10-1

1. Plan

Objectives

1 To graph quadratic functions of the form $y = ax^2$
2 To graph quadratic functions of the form $y = ax^2 + c$

Examples

1 Identifying a Vertex
2 Graphing $y = ax^2$
3 Comparing Widths of Parabolas
4 Graphing $y = ax^2 + c$
5 Real-World Problem Solving

Professional Development

Math Background

Formally, a parabola is the intersection of a right circular cone with a plane parallel to a line on the surface of the cone (a generating line).

More Math Background: p. 548C

Lesson Planning and Resources

See p. 548E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Exponents, Order of Operations
Lesson 1-2: Example 5
Extra Skills and Word Problem Practice, Ch. 1

Function Rules, Tables, Graphs
Lesson 5-3: Examples 1, 3
Extra Skills and Word Problem Practice, Ch. 5

10-1

Exploring Quadratic Graphs

What You'll Learn

- To graph quadratic functions of the form $y = ax^2$
- To graph quadratic functions of the form $y = ax^2 + c$

. . . And Why

To model a problem involving gravity, as in Example 5

Check Skills You'll Need GO for Help Lessons 1-2 and 5-3

Evaluate each expression for $h = 3, k = 2$, and $j = -4$.

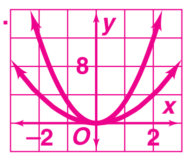
1. hkj **−24** 2. kh^2 **18** 3. hk^2 **12** 4. $kj^2 + h$ **35**

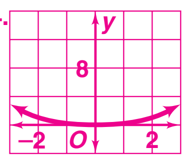
Graph each equation. **5–7. See back of book.**

5. $y = 2x - 1$ 6. $y = |x| + 1$ 7. $y = x^2 + 2$

New Vocabulary • quadratic function • standard form of a quadratic function • quadratic parent function • parabola • axis of symmetry • vertex • minimum • maximum

1 Graphing $y = ax^2$

1.


4.


Yes; the graph is wider than $y = x^2$.

Activity: Plotting Quadratic Curves

1. Graph the equations $y = x^2$ and $y = 3x^2$ on the same coordinate plane. **See left.**
2. a. Describe how the graphs are alike. **a–b. See back of book.**
   b. Describe how the graphs are different.
3. Predict how the graph of $y = \frac{1}{3}x^2$ will be similar to and different from the graph of $y = x^2$. **See back of book.**
4. Graph $y = \frac{1}{3}x^2$. Were your predictions correct? Explain. **See left.**

The functions shown above are quadratic functions.

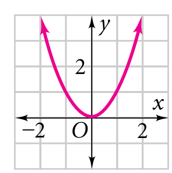
Key Concepts

Definition **Standard Form of a Quadratic Function**

A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form of a quadratic function.** **Examples** $y = 5x^2$ $y = x^2 + 7$ $y = x^2 - x - 3$

The simplest quadratic function, $f(x) = x^2$, or $y = x^2$, is the **quadratic parent function**.

The graph of a quadratic function is a U-shaped curve called a **parabola.** The graph of $y = x^2$, shown at the right, is a parabola.



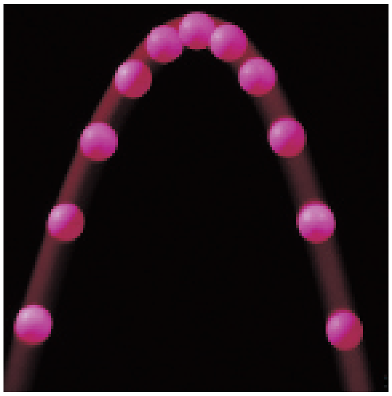
Differentiated Instruction Solutions for All Learners

Special Needs L1
Some students may have difficulty plotting points on a coordinate plane. For the activity, have them work with a partner to ensure accurate graphing as they determine (x, y) pairs of points

learning style: tactile

Below Level L2
Help students to see that quadratic functions have curved graphs by comparing tables of values for $y = x$ and $y = x^2$.

learning style: verbal



You can fold a parabola so that the two sides match exactly. This property is called *symmetry*. The fold or line that divides the parabola into two matching halves is called the **axis of symmetry.**

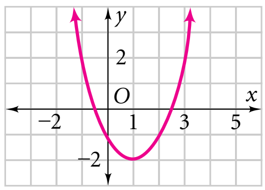
The highest or lowest point of a parabola is its **vertex,** which is on the axis of symmetry.

