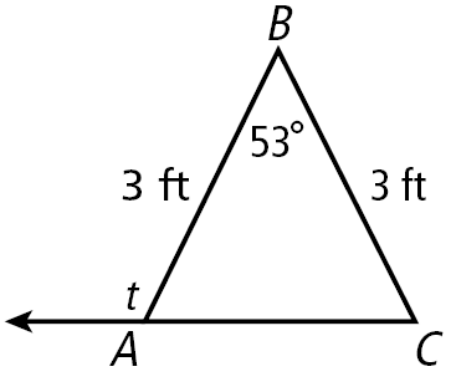
# 11-4 Inscribed Angles

## Warm Up

**Find each value.**




**1.** m∠*BCA* 63.5°

**2.** *t* 116.5°

**Solve for *x*.**

**3.** 58 – *x* = 4 (*x* + 7) 6

**4.** 2 (*x* – 8) = 8 12

# 11-4 Inscribed Angles

## Objectives

Find the measure of an inscribed angle.

Use inscribed angles and their properties to solve problems.

## Vocabulary

inscribed angle  intercepted arc

subtend

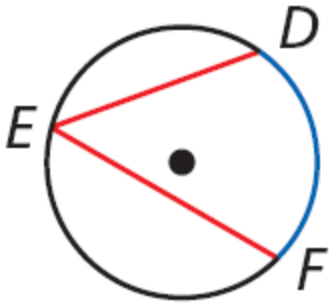
# 11-4 Inscribed Angles

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.

An **intercepted arc** consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them.

A chord or arc **subtends** an angle if its endpoints lie on the sides of the angle.

# 11-4 Inscribed Angles




$\angle DEF$ is an **inscribed angle**.

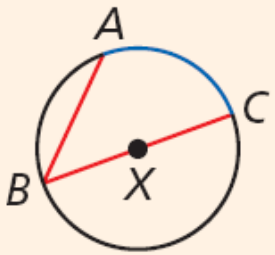
$\overset{\frown}{DF}$ is the **intercepted arc**.

$\overset{\frown}{DF}$ subtends $\angle DEF$.

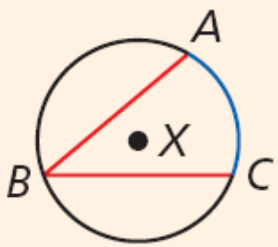
**Theorem 11-4-1** **Inscribed Angle Theorem**

The measure of an inscribed angle is half the measure of its intercepted arc.

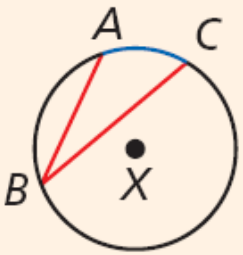
$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$$



