11-4

1. Plan

Objectives
1 To find the volume of a prism
2 To find the volume of a cylinder

Examples
1 Finding Volume of a Rectangular Prism
2 Finding Volume of a Triangular Prism
3 Finding Volume of a Cylinder
4 Finding Volume of a Composite Figure

Professional Development

Math Background

Integral calculus considers the area under a curve, which leads to computation of volumes of solids of revolution. Cavalieri's Principle is a forerunner of ideas formalized by Newton and Leibniz in calculus.

More Math Background: p. 596D

Lesson Planning and Resources

See p. 596E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Areas of Rectangles and Circles
Lesson 1-9: Examples 4, 5
Extra Skills, Word Problems, Proof Practice, Ch. 1

Area of a Triangle
Lesson 10-1: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 10

Volumes of Prisms and Cylinders

What You'll Learn
- To find the volume of a prism
- To find the volume of a cylinder

. . . And Why
To estimate the volume of a backpack, as in Example 4

Check Skills You'll Need GO for Help Lessons 1-9 and 10-1

Find the area of each figure. For answers that are not whole numbers, round to the nearest tenth.

1. a square with side length 7 cm **49 cm²**
2. a circle with diameter 15 in. **176.7 in.²**
3. a circle with radius 10 mm **314.2 mm²**
4. a rectangle with length 3 ft and width 1 ft **3 ft²**
5. a rectangle with base 14 in. and height 11 in. **154 in.²**
6. a triangle with base 11 cm and height 5 cm **27.5 cm²**
7. an equilateral triangle that is 8 in. on each side **27.7 in.²**

New Vocabulary • volume • composite space figure

1 Finding Volume of a Prism

Hands-On Activity: Finding Volume

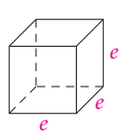
Explore the volume of a prism with unit cubes.

- Make a one-layer rectangular prism that is 4 cubes long and 2 cubes wide. The prism will be 4 units by 2 units by 1 unit.

1. How many cubes are in the prism? **8 cubes**

2. Add a second layer to your prism to make a prism 4 units by 2 units by 2 units. How many cubes are in this prism? **16 cubes**
3. Add a third layer to your prism to make a prism 4 units by 2 units by 3 units. How many cubes are in this prism? **24 cubes**
4. How many cubes would be in the prism if you added two additional layers of cubes for a total of 5 layers? **40 cubes**
5. How many cubes would be in the prism if there were 10 layers? **80 cubes**

Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches ($in.^3$), cubic feet (ft^3), or cubic centimeters (cm^3). The volume of a cube is the cube of the length of its edge, or $V = e^3$.

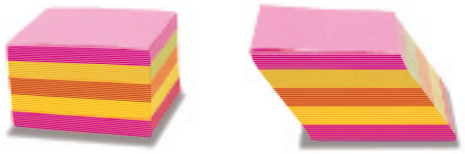


Differentiated Instruction Solutions for All Learners

Special Needs L1
In Example 2, some students may have trouble identifying the height because it is not vertical. Use a drawing at the board to show that the height of a prism is the perpendicular distance between the bases.
learning style: visual

Below Level L2
Before students work through Example 4, have them draw and label the cylinder used for the top of the backpack. This will clarify the formula in Step 3.
learning style: visual

Both stacks of paper below contain the same number of sheets.



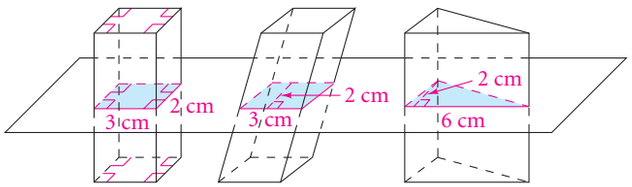
The first stack forms a right prism. The second forms an oblique prism. The stacks have the same height. The area of every cross section parallel to a base is the area of one sheet of paper. The stacks have the same volume. These stacks illustrate the following principle.

