The following multinomial equality holds:

$$\prod_{\mathrm{i=1}}^{M}(1-a_{\mathrm{i}}\ b_{\mathrm{i}})=\prod_{i=1}^{M}(1-a_i)+\sum_{k=1}^{M}\sum_{\mathrm{S}(k)\subseteq\{1,..,M\}}\left\{\prod_{i\in\mathrm{S}(k)}(1-b_i)a_i\prod_{i\notin\mathrm{S}(k)}(1-a_i)\right\} \tag{38}$$

where $\mathrm{S}(k)$ denotes a subset of $\{1,2,...,M\}$ with k different elements. Therefore, the cardinality of $\mathrm{S}(k)$ is $|\mathrm{S}(k)|=k$ and there are $\binom{M}{k}$ possible subsets with cardinality k.

Example: $(1-a_1\ b_1)(1-a_2\ b_2)(1-a_3\ b_3)=(1-a_1)(1-a_2)(1-a_3)+$

$$\underbrace{(1-b_1)\ a_1\ (1-a_2)\ (1-a_3)}_{S(1)=\{1\}}+\underbrace{(1-b_2)\ a_2\ (1-a_1)\ (1-a_3)}_{S(1)=\{2\}}+\underbrace{(1-b_3)\ a_3\ (1-a_1)\ (1-a_2)}_{S(1)=\{3\}}+$$
$$\underbrace{(1-b_1)\ a_1\ (1-b_2)\ a_2\ (1-a_3)}_{S(2)=\{1,2\}}+\underbrace{(1-b_2)\ a_2\ (1-b_3)\ a_3\ (1-a_1)}_{S(2)=\{2,3\}}+\underbrace{(1-b_1)\ a_1\ (1-b_3)\ a_3\ (1-a_2)}_{S(2)=\{1,3\}}+$$
$$\underbrace{(1-b_1)\ a_1\ (1-b_2)\ a_2\ (1-b_3)\ a_3}_{S(3)=\{1,2,3\}}$$

Proof 1 (Quick approach): It follows immediately from the fact that

$$(1-a_ib_i)=[(1-a_i)+(1-b_i)a_i]$$

□

Proof 2 (Brute-force approach): We can rewrite (38) as:

$$\prod_{\mathrm{i=1}}^{M}(1-a_i\ b_i)=\prod_{i=1}^{M}(1-a_{\mathrm{i}})+\sum_{k=1}^{M}\sum_{\binom{M}{k}}\left\{\prod_{n=1}^{k}(1-b_{(\mathrm{n})})a_{(\mathrm{n})}\prod_{n=k+1}^{M}(1-a_{(\mathrm{n})})\right\} \tag{39}$$

where the subscript notation (i) can be viewed as an *index of index* and denotes an integer number in $[1..M]$, with $a_{(i)}\equiv a_{(j)},\ b_{(i)}\equiv b_{(j)}$ if and only if $i=j$.

We prove it by rewriting the right-hand-side of the above equation.

$$\prod_{\mathrm{i=1}}^{M}(1-a_{\mathrm{i}}\ b_{\mathrm{i}})=\sum_{k=0}^{M}\ A_k^M+BA_k^M, \tag{40}$$

where A_k^M contains only products of $a_{(i)}$'s, while BA_k^M contains *mixed* products of $a_{(i)}$'s with $b_{(j)}$'s. It is obvious that $BA_{k=0}^M=0$.

Specifically, it can be shown that for integer $k\in[1,M]$ and integer $\lambda\in[1,k]$,

$$BA_k^M=\sum_{\binom{M}{1}}\ b_{(1)}f_{(1)}^k+\sum_{\binom{M}{2}}\ b_{(1)}b_{(2)}f_{(1)(2)}^k+\ldots+\sum_{\binom{M}{\lambda}}\ b_{(1)}b_{(2)}\ldots b_{(\lambda)}f_{(1)(2)\ldots(\lambda)}^k+\ldots$$
$$\ldots+\sum_{\binom{M}{k}}\ b_{(1)}b_{(2)}\ldots b_{(k)}f_{(1)(2)\ldots(k)}^k \tag{41}$$

