10.4 Mann-Whitney Test: n1, n2 small, same shape

GOALS:
1. Use the Mann-Whitney test to analyze data that is not normally distributed and from small samples.
2. Use overall ranking to weight the data in the 2 data sets while keeping the sets separate.
3. Compute the test statistic as the sum of the ranks of the first data set.
4. Use critical values as the criteria for hypothesis testing.
5. Find the right critical value using Table VI.
6. Find a left critical value using a formula and the right critical value.
6. Complete the testing procedure by using the critical value approach.

Study Ch. 10.4, # 105,106,107,111,112;
113-121, 2 of 123-127 [95-107,109 or 111,113]

10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anesthetic on reaction times. Two groups of individuals are compared, 8 with anesthesia and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.

Anaethesia	Rank	No Anaethesia	Rank
135		138	
171		230	
142		142	
144		156	
139		138	
165		191	
256			
244			

At the 5% significance level, does the anesthesia dull reaction time of patients?
Reaction times are not normally distributed in this experiment, and the sample sizes are small, so how do we proceed?

a) 86, b) 48, c) 45, 99

a) b) c)

10.4 Mann-Whitney Test: n1, n2 small, same shape

Mann- Whitney Test

Small n

Assumptions: 1. SRS, 2. Independent samples
3. Populations have same shape

Use calc to check 1. not nd 2. same shape

Step 1: H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

(Can also be used to test the median, H_0: $\eta_1 = \eta_2$)

Step 2: Decide α

$\eta_1 = \eta_2$

Step 3: Construct Table
* sort each sample separately
* find overall rank

Sample 1	overall Rank	Sample 2	overall Rank

Step 4: Find test statistic

$M_T = \sum$ ranks, population 1

Step 5: Find CV(s) **using Table VI and:**

$M_{1-\alpha} = n_1(n_1+n_2+1) - M_\alpha$

$M_{1-\alpha/2} = n_1(n_1+n_2+1) - M_{\alpha/2}$

or get M_R from Table VI

$M_L = n_1(n_1+n_2+1) - M_R$

Step 6: Decide whether to reject H_0 or not

Step 7: Verbal interpretation

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anesthetic on reaction times. Two groups of individuals are compared, 8 with anesthetic and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.
Reaction times are not normally distributed in this experiment, and the sample sizes are small, so we cannot use a 2 sample t test.
How do we proceed?

H_a: $\eta_1 < \eta_2$
H_0: $\eta_1 = \eta_2$
0.05

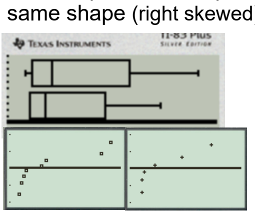
At the 5% significance level, does the anesthesia dull reaction time of patients?

To dull reaction time is to take longer

Anaethesia	Rank	No Anaethesia	Rank
135		138	
171		230	
142		142	
144		156	
139		138	
165		191	
256			
244			

1
2
3
4
5
6
7
8
9
10
11
12
13
14

Assumptions:
srs, independent samples,
same shape (right skewed)



Sequence data (calc) to avoid errors:

Anaethesia	Rank	No Anaethesia	Rank
135		138	
139		138	
142		142	
144		156	
165		191	
171		230	
244			
256			

Class Notes: Prof. G. Battaly, Westchester Community College, NY

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a) b) c)

10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anaesthetic on reaction times. Two groups of individuals are compared, 8 with anaesthetic and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.
Reaction times are not normally distributed in this experiment, and the sample sizes are small, so we cannot use a 2 sample t test.
How do we proceed?

At the 5% significance level, does the anaethesia dull reaction time of patients?

$H_0: \mu_A = \mu_N$

$H_a: \mu_A > \mu_N$

0.05 (right tail), M_R

Anaethesia	Rank	No Anaethesia	Rank
135	1	138	2.5
139	4	138	2.5
142	5.5	142	5.5
144	7	156	8
165	9	191	11
171	10	230	12
244	13		
256	14		

1	✓
2	2.5
3	2.5
4	✓
5	5.5
6	5.5
7	✓
8	✓
9	✓
10	✓
11	✓
12	✓
13	✓
14	✓

Class Notes: Prof. G. Battaly, Westchester Community College, NY

a) 86, b) 48, c) 45, 99

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10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anaesthetic on reaction times. Two groups of individuals are compared, 8 with anaesthetic and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.
Reaction times are not normally distributed in this experiment, and the sample sizes are small, so we cannot use a 2 sample t test.
How do we proceed?

At the 5% significance level, does the anaethesia dull reaction time of patients?
Reaction times are not normally distributed, and the sample sizes are small, so how do we proceed?

