

10. CONSOLIDATION

10.1 INFLUENCE OF DRAINAGE ON RATE OF SETTLEMENT

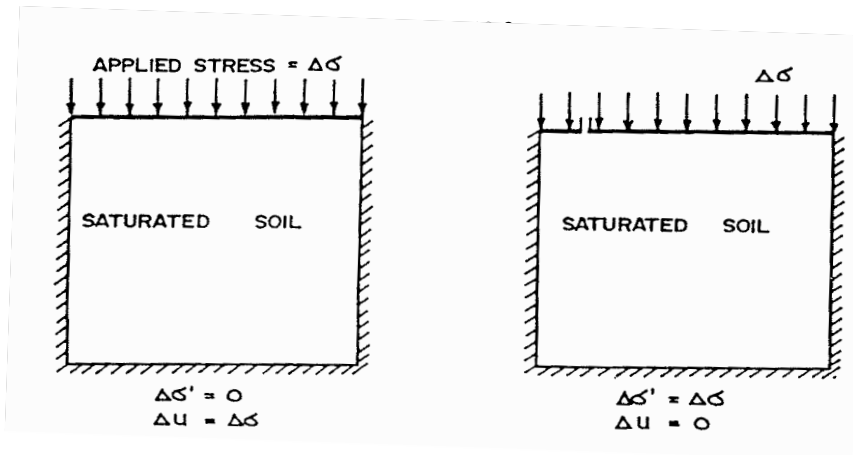
When a saturated stratum of sandy soil is subjected to a stress increase, such as that caused by the erection of a building on the ground surface, the pore water pressure is increased. This increase in pore pressure leads to drainage of some water from the voids of the soil. Because of the relatively high permeability of the sandy soil this drainage process will occur quite quickly. In other words the pore pressure increase will dissipate rapidly. As a consequence of the drainage of some water from the soil, volume change will occur and settlement will take place.

When a saturated stratum of clayey soil is subjected to a stress increase, the dissipation of the excess pore pressure generated will take place much more slowly because of the relatively low permeability of the clayey soil. This means that the settlement, caused by the drainage of some water from the voids of the soil, will take place gradually over a long period of time.

Fig. 10.1(a) represents a rigid but smooth walled container which is filled with saturated soil. The container is sealed by means of a membrane covering the upper surface of the soil. A uniform pressure of $\Delta\sigma$ is applied to the top of the soil. Since the soil is saturated and the container is rigid no settlement of the soil will be observed. If the pore pressure change within the soil was observed it would be found to equal the applied stress $\Delta\sigma$. Since the applied (total) stress and the pore pressure both increase by equal amounts, there will be no change in effective stress. The absence of any observed settlement is therefore consistent with the principle of effective stress, which requires that volume change will occur only as a result of an effective stress change.

In Fig. 10.1(b) an opening has been provided in the membrane to enable water to be expelled or drained from the container of soil. Under the effect of the increase in pore pressure $\Delta u (= \Delta\sigma)$, water will be expelled from the soil and this drainage of water will continue until the water pressure decreases to the equilibrium value prevailing before the stress change of $\Delta\sigma$ was applied to the soil. This means that the pore pressure change finally will be zero. Since the total stress has increased by $\Delta\sigma$ the effective stress will also increase by $\Delta\sigma$. In response to this effective stress change, settlement of the soil will occur, the amount depending upon the compressibility of the soil.

These observations illustrate that in a one dimensional compression situation for a saturated soil, settlement of the soil in response to an applied stress occurs only when water is allowed to be expelled from the soil.



(a) Sample sealed, drainage prevented (b) drainage permitted

Fig.10.1 Influence of Drainage upon stress changes

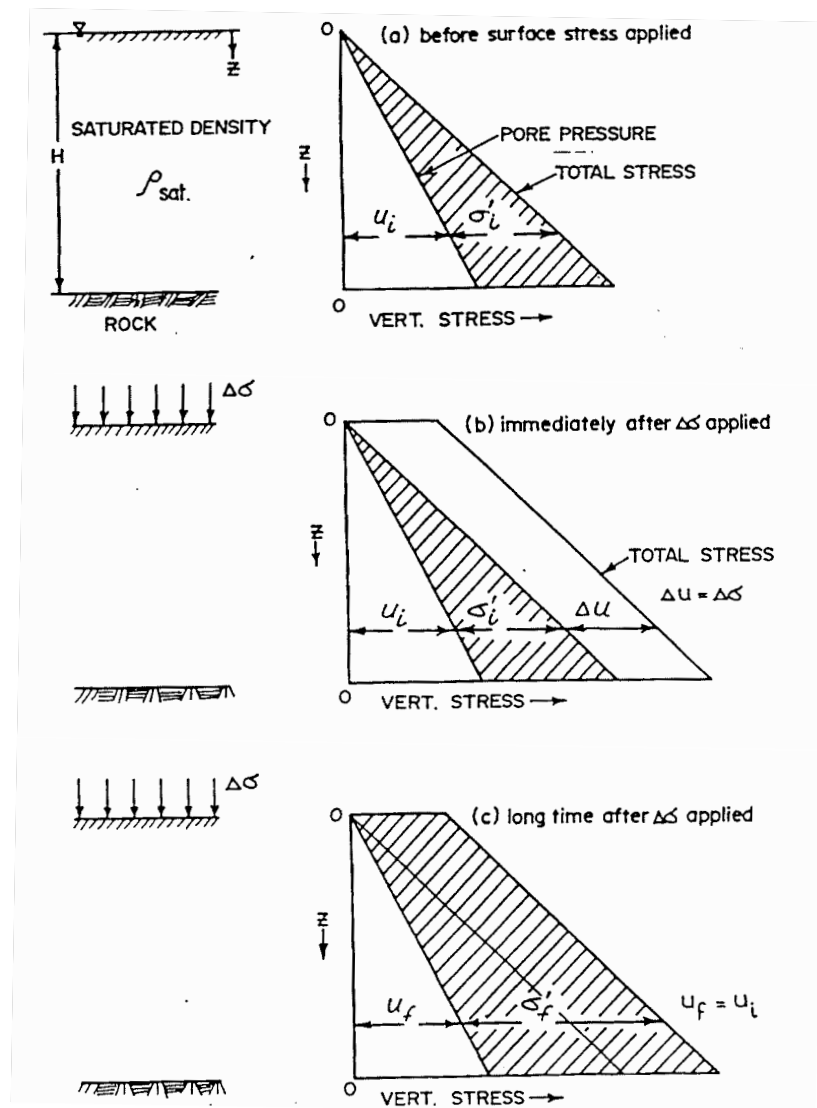


Fig 10.2 Vertical stress changes during consolidation

The stress changes throughout the depth of a soil layer in a one dimensional field situation are illustrated in Fig. 10.2. The initial conditions are represented in Fig. 10.2(a). Since the water table

is coincident with the ground surface the initial pore pressure u_i at any depth z below the ground surface is

$$u_i = \rho_w g z$$

the initial total vertical stress σ_i is

$$\sigma_i = \rho_{\text{sat}} g z$$

and the initial effective vertical stress σ'_i is

$$\begin{aligned} \sigma'_i &= \sigma_i - u_i \\ &= \rho_{\text{sat}} g z - \rho_w g z \\ &= \rho_b g z \end{aligned}$$

where ρ_b is the buoyant density of the soil. The distribution of this effective stress throughout the depth of soil is shown by the hatched area.

In Fig. 10.2(b) a stress of $\Delta\sigma$ has been applied over the ground surface. The stress diagram has been drawn for the instant following the application of load before any water has been expelled from the soil. The pore pressure will increase by an amount equal to the applied stress as in the case of Fig. 10.1(a). The pore pressure u at any depth z below the ground surface is

$$\begin{aligned} u &= u_i + \Delta u \\ &= \rho_w g z + \Delta\sigma \end{aligned}$$

and the effective stress is the same as that before the load application.

Under the effect of the additional (excess hydrostatic) pore pressure, water will be expelled from the soil. Water will be expelled through the upper boundary of the soil and, if the underlying rock is pervious, through the lower boundary as well. This drainage of water will continue until the pore pressure distribution coincides with that which existed before the surface stress was applied as shown in Fig. 10.2(c). This means that the stress $\Delta\sigma$ which was originally carried as a pore pressure Δu has now been transferred to effective stress. The final effective stress σ'_f at any depth z is

$$\begin{aligned} \sigma'_f &= \sigma'_i + \Delta\sigma \\ &= \rho_b g z + \Delta\sigma \end{aligned}$$

As a result of the increase $\Delta\sigma$ in effective stress the soil will undergo a volume decrease as a consequence of the expulsion of water from the soil and a time dependent settlement of the ground surface would be observed. The process of gradual transfer of stress from the pore pressure to effective stress with the associated volume change is referred to as consolidation. The rate at which the settlement occurs depends upon the rate at which water is expelled from the soil and this depends upon the total head gradient and the permeability of the soil.

6.2 USE OF A RHEOLOGICAL MODEL

An understanding of the time dependent nature of the settlement for a consolidating soil may be assisted by considering the consolidation process a rheological model. A simple model that is often used is the Kelvin model (Fig. 10.3) which consists of a linear spring and a dashpot in parallel. The spring constant (\bar{E}) and the dashpot constant (η) are defined as follows

$$\sigma_s = \bar{E} \varepsilon \quad (10.1)$$

$$\sigma_D = \eta (d\varepsilon/dt) \quad (10.2)$$

where

σ_s	=	stress in the spring
σ_D	=	stress in the dashpot
ε	=	strain
t	=	time

If a stress ($\Delta\sigma$) is applied to the model and remains constant

$$\begin{aligned} \Delta\sigma &= \sigma_s + \sigma_D \\ &= \bar{E} \varepsilon + \eta (d\varepsilon/dt) \end{aligned} \quad (10.3)$$

Assuming that the strain is zero at time zero, the solution to this equation is

$$\varepsilon = (\Delta\sigma/\bar{E}) (1 - e^{-(\bar{E}/\eta)t}) \quad (10.4)$$

which demonstrates the time dependency of the strain.