If $a > 0$ in $y = ax^2 + bx + c$	If $a < 0$ in $y = ax^2 + bx + c$
↓	↓
the parabola opens upward.	the parabola opens downward.
↓	↓
The vertex is the **minimum** point or lowest point of the parabola.	The vertex is the **maximum** point or highest point of the parabola.

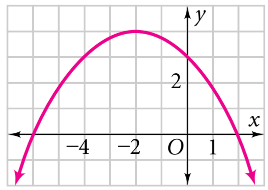
1 EXAMPLE Identifying a Vertex

Identify the vertex of each graph. Tell whether it is a minimum or maximum.

a.



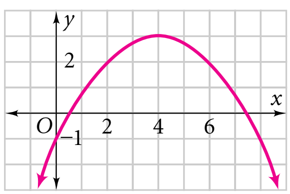
b.



The vertex is $(1, -2)$. It is a minimum. The vertex is $(-2, 4)$. It is a maximum.

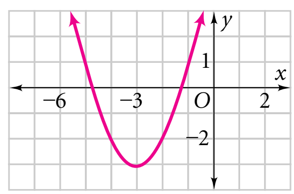
Quick Check 1 Identify the vertex of each graph. Tell whether it is a minimum or maximum.

a.



(4, 3); max.

b.



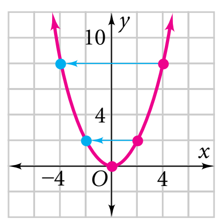
(−3, −3); min.

You can use the fact that a parabola is symmetric to graph it quickly. First find the coordinates of the vertex and several points on either side of the vertex. Then reflect the points across the axis of symmetry. For functions of the form $y = ax^2$, the vertex is at the origin.

2 EXAMPLE Graphing $y = ax^2$

Make a table of values and graph the quadratic function $y = \frac{1}{2}x^2$.

x	$y = \frac{1}{2}x^2$	(x, y)
0	$\frac{1}{2}(0)^2 = 0$	(0, 0)
2	$\frac{1}{2}(2)^2 = 2$	(2, 2)
4	$\frac{1}{2}(4)^2 = 8$	(4, 8)



Find the corresponding points on the other side of the axis of symmetry.

Quick Check 2 Make a table of values and graph the quadratic function $f(x) = -2x^2$. **See back of book.**

2. Teach

Guided Instruction

Activity
Make sure students understand that quadratic equations are graphed as curves connecting the points, not as segments connecting the points. Demonstrate by graphing $y = x^2$ on the board.

2 EXAMPLE Teaching Tip

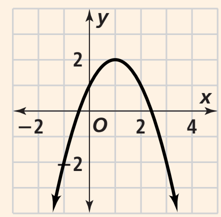
Since x^2 is multiplied by $\frac{1}{2}$, encourage students to choose even numbers for x.

PowerPoint

Additional Examples

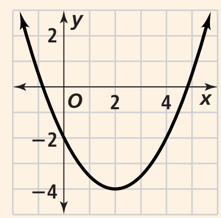
1 Identify the vertex of each graph. Tell whether the vertex is a minimum or a maximum.

a.



(1, 2); maximum

b.



(2, −4); minimum

2 Make a table of values and graph the quadratic function $y = \frac{1}{3}x^2$. **See back of book.**

Advanced Learners L4
Ask students to predict what kind of quadratic function would have a graph whose vertical axis of symmetry is either to the right or to the left of the origin.
learning style: verbal

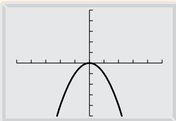
English Language Learners ELL
For students who may not be familiar with the word *bungee* have a volunteer explain how bungee jumping is done. Also point out that the letter *g* can be a hard *g* sound (as in *angle* or *English*) or a soft *g* sound like the letter *j* (as in *algebra* or *bungee*).
learning style: verbal

PowerPoint

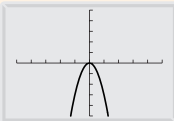
Additional Examples

3 Use the graphs below. Order the quadratic functions $f(x) = -x^2$, $f(x) = -3x^2$, and $f(x) = \frac{1}{2}x^2$ from widest to narrowest graph.