Key Concepts

Theorem 11-5 **Cavalieri's Principle**

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

The area of each shaded cross section below is 6 cm^2. Since the prisms have the same height, their volumes must be the same by Cavalieri's Principle.



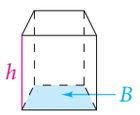
You can find the volume of a right prism by multiplying the area of the base by the height. Cavalieri's Principle lets you extend this idea to any prism.

Key Concepts

Theorem 11-6 **Volume of a Prism**

The volume of a prism is the product of the area of a base and the height of the prism.

$V = Bh$



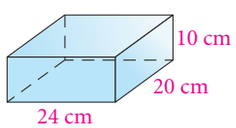
Online active math

For: Prism, Cylinder Activity
Use: Interactive Textbook, 11-4

1 EXAMPLE **Finding Volume of a Rectangular Prism**

Find the volume of the prism at the right.

$V = Bh$ **Use the formula for volume.**
$= 480 \cdot 10$ **$B = 24 \cdot 20 = 480$ cm^2**
$= 4800$ **Simplify.**



The volume of the rectangular prism is 4800 cm^3.

Quick Check

1 **Critical Thinking** Suppose the prism in Example 1 is turned so that the base is 20 cm by 10 cm and the height is 24 cm. Explain why the volume does not change.
Answers may vary. Sample: Multiplication is commutative.

Advanced Learners L4
Have students investigate how doubling the radius, diameter, or height affects the volume of a cylinder.
The volume increases by a factor of 4, 16, or 2.
learning style: verbal

English Language Learners ELL
Use a stack of index cards or coins to explain the term *cross section* and to illustrate Cavalieri's Principle. The coin model will help students see that this principle applies to cylinders as well as to prisms.
learning style: visual

2. Teach

Guided Instruction

Hands-On Activity

If you do not have enough cubes for each student, demonstrate the investigation, or have students use the isometric drawing techniques that they learned in Lesson 1-2 to simulate the activity.

Visual Learners

Illustrate *a cross section parallel to a base* as you discuss Cavalieri's Principle by removing a sheet from a stack of paper.

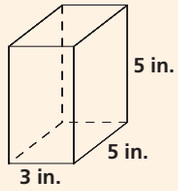
2 EXAMPLE **Error Prevention**

Students may have trouble identifying the height of a prism when its base is not horizontal. Remind them that height is the measure of an altitude perpendicular to a base.

PowerPoint

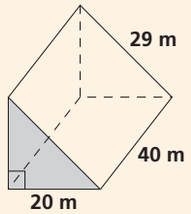
Additional Examples

1 Find the volume of the prism.



75 $in.^3$

2 Find the volume of the prism.



8400 m^3

Guided Instruction

Auditory Learners

Have students explain aloud why the formula for the volume of a prism is similar to the formula for the volume of a cylinder.

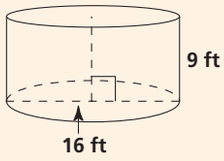
4 EXAMPLE **Math Tip**

Ask: *Why is the height of the prism 11 in.?* **The backpack's top is half of a cylinder with diameter 12 in., so the radius of the base is 6 in. The height of the prism is 17 in. − 6 in. = 11 in.**

PowerPoint

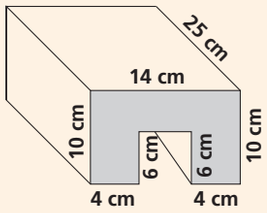
Additional Examples

3 Find the volume of the cylinder. Leave your answer in terms of π.



576π ft^3

4 Find the volume of the composite space figure.



2600 cm^3

Resources

- Daily Notetaking Guide 11-4 L3
- Daily Notetaking Guide 11-4—Adapted Instruction L1

Closure

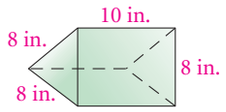
Ask students to solve the following exercise. A cube with 10-in. edges contains a cylinder 10 in. high. The cylinder's lateral surface touches four faces of the cube. Find the volume of the space between the cube and the cylinder to the nearest whole number. **215 in.^{3}**

2 EXAMPLE Finding Volume of a Triangular Prism

Test-Taking Tip

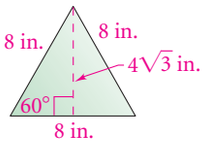
A volume question often requires you to find a base of a solid. A base does not have to be at the bottom (or top) of the solid.