$H_0: \mu_A = \mu_N$

$H_a: \mu_A > \mu_N$

Anaethesia	Rank	No Anaethesia	Rank
135	1	138	2.5
139	4	138	2.5
142	5.5	142	5.5
144	7	156	8
165	9	191	11
171	10	230	12
244	13		
256	14		
M=	63.5		

test

rej. 0.05, M_R

a) 86, b) 48, c) 4

TABLE VI
Values of M_α

n_2	α	3
3	0.10	14
	0.05	15
	0.025	—
	0.01	—
	0.005	—
4	0.10	16
	0.05	17
	0.025	18
	0.01	—
	0.005	—
5	0.10	18
	0.05	20
	0.025	21
	0.01	—
	0.005	—
6	0.10	21
	0.05	22
	0.025	23
	0.01	24
	0.005	—
7	0.10	23
	0.05	24
	0.025	26
	0.01	27
	0.005	—
8	0.10	25
	0.05	27
	0.025	28
	0.01	29
	0.005	30
9	0.10	27
	0.05	29
	0.025	31
	0.01	32
	0.005	33
10	0.10	29
	0.05	31
	0.025	33
	0.01	34
	0.005	36

$M_T = 63.5 < M_R = 23$

∴ do not rej. H_0.

The data is not sufficient to claim that anaesthesia dulls responses.

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

$M_L = n_1(n_1+n_2+1) - M_R$

(get M_R from Table VI)

Finding Critical Values

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05

F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test

Use Table VI, Appendix A

a) b) c)

TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6	7	8	9	10
3	0.10	14	20	27	36	45	55	66	78
	0.05	15	21	29	37	46	57	68	80
	0.025	—	22	30	38	48	58	70	82
	0.01	—	—	—	39	49	59	71	83
	0.005	—	—	—	—	—	60	72	85
7	0.10	23	31	41	52	63	76	89	104
	0.05	24	33	43	54	66	79	93	107
	0.025	26	35	45	56	68	81	95	110
	0.01	27	36	47	58	71	84	98	114
	0.005	—	37	48	60	72	86	101	116
8	0.10	25	34	44	56	68	81	95	110
	0.05	27	36	47	58	71	84	99	114
	0.025	28	38	49	61	73	87	102	117
	0.01	29	39	51	63	76	90	105	121
	0.005	30	40	52	65	78	92	108	124
9	0.10	27	37	48	60	72	86	101	116
	0.05	29	39	50	63	76	90	105	121
	0.025	31	41	53	65	78	93	108	124
	0.01	32	43	55	68	81	96	112	129
	0.005	33	44	56	70	84	99	114	131
10	0.10	29	40	51	64	77	91	106	123
	0.05	31	42	54	67	80	95	111	127
	0.025	33	44	56	69	83	98	114	131
	0.01	34	46	59	72	87	102	119	136
	0.005	36	48	61	74	89	105	121	139

10.4 Mann-Whitney Test: n1, n2 small, same shape

$M_L = n_1(n_1+n_2+1) - M_R$

(get M_R from Table VI)

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05

F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test

Use Table VI, Appendix A

a)

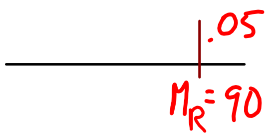


TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6	7	8
3	0.10	14	20	27	36	45	55
	0.05	15	21	29	37	46	57
	0.025	—	22	30	38	48	58
	0.01	—	—	—	39	49	59
	0.005	—	—	—	—	—	60
7	0.10	23	31	41	52	63	76
	0.05	24	33	43	54	66	79
	0.025	26	35	45	56	68	81
	0.01	27	36	47	58	71	84
	0.005	—	37	48	60	72	86
8	0.10	25	34	44	56	68	81
	0.05	27	36	47	58	71	84
	0.025	28	38	49	61	73	87
	0.01	29	39	51	63	76	90
	0.005	30	40	52	65	78	92
9	0.10	27	37	48	60	72	86
	0.05	29	39	50	63	76	90
	0.025	31	41	53	65	78	93
	0.01	32	43	55	68	81	96
	0.005	33	44	56	70	84	99
10	0.10	29	40	51	64	77	91
	0.05	31	42	54	67	80	95
	0.025	33	44	56	69	83	98
	0.01	34	46	59	72	87	102
	0.005	36	48	61	74	89	105

a) 86, b) 48, c) 45, 99

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05
F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test
Use Table VI, Appendix A