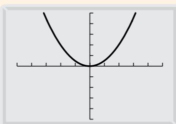
$f(x) = -x^2$



$f(x) = -3x^2$



$f(x) = \frac{1}{2}x^2$



$f(x) = \frac{1}{2}x^2$, $f(x) = -x^2$, $f(x) = -3x^2$

4 EXAMPLE Tactile Learners

After students have graphed $y = 2x^2$, have them bend a paper clip into the shape of the graph. Then instruct them to move the paper clip around to model translating the graph.

PowerPoint

Additional Examples

4 Graph the quadratic functions $y = 3x^2$ and $y = 3x^2 - 2$. Compare the graphs. **The graph of $y = 3x^2 - 2$ has the same shape as the graph of $y = 3x^2$, but it is shifted down 2 units. See back of book.**

5 A monkey drops an orange from a branch 26 ft above the ground. The force of gravity causes the orange to fall toward the Earth. The function $h = -16t^2 + 26$ gives the height of the orange h in feet after t seconds. Graph this quadratic function. **See back of book.**

Resources

- Daily Notetaking Guide 10-1 L3
- Daily Notetaking Guide 10-1—Adapted Instruction L1

Closure

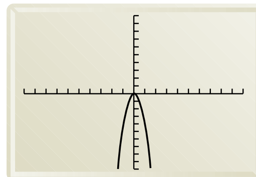
Ask: *What is the shape of a quadratic graph?* **U-shaped** *How do a and c affect a quadratic graph?* **The greater the absolute value of a, the narrower the graph. If a is positive, the graph opens upward. If a is negative, the graph opens downward. The value of c determines the number of units, and also in which direction, the graph is shifted vertically.**

The value of a, the coefficient of the x^2 term in a quadratic function, affects the width of a parabola as well as the direction in which it opens.

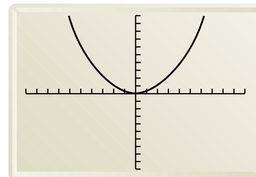
3 EXAMPLE Comparing Widths of Parabolas

Use the graphs below. Order the quadratic functions $f(x) = -4x^2$, $f(x) = \frac{1}{4}x^2$, and $f(x) = x^2$ from widest to narrowest graph.

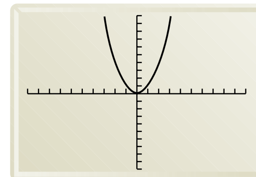
$y = -4x^2$



$y = \frac{1}{4}x^2$



$y = x^2$



Of the three graphs, $f(x) = \frac{1}{4}x^2$ is the widest and $f(x) = -4x^2$ is the narrowest. So, the order from widest to narrowest is $f(x) = \frac{1}{4}x^2$, $f(x) = x^2$, and $f(x) = -4x^2$.

Quick Check 3 Order the quadratic functions $y = x^2$, $y = \frac{1}{2}x^2$, and $y = -2x^2$ from widest to narrowest graph. **$y = \frac{1}{2}x^2$, $y = x^2$, $y = -2x^2$**

When $|m| < |n|$, the graph of $y = mx^2$ is wider than the graph of $y = nx^2$.

2 Graphing $y = ax^2 + c$

The y-axis is the axis of symmetry for functions in the form $y = ax^2 + c$. The value of c translates the graph up or down.

4 EXAMPLE Graphing $y = ax^2 + c$

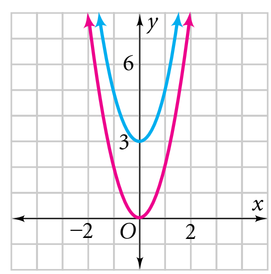
Multiple Choice How is the graph of $y = 2x^2 + 3$ different from the graph of $y = 2x^2$?

A It is shifted 3 units up.
B It is shifted 3 units down.
C It is shifted 3 units to the right.
D It is shifted 3 units to the left.

Test-Taking Tip

You can use symmetry of the parabola to check calculated (x, y) coordinates or points on the graph.

x	$y = 2x^2$	$y = 2x^2 + 3$
−2	8	11
−1	2	5
0	0	3
−1	2	5
−2	8	11

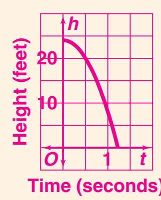


The graph of $y = 2x^2 + 3$ has the same shape as the graph of $y = 2x^2$, but it is shifted up 3 units. So A is the correct answer.