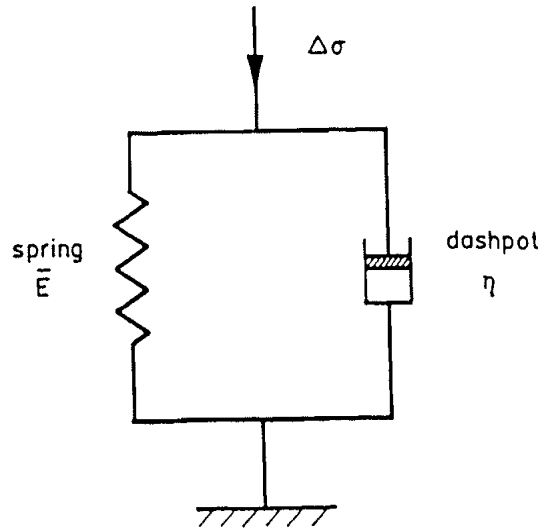


Fig. 10.3 Kelvin Model

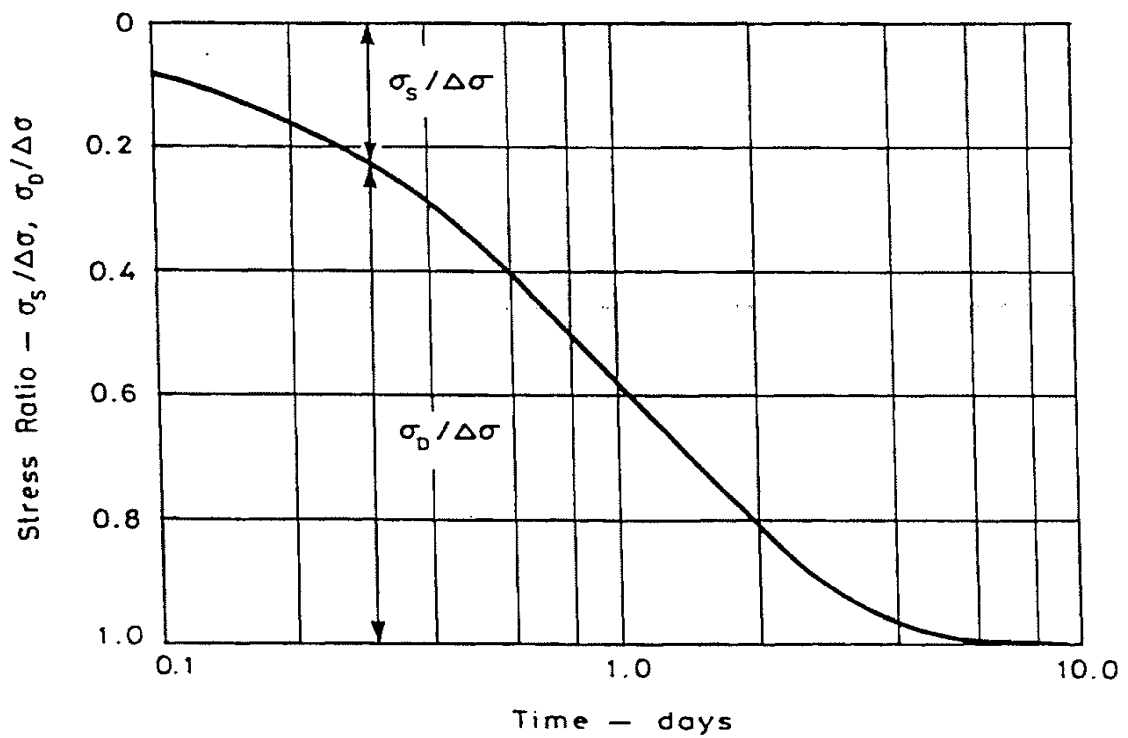


Fig. 10.4

In the analogy provided by use of the Kelvin model the stress in the spring (σ_s) can be interpreted as effective stress in the soil and the stress in the dashpot (σ_D) may be interpreted as the pore water pressure.

EXAMPLE

In a Kelvin model evaluate the stresses in the spring and in the dashpot (as proportions of the applied stress) as a function of time. The spring constant (\bar{E}) is 1 MPa and the dashpot constant (η) is 10^{11} Ns/m².

From equations (10.4) and (10.1)

$$\sigma_s = \bar{E} e = \Delta\sigma (1 - e^{-(\bar{E}/\eta)t})$$

and from equation (10.3)

$$\sigma_D = \Delta\sigma - \sigma_s = \Delta\sigma e^{-(\bar{E}/\eta)t}$$

Expressing σ_s and σ_D as proportions of $\Delta\sigma$

$$\sigma_s / \Delta\sigma = 1 - e^{-(\bar{E}/\eta)t} \quad (10.5)$$

$$\sigma_D / \Delta\sigma = e^{-(\bar{E}/\eta)t} \quad (10.6)$$

$$= 1 - (\sigma_s / \Delta\sigma)$$

The time variations in σ_s and σ_D may be found from equations (10.5) and (10.6) following substitution for the given values of \bar{E} and η . The single curve representing variations in both stresses has been plotted in Fig. 10.4.

10.3 CONSOLIDATION AS A SEEPAGE PROBLEM

The seepage of water from the soil during consolidation may be represented by means of a head diagram of the type shown in Fig. 5.5. The problem will be illustrated for the situation shown in Fig. 10.5., in which a compressible clay is sandwiched between two relatively incompressible sand layers, the water table being at the ground surface.

Before the application of the surface pressure a hydrostatic pore pressure distribution prevails throughout the water in the voids of the soils. In other words the pressure head line is represented by line ACFB. The elevation head line from the arbitrarily chosen datum is given by line GH. The total head line is therefore HIKB. Since the total head has a constant value (equal to GB) throughout the depth of soil considered, no seepage will take place.

As discussed in Section 10.1 immediately following the application of a wide surface pressure $\Delta\sigma$ to the ground surface the pore pressure in the saturated clay will rise by an amount Δu where

$$\Delta u = \Delta\sigma$$

In other words the pressure head in the clay will increase by an amount $(\Delta u/\rho_w g)$. Because of the relatively high permeability of the sand the pore pressure increase will dissipate very rapidly in the sand. The pressure in the sand will be transferred to effective stress almost immediately. For this reason no pressure head increase has been drawn for the pore water in the sand. The new pressure head diagram is now represented by ACDEFB.

If the elevation head is added to the pressure head the total head line HIJLKB is obtained. This line shows that there are sudden changes in total head at the upper and lower boundaries of the clay layer. Because of the very large total head gradients (theoretically infinity) at these locations water will be expelled from the clay into the sand layers. This expulsion of water commences at the two boundaries of the clay layer and works progressively in towards the centre of the clay. As the water is expelled the excess hydrostatic pore pressure decreases, the total head decreases, the total head gradient decreases and consequently the rate at which the water is expelled decreases. This unsteady seepage situation is represented by the total head lines for various times which are drawn for the upper portion of the clay in the IJK portion of the diagram in Fig. 10.5. The total head line gradually approaches line IK as time elapses and finally coincides with it as the total head gradient becomes zero and the expulsion of water ceases. At this stage the original excess hydrostatic pore pressure (Δu) has been fully transferred to effective stress.

The rate at which this process of consolidation proceeds depends upon a number of factors such as the soil properties, the layer thickness and the boundary conditions. These are examined qualitatively in Section 10.4 and quantitatively in Section 10.5.

10.4 FACTORS AFFECTING THE RATE OF CONSOLIDATION

10.4.1 Permeability

An increase in permeability of the consolidating soil would lead to an increase in the rate of seepage flow, other factors remaining constant. With the greater rate of expulsion of