Case 1



Case 2

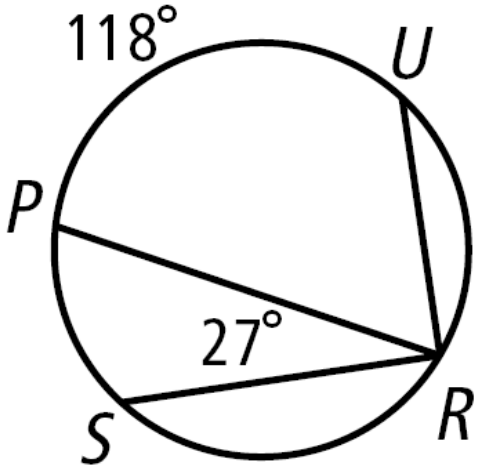


Case 3

## Example 1A: Finding Measures of Arcs and Inscribed Angles

**Find each measure.**

$m\angle PRU$ $m\overset{\frown}{SP}$



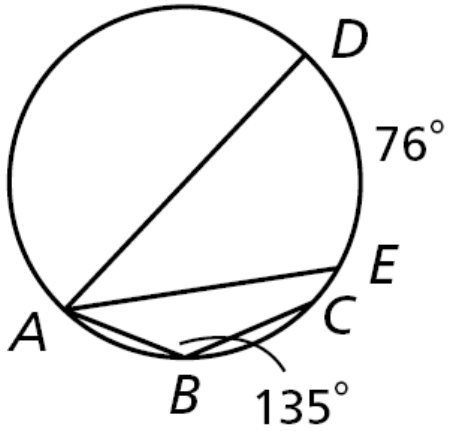

$$m\angle PRU = \frac{1}{2}m\overset{\frown}{PU} = \frac{1}{2}(118°) = 59°$$

$$m\angle SRP = \frac{1}{2}m\overset{\frown}{SP} \quad 27° = \frac{1}{2}m\overset{\frown}{SP} \quad m\overset{\frown}{SP} = 54°$$

## Check It Out! Example 1a

**Find each measure.**

$m\overset{\frown}{ADC}$ $m\angle DAE$



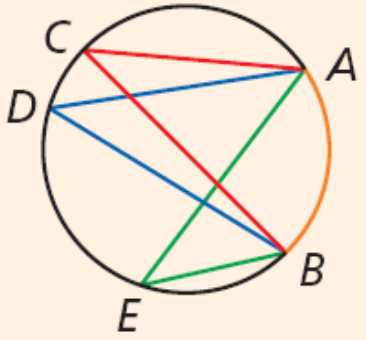

$$m\angle ABC = \frac{1}{2}m\overset{\frown}{ADC} \quad 135° = \frac{1}{2}m\overset{\frown}{ADC}$$

$$270° = m\overset{\frown}{ADC}$$

$$m\angle DAE = \frac{1}{2}m\overset{\frown}{DE} \quad = \frac{1}{2}(76°) = 38°$$

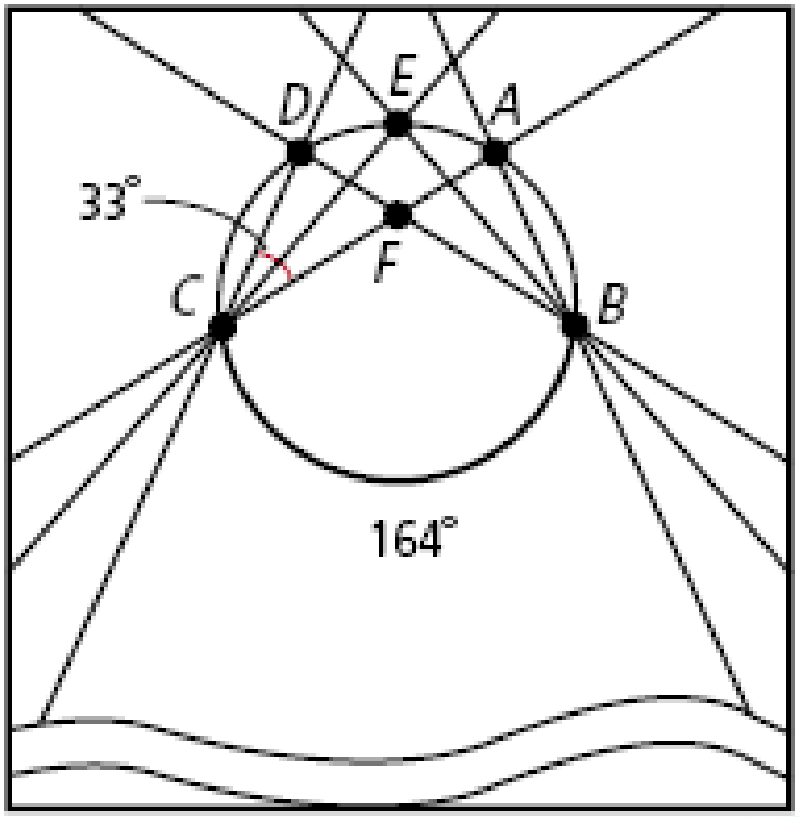
# 11-4 Inscribed Angles

## Corollary 11-4-2

| COROLLARY | HYPOTHESIS | CONCLUSION |
|---|---|---|
| If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent. |   $\angle ACB$, $\angle ADB$, and $\angle AEB$ intercept $\overset{\frown}{AB}$. | $\angle ACB \cong \angle ADB \cong \angle AEB$ (and $\angle CAE \cong \angle CBE$) |