Quick Check 4 a. Graph $y = x^2$ and $y = x^2 - 4$. Compare the graphs. **See back of book.**
b. **Critical Thinking** Describe what positive and negative values of c do to the position of the vertex. **Positive values of c shift the vertex up. Negative values of c shift the vertex down.**

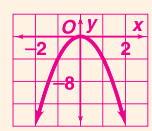
page 553 Quick Check

5a.



pages 553–556 Exercises

4.

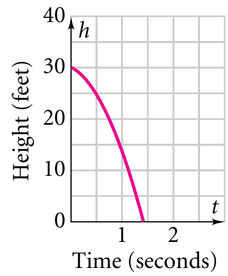


You can model the height of an object moving under the influence of gravity using a quadratic function. As an object falls, its speed continues to increase. Ignoring air resistance, you can find the approximate height of a falling object using the function $h = -16t^2 + c$. The height h is in feet, the time t is in seconds, and the initial height of the object c is in feet.

5 EXAMPLE Real-World Problem Solving

Nature Suppose you see an eagle flying over a canyon. The eagle is 30 ft above the level of the canyon's edge when it drops a stick from its claws. The force of gravity causes the stick to fall toward Earth. The function $h = -16t^2 + 30$ gives the height of the stick h in feet after t seconds. Graph this quadratic function.

t	$h = -16t^2 + 30$
0	30
1	14
2	−34



Height h is dependent on time t. Graph t on the x-axis and h on the y-axis. Use nonnegative values for t.

Quick Check 5 **a.** Suppose a squirrel is in a tree 24 ft above the ground. She drops an acorn. The function $h = -16t^2 + 24$ gives the height of the acorn in feet after t seconds. Graph this function. **See margin.**

b. **Critical Thinking** Describe a reasonable domain and range for the function in Example 5. **Domain: 0 to about 1.5 seconds; Range: 0 to 30 feet**

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

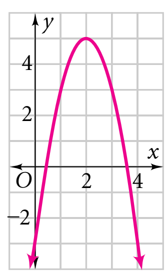
A Practice by Example

GO for Help

Example 1 (page 551)

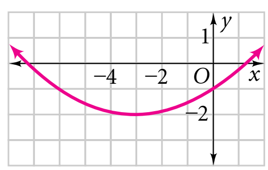
Identify the vertex of each graph. Tell whether it is a minimum or maximum.

1.



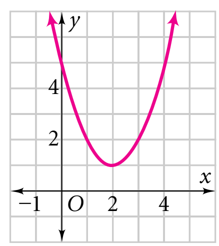
(2, 5); max.

2.



(−3, −2); min.

3.



(2, 1); min.

Example 2 (page 551)

Graph each function. 4–9. See margin.

4. $y = -4x^2$ **5.** $f(x) = 1.5x^2$ **6.** $y = \frac{2}{3}x^2$

7. $f(x) = -\frac{1}{2}x^2$ **8.** $y = -\frac{1}{3}x^2$ **9.** $f(x) = 3x^2$

Example 3 (page 552)

Order each group of quadratic functions from widest to narrowest graph. **10–13. See margin.**

10. $y = 3x^2, y = \frac{1}{2}x^2, y = 4x^2$ **11.** $f(x) = 5x^2, f(x) = \frac{1}{3}x^2, f(x) = x^2$

12. $y = -\frac{1}{2}x^2, y = 5x^2, y = -\frac{1}{4}x^2$ **13.** $f(x) = -2x^2, f(x) = -\frac{2}{3}x^2, f(x) = -4x^2$

3. Practice

Assignment Guide

1 A B 1-13, 27-30, 34-37, 40-43, 45

2 A B 14-26, 31-33, 38-39, 44

C Challenge 46-49

Test Prep 50-53
Mixed Review 54-66

Homework Quick Check

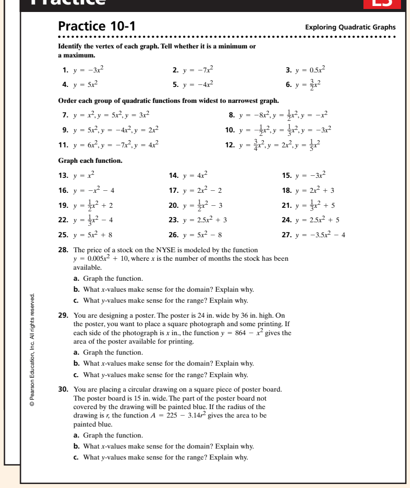
To check students' understanding of key skills and concepts, go over Exercises 4, 18, 28, 39, 44.