Multiple Choice Find the approximate volume of the triangular prism at the right.



(A) 188 in.^{3} (B) 277 in.^{3}
(C) 295 in.^{3} (D) 554 in.^{3}

Each base of the triangular prism is an equilateral triangle. An altitude of the triangle divides it into two 30°-60°-90° triangles. The area of the base is $\frac{1}{2} \cdot 8 \cdot 4\sqrt{3}$, or $16\sqrt{3}$ in.^{2}.

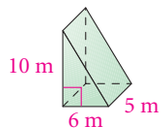


$V = Bh$ **Use the formula for the volume of a prism.**
$= 16\sqrt{3} \cdot 10$ **Substitute.**
$= 160\sqrt{3}$ **Simplify.**
$= 277.12813$ **Use a calculator.**

- The volume of the triangular prism is about 277 in.^{3}. The answer is B.

Quick Check

2 Find the volume of the triangular prism at the right.
150 m^3



2 Finding Volume of a Cylinder

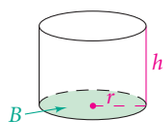
To find the volume of a cylinder, you use the same formula $V = Bh$ that you use to find the volume of a prism. Now, however, B is the area of the circle, so you use the formula $B = \pi r^2$ to find its value.

Key Concepts

Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$V = Bh$, or $V = \pi r^2 h$

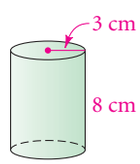


GO Online

Video Tutor Help
Visit: PHSchool.com
Web Code: aue-0775

3 EXAMPLE Finding Volume of a Cylinder

Find the volume of the cylinder at the right. Leave your answer in terms of π.

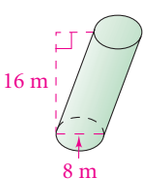


$V = \pi r^2 h$ **Use the formula for the volume of a cylinder.**
$= \pi(3)^2(8)$ **Substitute.**
$= \pi(72)$ **Simplify.**

- The volume of the cylinder is 72π cm^3.

Quick Check

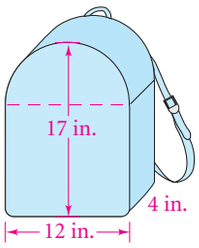
3 The cylinder at the right is oblique.
a. Find its volume in terms of π. **256π m^3**
b. Find its volume to the nearest tenth of a cubic meter. **804.2 m^3**



A **composite space figure** is a three-dimensional figure that is the combination of two or more simpler figures. A space probe, for example, might begin as a composite figure—a cylindrical rocket engine in combination with a nose cone.

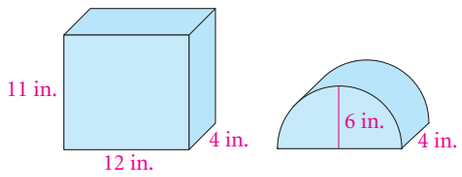
You can find the volume of a composite space figure by adding the volumes of the figures that are combined.

4 EXAMPLE Finding Volume of a Composite Figure



Estimation Use a composite space figure to estimate the volume of the backpack shown at the left.

Step 1: You can use a prism and half of a cylinder to approximate the shape, and therefore the volume, of the backpack.