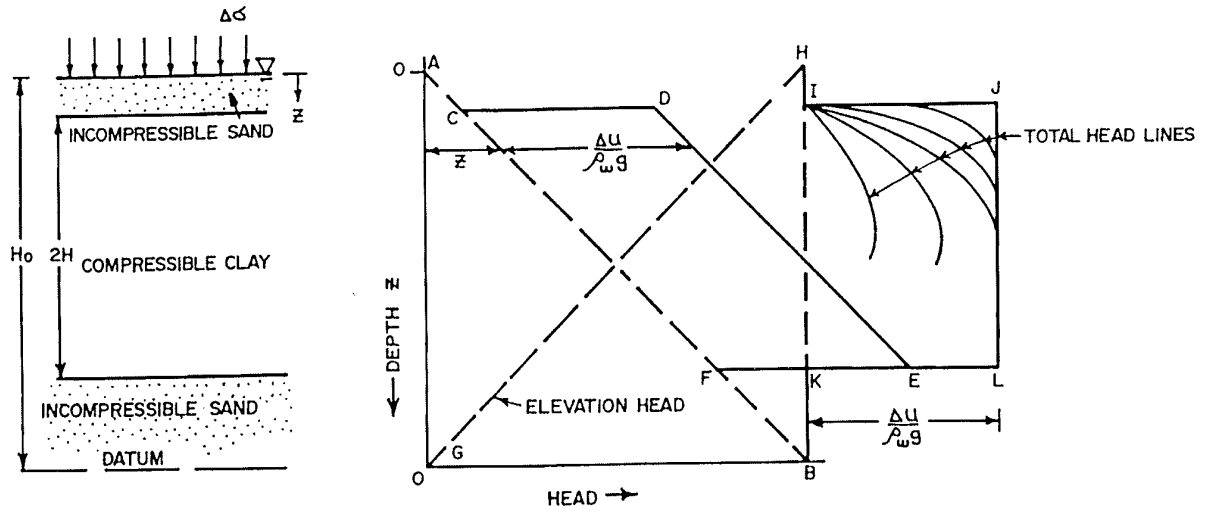
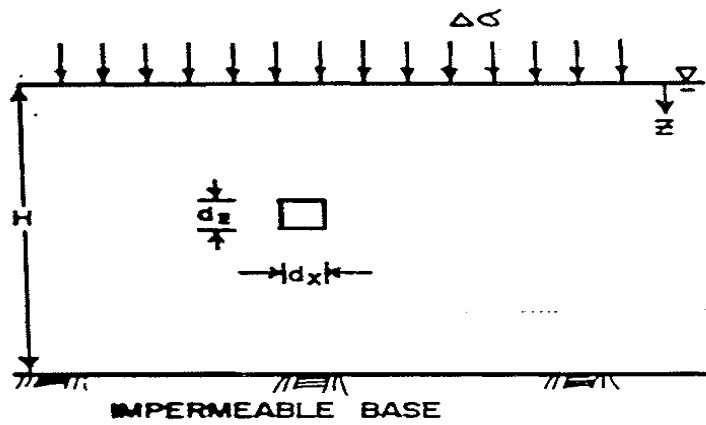
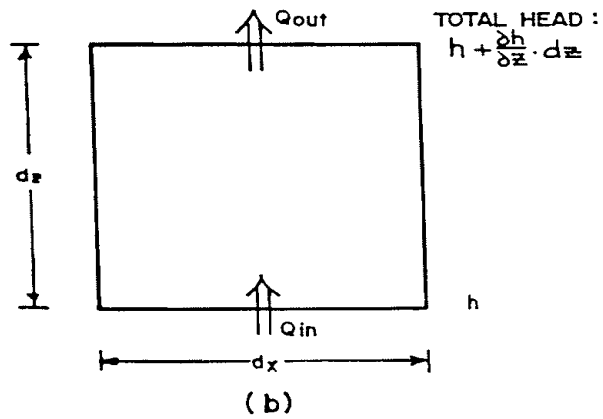


Fig. 10.5 Head Diagram for Consolidation



(a)



(b)

Fig. 10.6 Element of Soil in a Consolidating Layer

water from the soil the pore pressures will dissipate more rapidly. This means that a more rapid rate of consolidation occurs.

10.4.2 Compressibility

A greater compressibility leads to a greater decrease in the void space of the soil for a particular stress change. This means that a greater volume of water must be expelled from the soil and this will require a longer time. Consequently a lower rate of consolidation will result.

10.4.3 Layer Thickness

An increase in the layer thickness leads to a decrease in the total head gradient during the stage of pore water expulsion. It also means an increase in the volume of water to be expelled and both of these effects lead to a lower rate of consolidation.

10.4.4 Boundary Conditions

The presence of drainage boundaries through which water may be expelled has a significant effect on the rate of consolidation. If drainage layers exist on both sides of a consolidating layer (doubly drained) the rate of expulsion of water will be greater than in the case where one drainage layer only exists, the other side being an impermeable layer (singly drained). Consequently, a consolidating layer which is doubly drained will consolidate at a faster rate than one which is singly drained.

10.5 TERZAGHI THEORY OF ONE DIMENSIONAL CONSOLIDATION

A layer of soil undergoing consolidation is represented in Fig. 10.6(a). The soil is underlain by an impermeable base so in this case the flow of water is in the upward direction towards the drainage boundary at the ground surface. An element of soil measuring dx , dy , dz has been selected for development of the consolidation equation and this element is enlarged in Fig. 10.6(b).

The rate of water flow into the element is indicated by Q_{in} and the rate of flow out of the element is indicated by Q_{out} . Since the element is decreasing in volume during consolidation, Q_{in} and Q_{out} will not be equal.

The ratio of flow in and out of the element will be given by the Darcy equation (see section 5.1).

$$Q_{in} = k i A \quad (5.4)$$

$$\begin{aligned}
 &= k \frac{\partial h}{\partial z} dx dy \\
 \text{and } Q_{\text{out}} &= k \frac{\partial}{\partial z} \left(h + \frac{\partial h}{\partial z} dz \right) dx dy \\
 &= k \left(\frac{\partial h}{\partial z} + \frac{\partial^2 h}{\partial z^2} dz \right) dx dy
 \end{aligned}$$

$$\begin{aligned}
 \text{Rate of storage of water} &= Q_{\text{in}} - Q_{\text{out}} \\
 &= -k \frac{\partial^2 h}{\partial z^2} dx dy dz \quad (10.7)
 \end{aligned}$$

$$\text{Now volume of element} = dx dy dz$$

$$\text{Pore volume} = dx dy dz \frac{e}{1+e} = dx dy dz n$$

where e indicates the void ratio, and n is the porosity of the soil

$$\text{Rate of change of pore volume} = - dx dy dz \frac{\partial n}{\partial t} \quad (10.8)$$

In order to satisfy continuity the rate of change of pore volume must equal the rate of storage of water. Equating these two rates as given by equations (10.7) and (10.8).

$$k \frac{\partial^2 h}{\partial z^2} dx dy dz = dx dy dz \frac{\partial n}{\partial t}$$

$$\text{or } k \frac{\partial^2 h}{\partial z^2} = \frac{\partial n}{\partial t} \quad (10.9)$$

$$\begin{aligned}
 \text{Now } \frac{\partial n}{\partial t} &= \frac{\partial n}{\partial \sigma'} \frac{\partial \sigma'}{\partial t} \\
 &= - m_v \frac{\partial \sigma'}{\partial t} \text{ from equation (9.12)} \quad (10.10)
 \end{aligned}$$

where m_v is the one dimensional compressibility.

Since $\sigma' = \sigma - (u_i + u)$

where u_i is the hydrostatic pore water pressure which does not vary with time

u is the excess hydrostatic pore water pressure which varies with time.

$$\frac{\partial \sigma'}{\partial t} = \frac{\partial \sigma}{\partial t} - \frac{\partial u}{\partial t}$$

and substituting into equation (10.10)

$$\frac{\partial n}{\partial t} = m_v \left(\frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t} \right)$$

and substituting into equation (10.9)

$$k \frac{\partial^2 h}{\partial z^2} = m_v \left(\frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t} \right) \quad (10.11)$$

The total head h is the sum of the elevation and pressure heads

$$\begin{aligned} h &= h_e + h_p \\ &= h_e + \left(\frac{u_i}{\rho_w g} + \frac{u}{\rho_w g} \right) \end{aligned}$$

in which u_i (hydrostatic pore water pressure) varies linearly with z

u (excess hydrostatic pore water pressure) varies non-linearly with z

Therefore
$$\frac{\partial^2 h}{\partial z^2} = \frac{1}{\rho_w g} \frac{\partial^2 u}{\partial z^2}$$

Substituting into equation (10.11)

$$\frac{k}{\rho_w g m_v} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t}$$

or
$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t} \quad (10.12)$$

$$\text{where } c_v = \frac{k}{\rho_w g m_v} \quad (10.13)$$

c_v is the coefficient of consolidation and is a measure of the rate at which the consolidation process proceeds.

In many consolidation problems in which the total stress σ remains constant throughout consolidation, equation (10.12) simplifies to

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad (10.14)$$

This is the basic differential equation for one dimensional consolidation which was developed by Terzaghi (1943). The solutions to equation (10.14) for various boundary conditions have been described in detail by Taylor (1948). The solution for the case of a constant initial excess hydrostatic pore pressure (Δu) may be expressed as follows.

$$u = \sum_{m=0}^{m=\infty} \frac{2 \Delta u}{M} (\sin (Mz/H)) e^{-M^2 T} \quad (10.15)$$

where u = pore pressure (excess hydrostatic) at particular values of depth (z) and time (t)

Δu = initial value of excess hydrostatic pore pressure

M = $(\pi/2) (2 m + 1)$

m = integer

H = thickness of a singly drained layer

T = dimensionless time factor

= $c_v t/H^2$

The progress of consolidation is usually indicated by a variable known as the degree of consolidation (U) and this is defined as follows

$$U(z,t) = 1 - (u/\Delta u) \quad (10.16)$$

In terms of the degree of consolidation, the solution to the differential equation (10.14) becomes

$$U(z,t) = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M} (\sin(Mz/H)) e^{-M^2 T} \quad (10.17)$$

Equation (10.17) is illustrated graphically in Fig. 10.7, in which the lines of equal time factor are known as isochrones. These lines represent the degrees of consolidation at particular times and at particular locations throughout the thickness of the consolidating layer. For example, at point P at a time corresponding to a time factor T of 0.4 the pore pressure (excess hydrostatic) has dissipated to 40% of the initial value which is the same as saying that the degree of consolidation is 0.6.

Fig. 10.7 shows that the soil adjacent to the drainage boundary consolidates quickly whereas the soil adjacent to the impermeable boundary consolidates much more slowly.

For a soil layer that is drained at both the upper and lower boundaries the value of H must be made equal to one half of the total layer thickness. The isochrones in Fig. 10.7 would apply to the upper half of the layer and their mirror images would apply to the lower half.