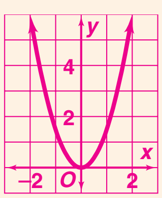
Error Prevention!

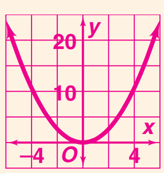
Exercises 10–13 Remind students to use the absolute value of *a* when comparing graphs.

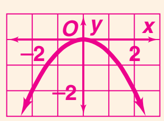
Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3


Practice 10-1 Exploring Quadratic Graphs

5.


6.


7.


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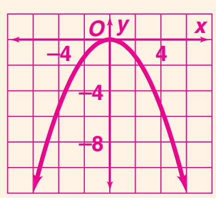
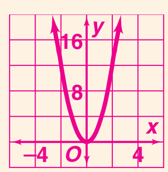


9.


10. $y = \frac{1}{2}x^2, y = 3x^2, y = 4x^2$

11. $f(x) = \frac{1}{3}x^2, f(x) = x^2, f(x) = 5x^2$

12. $y = -\frac{1}{4}x^2, y = -\frac{1}{2}x^2, y = 5x^2$

13. $f(x) = -\frac{2}{3}x^2, f(x) = -2x^2, f(x) = -4x^2$

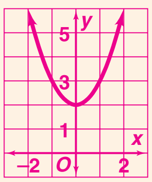
Tactile Learners

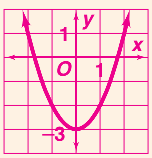
Exercises 34–35 Suggest to students that they fold each traced graph to find the axis of symmetry.

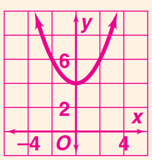
Careers

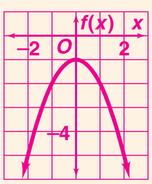
Exercise 47 Tell students that a landscape designer analyzes the needs and preferences of clients and then plans exterior spaces accordingly. The designer selects and integrates appropriate plant and non-plant elements to create the space. One task of the designer that involves mathematics is to develop a plot plan to scale on grid paper.

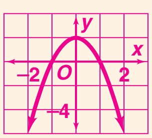
pages 553–556 Exercises

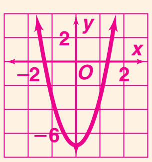
14. 

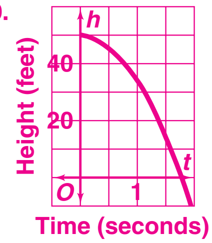
15. 

16. 

17. 

18. 

19. 

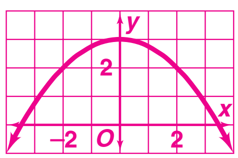
20. 

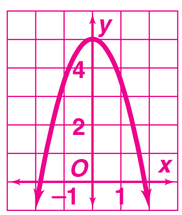
27. The graph of $y = 2x^2$ is narrower.

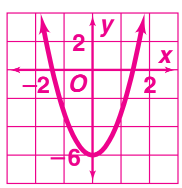
28. The graph of $y = -x^2$ opens downward.

29. The graph of $y = 1.5x^2$ is narrower.

30. The graph of $y = \frac{1}{2}x^2$ is wider.

31. 

32. 

33. 

Example 4 (page 552)

Graph each function. 14–19. See margin.

14. $f(x) = x^2 + 2$ **15.** $y = x^2 - 3$ **16.** $y = \frac{1}{2}x^2 + 4$

17. $f(x) = -x^2 - 1$ **18.** $y = -2x^2 + 2$ **19.** $f(x) = 4x^2 - 7$

Example 5 (page 553)

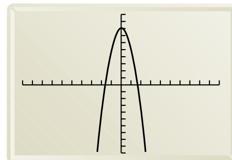
20. A gull drops a clam shell onto some rocks from a height of 50 ft. The function $h = -16t^2 + 50$ gives the shell's approximate height h in feet after t seconds. Graph the function. See left.