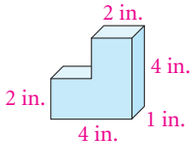
Step 2: Volume of the prism $= Bh = (12 \cdot 4)11 = 528$

Step 3: Volume of the half cylinder $= \frac{1}{2}(\pi r^2 h) = \frac{1}{2}\pi(6)^2(4)$
$= \frac{1}{2}\pi(36)(4) \approx 226$

Step 4: Sum of the two volumes $= 528 + 226 = 754$

The approximate volume of the backpack is 754 in.^{3}.

Quick Check 4 Find the volume of the composite space figure. **12 in.^{3}**



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

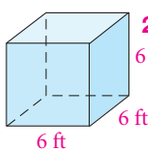
Practice and Problem Solving

A Practice by Example

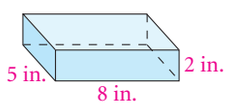
Example 1 (page 625)

GO for Help

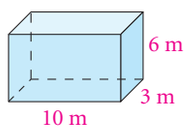
In Exercises 1–8, find the volume of each prism.

1. 

216 ft^3

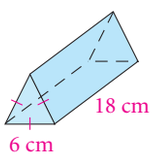
2. 

80 in.^{3}

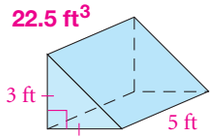
3. **180 m^3**


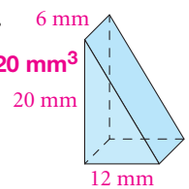
4. The base is a square, 2 cm on a side. The height is 3.5 cm. **14 cm^3**

Example 2 (page 626)

5. 

about 280.6 cm^3

6. **22.5 ft^3**


7. 

8. The base is a 45°-45°-90° triangle with a leg of 5 in. The height is 1.8 in. **22.5 in.^{3}**

3. Practice

Assignment Guide

1	A B	1-8, 14, 16, 18-22, 24, 25, 29, 36
2	A B	9-13, 15, 17, 12, 26-28, 30-35
C	Challenge	37-40
	Test Prep	41-45
	Mixed Review	46-51

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 10, 12, 18, 24, 29.

Connection to Algebra

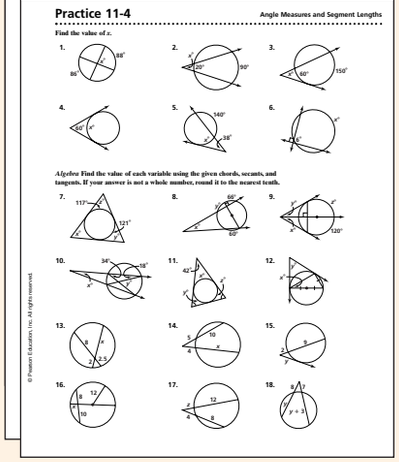
Exercises 1–11 Use these exercises to assess whether students substitute correctly for variables.

Alternative Method

Exercise 12 This figure is a prism whose vertical bases are a combination of shapes. Ask: *Which letter best describes the shape of the base?* L Have students use the area of this base to find the volume of the prism.

Differentiated Instruction Resources

- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3


Practice 11-4 Angle Measures and Segment Lengths

Exercise 14 Discuss why the weights of fluids and gases are given per unit of volume.

Exercise 16 Remind students that polygons with equal areas need not have equal perimeters. Similarly, space figures with equal volumes need not have equal surface areas.

Connection to Ecology

Exercise 21 Have students investigate how plants can improve the quality of indoor air.

Error Prevention!

Exercise 23 Remind students who multiply by 12 to convert cubic feet to cubic inches that ft^3 means ft · ft · ft, so 1 ft^3 = 12 in. · 12 in. · 12 in. or 1728 $in.^3$

Exercise 28 Some students may incorrectly substitute the 9-in. diameter instead of the 4.5-in. radius in $V = \pi r^2 h$.

Example 3 (page 626)

Find the volume of each cylinder in terms of π and to the nearest tenth.