A comparison of the total head lines in Fig. 10.5 with the isochrones of Fig. 10.7 shows that they are identical in the sense that they both represent the distribution of excess hydrostatic pore pressure throughout the thickness of the layer at various times.

EXAMPLE

A 10m thick submerged clay layer which is drained at both the upper and lower boundaries is subjected to a wide surface pressure of 50kN/m². The water table is coincident with the top of the clay layer at the ground surface. If the coefficient of consolidation of the clay is 1.16 x 10⁻² cm²/sec determine the pore pressure at the mid depth of the layer 50 days after the surface pressure was applied.

For this problem the value of H will be equal to half of the overall layer thickness.

$$H = 5\text{m}$$

From the information provided the dimensionless time factor may be calculated

$$\begin{aligned}
 T &= \frac{c_v t}{H^2} \\
 &= \frac{1.16 \times 10^{-2} \times 50 \times 24 \times 3600}{500^2} \\
 &= 0.2
 \end{aligned}$$

The mid depth of this clay layer is represented by the value of (z/H) of 1.0 in Fig. 10.7. For a time factor of 0.2 the degree of consolidation is 0.23. That is

$$1 - \frac{u}{\Delta u} = 0.23$$

$$\therefore \frac{u}{\Delta u} = 0.77$$

$$\begin{aligned}
 u &= 0.77 \times \Delta u \\
 &= 0.77 \times 50 \\
 &= 38.5 \text{ kN/m}^2
 \end{aligned}$$

This is the excess hydrostatic pore pressure, the total pore pressure being found by the addition of the hydrostatic pore pressure.

$$\begin{aligned}
 \text{total pore pressure} &= \rho_w g z + u \\
 &= 1.0 \times 9.81 \times 5 + 38.5 \\
 &= 49.0 + 38.5 \\
 &= 87.5 \text{ kN/m}^2
 \end{aligned}$$

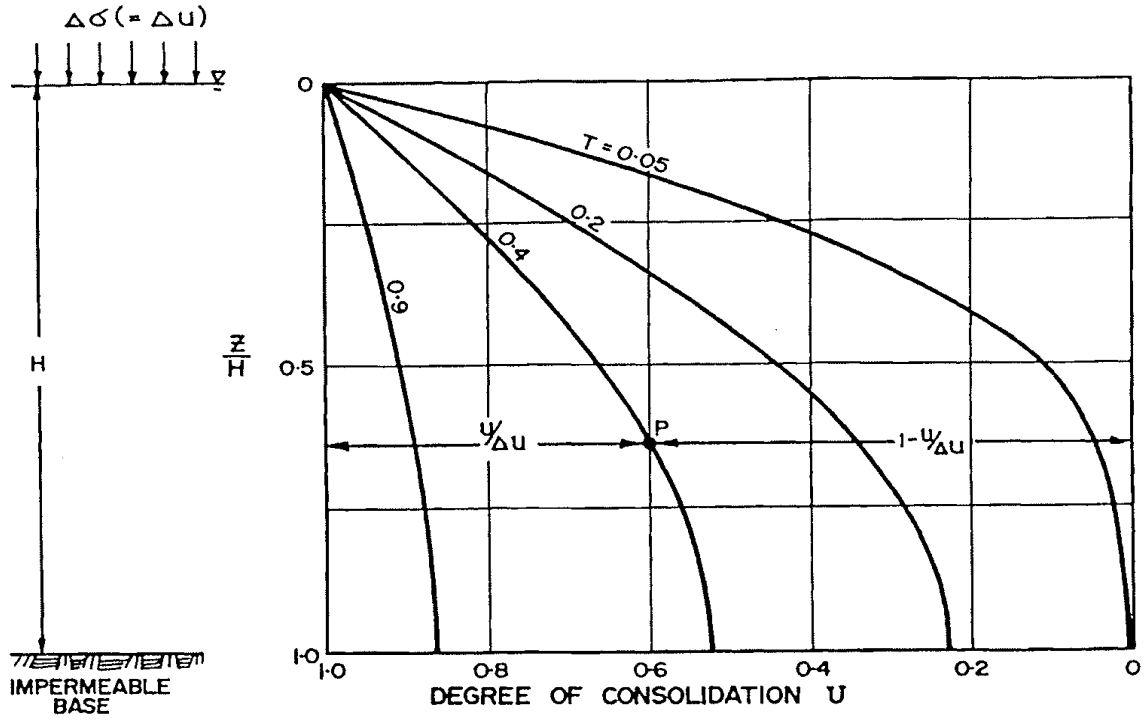


Fig. 10.7 Isochrones for One Dimensional Consolidation

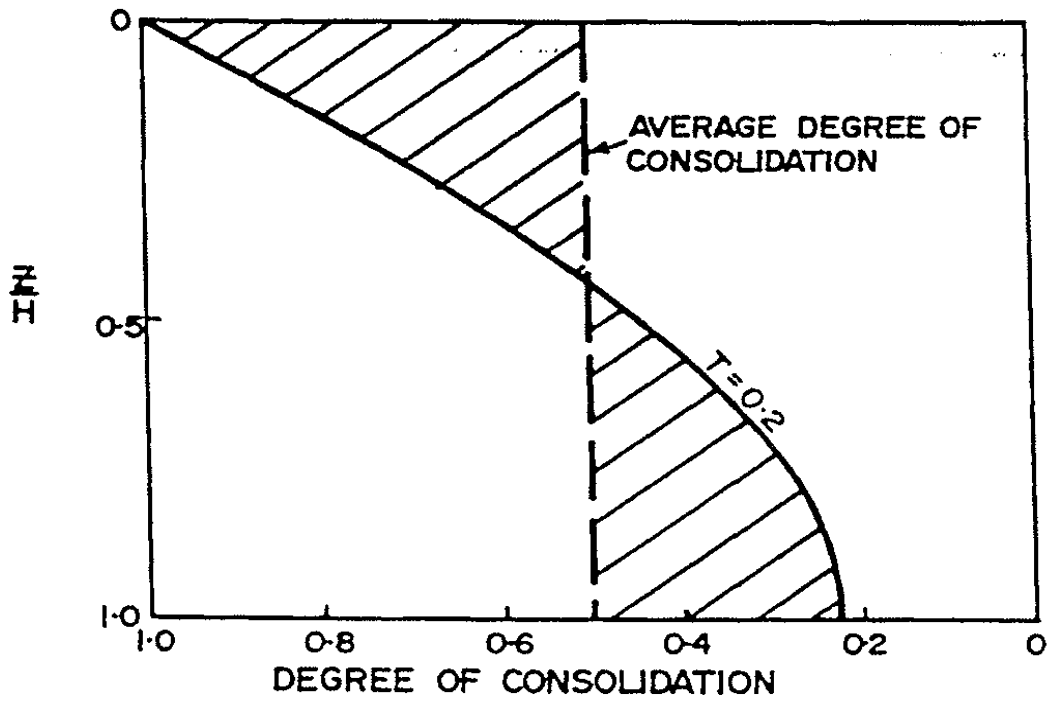


Fig. 10.8 Determination of Average Degree of Consolidation

10.6 RATE OF SETTLEMENT

The overall behaviour of a consolidating layer with time may be studied by means of average degrees of consolidation (U_{av}), which can be evaluated by means of integration over the thickness of the layer.

$$U_{av} = 1 - \frac{\int_0^H u \, dz}{\int_0^H \Delta u \, dz} \quad (10.18)$$

If equation (10.18) is applied to the theoretical solution for u for the case of Δu being constant with depth, the following expression is obtained

$$U_{av} = 1 - \sum_{m=0}^{m=\alpha} \frac{2}{M^2} e^{-M^2 T} \quad (10.19)$$

This is illustrated graphically in Fig. 10.8 for the $T = 0.2$ curve. The average degree of consolidation corresponding to this value of the time factor is selected such that the hatched areas are equal. The resulting relationship between average degree of consolidation and time factor is shown in Fig. 10.9 and in Table 10.1. This relationship holds for a constant initial excess hydrostatic pore pressure (Δu) throughout the layer thickness. Different relationships between U and T have been determined for other initial pore pressure distributions (Taylor (1984), Das (1985)). It may be noted that the use of the Kelvin model (Section 10.2) produces “consolidation” curves that are not exactly the same shape as that for the Terzaghi theory (Fig. 10.9).

TABLE 10.1

Variation of Time Factor with Average Degree of Consolidation

U	T	U	T
0.1	0.008	0.6	0.287
0.2	0.031	0.7	0.403
0.3	0.071	0.8	0.567
0.4	0.126	0.9	0.848
0.5	0.197	1.0	—

The average degree of consolidation is a “degree of pore pressure dissipation” according to the definition in equation (10.18). In order to examine the time rate of settlement, it is necessary to determine the relationship between the degree of consolidation (U) and the “degree of settlement” (U_s) where

$$U_s = \rho_t / \rho_{\text{final}} \quad (10.20)$$

ρ_t and ρ_{final} being the settlement of the ground surface at any time t and the settlement that finally occurs respectively.

$$U_s = \frac{\rho_t}{\rho_{\text{final}}} = \frac{m_v \Delta\sigma'_t H}{m_v \Delta\sigma'_f H} = \frac{\Delta\sigma'_t}{\Delta\sigma'_f} \quad (10.21)$$

where $\Delta\sigma'_t$ is the average change in vertical effective stress at any time t

$\Delta\sigma'_f$ is the final change in the vertical effective stress at the end of consolidation.