n_2	α	3	4	5	6	7	8	9	10
3	0.10	14	20	27	36	45	55	66	78
	0.05	15	21	29	37	46	57	68	80
	0.025	—	22	30	38	48	58	70	82
	0.01	—	—	—	39	49	59	71	83
	0.005	—	—	—	—	—	60	72	85
7	0.10	23	31	41	52	63	76	89	104
	0.05	24	33	43	54	66	79	93	107
	0.025	26	35	45	56	68	81	95	110
	0.01	27	36	47	58	71	84	98	114
	0.005	—	37	48	60	72	86	101	116
8	0.10	25	34	44	56	68	81	95	110
	0.05	27	36	47	58	71	84	99	114
	0.025	28	38	49	61	73	87	102	117
	0.01	29	39	51	63	76	90	105	121
	0.005	30	40	52	65	78	92	108	124
9	0.10	27	37	48	60	72	86	101	116
	0.05	29	39	50	63	76	90	105	121
	0.025	31	41	53	65	78	93	108	124
	0.01	32	43	55	68	81	96	112	129
	0.005	33	44	56	70	84	99	114	131
10	0.10	29	40	51	64	77	91	106	123
	0.05	31	42	54	67	80	95	111	127
	0.025	33	44	56	69	83	98	114	131
	0.01	34	46	59	72	87	102	119	136
	0.005	36	48	61	74	89	105	121	139

b)

.05

$M_L = 54$

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

$M_L = 8(8+9+1) - 90$
$= 144-90 = 54$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

a) 86, b) 48, c) 45, 99

Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05
F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test
Use Table VI, Appendix A

4

c)

.025 .025

$M_L = 51$ $M_R = 93$

n_2	α	3	4	5	6	7	8	9	10
3	0.10	14	20	27	36	45	55	66	78
	0.05	15	21	29	37	46	57	68	80
	0.025	—	22	30	38	48	58	70	82
	0.01	—	—	—	39	49	59	71	83
	0.005	—	—	—	—	—	60	72	85
7	0.10	23	31	41	52	63	76	89	104
	0.05	24	33	43	54	66	79	93	107
	0.025	26	35	45	56	68	81	95	110
	0.01	27	36	47	58	71	84	98	114
	0.005	—	37	48	60	72	86	101	116
8	0.10	25	34	44	56	68	81	95	110
	0.05	27	36	47	58	71	84	99	114
	0.025	28	38	49	61	73	87	102	117
	0.01	29	39	51	63	76	90	105	121
	0.005	30	40	52	65	78	92	108	124
9	0.10	27	37	48	60	72	86	101	116
	0.05	29	39	50	63	76	90	105	121
	0.025	31	41	53	65	78	93	108	124
	0.01	32	43	55	68	81	96	112	129
	0.005	33	44	56	70	84	99	114	131
10	0.10	29	40	51	64	77	91	106	123
	0.05	31	42	54	67	80	95	111	127
	0.025	33	44	56	69	83	98	114	131
	0.01	34	46	59	72	87	102	119	136
	0.005	36	48	61	74	89	105	121	139

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

$M_L = 8(8+9+1) - 93$
$= 144 - 93 = 51$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

a) 86, b) 48, c) 45, 99

Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

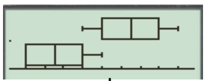
$M_L = n_1(n_1+n_2+1) - M_R$ (get M_R from Table VI)

Six people were asked to taste Brand A coffee, and 6 different people asked to taste Brand B. They were then asked to rank how likely it was that they would purchase that Brand, using a scale of 1 to 10, with 10 being most likely.

At the 5% significance level did they like both brands equally?

Ranked, not continuous.

Brand A	Brand B
9	3
7	4
5	2
10	6
6	2
8	5

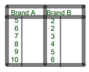


same shape

TABLE VI
Values of M_α

n_2	α	3	4
	0.10	14	20
	0.05	15	21
3	0.025	—	22
	0.01	—	—
	0.005	—	—
	0.10	16	23
	0.05	17	24
4	0.025	18	25
	0.01	—	26
	0.005	—	—
	0.10	18	26
	0.05	20	27
5	0.025	21	28
	0.01	—	30
	0.005	—	—
	0.10	21	29
	0.05	22	30
6	0.025	23	32
	0.01	24	33
	0.005	—	34

1 2 3 4 5 6 7 8 9 10 11 12



10.4 Mann-Whitney Test: n1, n2 small, same shape

$M_L = n_1(n_1+n_2+1) - M_R$ (get M_R from Table VI)

Six people were asked to taste Brand A coffee, and 6 different people asked to taste Brand B. They were then asked to rank how likely it was that they would purchase that Brand, using a scale of 1 to 10, with 10 being most likely.

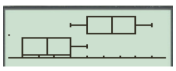
Ranked data, not continuous.

At the 5% significance level did they like both brands equally?