with

$$
\begin{aligned}
f^k_{(1)(2)\ldots(\lambda)} = (-1)^k\ a_{(1)}\ a_{(2)}\ldots a_{(\lambda)}\Big[&(-1)^{k-\lambda}\sum_{\binom{M-\lambda}{k-\lambda}} 1\ \underbrace{a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(k)}}_{k-\lambda\quad terms}\ + \\
&+(-1)^{k-\lambda+1}\binom{k-\lambda+1}{k-\lambda}\sum_{\binom{M-\lambda}{k-\lambda+1}}\ \underbrace{a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(k+1)}}_{k-\lambda+1\quad terms}\ + \\
&+\ldots+ \\
&+(-1)^{\mu-\lambda}\binom{\mu-\lambda}{k-\lambda}\sum_{\binom{M-\lambda}{\mu-\lambda}}\ \underbrace{a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(\mu)}}_{\mu-\lambda\quad terms}\ + \\
&+\ldots+ \\
&+(-1)^{M-\lambda-1}\binom{M-\lambda-1}{k-\lambda}\sum_{\binom{M-\lambda}{M-\lambda-1}}\ \underbrace{a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(M-1)}}_{M-\lambda-1\quad terms}+ \\
&+(-1)^{M-\lambda}\binom{M-\lambda}{k-\lambda}\ \underbrace{a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(M)}}_{M-\lambda\quad terms}\Big]
\end{aligned}
\tag{42}
$$

Similarly, for integer $\nu \in [k, M]$,

$$
\begin{aligned}
A^M_k &= \sum_{\binom{M}{k}}\ a_{(1)}\ a_{(2)}\ldots a_{(k)}\ (1-a_{(k+1)})\ (1-a_{(k+2)})\ldots(1-a_{(M)}) \\
&= (-1)^0\binom{k}{k}\sum_{\binom{M}{k}}\ a_{(1)}\ a_{(2)}\ldots\ a_{(k)}\ + \\
&\quad +(-1)^1\binom{k+1}{k}\sum_{\binom{M}{k+1}}\ a_{(1)}\ a_{(2)}\ldots\ a_{(k+1)}\ + \\
&\quad +\ldots+ \\
&\quad +(-1)^{\nu-k}\binom{\nu}{k}\sum_{\binom{M}{\nu}}\ a_{(1)}\ a_{(2)}\ldots a_{(k)}\ a_{(k+1)}\ldots a_{(\nu)}\ + \\
&\quad +\ldots+ \\
&\quad +(-1)^{M-1-k}\binom{M-1}{k}\sum_{\binom{M}{M-1}}\ a_{(1)}\ a_{(2)}\ldots a_{(M-1)}\ + \\
&\quad +(-1)^{M-k}\binom{M}{k}\ a_{(1)}\ a_{(2)}\ldots a_{(M)}
\end{aligned}
\tag{43}
$$

From equation (41), we see that the term $b_{(1)}b_{(2)}\ldots b_{(\lambda)}$ in $\sum_k BA^M_k$, is multiplied by the

following term:

$$f^{k=\lambda}_{(1)(2)\ldots(\lambda)} + f^{k=\lambda+1}_{(1)(2)\ldots(\lambda)} + f^{k=\lambda+2}_{(1)(2)\ldots(\lambda)} + \ldots + f^{k=M}_{(1)(2)\ldots(\lambda)} =$$

$$(-1)^{\lambda}\ a_{(1)}\ a_{(2)}\ldots a_{(\lambda)} \quad \Big[\ 1+$$

$$(-1+1) \sum_{\binom{M-\lambda}{1}} a_{(\lambda+1)} \quad +$$

$$(\binom{2}{0} - \binom{2}{1} + \binom{2}{2}) \sum_{\binom{M-\lambda}{2}} a_{(\lambda+1)}\ a_{(\lambda+2)} \quad +$$

$$+\ldots+$$

$$\underbrace{(\binom{\mu-\lambda}{0} - \binom{\mu-\lambda}{1} + \ldots + (-1)^{\mu-\lambda}\binom{\mu-\lambda}{\mu-\lambda})}_{=0} \sum_{\binom{M-\lambda}{\mu-\lambda}} a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots a_{(\mu)} \quad +$$

$$+\ldots+$$

$$(\binom{M-\lambda}{0} - \binom{M-\lambda}{1} + \ldots + (-1)^{M-\lambda}\binom{M-\lambda}{M-\lambda})\ a_{(\lambda+1)}\ a_{(\lambda+2)}\ldots\ a_{(M)}\ \Big] =$$

$$= (-1)^{\lambda}\ a_{(1)}\ a_{(2)}\ldots a_{(\lambda)} \qquad (44)$$

since, $\sum_{k=0}^{n}(-1)^k\binom{n}{k} \equiv (1-1)^n = 0$.