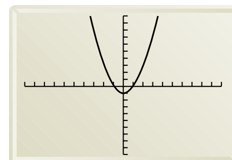
B Apply Your Skills

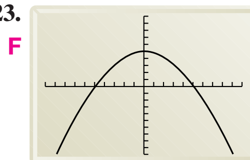
Match each graph with its function.

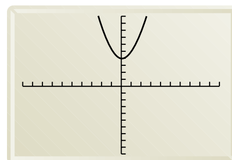
A. $f(x) = x^2 - 1$ **B.** $f(x) = x^2 + 4$ **C.** $f(x) = -x^2 + 2$

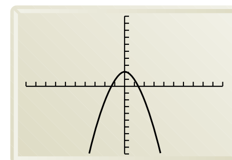
D. $f(x) = 3x^2 - 5$ **E.** $f(x) = -3x^2 + 8$ **F.** $f(x) = -0.2x^2 + 5$

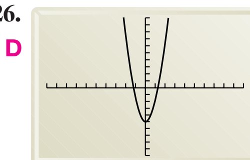
21. E 

22. A 

23. F 

24. B 

25. C 

26. D 

Writing **Without graphing, describe how each graph differs from the graph of $y = x^2$.**

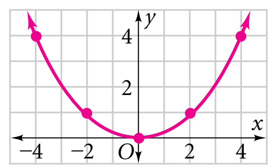
27. $y = 2x^2$ **28.** $y = -x^2$ **29.** $y = 1.5x^2$ **30.** $y = \frac{1}{2}x^2$

27–30. See left.

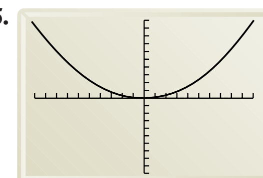
Graph each function. 31–33. See left.

31. $y = -\frac{1}{4}x^2 + 3$ **32.** $f(x) = -1.5x^2 + 5$ **33.** $y = 3x^2 - 6$

Trace each parabola on a sheet of paper and draw its axis of symmetry.

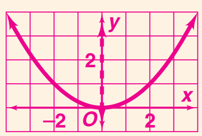
34. 

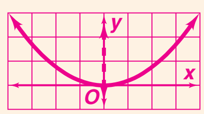
34–37. See margin.

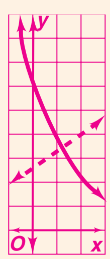
35. 

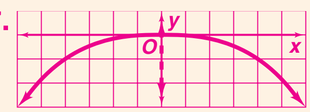
36. 

37. 

34. 

35. 

36. 

37. 

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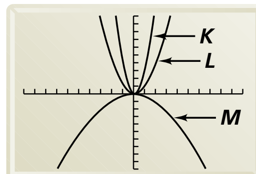
38. A bungee jumper dives from a platform. The function $h = -16t^2 + 200$ gives her approximate height h in feet after t seconds.
a. Graph the function. Graph t on the x-axis and h on the y-axis. **See back of book.**
b. What will the jumper's height be after 1 second? **184 ft**
c. What will the jumper's height be after 3 seconds? **56 ft**

39. **Geometry** Suppose that a pizza must fit into a box with a base that is 12 in. long and 12 in. wide. You can use the quadratic function $A = \pi r^2$ to find the area of a pizza in terms of its radius.

a. What values of r make sense for the function? **$0 < r < 6$**
b. What values of A make sense for the function?
c. Graph the function. Round values of A to the nearest tenth. **See margin.**

39b. $0 < A < 36\pi \approx 113.1$

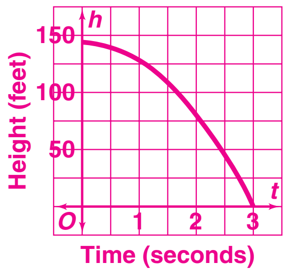
Three graphs are shown below. For Exercises 40–43, identify the graph(s) that fit each description.