H_o: $\mu1 = \mu2$

H_a: $\mu1 \neq \mu2$

Brand A	Brand B
9	3
7	4
5	2
10	6
6	2
8	5



Brand A		Brand B	
5	5.5	2	1.5
6	7.5	2	1.5
7	9	3	3
8	10	4	4
9	11	5	5.5
10	12	6	7.5

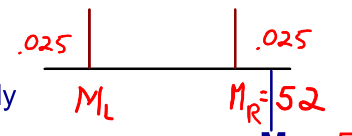
$M_T = 55$

1 2 3 4 5 6 7 8 9 10 11 12

TABLE VI
Values of M_α

n_2	α	3	4	5
	0.10	14	20	27
	0.05	15	21	29
3	0.025	—	22	30
	0.01	—	—	—
	0.005	—	—	—
	0.10	16	23	31
	0.05	17	24	32
4	0.025	18	25	33
	0.01	—	26	35
	0.005	—	—	—
	0.10	18	26	34
	0.05	20	27	36
5	0.025	21	28	37
	0.01	—	30	39
	0.005	—	—	40
	0.10	21	29	38
	0.05	22	30	40
6	0.025	23	32	41
	0.01	24	33	43
	0.005	—	34	44

$M_T = 55 > 52 = M_R$ Reject H_O



$M_T = 55$

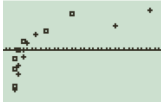
Conclude that the coffees were not equally liked.
(Note: Have stronger evidence than 5% s.l.; actually have 0.5% significance, or 99.5% confidence.)

10.4 Mann-Whitney Test: n1, n2 small, same shape

Ages of Statistics students: independent samples

At the 5% significance level, do the ages of the students in the two groups differ?

Ages of Students			
Group 1	Rank	Group 2	Rank
18		49	
18		19	
19		41	
31		21	
25		20	
18		23	
20		20	
		18	
		19	



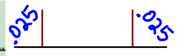


TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6
	0.10	14	20	27	36
	0.05	15	21	29	37
3	0.025	—	22	30	38
		—	—	—	39
		—	—	—	—
		16	23	31	40
		17	24	32	41
		18	25	33	43
		—	26	35	44
	0.005	—	—	—	45
	0.10	18	26	34	44
	0.05	20	27	36	46
5	0.025	21	28	37	47
	0.01	—	30	39	49
	0.005	—	—	40	50
	0.10	21	29	38	48
	0.05	22	30	40	50
6	0.025	23	32	41	52
	0.01	24	33	43	54
	0.005	—	34	44	55
	0.10	23	31	41	52
	0.05	24	33	43	54
7	0.025	26	35	45	56
	0.01	27	36	47	58
	0.005	—	37	48	60
	0.10	25	34	44	56
	0.05	27	36	47	58
8	0.025	28	38	49	61
	0.01	29	39	51	63
	0.005	30	40	52	65
	0.10	27	37	48	60
	0.05	29	39	50	63
9	0.025	31	41	53	65
	0.01	32	43	55	68
	0.005	33	44	56	70
	0.10	29	40	51	64
	0.05	31	42	54	67
10	0.025	33	44	56	69
	0.01	34	46	59	72
	0.005	36	48	61	74

Ages of Students			
Group 1	Rank	Group 2	Rank
18		18	
18		19	
18		19	
19		20	
20		20	
25		21	
31		23	
		41	
		49	

$M_T = 63.5 < M_e = 23$

∴ do NOT rej. H_0.

The data is not sufficient to claim that anaesthesia dulls responses.

Class Notes: Prof. G. Battaly, Westchester

Statistics Home Page

Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

Ages of Statistics students: independent samples

At the 5% significance level, do the ages of the students in the two groups differ?

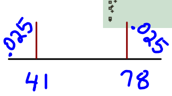
Ages of Students			
Group 1	Rank	Group 2	Rank
18	2.5	18	2.5
18	2.5	19	6
18	2.5	19	6
19	6	20	9
20	9	20	9
25	13	21	11
31	14	23	12
		41	15
		49	16

M =	49.5

n	7	9

1, 2, 3, 4 — 2.5
5, 6, 7 — 6
8, 9, 10 — 9
11 ✓
12 ✓
13 ✓
14 ✓
15 ✓
16 ✓

M



$M_L = n_1(n_1+n_2+1) - M_R$

(get M_R from Table VI)

$M_L = 7(7+9+1) - 78 = 119-78 = 41$

$M_R =$ 78

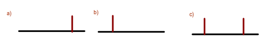
$M_L = 41 < 49.5 < 78 = M_R$

Do NOT reject H_o

Test statistic, 49.5, is not in either tail. Conclude: the ages of the 2 groups is the same.