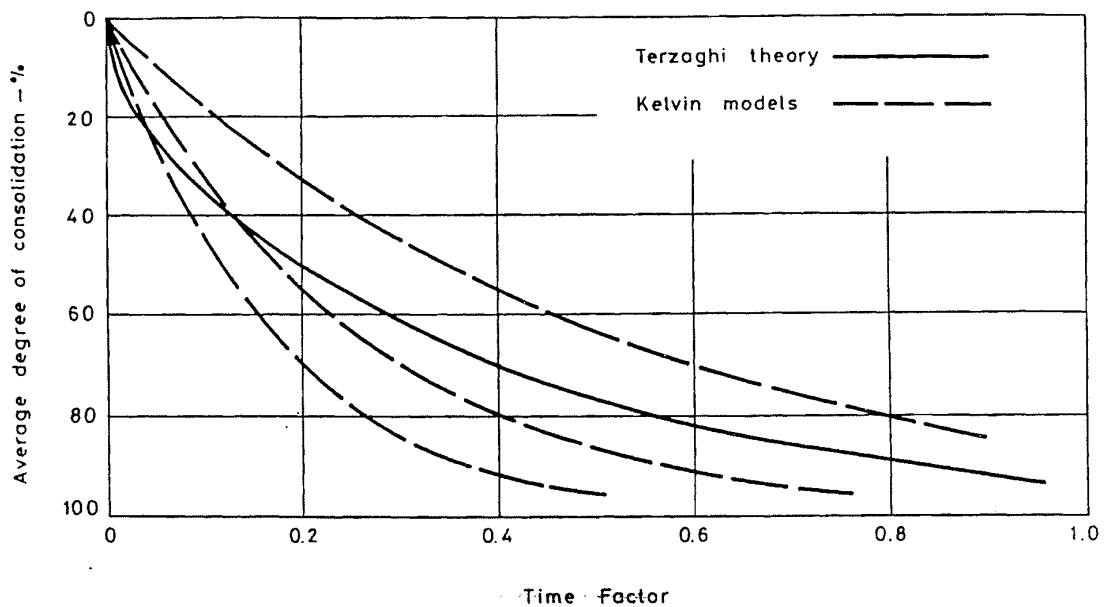


Fig. 10.9 Consolidation Curve from Terzaghi Theory

If $\Delta\sigma$ is the total applied stress at $t = 0$ then the initial pore pressure (excess hydrostatic) Δu is

$$\Delta u = \Delta\sigma$$

and $\Delta\sigma'_{(t=0)} = 0$

At any time t during consolidation if the average excess hydrostatic pore pressure is indicated by u then

$$\Delta\sigma'_t = \Delta\sigma - u$$

$$= \Delta u - u$$

and finally when the pore pressure has fully dissipated

$$u = 0$$

and $\Delta\sigma'_f = \Delta\sigma = \Delta u$

Substituting these values into equation (6.21)

$$U_s = \frac{\sigma'_t}{\sigma'_f} = \frac{\Delta u - u}{\Delta u} = 1 - \frac{u}{\Delta u}$$

$$= U$$

so the average degree of consolidation is equal to the degree of settlement. In other words, the settlement of the consolidating layer takes place at the same rate as that of the average pore pressure dissipation for the case of one dimensional consolidation.

EXAMPLE

A layer of submerged soil 8m thick is drained at its upper surface but is underlain by an impermeable shale. The soil is subjected to a uniform vertical stress which is produced by the construction of an extensive embankment on the ground surface. If the coefficient of consolidation for the soil is $2 \times 10^{-3} \text{ cm}^2/\text{sec}$ calculate the times when 50% and 90% respectively of the final settlement will take place.

Since this soil layer is singly drained

$$H = 8\text{m}$$

When 50% of the settlement has taken place the degree of consolidation U will be 0.5. From Table 10.1 the corresponding time factor T_{50} is 0.197.

$$\begin{aligned} T_{50} &= \frac{c_v t_{50}}{H^2} \\ \therefore t_{50} &= \frac{T_{50} H^2}{c_v} \\ &= \frac{0.197 \times 8^2}{2 \times 10^{-3} \times 10^{-4}} \text{ sec} \\ &= 2.0 \text{ yr} \end{aligned}$$

Similarly for 90% consolidation

$$\begin{aligned} T_{90} &= 0.848 = \frac{c_v t_{90}}{H^2} \\ \therefore t_{90} &= \frac{0.848 \times 8^2}{2 \times 10^{-3} \times 10^{-4}} \text{ sec} \\ &= 8.6 \text{ yr} \end{aligned}$$

10.7 LABORATORY DETERMINATION OF THE COEFFICIENT OF CONSOLIDATION

When it is required to predict the time rate of settlement of soil in the field, it is necessary to know the coefficient of consolidation, c_v and the appropriate boundary conditions. The oedometer test (described in Section 5.4) with vertical flow of water only is applicable to one dimensional consolidation problems which are encountered in situations where a wide surface load is placed over a relatively thin compressible stratum. There are two commonly used methods for the determination of the coefficient of consolidation from oedometer data. These are known as the logarithm of time fitting method and the square root of time fitting method. With these methods the experimental deflection - time plots are fitted to the theoretical degree of consolidation - time factor curves.

10.7.1 Log Time Method

With this method the experimental data for a particular load increment is presented on a deflection - log (time) plot as illustrated in Fig. 10.10(b). The theoretical degree of consolidation - time factor curve is plotted in a similar fashion as shown in Fig. 10.10(a). With the theoretical curve the initial and final points ($U = 0$ and 1.0 respectively) are known but this cannot be said for the experimental curve. With this fitting method the theoretical and experimental curves are compared to facilitate selection of the $U = 0$ and $U = 1.0$ points on the experimental plot. The initial dial gauge reading at zero time does not necessarily correspond with $U = 0$. Similarly the final reading taken does not necessarily correspond with $U = 1.0$.

Since the initial portion of the curve is approximately parabolic the zero point may be estimated by means of the construction showing Fig. 10.10(a). The difference in ordinates (a) between two time factors in the ratio of 4 to 1 is measured above the upper point. This procedure has been repeated on the experimental curve in Fig. 10.10(b) to enable the determination of the dial gauge reading corresponding to the beginning of consolidation (ie. $U = 0.0$).

The theoretical $U = 1.0$ point corresponds with the intersection of the tangent through the point of inflexion and the asymptote to infinite time factor as shown in Fig. 10.10(a). On the experimental curve the asymptote is sometimes not horizontal but the point of intersection still provides a reasonable estimate of the ordinate corresponding to $U = 1.0$. The compression which takes place between ordinates $U = 0$ and $U = 1.0$ is referred to as primary compression to distinguish it from the secondary compression which occurs after consolidation is complete as shown in Fig. 10.10(b).

Once the ordinates corresponding to $U = 0.0$ and $U = 1.0$ are known, intermediate values may be determined. The $U = 0.5$ (or 50%) point on the curve is normally selected for the calculation of coefficient of consolidation.

The time factor T_{50} corresponding to 50% consolidation is (from Fig. 10.10(a)) 0.197. The actual time t_{50} corresponding to 50% consolidation may be read from Fig. 10.10(b). The coefficient of consolidation c_v , may then be calculated from the equation defining the time factor

$$c_v = \frac{T_{50} H^2}{t_{50}} = \frac{0.197 H^2}{t_{50}} \quad (10.22)$$

10.7.2 Square Root of Time Method

When the theoretical degree of consolidation U is plotted against the square root of the time factor T the curve shown in Fig. 10.11(a) is obtained. The initial portion of the curve is a straight line.

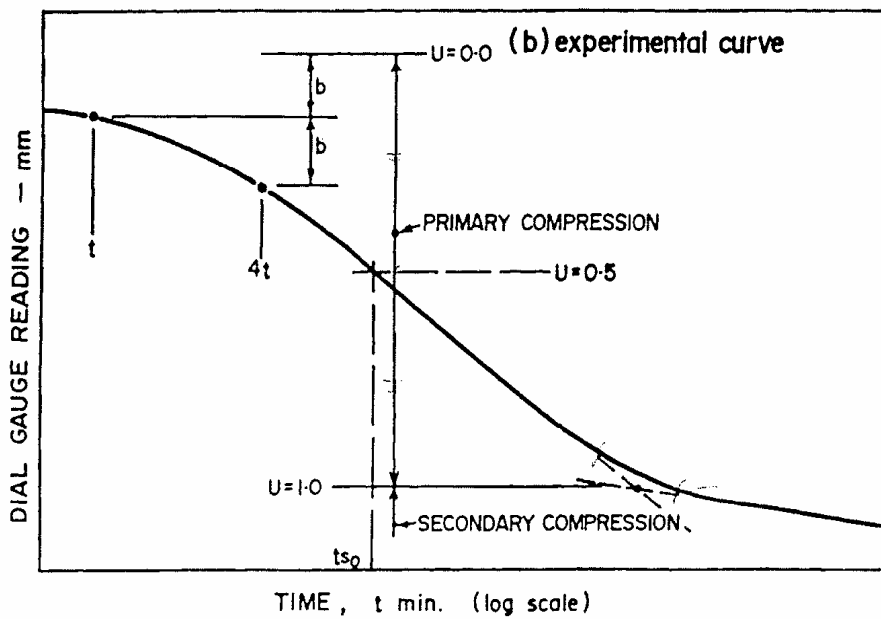
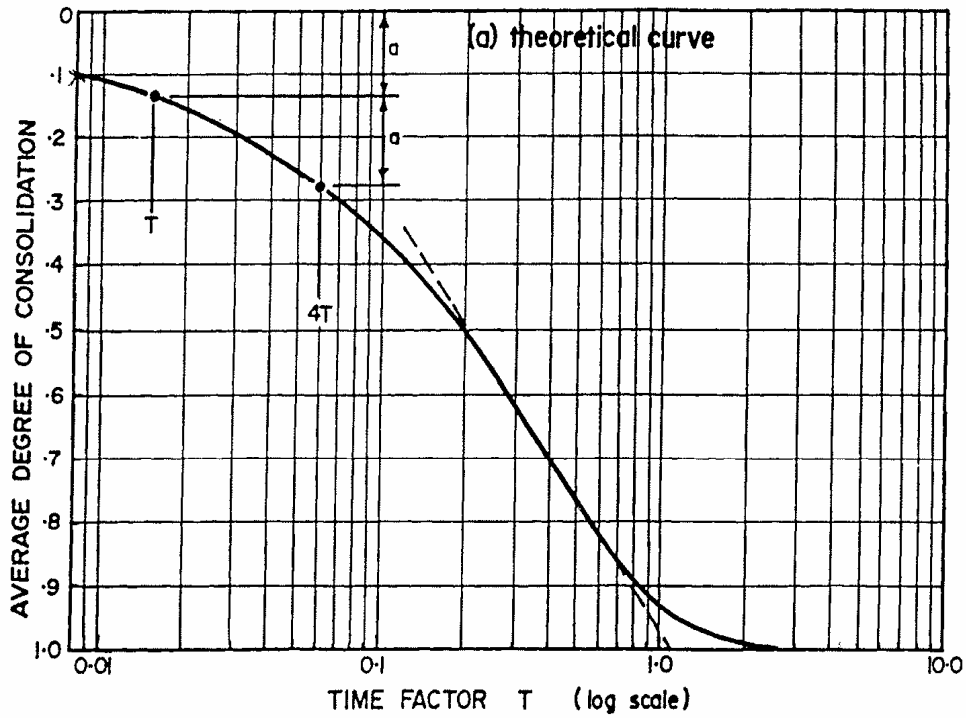