40. $a > 0$ **K, L**
41. $a < 0$ **M**
42. $|a|$ has the greatest value. **K**
43. $|a|$ has the least value. **M**

44. **Gravity** Suppose a person is riding in a hot-air balloon, 144 feet above the ground. He drops an apple. The height of the apple above the ground is given by the formula $h = -16t^2 + 144$, where h is height in feet and t is time in seconds.
a. Graph the function. **See left.**
b. How far has the apple fallen from time $t = 0$ to $t = 1$? **16 ft**
c. **Critical Thinking** Does the apple fall as far from time $t = 1$ to $t = 2$ as it does from time $t = 0$ to $t = 1$? Explain. **See left.**

44a.



c. No; the apple falls 48 ft from $t = 1$ to $t = 2$, because it is accelerating.

45. **Multiple Choice** Which function has a graph that is the same as the graph of $f(x) = x^2 + 1$ shifted 4 units down? **B**
(A) $f(x) = x^2 + 5$
(B) $f(x) = x^2 - 3$
(C) $f(x) = x^2 - 4$
(D) $f(x) = x^2 + 4$

Challenge

46. **Critical Thinking** Complete each statement. Assume $a \neq 0$. **a–b. See left.**
a. The graph of $y = ax^2 + c$ intersects the x-axis in two places when _?_.
b. The graph of $y = ax^2 + c$ does not intersect the x-axis when _?_.

46a. $c \neq 0$ and a and c have opp. signs.
b. $c \neq 0$ and a and c have the same signs.

47. **Landscaping** The plan for a 20 ft-by-12 ft patio has a square garden in the middle of it. If each side of the garden is x ft, the function $y = 240 - x^2$ gives the area of the patio without the garden in square feet.
a. Graph the function. **See back of book.**
b. What values make sense for the domain? Explain. **b–c. See margin.**
c. What is the range of the function? Explain.
d. Use the graph to estimate the side length of the garden if the area of the patio is 200 ft^2. **about 6 ft**

48. Consider the graphs of $y = ax^2$ and $y = (ax)^2$. Assume $a \neq 0$.
a. For what values of a will both graphs lie in the same quadrants? **$a > 0$**
b. For what values of a will the graph of $y = ax^2$ be wider than the graph of $y = (ax)^2$? **$|a| > 1$**

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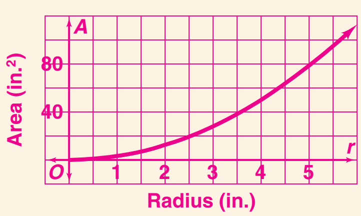
39c.



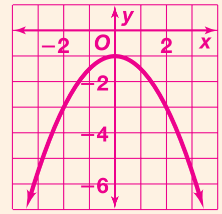
47b. $0 < x < 12$; the side length of the square garden must be less than the width of the patio.

c. $96 < A < 240$; as the side length of the garden increases from 0 to 12, the area of the patio decreases from 240 to 96.

4. Assess & Reteach

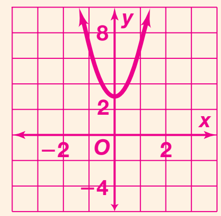
Lesson Quiz

1. a. Graph $y = -\frac{1}{2}x^2 - 1$.



b. Identify the vertex. Tell whether it is a maximum or a minimum. **(0, −1); maximum**
c. Compare this graph to the graph of $y = -x^2$. **This graph is wider, opens downward, and is shifted 1 unit down.**

2. a. Graph $y = 4x^2 + 3$.



b. Identify the vertex. Tell whether it is a maximum or a minimum. **(0, 3); minimum**
c. Compare this graph to the graph of $y = x^2$. **This graph is narrower and shifted 3 units up.**

3. Order the quadratic functions $y = -4x^2$, $y = \frac{1}{4}x^2$, and $y = 2x^2$ from widest to narrowest graph.
$y = \frac{1}{4}x^2, y = 2x^2, y = -4x^2$

Alternative Assessment

Direct all students to stand. Instruct them to model the function $y = x^2$ with their arms. Write a quadratic function in the form of $y = ax^2 + c$ on the board. Tell students to move their arms wider or narrower to represent a, and curve their arms downward to model negative a. They can raise their shoulders and stand on their toes, or bend their knees to model the value of c. Repeat with various quadratic functions.