TABLE VI
Values of M_α

n_2	α	3	4
	0.10	14	20
	0.05	15	21
3	0.025	—	22
	0.01	—	—
	0.005	—	—
	0.10	16	23
	0.05	17	24
4	0.025	18	25
	0.01	—	26
	0.005	—	—
	0.10	18	26
	0.05	20	27
5	0.025	21	28
	0.01	—	30
	0.005	—	—
	0.10	21	29
	0.05	22	30
6	0.025	23	32
	0.01	24	33
	0.005	—	34
	0.10	23	31
	0.05	24	33
7	0.025	26	35
	0.01	27	36
	0.005	—	37
	0.10	25	34
	0.05	27	36
8	0.025	28	38
	0.01	29	39
	0.005	30	40
	0.10	27	37
	0.05	29	39
9	0.025	31	41
	0.01	32	43
	0.005	33	44
	0.10	29	40
	0.05	31	42
10	0.025	33	44
	0.01	34	46
	0.005	36	48



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Statistics Home Page

Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: H_a: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test

Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

Sample 1	overall Rank	Sample 2	overall Rank

TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6	7
	0.10	14	20	27	36	4
	0.05	15	21	29	37	4
3	0.025	—	22	30	38	4
	0.01	—	—	—	39	4
	0.005	—	—	—	—	—
	0.10	16	23	31	40	4
	0.05	17	24	32	41	5
4	0.025	18	25	33	43	5
	0.01	—	26	35	44	5
	0.005	—	—	—	45	5
	0.10	18	26	34	44	5
	0.05	20	27	36	46	5
5	0.025	21	28	37	47	5
	0.01	—	30	39	49	6
	0.005	—	—	40	50	6
	0.10	21	29	38	48	5
	0.05	22	30	40	50	6
6	0.025	23	32	41	52	6
	0.01	24	33	43	54	6
	0.005	—	34	44	55	6
	0.10	23	31	41	52	6
	0.05	24	33	43	54	6
7	0.025	26	35	45	56	6
	0.01	27	36	47	58	7
	0.005	—	37	48	60	7
	0.10	25	34	44	56	6
	0.05	27	36	47	58	7
8	0.025	28	38	49	61	7
	0.01	29	39	51	63	7
	0.005	30	40	52	65	7
	0.10	27	37	48	60	7
	0.05	29	39	50	63	7
9	0.025	31	41	53	65	7
	0.01	32	43	55	68	8
	0.005	33	44	56	70	8
	0.10	29	40	51	64	7
	0.05	31	42	54	67	8
10	0.025	33	44	56	69	8
	0.01	34	46	59	72	8
	0.005	36	48	61	74	8

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

$H_0: \mu_A = \mu_N$
$H_a: \mu_A < \mu_N$

reject
.10

G: H_a: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test

Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

Find test statistic: $M_T = \Sigma$ ranks, population 1

Sample 1	overall Rank	Sample 2	overall Rank
5	3	3	1
5	3	6	5.5
5	3	7	7
6	5.5	8	8
10	9		

1 ✓
2, 3, 4 → 3
5, 6 → 5.5
7 ✓
8 ✓
9 ✓

$M_T = 23.5$

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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10.4 Mann-Whitney Test: n1, n2 small, same shape

G: H_a: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test

Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

$H_0: \mu_A = \mu_N$
$H_a: \mu_A < \mu_N$

reject .10 19 23.5

Sample 1	overall Rank	Sample 2	overall Rank
5	3	3	1
5	3	6	5.5
5	3	7	7
6	5.5	8	8
10	9		

1 ✓
2, 3, 4 → 3
5, 6 → 5.5
7 ✓
8 ✓
9 ✓

M_T = 23.5

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

31

$M_L = 5(5+4+1) - 31 = 19$

M_T =23.5 > M_L =19

Do NOT rej.

TABLE VI
Values of M_α

		n_1					
n_2	α	3	4	5	6	7	8
	0.10	14	20	27	36	45	5
	0.05	15	21	29	37	46	5
3	0.025	—	22	30	38	48	5
	0.01	—	—	—	39	49	5
	0.005	—	—	—	—	—	6
	0.10	16	23	31	40	49	6
	0.05	17	24	32	41	51	6
4	0.025	18	25	33	43	53	6
	0.01	—	26	35	44	54	6
	0.005	—	—	—	45	55	6
	0.10	18	26	34	44	54	6
	0.05	20	27	36	46	56	6
5	0.025	21	28	37	47	58	7
	0.01	—	30	39	49	60	7
	0.005	—	—	40	50	61	7
	0.10	21	29	38	48	59	7
	0.05	22	30	40	50	61	7
6	0.025	23	32	41	52	63	7
	0.01	24	33	43	54	65	7
	0.005	—	34	44	55	67	8
	0.10	23	31	41	52	63	7
	0.05	24	33	43	54	66	7
7	0.025	26	35	45	56	68	8
	0.01	27	36	47	58	71	8
	0.005	—	37	48	60	72	8
	0.10	25	34	44	56	68	8
	0.05	27	36	47	58	71	8
8	0.025	28	38	49	61	73	8
	0.01	29	39	51	63	76	9
	0.005	30	40	52	65	78	9
	0.10	27	37	48	60	72	8
	0.05	29	39	50	63	76	9
9	0.025	31	41	53	65	78	9
	0.01	32	43	55	68	81	9
	0.005	33	44	56	70	84	9
	0.10	29	40	51	64	77	9
	0.05	31	42	54	67	80	9
10	0.025	33	44	56	69	83	9
	0.01	34	46	59	72	87	10
	0.005	36	48	61	74	89	10

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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10.4 Mann-Whitney Test: n1, n2 small, same shape

Avoid left-tailed by switching samples.