Fig.10.10 Log Time Fitting Method

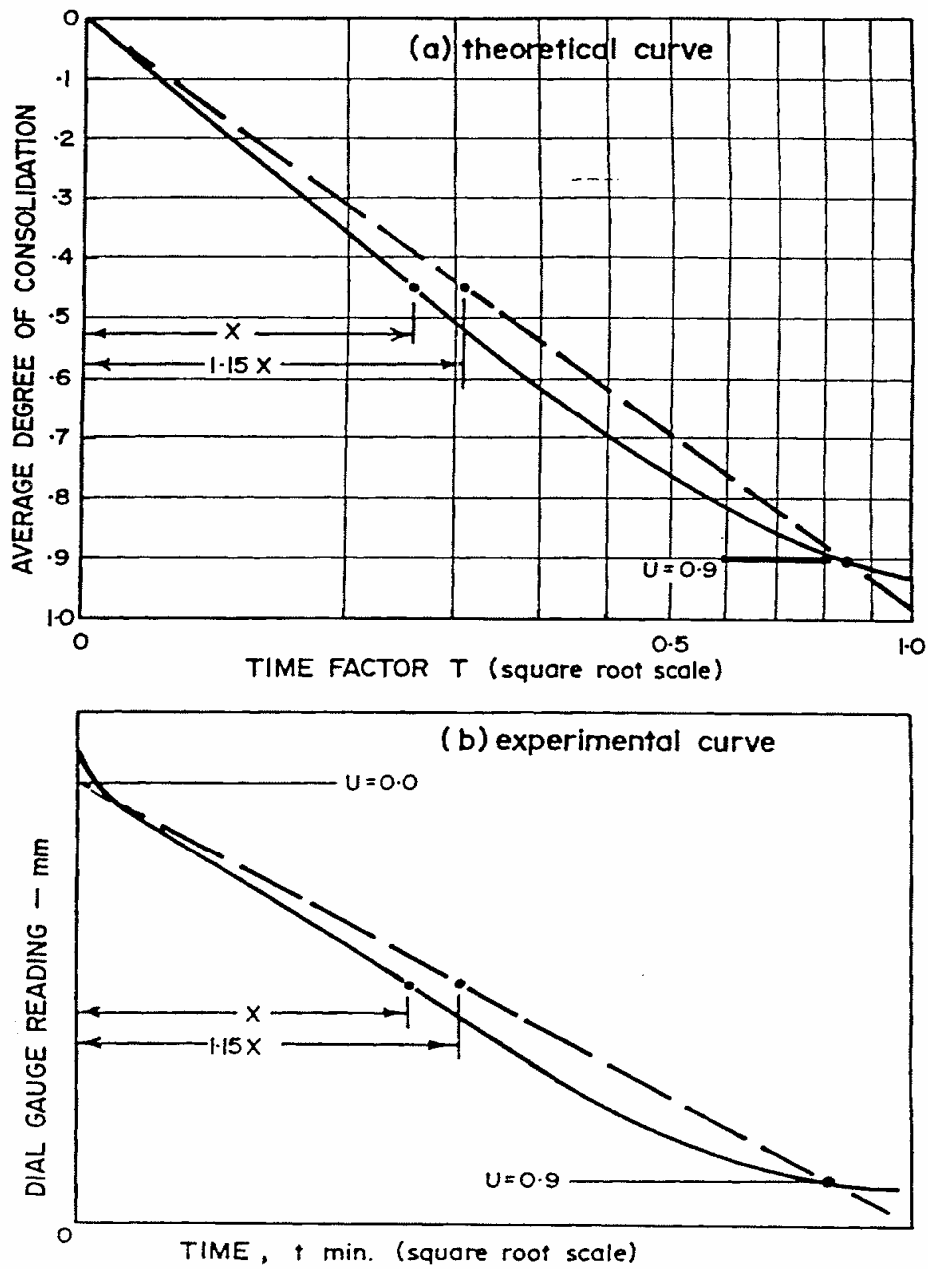


Fig.10.11 Square Root Fitting Method

With the experimental curve, which is plotted in Fig. 10.11(b) an initial curvature is often present before the straight line portion. This curvature is attributed to compression of air in the voids of the soil and the corrected origin is found by backward projection of the straight line portion to zero time.

If a line (shown dashed in Fig. 10.11(a)) is drawn from the origin with abscissa equal to 1.15 times that of the theoretical curve the two lines intersect at $U = 0.9$. This characteristic is used to locate the 90% consolidation point on the experimental curve which is plotted in Fig. 10.11(b). The time, t_{90} corresponding to $U = 0.9$ is read from the experimental curve and the coefficient of consolidation is calculated from

$$c_v = \frac{T_{90} H^2}{t_{90}} = \frac{0.848 H^2}{t_{90}} \quad (10.23)$$

Typical values of the coefficient of consolidation are given in Table 10.2.

TABLE 10.2

Typical Values of Coefficient of Consolidation

Soil	c_v (cm²/sec) x 10⁻⁴
Mexico City Clay (MH) (Leonards & Girault, 1961)	0.9 - 1.5
Soft blue clay (CL - CH) (Wallace & Otto, 1964)	1.6 - 26
Organic Silt (OH) (Lowe, Zaccheo & Feldman, 1964)	5 - 170
Chicago Silty Clay (CL) (Terzaghi & Peck, 1967)	8 - 11
Sandy silty clay (ML - CL) dredge spoil (Van Tol et al, 1985)	5 - 20
Organic Silts and Clays (OH) (Sivakugan, 1990)	1 - 10

EXAMPLE

The following time-compression data was obtained from an oedometer test during consolidation following the application of a load increment:

Dial Gauge Reading (mm)	Time
8.99	0
9.10	6 sec
9.14	12 sec
9.21	30 sec
9.29	1 min
9.39	2 min
9.50	4 min
9.65	8 min
9.74	20 min
9.77	40 min
9.79	100 min

If the thickness of the doubly drained sample is 17.0mm calculate the coefficient of consolidation.

The log time fitting method will be used to determine the coefficient of consolidation and the appropriate plot of the experimental data is presented in Fig. 10.12.

The dial gauge reading corresponding to $U = 0.0$ is found by the procedure outlined in Section 10.7.1. Several estimates have been made and the average has been selected to represent $U = 0.0$. The corresponding dial gauge reading is 9.018mm.

The intersection of the tangent and the asymptote portions of the curve yields a dial gauge reading of 9.748mm for $U = 1.0$ as shown in Fig. 10.12. Some secondary compression is occurring with this sample.