## Example 2: Hobby Application

**An art student turns in an abstract design for his art project.**




**Find m∠*DFA*.**

$$m\angle DFA = m\angle DCF + m\angle CDF$$

$$= m\angle DCF + \frac{1}{2}m\overset{\frown}{BC}$$

$$= 33° + \frac{1}{2}(164°)$$

$$= 115°$$

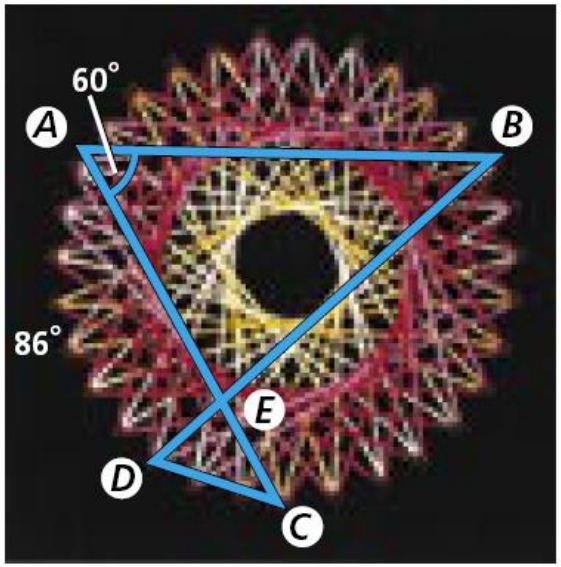
## Check It Out! Example 2

**Find $m\angle ABD$ and $m\overset{\frown}{BC}$ in the string art.**

$$m\angle ABD = \frac{1}{2}m\overset{\frown}{DA}$$

$$= \frac{1}{2}(86°)$$

$$= 43°$$




$$m\angle CAB = \frac{1}{2}m\overset{\frown}{BC}$$

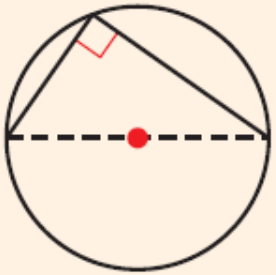
$$60° = \frac{1}{2}m\overset{\frown}{BC}$$