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

G: Ha: $\mu_1 > \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test

Sample **2** 5 5 6 5 10
Sample **1** 6 8 7 3

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 > \mu_2$

reject

Find test statistic: $M_T = \Sigma$ ranks, population 1

Sample 1	overall Rank	Sample 2	overall Rank
3	1	5	3
6	5.5	5	3
7	7	5	3
8	8	6	5.5
		10	9

1 ✓
2, 3, 4 → 3
5, 6 → 5.5
7 ✓
8 ✓
9 ✓

M_T = 21.5

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

Avoid left-tailed by switching samples.

$M_L = n_1(n_1+n_2+1) - M_R$ (get M_R from Table VI)

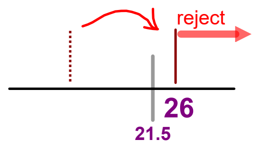
G: Ha: $\mu_1 > \mu_2$ Significance Level = 0.10

F: Mann-Whitney Test

Sample **2**	5	5	6	5	10
Sample **1**	6	8	7	3	

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$



Sample 1	overall Rank	Sample 2	overall Rank
3	1	5	3
6	5.5	5	3
7	7	5	3
8	8	6	5.5
		10	9

1 ✓
2, 3, 4 → 3
5, 6 → 5.5
7 ✓
8 ✓
9 ✓

M_T = 21.5

TABLE VI
Values of M_α

n_2	α	3	4
3	0.10	14	20
	0.05	15	21
	0.025	—	22
	0.01	—	—
	0.005	—	—
4	0.10	16	23
	0.05	17	24
	0.025	18	25
	0.01	—	26
	0.005	—	—
5	0.10	18	26
	0.05	20	27
	0.025	21	28
	0.01	—	30
	0.005	—	—
6	0.10	21	29
	0.05	22	30
	0.025	23	32
	0.01	24	33
	0.005	—	34
	0.10	23	31

Compare and decide:

M_T = 21.5 < M_R = 26

Do NOT rej.

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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10.4 Mann-Whitney Test: n1, n2 small, same shape

G: Independent random samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North	16.2	11.2	24.8	36.4	15.0	23.6	12.1
South	9.8	18.7	26.0	7.4	15.0		

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$

$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

North n=7	Rank	South n=5	Rank

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

Σ ranks of 1st sample: M_t = _____

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Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: Independent random samples of picoplankton from North and South sections of San Francisco Bay. (~ same shape)
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

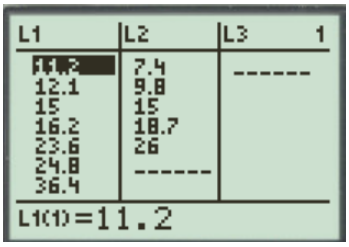
$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025

calc

North n = 7	Rank	South n = 5	Rank

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)



Σ ranks of
1st sample: M_t = _____

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Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

7
5
12

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025

North n = 7	Rank	South n = 5	Rank
11.2		7.4	
12.1		9.8	
15		15	
16.2		18.7	
23.6		26	
24.8			
36.4			

1
2
3
4
5
6
7
8
9
10
11
12

Σ ranks of
1st sample: M_t = ______

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

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10.4 Mann-Whitney Test: n1, n2 small, same shape

7
5
12

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_s$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_s$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025

North $n=7$	Rank	South $n=5$	Rank
11.2	3	7.4	1
12.1	4	9.8	2
15	5.5	15	5.5
16.2	7	18.7	8
23.6	9	26	11
24.8	10		
36.4	12		

1 ✓
2 ✓
3 ✓
4 ✓
5 | 5.5
6 |
7 ✓
8 ✓
9 ✓
10 ✓
11 ✓
12 ✓

Σ ranks of
1st sample: $\mathbf{M_t}$ = ______

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

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10.4 Mann-Whitney Test: n1, n2 small, same shape

7
5
12

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_s$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_s$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025

North $n=7$	Rank	South $n=5$	Rank
11.2	3	7.4	1
12.1	4	9.8	2
15	5.5	15	5.5
16.2	7	18.7	8
23.6	9	26	11
24.8	10		
36.4	12		

		n_1				
n_2	α	3	4	5	6	7
	0.10	14	20	27	36	45
	0.05	15	21	29	37	46
3	0.025	—	22	30	38	48
	0.01	—	—	—	39	49
	0.005	—	—	—	—	—
	0.10	16	23	31	40	49
	0.05	17	24	32	41	51
4	0.025	18	25	33	43	53
	0.01	—	26	35	44	54
	0.005	—	—	—	45	55
	0.10	18	26	34	44	54
	0.05	20	27	36	46	56
5	0.025	21	28	37	47	58
	0.01	—	30	39	49	60
	0.005	—	—	40	50	61