The dial gauge reading corresponding to $U = 0.5$ may now be calculated

$$\begin{aligned}
 &= \frac{9.018 + 9.748}{2} \\
 &= 9.383\text{mm}
 \end{aligned}$$

If the time corresponding to $U = 0.5$ is read from the experimental plot

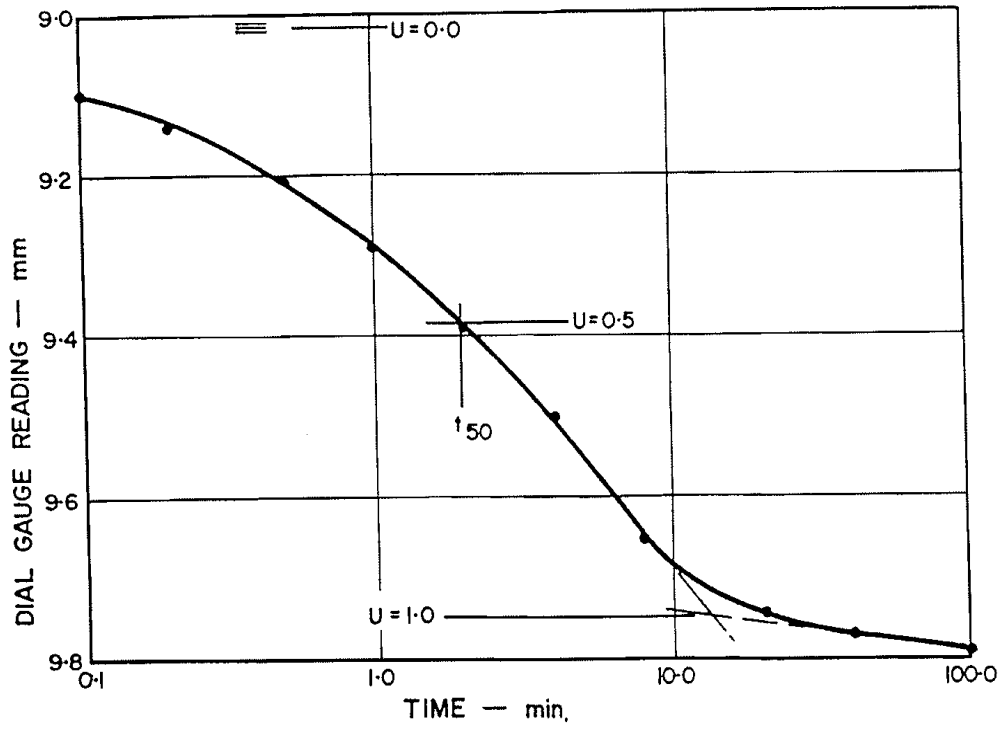
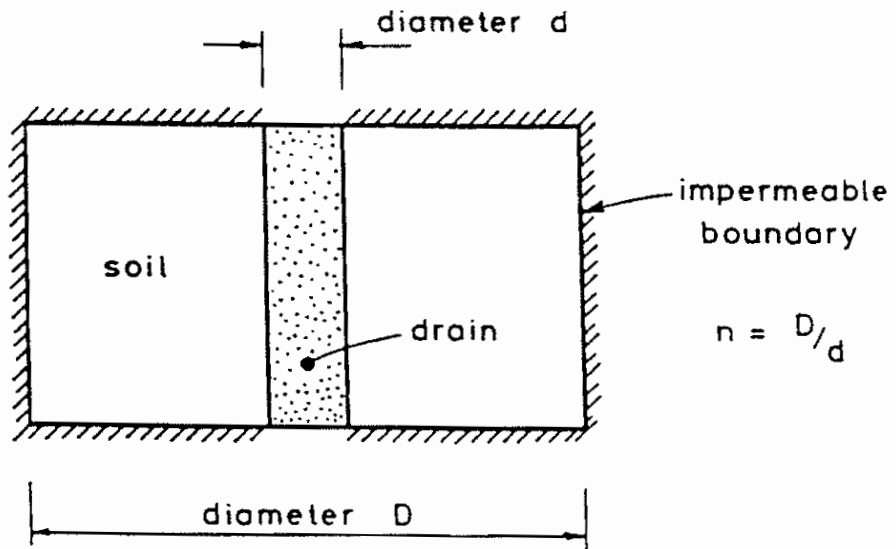


Fig. 10.12



$$t_{50} = 1.95 \text{ min}$$

Since the sample is drained at both upper and lower boundaries

$$\begin{aligned} H &= \frac{17.0}{2} \\ &= 8.5 \text{ mm} \end{aligned}$$

Therefore, the coefficient of consolidation is

$$\begin{aligned} c_v &= \frac{T_{50} H^2}{t_{50}} \\ &= \frac{0.197 \times 8.5^2}{1.95 \times 60} \\ &= 0.122 \text{ mm}^2/\text{sec}. \end{aligned}$$

10.8 OTHER CONSOLIDATION SOLUTIONS

It should be remembered that the Terzaghi consolidation theory discussed above applies only to cases of one dimensional drainage in which the parameters c_v and m_v are constant throughout the soil layer. Other solutions have been developed for cases where the parameters c_v and m_v vary with depth; where there is more than one consolidating layer; and where the boundary conditions are such that the drainage is not purely one dimensional (for example see Biot, 1941 and Gibson and Lumb, 1953).

For the case of purely radial drainage under vertical loading of a cylindrical block of soil of diameter D to a central axial drain of diameter d , Barron (1948) has produced a free strain consolidation solution. The model is illustrated in Fig. 10.13 and the solutions for various values of n (D/d) are shown in Fig. 10.14. The time factor (T_r) for radial flow is defined as

$$T_r = c_r t/D^2 \quad (10.24)$$

where $c_r =$ radial coefficient of consolidation

$$= k_r / m_v \rho_w g$$

$k_r =$ radial permeability

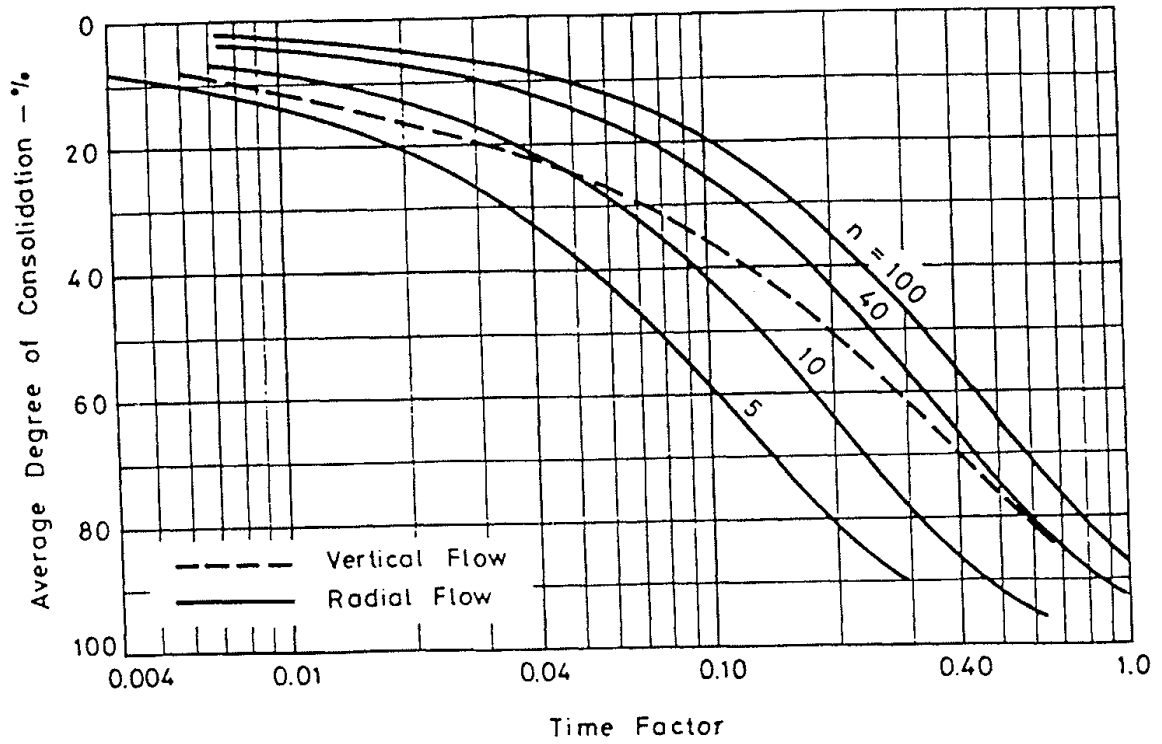


Fig. 10.14 Consolidation Rates for Radial Flow
(after Barron, 1948)

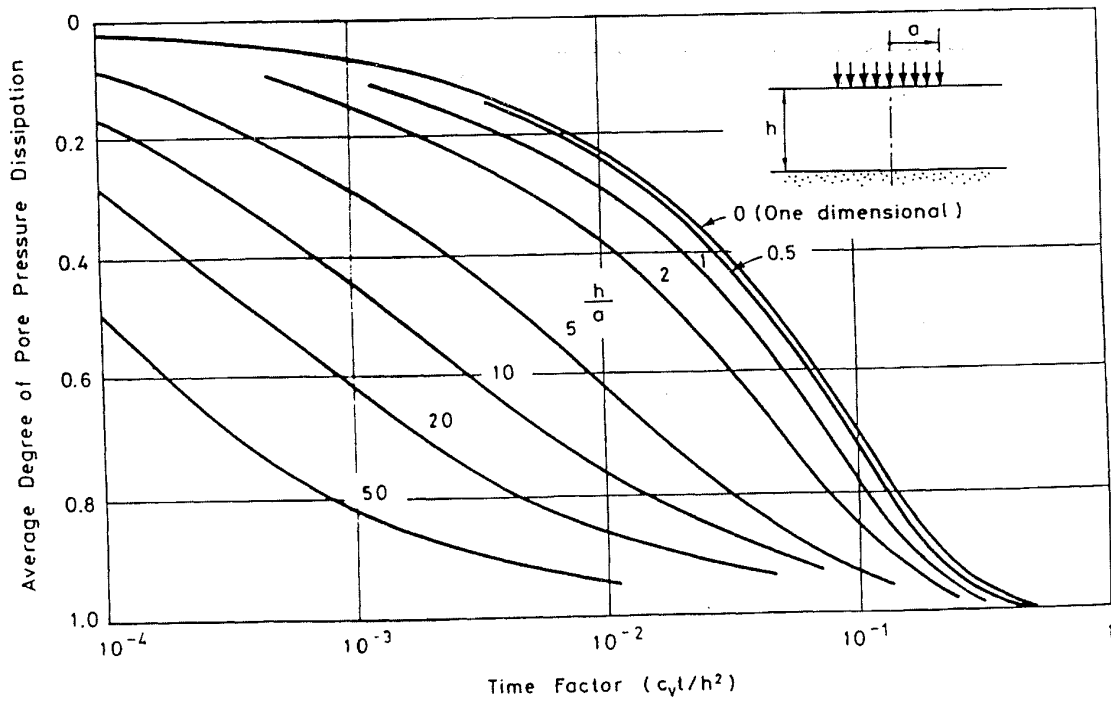


Fig. 10.15 Circular Footing, Permeable Top, Permeable Base
(after Davis & Poulos, 1972)

For comparative purposes the Terzaghi one dimensional solution for vertical flow has been superimposed on Fig. 10.14 and for this curve the time factor is as defined in section 10.5. Fig. 10.14 may be used for problems in which sand drains are used to accelerate the consolidation of a soil layer which has a high ratio of horizontal to vertical permeability.