Test Prep

Resources
For additional practice with a variety of test item formats:

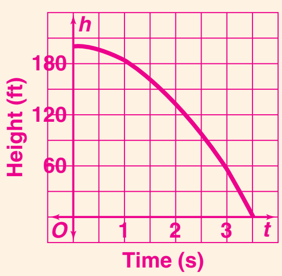
- Standardized Test Prep, p. 611
- Test-Taking Strategies, p. 606
- Test-Taking Strategies with Transparencies

Exercise 52 Point out to students that they can logically eliminate answer choices B and C because these choices are reflections over the *y*-axis. They are either both on, or both not on, the graph. Only one answer choice is not on the graph. Therefore, they both must be on the graph.

pages 553–556 Exercises

53. [4] a.

t	h
0	200
1	184
2	136
3	56
4	−56



b. 3.5 s

[3] estimate incorrect or missing

[2] error in table or graph

[1] table OR reasonable graph only

49. Architecture An architect wants to design an archway with the following requirements.

- The archway is 6 ft wide and has vertical sides 7 ft high.
- The top of the archway is modeled by the function $y = -\frac{1}{3}x^2 + 10$.

a. Sketch the archway by drawing vertical lines 7 units high at $x = -3$ and $x = 3$ and graphing the portion of the quadratic function that lies between $x = -3$ and $x = 3$. **See back of book.**

b. The plan for the archway is then changed so that the top is modeled by the function $y = -0.5x^2 + 11.5$. Make a revised sketch of the archway. **See back of book.**

Test Prep

Multiple Choice

50. Which of the following equations has a graph that is narrower than the graph of $y = 2x^2 + 3$? **B**

A. $y = 2x^2 - 3$ **B.** $y = -3x^2 + 2$
C. $y = 0.5x^2 + 3$ **D.** $y = -0.5x^2 - 2$

51. Which of the following equations has a graph that crosses the *y*-axis at a point lower than the graph of $y = -2x^2 - 1$? **G**

F. $y = -3x^2 - 1$ **G.** $y = 3x^2 - 3$
H. $y = -3x^2 + 1$ **J.** $y = -3x^2 + 3$

52. The graph of $y = 4x^2 + 3$ does NOT pass through which of the following points? **D**

A. (0, 3) **B.** (1, 7) **C.** (−1, 7) **D.** (3, 27)

Extended Response

53. A construction worker drops a tool from the top of a building that is 200 ft high. The height of the tool above the ground can be modeled by $h = -16t^2 + 200$, where h is height in feet and t is time in seconds.

a. Make a table and graph this function. **a–b. See margin.**

b. Use your graph to estimate the amount of time it takes for the tool to hit the ground. Round to the nearest tenth of a second.

Mixed Review

GO for Help

Lesson 9-8 **Factor each expression.**

54. $x^3 - 4x^2 + 2x - 8$ **$(x^2 + 2)(x - 4)$**
55. $15a^3 - 18a^2 - 10a + 12$ **$(3a^2 - 2)(5a - 6)$**
56. $7b^3 + 14b^2 + b + 2$ **$(7b^2 + 1)(b + 2)$**
57. $y^3 + 3y^2 - 4y - 12$ **See left.**
58. $2n^3 - 2n^2 - 24n$ **$2n(n + 3)(n - 4)$**
59. $30m^3 + 51m^2 + 9m$ **$3m(2m + 3)(5m + 1)$**

57. $(y + 2)(y - 2)(y + 3)$

Lesson 9-2 **Simplify each expression.**

60. $5x(3x - 4)$ **$15x^2 - 20x$**
61. $(n - 7)9n$ **$9n^2 - 63n$**
62. $-2t^2(6t - 11)$ **$-12t^3 + 22t^2$**
63. $4m^2(3m^4 - m^3 + 5)$ **$12m^6 - 4m^5 + 20m^2$**
64. $-5y(3y^5 + 2y^3 - 4)$ **$-15y^6 - 10y^4 + 20y$**
65. $3c^3(-4c^2 + 7c - 8)$ **$-12c^5 + 21c^4 - 24c^3$**

66. Business The City Council invites your art club to sell helium balloons during a citywide celebration. The rental of the helium tank is \$27.00 for the day. Each balloon costs \$.20. If the balloons sell for \$2.00 each, how many will your art club have to sell to break even? **15 balloons**