Σ ranks of
1st sample: $\mathbf{M_t}$ = 50.5

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

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∴ Do Not rej. H_0.
Concl. There is no diff. in phytoplankton concentr. in N and S

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

.025 | 33 ——— 58 | .025

North n = 7	Rank	South n = 5	Rank
11.2	3	7.4	1
12.1	4	9.8	2
15	5.5	15	5.5
16.2	7	18.7	8
23.6	9	26	11
24.8	10		
36.4	12		

1 ✓ 2 ✓ 3 ✓ 4 ✓ 5 | 5. 6 | 7 ✓ 8 ✓ 9 ✓ 10 ✓ 11 ✓ 12 ✓

Σ ranks of 1st sample: M_t = 50.5

n_2	α	n_1 = 3	4	5	6	7
3	0.10	14	20	27	36	45
	0.05	15	21	29	37	46
	0.025	—	22	30	38	48
	0.01	—	—	—	39	49
	0.005	—	—	—	—	—
4	0.10	16	23	31	40	49
	0.05	17	24	32	41	51
	0.025	18	25	33	43	53
	0.01	—	26	35	44	54
	0.005	—	—	—	45	55
5	0.10	18	26	34	44	54
	0.05	20	27	36	46	56
	0.025	21	28	37	47	58
	0.01	—	30	39	49	60
	0.005	—	—	40	50	61

$M_L = n_1(n_1+n_2+1) - M_R$
(get M_R from Table VI)

$M_L = 7(7+5+1) - 58$
$= 91-58 = 33$

$M_L=33 < M_T=50.5 < M_R = 58$

Do NOT reject. Conclude: No difference in picoplankton concentrations at two sites.

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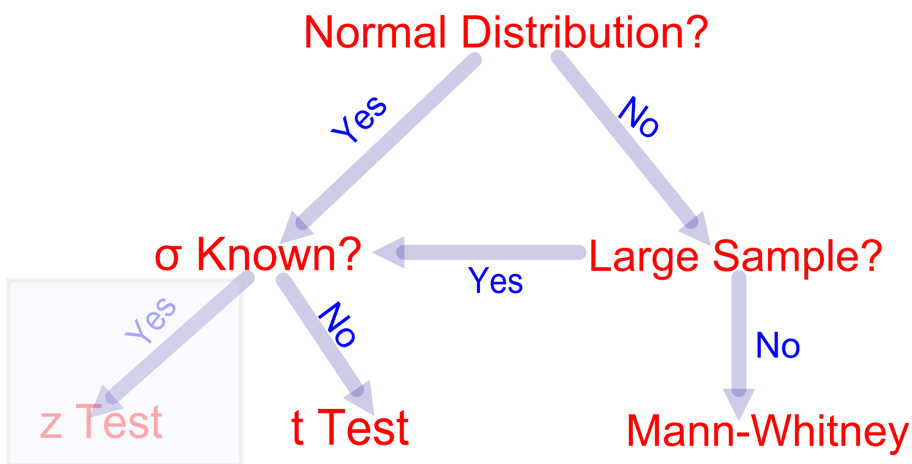
10.4 Mann-Whitney Test: n1, n2 small, same shape



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10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $\alpha = 0.10$ F: Does % of S exceed % of H

S	H
17	10.1
21	11.7
25	15.2
25	15.6
27	23.9
27	30.3
38	41.1
39	
40	

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

$H_0: \mu_S = \mu_H$

$H_a: \mu_S > \mu_H$

$M_e = 89$

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10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $\alpha = 0.10$ F: Does % of S exceed % of H

S	rank	H	rank
17	5	10.1	1
21	6	11.7	2
25	8.5	15.2	3
25	8.5	15.6	4
27	10.5	23.9	7
27	10.5	30.3	12
38	13	41.1	16
39	14		
40	15		
Σ	$91.0 = M_T$		