For evaluating the rate of settlement of circular and strip footings on a soil layer, Davis and Poulos (1972) have produced a number of solutions. Fig. 10.15 gives solutions for a circular footing on a soil layer with a permeable upper surface and a permeable base. Fig. 10.16 gives solutions for a circular footing on a soil layer with a permeable upper surface and an impermeable base. Fig. 10.17 gives solutions for a strip footing on a soil layer with a permeable upper surface and a permeable base. With these three figures, the time factor (T) is defined as

$$T = c_v t / h^2 \quad (10.25)$$

where c_v = one dimensional coefficient of consolidation for vertical drainage

$$h = \text{thickness of soil layer}$$

The vertical axis of Figs. 10.15, 10.16 and 10.17 is average degree of pore pressure dissipation (U_p). This is calculated on any vertical line and is defined as

$$U_p = 1 - \left(\int u \, dz / \int \Delta u \, dz \right) \quad (10.26)$$

where u and Δu are as defined in equation (10.15). The degree of pore pressure dissipation (U_p) is approximately equal to the degree of settlement (U_s).

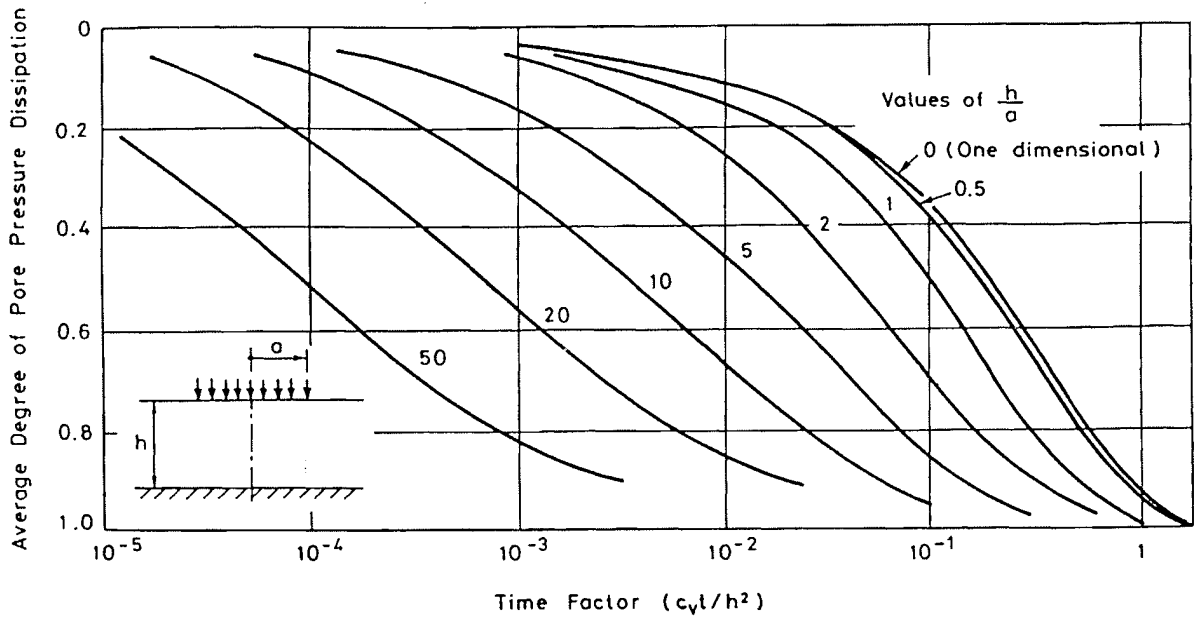


Fig. 10.16 Circular Footing, Permeable Top, Impermeable Base
(after Davis & Poulos, 1972)

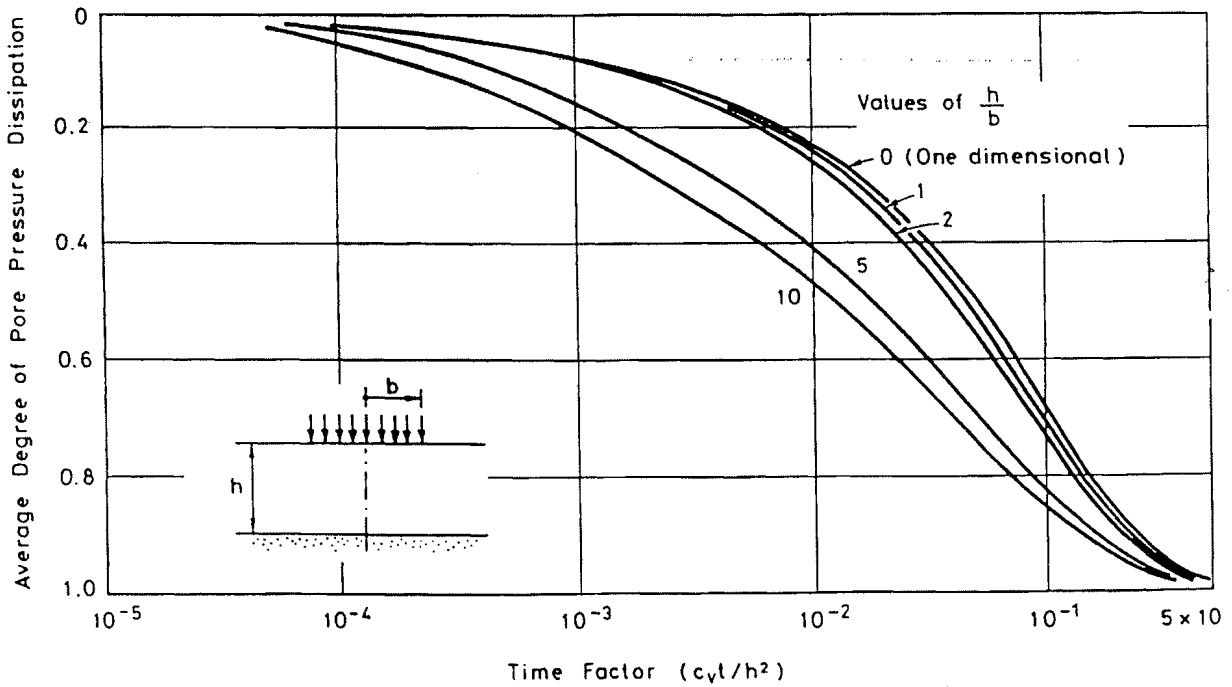


Fig. 10.17 Strip Footing, Permeable Top, Permeable Base (after Davis & Poulos, 1972)

REFERENCES

- Barron, R.A., (1948), "Consolidation of Fine Grained Soils by Drain Wells", Trans. ASCE., Vol. 113, pp 718-742.
- Biot, M.A., (1941), "General Theory of Three Dimensional Consolidation", Jnl. Applied Physics, Vol. 12, pp 155-164.
- Das, B.M., (1985), "Principles of Geotechnical Engineering", PWS Publishers, Boston, 571 p.
- Davis, E.H. and Poulos, H.G., (1972), "Rate of Settlement under Two-and-Three-Dimensional Conditions", Geotechnique Vol. 22, No. 1, pp 95-114.
- Gibson, R.F. and Lumb, P., (1953), "Numerical Solution of some Problems in the Consolidation of Clay", Jnl. Instn. Civil Engrs., London, Vol. 2, Part I, pp 182-198.
- Leonards, G.A. and Girault, P. (1961), "A Study of the One-Dimensional Consolidation Test", Proc. Fifth Int. Conf. on Soil Mechanics and Found. Eng., Paris, Vol. 1, pp 116-130.
- Lowe, J, Zaccheo, P.F. and Feldman, H.S. (1964), "Consolidation Testing with Back Pressure", Jnl. Soil Mechanics and Found. Div., ASCE, Vol. 90, No. SM5, pp 69-86.
- Sivakugan, N., (1990), "Development of Marshy Areas in Colombo, Sri Lanka", Proc. Tenth Southeast Asian Geot. Conf., Taipei, Vol. 1, pp 469-472.
- Taylor, D.W., (1948), "Fundamentals of Soil Mechanics", John Wiley & Sons, New York, 700 p.
- Terzaghi, K., (1943) "Theoretical Soil Mechanics", John Wiley & Sons, New York, 510 p.
- Terzaghi, K. and Peck, R.B., (1967), "Soil Mechanics in Engineering Practice", John Wiley & Sons, New York, 729 pp.
- Van Tol, A.F., Brassinga, H.E., Elprama, R. and Van Rijt, C., (1985), "Consolidation of Soft Fine-Grained Dredged Material", Proc. Eleventh Int. Conf. on Soil Mechanics and Found. Eng., San Francisco, Vol. 2, pp 687-690.
- Wallace, G.B and Otto, W.C. (1964), "Differential Settlement at Selfridge Air Force Base", Jnl. Soil Mechanics and Found. Div., ASCE, Vol. 90, No. SM5, pp 197-220.