9. 8 in., 6 in. — **288π $in.^3$, 904.8 $in.^3$**

10. 4 cm, 10 cm — **40π cm^3, 125.7 cm^3**

11. 5 m, 6 m — **37.5π m^3, 117.8 m^3**

Example 4 (page 627)

Find the volume of each composite space figure to the nearest whole number.

12. 2 cm, 3 cm, 4 cm, 2 cm, 8 cm, 6 cm — **144 cm^3**

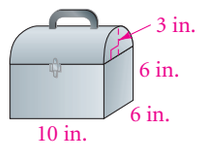
13. 10 in., 12 in., 24 in. — **3445 $in.^3$**

B Apply Your Skills

14. a. What is the volume of a waterbed mattress that is 7 ft by 4 ft by 1 ft? **28 ft^3**

b. To the nearest pound, what is the weight of the water in a full mattress? (Water weighs 62.4 lb/ft^3.) **1747 lb**

15. Find the volume of the lunch box shown at the right to the nearest cubic inch. **501 $in.^3$**



16. Open-Ended Give the dimensions of two rectangular prisms that have volumes of 80 cm^3 each but also have different surface areas.
Answers may vary. Sample: 2 cm by 4 cm by 10 cm; 4 cm by 4 cm by 5 cm

Find the height of each figure with the given volume.

17. h, 9 cm — **$\frac{26}{9}$ cm**; $V = 234\pi$ cm^3

18. h, 5 in., 5 in. — **5 in.**; $V = 125$ $in.^3$

19. 3 ft, h — **6 ft**; $V = 27$ ft^3

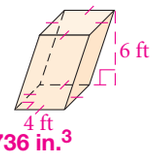
Real-World Connection

Careers An ecologist studies living organisms and their environments.

20. Ecology The isolation cube at the left measures 27 in. on each side. What is its volume in cubic feet? **19,683 ft^3**

21. Environmental Engineering A scientist suggests keeping indoor air relatively clean as follows: Provide two or three pots of flowers for every 100 square feet of floor space under a ceiling of 8 feet. If your classroom has an 8-ft ceiling and measures 35 ft by 40 ft, how many pots of flowers should it have? **28–42 pots**

22. Find the volume of the oblique prism pictured at the right. **96 ft^3**



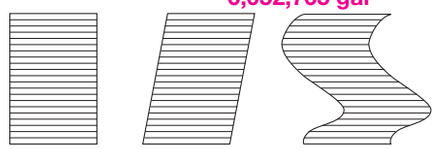
23. Tank Capacity The main tank at an aquarium is a cylinder with diameter 203 ft and height 25 ft. **809,137 ft^3**

a. Find the volume of the tank to the nearest cubic foot.

b. Convert your answer to part (a) to cubic inches. **1,398,188,736 $in.^3$**

c. If 1 gallon ≈ 231 $in.^3$, about how many gallons does the tank hold? **6,052,765 gal**

24. Writing The figures at the right can be covered by equal numbers of straws that are the same length. Describe how Cavalieri's Principle could be adapted to compare the areas of these figures. **Answers may vary. Sample: "If two plane figures have the same height and the same width at every level, then they have the same area."**



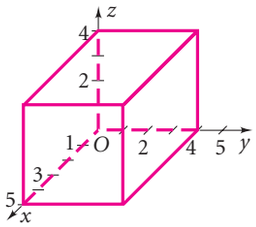
GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-1104

628

Problem Solving Hint

In Exercise 25, find the length, width, and height along the axes.

