Further Maths A-Level Starter Activity

Jethwa Maths

Topic: Volumes of Revolution around the Y-axis (1) Chapter Reference: Core Pure 1, Chapter 5	10 minutes

1. Fig. 1 shows the curve $y = \sqrt{1 + e^{2x}}$.

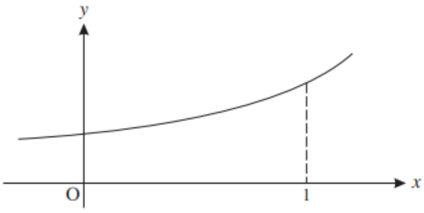

Fig. 1

The region bounded by the curve, the x-axis, the y-axis and the line $x = 1$ is rotated through 360° about the x-axis. Show that the volume of the solid of revolution produced is $\frac{1}{2}\pi(1 + e^2)$. **(4)**

2. Fig. 2 shows a sketch of the region enclosed by the curve $1 + e^{-2x}$, the x-axis, the y-axis and the line $x = 1$.

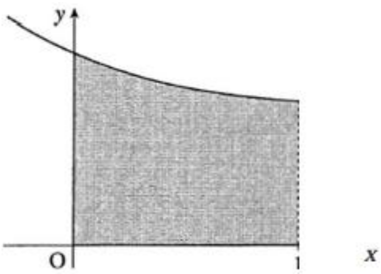

Fig. 2

Find the volume of the solid generated when this region is rotated through 360° about the x-axis. Give your answer in an exact form. **(5)**

Jethwa Maths

Solutions

1.

$V=\int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x})dx$	**M1**
$=\pi\left[x+\frac{1}{2}e^{2x}\right]_0^1$	**B1**
$=\pi(1+\frac{1}{2}e^2-\frac{1}{2})$	**M1**
$=\frac{1}{2}\pi(1+e^2)$ *	**E1**

2.

$V=\int \pi y^2 dx$	**M1**
$=\int_0^1 \pi(1+e^{-2x})dx$	**M1**
$=\pi\left[x-\frac{1}{2}e^{-2x}\right]_0^1$	**B1**
$=\pi(1-½e^{-2}+½)$	**M1**
$=\pi(1½-½e^{-2})$	**A1**

Further Maths A-Level Starter Activity

Topic: Volumes of Revolution around the Y-axis (2)

Chapter Reference: Core Pure 1, Chapter 5

10 minutes

1. Fig. 1 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

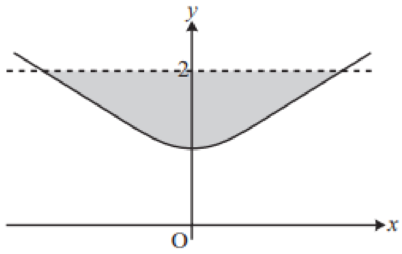

Fig. 1

This region is rotated about the y-axis. Find the volume of revolution formed, giving your answer as a multiple of π. **(6)**

Solutions

1.

$y=(1+2x^2)^{\frac{1}{3}} \Rightarrow y^3 = 1+2x^2$ $\Rightarrow x^2 = \frac{1}{2}(y^3-1)$	**M1**
$V = \int_1^2 \pi x^2 \, dy = \frac{1}{2}\pi\int_1^2 (y^3-1)\, dy$	**M1**
www For A1 it must be correct with correct limits 1 and 2, but they may appear later	**A1**
$1/2[y^4/4 - y]$ independent of π and limits	**B1**
$= \frac{1}{2}\pi\left[\frac{1}{4}y^4 - y\right]_1^2 = \frac{1}{2}\pi(2+\frac{3}{4})$	**M1**
$= \frac{11}{8}\pi$	**A1**

Further Maths A-Level Starter Activity

Topic: Adding and Subtracting Volumes (3)

Chapter Reference: Core Pure 1, Chapter 5

10 minutes

1. The region R is bounded by the curve with equation $y = x^3 - 3$, the line $y = -2x$ and the x-axis. The two lines intercept at point A.

a. Verify that the coordinates of A are (1, -2). **(2)**

A solid is created by rotating the region 360° about the x-axis.

b. Find the volume of this solid. Give your answer to three significant figures. **(6)**

Solutions

1a.

$-2x = x^3 - 3$	**M1**
Obtains (1, -2) using any valid method	**A1**

1b.

Attempt to split, find V_1 and V_2	**M1**
Use of correct limits [1, 1.442…]	**M1**
A volume of $\frac{4}{3}\pi$ obtained	**A1**
A volume of 1.5552… obtained	**A1**
$V_1 + V_2 = \frac{4}{3}\pi + 1.5552 \ldots = 5.74$ (award one mark if not 3 s.f.)	**A2**

Topic: Modelling with Volumes of Revolution (4)

Chapter Reference: Core Pure 1, Chapter 5

10 minutes

1. A bowl has a height of 5cm. The shape of the bowl is modelled by rotating the curve with equation $y = 0.05x^2$ through 2π radians about the y-axis.

a. Find the diameter of the bowl. **(2)**

b. Find the maximum volume of liquid that can be contained within the bowl. **(4)**

Solutions

1a.

Equates $y = 5$ and $y = 0.05x^2$	**M1**
$= 20$	**A1**

1b.

Attempts to integrate	**M1**
Uses limits of $y = 0$ and $y = 5$	**A1**
250π (award one mark if π omitted)	**A2**

Further Maths A-Level Starter Activity

Topic: Modelling with Volumes of Revolution (5) Chapter Reference: Core Pure 1, Chapter 5	**10 minutes**

1. The region R is the area between the curve $x = y^2 - 6y + 10$, the lines $y = 1$, $y = 4$ and the y-axis.

a. Find the area of the region R. **(3)**

A ceiling lampshade is modelled by rotating region R through 360° about the y-axis.

b. Use integration to find an exact value for the volume of the lampshade. **(5)**

Solutions

1a.

Attempts integration wrt y	**M1**
Uses limits of $y = 1$ and $y = 4$	**A1**
$= 6$	**A1**

1b.

Attempts integration	**M1**
Uses limits of $y = 1$ and $y = 4$	**A1**
$\pi\left(\left(\frac{1024}{5}+\frac{3584}{3}-1328\right)-\left(\frac{1}{5}+\frac{56}{3}+37\right)\right)$ (or correct integration)	**M1**
$= \frac{78}{5}\pi$ (award one mark if π omitted)	**A2**