$$m\overset{\frown}{BC} = 120°$$

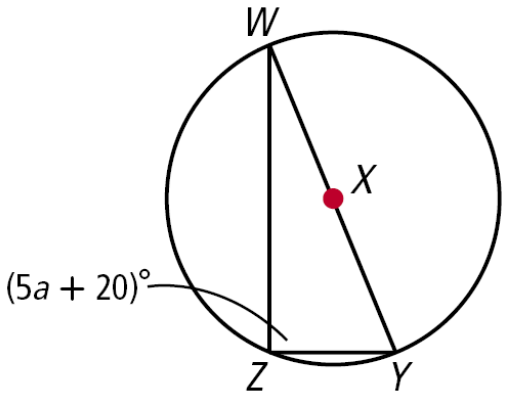
# 11-4 Inscribed Angles

## Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



## Finding Angle Measures in Inscribed Triangles




**Find *a*.**

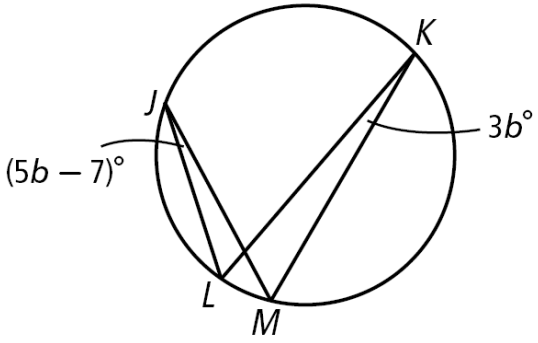
$\angle WZY$ is a right angle

$m\angle WZY = 90°$

$5a + 20 = 90$

$5a = 70$

$a = 14$




**Find $m\angle LJM$.**

$m\angle LJM = m\angle LKM$

$5b - 7 = 3b$

$2b - 7 = 0$

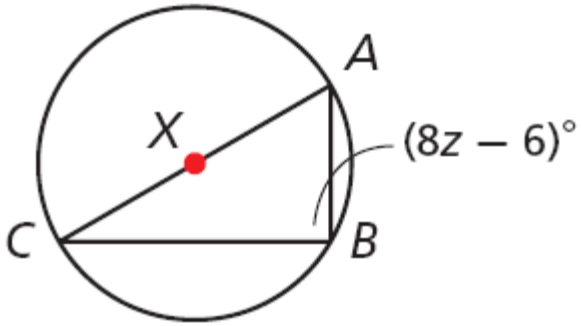
$2b = 7$

$b = 3.5$

$m\angle LJM = 5(3.5) - 7 = 10.5°$

# 11-4 Inscribed Angles

**Find *z*.**




$\angle ABC$ is a right angle

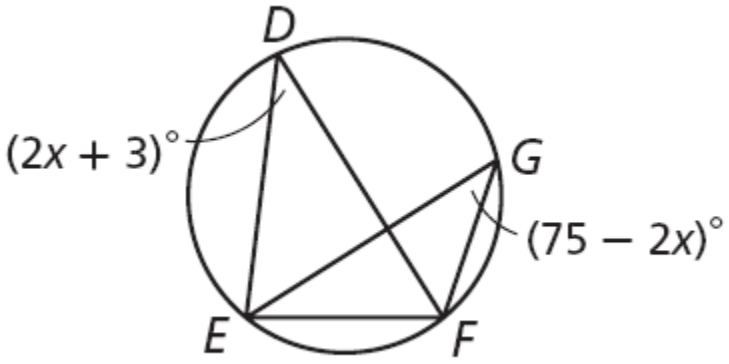
$m\angle ABC = 90°$

$8z - 6 = 90$

$8z = 96$

$z = 12$

**Find $m\angle EDF$.**




$m\angle EDF = m\angle EGF$

$2x + 3 = 75 - 2x$

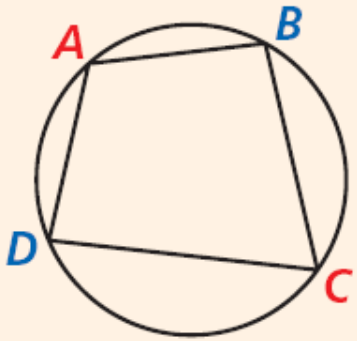
$4x = 72$

$x = 18$

$m\angle EDF = 2(18) + 3 = 39°$

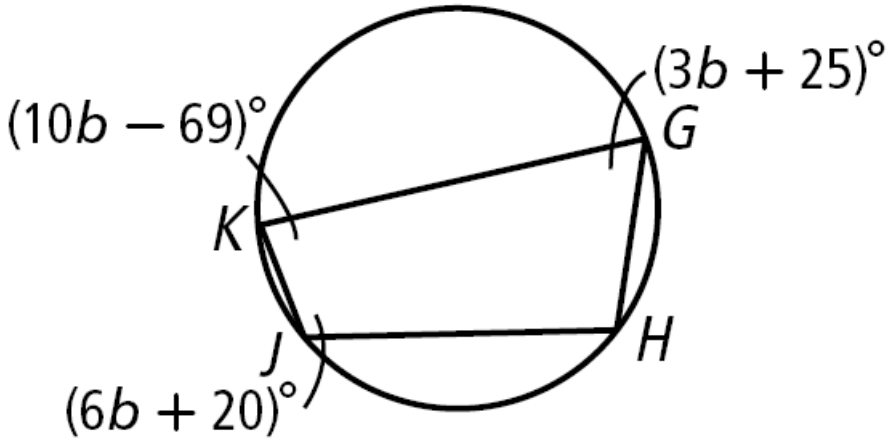
# 11-4 Inscribed Angles

## Theorem 11-4-4

| THEOREM | HYPOTHESIS | CONCLUSION |
| --- | --- | --- |
| If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. |   *ABCD* is inscribed in ⊙*E*. | ∠*A* and ∠*C* are supplementary. ∠*B* and ∠*D* are supplementary. |