25. Coordinate Geometry Find the volume of the rectangular prism at the right. **80 units^3**

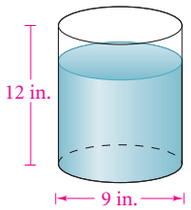


26. The volume of a cylinder is 600π cm^3. The radius of a base of the cylinder is 5 cm. What is the height of the cylinder? **24 cm**

27. The volume of a cylinder is 135π cm^3. The height of the cylinder is 15 cm. What is the radius of a base of the cylinder? **3 cm**

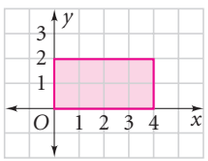
28. Multiple Choice A cylindrical water tank has a diameter of 9 inches and a height of 12 inches. The water surface is 2.5 inches from the top. About how much water is in the tank? **A**

(A) 604 in.^{3} (B) 636 in.^{3}
(C) 668 in.^{3} (D) 763 in.^{3}



29. Landscaping To landscape her 70 ft-by-60 ft rectangular backyard, Joy is planning first to put down a 4-in. layer of topsoil. She can buy bags of topsoil at \$2.50 per 3-ft^3 bag, with free delivery. Or, she can buy bulk topsoil for \$22.00/yd^3, plus a \$20 delivery fee. Which option is less expensive? Explain. **See left.**

Visualization **The plane region is revolved completely about the given line to sweep out a solid of revolution. Describe the solid and find its volume in terms of π.** **30–33. See left.**

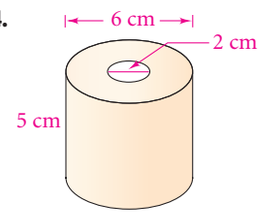


30. the x-axis **31.** the y-axis

32. the line $y = 2$ **33.** the line $x = 5$

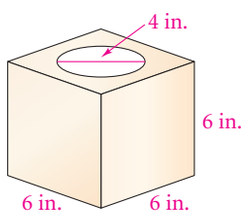
A cylinder has been cut out of each solid. Find the volume of the remaining solid. Round your answer to the nearest tenth.

34.



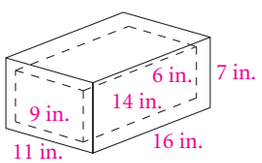
125.7 cm^3

35.



140.6 in.^{3}

36. A closed box is 9 in. by 14 in. by 6 in. on the inside and 11 in. by 16 in. by 7 in. on the outside. Find each measurement.



a. the outside surface area **730 in.^{2}**
b. the inside surface area **528 in.^{2}**
c. the inside volume **756 in.^{3}**
d. the volume of the material needed to make the box **476 in.^{3}**

37. Any rectangular sheet of paper can be rolled into a right cylinder in two ways.
a. Use ordinary sheets of paper to model the two cylinders. Compute the volume of each cylinder. How do they compare? **a–b. See left.**
b. Of all sheets of paper with perimeter 39 in., which size can be rolled into a right cylinder with greatest volume? (*Hint:* See Activity Lab, page 616.)

Challenge

29. Bulk; cost of bags = \$1167.50, cost of bulk is ≈\$1164.

30. cylinder with $r = 2$ and $h = 4$; 16π units^3

31. cylinder with $r = 4$ and $h = 2$; 32π units^3

32. cylinder with $r = 2$ and $h = 4$; 16π units^3

33. cylinder with $r = 5$, $h = 2$, and a hole of radius 1; 48π units^3

37a. circumference $8\frac{1}{2}$ in. and height 11 in.: $V \approx 63.2\pi$ in.^{3}; circumference 11 in. and height $8\frac{1}{2}$ in.: $V \approx 81.8\pi$ in.^{3}; one is about 0.8 times the volume of the other.

b. about 6.5 in. by 13.0 in.

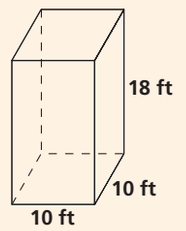
4. Assess & Reteach

PowerPoint

Lesson Quiz

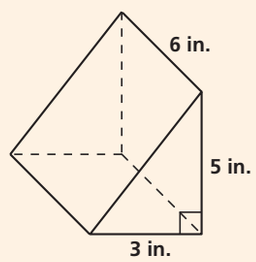
Find the volume of each figure to the nearest whole number.

1.



