10th Grade Algebra 2 with Trigonometry
4th Nine Weeks Curriculum Guide
Week 1- Rational Functions

Suggested Pacing: 1 day (50 min class)

ACOS Standards:
A.CED.2 & 4
- Create equations in two or more variables to represent relationships between quantities
- Graph equations on coordinate plane with appropriate axis labels and scales

Learning Objectives:
- To recognize and use inverse variation
- To use joint and other variations

Essential Understanding(s):
- If a product is constant, a decrease in the value of one factor must accompany an increase in the value of the other factor.
- In direct variations, two positive quantities, either increase or decrease together.
- In inverse variations, one quantity increases as another decreases.
- Quantities x and y are inversely proportional only if increasing x by factor a ($a \neq 0$) means shrinking y by the factor of $\frac{1}{\square}$.
    - Example: $\square = \frac{\square}{\square}$ $\square\square\ \square = \frac{\square}{\square}$ (a *or* $k \neq 0$)

Key Vocabulary: What key terms will students need to know to understand?
Direct variation, inverse variation, combined variation, joint variation, product, inverse, proportional

Learning Activities:
Before (I Do): Teacher facilitates discussion about variations; present students with real world data showing but direct and indirect variations. Students are then asked to compare and contrast the trends in each set of data. How does one variable "respond" to the other? If the data were plotted, predict what the general appearance and characteristics of each graph?
(Approx. 5 minutes)

During (We Do): Guided practice using examples focusing on the following skills
- Graphing Data using coordinate plane with appropriately scaled and labeled axes identifying direct and inverse variations (Ex. Problems #1 & 2)
- Modeling an Inverse Variation (Ex. Problem #3)
- Exploring and Appling concepts related to combined variations (Ex. Problems #4 & 5)

(Approx. 25 minutes)

After (They Do):
- Students will work in small groups applying skills taught in examples and mathematical practices to solve a problem using data (Example - Problems #22 and 24)
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(Approx. 20 minutes)

Assessment Plan: How will I assess prior knowledge? How will I know students mastered the standard? (Formative, Summative, Other Evidence): Formative Pencil-paper Assessment and 3,2,1 Strategy

Differentiation/Accommodations:

Technology Integration:
www.desmos.com

Teacher Notes and Resouces:
Before: Students should recognize that in direct variation data, as one variable increases so does the other. Conversely, in the inverse variation data, as one variable increases, the other decrease.

http://cnx.org/content/m18281/latest/?collection=col10624

Suggested Pacing: 1 day 50 min class

ACOS Standards:

11 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

19 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.

21 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

34 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

B.1.h Apply previously learned algebraic and geometric concepts to more advanced problems.
E.2.b Use transformations to draw the graph of a relation and determine a relation that fits a graph.

Learning Objectives:

The reciprocal function family
Students will graph reciprocal functions.
Students will graph translations of functions.

Key Vocabulary:

reciprocal function
branch
transformation
stretch
shrink

Assessment Plan:

Guided Questions
Guided Practice
Student Activity Pages

Learning Activities:

Tuesday

Before:

Teacher will draw the graphs of the function $f(x)=\frac{1}{x-1}+2$ and $g(x)=\frac{1}{x+3}-4$ on the board. The students will identify the x-values that are not included in the domain, and the students will identify the y-values that are not included in the range. For f(x), the x-value not included in the domain is x = 1 and the y-value not included in the range is y = 2. For g(x), the x-value not included in the domain is x = -3 and the y-value not included in the range is -4.

During:

- The students will complete the Student Activity Pages of Transformation of Rational Functions.
- The teacher will monitor the students and answer questions as they work through the worksheet.
- Each LTF lesson comes with Teaching Strategies that need to be read before implementing the lesson.
- This lesson has students rewriting equations to practice algebraic skills in order to graph the functions.
- The teacher will need to model at least one algebraic rewrite for the students from the worksheet.
- The teacher will make sure that students understand how the graphs translate horizontally and vertically.
- The teacher will also make that students will understand that if a factor cancels from the numerator and denominator, then, the x value is a hole in the graph and not a vertical asymptote.

After:

Students will pair up with a partner. They both will write (create) a function in the form of $f(x)=\frac{a}{x-h}+k$. They will pass of the function to each other and graph it on graph paper. After graphing it, they will hand the graph back to their partner for them to check. The completed graphs will be turned into the teacher.

Materials: Transformation of Rational Functions Worksheet, graphing calculators or www.desmos.com on the computer, and graph paper. This activity can be done without the use of technology, but it helps to speed up the process of graphing and discovering. The LTF worksheet is attached to this file as the Rational Functions worksheet.