$H_0: \mu_S = \mu_H$

$H_a: \mu_S > \mu_H$

M_T

$M_e = 89$

$M_T = 91.0 > 89 = M_R$

$\therefore$ rej. H_0 conc. that % S exceeds % H

8
9 8.5
10
11

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TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6	7	8	9	10
3	0.10	14	20	27	36	45	55	66	78
	0.05	15	21	29	37	46	57	68	80
	0.025	—	22	30	38	48	58	70	82
	0.01	—	—	—	39	49	59	71	83
	0.005	—	—	—	—	—	60	72	85
4	0.10	16	23	31	40	49	60	72	85
	0.05	17	24	32	41	51	62	74	87
	0.025	18	25	33	43	53	64	76	89
	0.01	—	26	35	44	54	65	78	91
	0.005	—	—	—	45	55	66	79	93
5	0.10	18	26	34	44	54	65	78	91
	0.05	20	27	36	46	56	68	80	94
	0.025	21	28	37	47	58	70	83	96
	0.01	—	30	39	49	60	72	85	99
	0.005	—	—	40	50	61	73	86	101
6	0.10	21	29	38	48	59	71	84	98
	0.05	22	30	40	50	61	73	87	101
	0.025	23	32	41	52	63	76	89	103
	0.01	24	33	43	54	65	78	92	106
	0.005	—	34	44	55	67	80	94	108
7	0.10	23	31	41	52	63	76	89	104
	0.05	24	33	43	54	66	79	93	107
	0.025	26	35	45	56	68	81	95	110
	0.01	27	36	47	58	71	84	98	114
	0.005	—	37	48	60	72	86	101	116
8	0.10	25	34	44	56	68	81	95	110
	0.05	27	36	47	58	71	84	99	114
	0.025	28	38	49	61	73	87	102	117
	0.01	29	39	51	63	76	90	105	121
	0.005	30	40	52	65	78	92	108	124
9	0.10	27	37	48	60	72	86	101	116
	0.05	29	39	50	63	76	90	105	121
	0.025	31	41	53	65	78	93	108	124
	0.01	32	43	55	68	81	96	112	129
	0.005	33	44	56	70	84	99	114	131
10	0.10	29	40	51	64	77	91	106	123
	0.05	31	42	54	67	80	95	111	127
	0.025	33	44	56	69	83	98	114	131
	0.01	34	46	59	72	87	102	119	136
	0.005	36	48	61	74	89	105	121	139

TABLE VI
Values of M_α

n_2	α	n_1 = 3	4	5	6	7	8	9
	0.10	14	20	27	36	45	55	66
	0.05	15	21	29	37	46	57	68
3	0.025	—	22	30	38	48	58	70
	0.01	—	—	—	39	49	59	71
	0.005	—	—	—	—	—	60	72
	0.10	16	23	31	40	49	60	72
	0.05	17	24	32	41	51	62	74
4	0.025	18	25	33	43	53	64	76
	0.01	—	26	35	44	54	65	78
	0.005	—	—	—	45	55	66	79
	0.10	18	26	34	44	54	65	78
	0.05	20	27	36	46	56	68	80
5	0.025	21	28	37	47	58	70	83
	0.01	—	30	39	49	60	72	85
	0.005	—	—	40	50	61	73	86
	0.10	21	29	38	48	59	71	84
	0.05	22	30	40	50	61	73	87
6	0.025	23	32	41	52	63	76	89
	0.01	24	33	43	54	65	78	92
	0.005	—	34	44	55	67	80	94
	0.10	23	31	41	52	63	76	89
	0.05	24	33	43	54	66	79	93
7	0.025	26	35	45	56	68	81	95
	0.01	27	36	47	58	71	84	98
	0.005	—	37	48	60	72	86	101
	0.10	25	34	44	56	68	81	95
	0.05	27	36	47	58	71	84	99
8	0.025	28	38	49	61	73	87	102
	0.01	29	39	51	63	76	90	105
	0.005	30	40	52	65	78	92	108
	0.10	27	37	48	60	72	86	101
	0.05	29	39	50	63	76	90	105
9	0.025	31	41	53	65	78	93	108
	0.01	32	43	55	68	81	96	112
	0.005	33	44	56	70	84	99	114
	0.10	29	40	51	64	77	91	106
	0.05	31	42	54	67	80	95	111
10	0.025	33	44	56	69	83	98	114
	0.01	34	46	59	72	87	102	119
	0.005	36	48	61	74	89	105	121

Class Notes: Prof. G. Battaly, Westchester Community College, NY
Statistics Home Page Class Notes Homework

10.4 Mann-Whitney Test: n1, n2 small, same shape

Number of days of ice on lake

At 10% s.l. $\alpha = 0.10$ is there a diff. in # Days of Ice on the 2 Lakes?

Me	rank	Mo	rank
47	1.5	47	1.5
53	4	52	3
74	5	83	6
87	7	91	8
102	10	96	9
108	13	107	11
115	15	108	13
119	16	108	13
	71.5		

Days: 1, 2 (1.5); 3, 4, 5, 6, 7, 8, 9, 10, 11; 12, 13, 14 (13); 15, 16

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

0.05 | M_L = 52 ... 71.5 ... M_R = 84 | 0.05

$M_T = 71.5$

$M_L = n_1(n_1 + n_2 + 1) - M_R$
$= 8(8+8+1) - 84 =$
$136 - 84 = 52$

$52 < 71.5 < 84$

DO NOT rej.

Concl.: there is no diff. in # days of ice on the 2 lakes.