Therefore:

$$\sum_k BA^M_k = -\sum_{\binom{M}{1}} a_{(1)}\ b_{(1)} + \sum_{\binom{M}{2}} a_{(1)}\ b_{(1)}a_{(2)}\ b_{(2)} + \ldots +$$

$$(-1)^{M-1}\sum_{\binom{M}{M-1}} a_{(1)}\ b_{(1)}a_{(2)}\ b_{(2)}\ldots a_{(M-1)}\ b_{(M-1)} + (-1)^M\ a_1\ b_1a_2\ b_2\ldots a_M\ b_M \qquad (45)$$

Similarly, from equation (44), we see that the term $a_{(1)}\ a_{(2)}\ldots a_{(k)}\ a_{(k+1)}\ldots a_{(\nu)}$, appears in each A^M_k (for $k = 0..M$) with a multiplying term $(-1)^{\nu-k}\binom{\nu}{k}$. Therefore, $a_{(1)}\ a_{(2)}\ldots a_{(k)}\ a_{(k+1)}\ldots a_{(\nu)}$ in $\sum_k A^M_k$, is multiplied by the following term:

$$\sum_{k=0}^{\nu}(-1)^{\nu-k}\binom{\nu}{k} \equiv (1-1)^{\nu} = 0$$

The only term that does not cancel out in $\sum_k A^M_k$, is the term that does not include any a'_is. That is the unit term, coming from A^M_0. In short,

$$\sum_k A^M_k = 1. \qquad (46)$$

Equations (45),(46) show that equation (40) is indeed true, concluding the proof. □

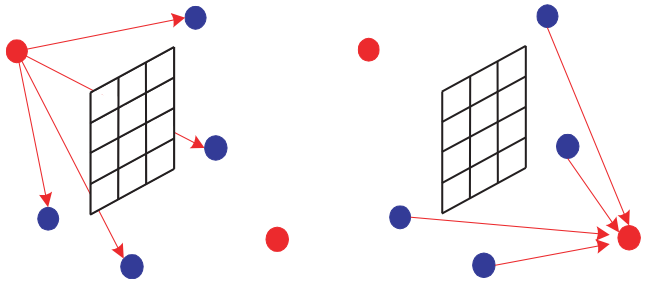

Fig. 1. Scenario addressed in this work: source and destination are blocked or have *poor* connection. Relays forward information in the simplest two-stage scheme and different relay strategies are compared.

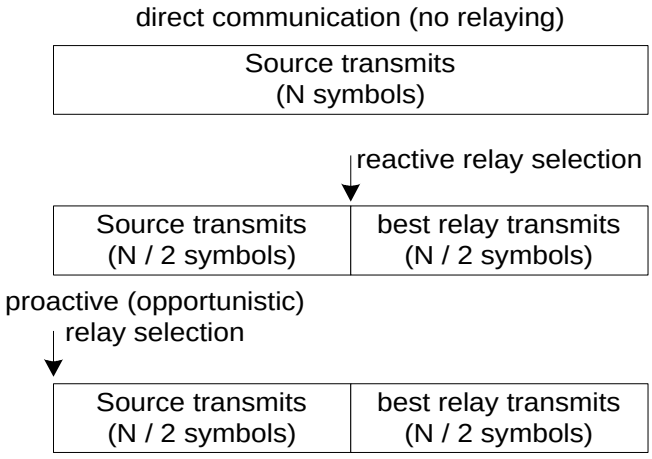

Fig. 2. Standard two-stage cooperation. During the first stage source transmits and relays listen, while during the second stage, the relays transmit. Relay selection can be performed *proactively*, at the beginning of the block, before source transmission, or *reactively*, after source transmission. Single Relay forwarding is compared to all-relays transmit, based on distributed space-time coding and decode-and-forward or amplify-and-forward, in this two-stage protocols.

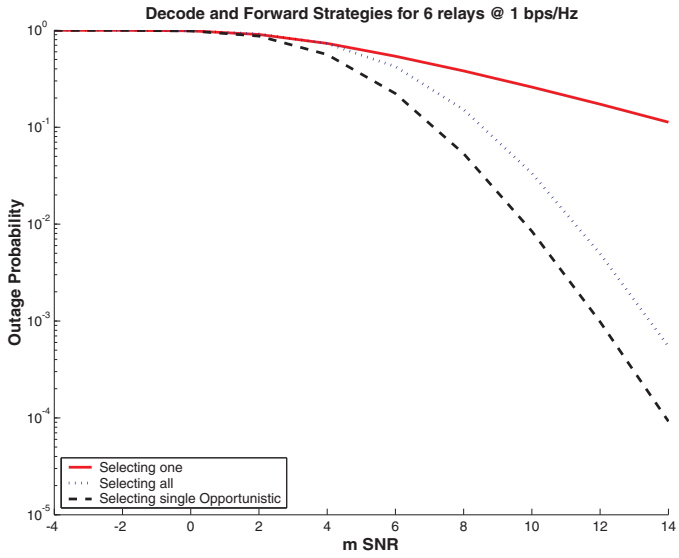

Fig. 3. Outage event probability as a function of SNR. Opportunistic, single relay transmission outperforms simultaneous transmissions with distributed space-time coding (”All relays”) or single relay based on average channel conditions.