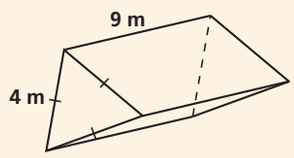
1800 ft^3

2.



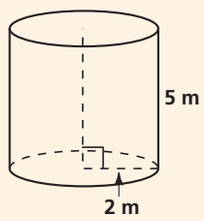
45 in.^{3}

3.



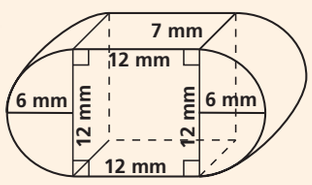
62 m^3

4.



63 m^3

5.



1800 mm^3

Alternative Assessment

Have each student bring in one cylindrical food container and one shaped like a prism. Distribute one cylinder and one prism to each student, and have them calculate the volume of each container and explain their calculations.

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 657
- Test-Taking Strategies, p. 652
- Test-Taking Strategies with Transparencies

38. The outside diameter of a pipe is 5 cm. The inside diameter is 4 cm. The pipe is 4 m long. What is the volume of the material used for this length of pipe? Round your answer to the nearest cubic centimeter. **2827 cm^3**

39. A cube has a volume of $2M$ cubic units and a total surface area of $3M$ square units. Find the length of an edge of the cube. **4 units**

40. The radius of cylinder B is twice the radius of cylinder A. The height of cylinder B is half the height of cylinder A. Compare their volumes.
The volume of *B* is twice the volume of *A*.

Test Prep

Multiple Choice

41. What is the volume of a rectangular prism whose edges measure 2 ft, 2 ft, and 3 ft? **B**
A. 7 ft^3 **B.** 12 ft^3 **C.** 14 ft^3 **D.** 16 ft^3

42. One gallon fills about 231 in.^{3}. A right cylindrical carton is 12 in. tall and holds 9 gal when full. Find the radius of the carton to the nearest tenth of an inch. **G**
F. 0.5 in. **G.** 7.4 in. **H.** 37.7 in. **J.** 55.1 in.

43. The height of a triangular prism is 8 feet. One side of the base measures 6 feet. What additional information do you need to find the volume? **C**
A. the perimeter of the base
B. the length of a second side of the base
C. the altitude of the base to the 6-foot side
D. the area of each rectangular face of the prism

44. A rectangular prism has a volume of 100 ft^3. If the base measures 5 ft by 8 ft, what is the height of the prism? **F**
F. 2.5 ft **G.** 12.5 ft **H.** 20 ft **J.** 40 ft

Short Response

45. How is the formula for finding the lateral area of a cylinder like the formula for finding the area of a rectangle? **See margin.**

Mixed Review

Lesson 11-3

GO for Help

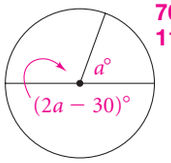
Find the lateral area of each figure to the nearest tenth.

46. a right circular cone with height 12 mm and radius 5 mm **204.2 mm^2**

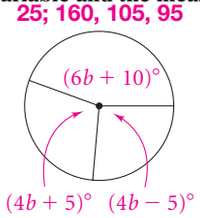
47. a regular hexagonal pyramid with base edges 9.2 ft long and slant height 17 ft **469.2 ft^2**

Lesson 10-6 x^2 **Algebra** **Find the value of each variable and the measure of each labeled angle.**

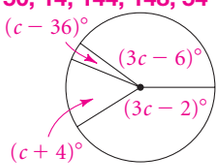
48. **70; 110, 70**



49. **25; 160, 105, 95**



50. **50; 14, 144, 148, 54**



Lesson 7-3

51. You want to find the height of a tree near your school. Your shadow is three-fourths of your height. The tree's shadow is 57 feet. How tall is the tree? **76 ft**

45. **[2] L.A. = $2\pi rh$ and $A = bh$; $2\pi r$ is the length of the base when the cylinder is unwrapped.**

[1] correct formulas are given, but comparison is unclear