Differentiation/Accommodations:
The teachers will have to take away questions or make certain questions part of whole class instruction. Allow students to work independently as it is feasible. The worksheet can be modified to fit your students' needs.

Technology Integration:

The students will probably need to use a graphing calculator or www.desmos.com .

Teacher Notes:

LTF is a program that takes patience to implement and teachers do have to model some of the questions, as they are more rigorous.

Suggested Pacing: 2 days with one day being a 90 min class and one day being a 50 min class

ACOS Standards:
12 Interpret expressions that represent a quantity in terms of its context.*
a. Interpret parts of an expression such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

29 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

31 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Learning Objectives:
Long-run behavior
Rational Functions and their Graphs
The students will identify properties of rational functions.
The students will graph rational functions.

Key Vocabulary:
rational function
continuous graph
point of discontinuity
removable discontinuity
non-removable discontinuity
vertical asymptote
horizontal asymptote
slant or oblique asymptote

Assessment Plan:
Guided questions
Guided practice
Student Activity Pages

Learning Activities:

Wednesday/ Thursday

Before:

The teacher will write the trinomials $x^2 + x - 12$ and $x^2 - 9$ and have the students factor them. The correct answer for the first expression is $(x+4)(x-3)$. The correct answer for the second expression is $(x-3)(x+3)$.

During:

- The students will complete the Student Activity Pages Rational Functions- Long Run Behavior.
- The teacher will monitor students as they work through the student activity pages.
- Each LTF lesson comes with Teaching Strategies that need to be read before implementing the lesson.
- After the activity, the teacher will make sure that the students understand the long run behavior of rational functions. If the function is top heavy, then the horizontal asymptote is $y = 0$. If the function has the same heaviness, then the horizontal asymptote is the ratio of the leading coefficients, which is $y = \frac{a}{b}$. If the function is top heavy, then there is no horizontal asymptote, but only a slant or oblique asymptote.

After:

The students will turn in an exit slip. On the exit slip, the students will write the three rules of horizontal asymptotes that are listed above. Then the students will write down one question that they still have.

Friday

Before:

The teacher will graph the function $f(x) = \frac{x^2 - 7x + 10}{x^2 - 25}$ on the board. Have the students identify any horizontal or vertical asymptotes, holes in the graph, x-

intercepts, and y-intercepts. These items should be labeled on the graph on the board and easily identifiable.

During:

- The teacher will bring all the concepts together on graphing rational functions.
- First, the teacher will model, "I do," with the function $f(x)=\frac{x^2-7x+10}{x^2-25}$.
- The teacher will begin by factoring the function into $f(x)=\frac{(x-5)(x-2)}{(x-5)(x+5)}$.
- The teacher will cancel the $x-5$ factor.
- The resulting equation, sister function, is $f(x)=\frac{x-2}{x+5}; x\neq 5$.
- The teacher will remind the students that $x=5$ is a hole in the graph.
- The teacher will set $x+5=0$ and get the vertical asymptote $x=-5$.
- The teacher will set $x-2=0$ and get the x-intercept $x=2$ or the point $(2,0)$.
- The teacher will show the students that the sister function has the same heaviness, making the ratio $y=\frac{1}{1}=1$, he horizontal asymptote.
- The teacher will substitute zero for the x-values in the sister function to get $f(0)=\frac{0-2}{0-5}=\frac{2}{5}$. Therefore, the y-intercept is $\left(0,\frac{2}{5}\right)$.
- Put all of these values on the graph and draw the branches on the graph. If help is needed to see the graph, put the original function into www.desmos.com to see what it looks like.
- Next, the teacher should do a "we do" with the students with the function $g(x)=\frac{x^2-2x-63}{x^3+5x^2-14x}$.
- Do not forget to lead the students to factor first.
- There will be 2 vertical asymptotes: $x=0, x=2$
- 1 hole in the graph: $\left(-7,-\frac{16}{63}\right)$
- 1 horizontal asymptote: $y=0$
- Have the students, "you do," graph the function $h(x)=\frac{x^2-2x}{x^2-4}$.

After:

Have the students journal about the process of finding the sister function, the hole in the graph, the vertical asymptotes, horizontal asymptotes, x-intercepts and y-intercepts. Have the students write any question that they might still have and turn their journal entry in to you.

Materials:
Student Activity pages
Graphing Calculators
www.desmos.com

Differentiation/Accommodations:
The teachers will have to take away questions or make certain questions part of whole class instruction. Allow students to work independently as it is feasible. The worksheet can be modified to fit your students' needs.

Technology Integration:
Graphing Calculator
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Teacher Notes:
LTF is a program that takes patience to implement and teachers do have to model some of the questions, as they are more rigorous.