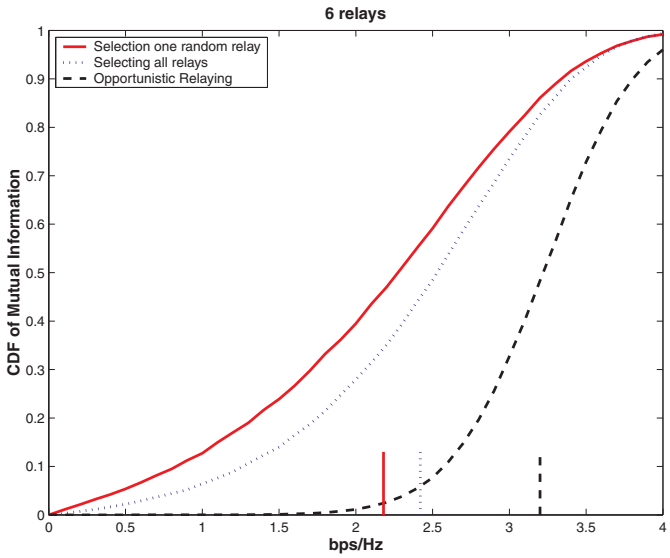

Fig. 4. Cumulative Distribution Function (CDF) of mutual information (eq. 33), for SNR=20 dB. Notice that the CDF function provides the outage probability. Average values (in bps/Hz) are also depicted

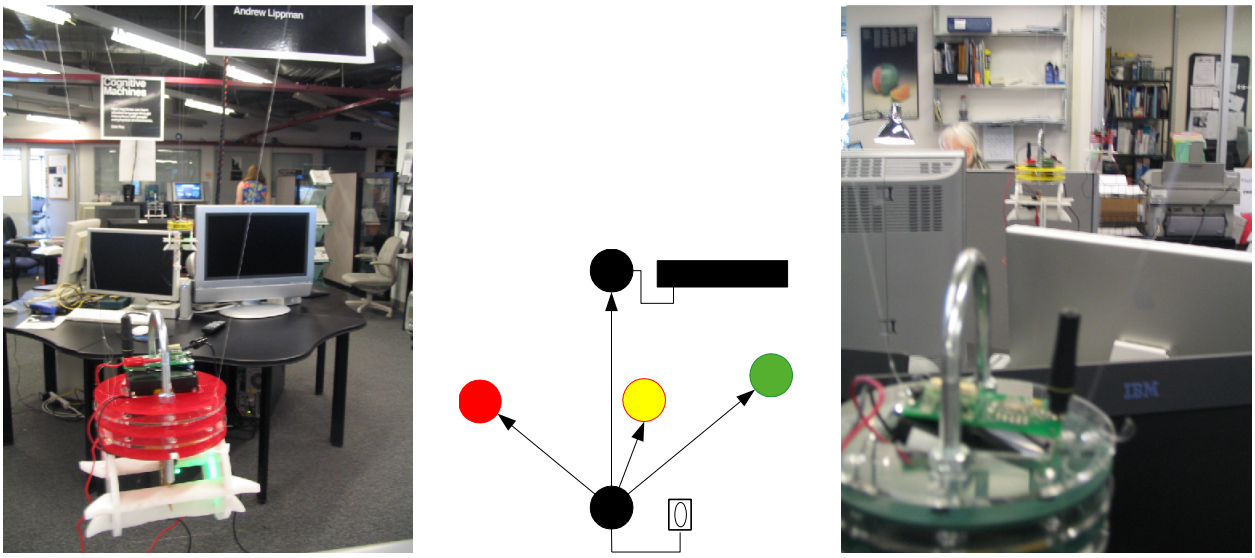

Fig. 5. Three ”colored” relays (”red”, ”yellow”, ”blue”) are depicted. The relays are willing to assist a single source-destination pair (not depicted). The source is connected to a weather report service (through a PDA) and the destination is connected to a large store display.

Fig. 6. A single, ”best” relay is chosen based on the end-to-end channel conditions, among all relays, in a distributed manner. The selection adapts to the wireless channel changes. For example, when ”red” relay path is blocked, ”yellow” path is chosen (left figure) and vice versa, when the ”red” path is chosen (right figure). The text color at the store display shows the best path, currently used.