## Example 4: Finding Angle Measures in Inscribed Quadrilaterals

Find the angle measures of *GHJK*.

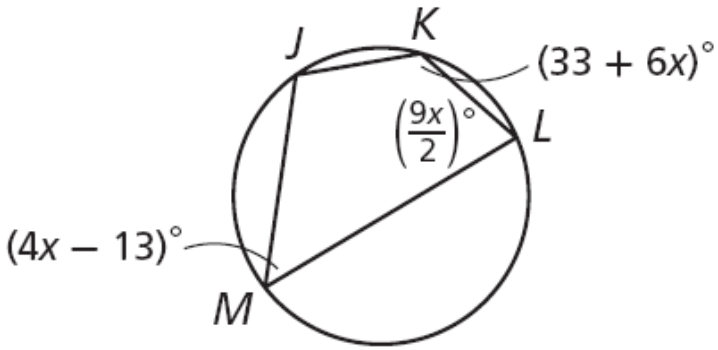



**Step 1** Find the value of *b*.

**Step 2** Find the measure of each angle.

## Check It Out! Example 4

**Find the angle measures of *JKLM*.**

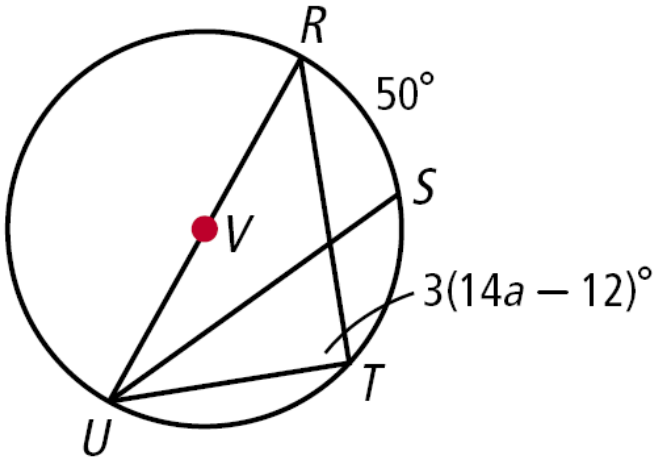


## Lesson Quiz: Part I

**Find each measure.**

**1.** $\angle RUS$ 25°

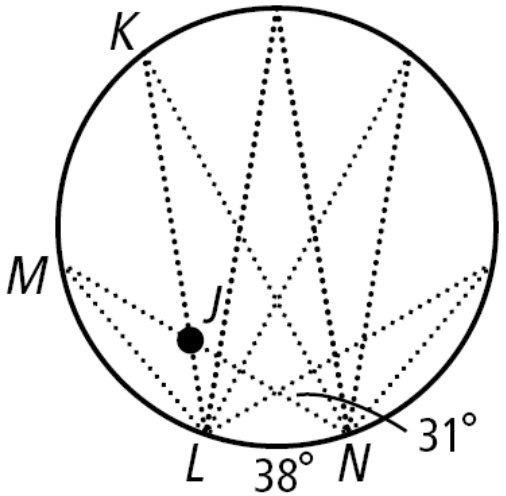
**2.** $a$ 3



## Lesson Quiz: Part II

**3.** A manufacturer designs a circular ornament with lines of glitter as shown. Find m∠*KJN*.

130°




**4.** Find the angle measures of *ABCD*.

m∠ *A* = 95°
m∠ *B* = 85°
m∠ *C* = 85°
m∠ *D* = 95°