1. Plan

Objectives
1 To find relationships between the ratios of the areas and volumes of similar solids

Examples
1 Identifying Similar Solids
2 Finding the Similarity Ratio
3 Using a Similarity Ratio
4 Real-World Connection

Professional Development

Math Background

The surface areas and volumes of similar solids are directly proportional, respectively, to the square and cube of their similarity ratios. The weight of a prism is twice the weight of a similar prism made of the same material when the ratio of corresponding lengths is $\sqrt[3]{2} : 1 \approx 5 : 4$.

More Math Background: p. 596D

Lesson Planning and Resources

See p. 596E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

✓ **Check Skills You'll Need**
For intervention, direct students to:

Determining Similarity
Lesson 7-2: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 7

Finding Volumes of Prisms and Cylinders
Lesson 11-4: Examples 1, 3
Extra Skills, Word Problems, Proof Practice, Ch. 11

11-7

Areas and Volumes of Similar Solids

What You'll Learn
• To find relationships between the ratios of the areas and volumes of similar solids

. . . And Why
To use similarity ratios to find the weight of an object, as in Example 4

✓ **Check Skills You'll Need**   **GO for Help** Lessons 8-2 and 11-4

Are the figures similar? Explain. Include the similarity ratio as appropriate.

1. two squares, one with 3-in. sides and the other with 1-in. sides **See back of book.**
2. two isosceles right triangles, one with a 3-cm hypotenuse and the other with a 1-cm leg **Yes; all corr. ∠ are ≅ and corr. sides are prop.; 3 : √2.**

Find the volume of each space figure.

3. a cube with a 3-in. edge **27 in.³**
4. a 3 m-by-5 m-by-9 m rectangular prism **135 m³**
5. a cylinder with radius 4 cm and height 8 cm **about 402.1 cm³**

🔊 **New Vocabulary** • similar solids • similarity ratio

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1 Finding Relationships in Area and Volume

Similar solids have the same shape, and all their corresponding dimensions are proportional. The ratio of corresponding linear dimensions of two similar solids is the **similarity ratio.** Any two cubes are similar, as are any two spheres.
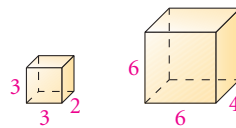
1 EXAMPLE **Identifying Similar Solids**

Are the two rectangular prisms similar? If so, give the similarity ratio.

a.

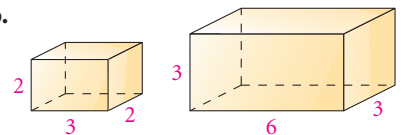$\frac{3}{6} = \frac{3}{6} = \frac{2}{4}$

The rectangular prisms are similar because the ratios of the corresponding linear dimensions are equal.
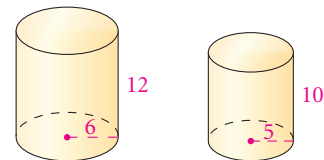
The similarity ratio is $\frac{1}{2}$.

b.

$\frac{2}{3} \neq \frac{3}{6}$

The rectangular prisms are not similar because the ratios of corresponding linear dimensions are not equal.

Real-World Connection

These Russian nesting dolls suggest similar solids.

✓ **Quick Check** ❶ Are the two cylinders similar? If so, give the similarity ratio.
yes; 6 : 5

Differentiated Instruction **Solutions for All Learners**

Special Needs **L1**
Note that *similar solids* have the same shape and proportional size, or corresponding dimensions are proportional. Have students draw the solids studied in Chapter 11 and list the conditions for two of each solid to be similar.
learning style: visual

Below Level **L2**
Bring in sample 8-oz. and 26-oz. cans of tomato sauce. Have students calculate the similarity ratio, and measure the volumes. The ratio of the volumes will illustrate Theorem 11-12 (2).
**learning style: visual**

The two similar prisms shown here suggest two important relationships for similar solids.

The ratio of the side lengths is 1 : 2.
The ratio of the surface areas is 22 : 88, or 1 : 4.
The ratio of the volumes is 6 : 48, or 1 : 8.

The ratio of the surface areas equals the square of the similarity ratio. The ratio of the volumes equals the cube of the similarity ratio. These two facts apply to all similar solids.
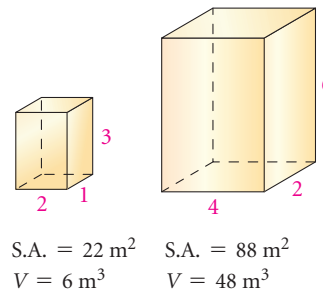
S.A. = 22 m^2 S.A. = 88 m^2
V = 6 m^3 V = 48 m^3

Key Concepts

Theorem 11-12	Areas and Volumes of Similar Solids

If the similarity ratio of two similar solids is $a : b$, then
(1) the ratio of their corresponding areas is $a^2 : b^2$, and
(2) the ratio of their volumes is $a^3 : b^3$.

2 EXAMPLE Finding the Similarity Ratio

Find the similarity ratio of two cubes with volumes of 729 cm^3 and 1331 cm^3.

$\dfrac{a^3}{b^3} = \dfrac{729}{1331}$ **The ratio of the volumes is $a^3 : b^3$.**

$\dfrac{a}{b} = \dfrac{9}{11}$ **Take cube roots.**

● The similarity ratio is 9 : 11.

Quick Check **2** Find the similarity ratio of two similar prisms with surface areas 144 m^2 and 324 m^2.
2 : 3

3 EXAMPLE Using a Similarity Ratio

Paint Cans The lateral areas of two similar paint cans are 1019 cm^2 and 425 cm^2. The volume of the small can is 1157 cm^3. Find the volume of the large can.

First find the similarity ratio $a : b$.

$\dfrac{a^2}{b^2} = \dfrac{1019}{425}$ **The ratio of the surface areas is $a^2 : b^2$.**

$\dfrac{a}{b} = \dfrac{\sqrt{1019}}{\sqrt{425}}$ **Take square roots.**

Use the similarity ratio to find the volume.

$\dfrac{V_{large}}{V_{small}} = \dfrac{\sqrt{1019}^3}{\sqrt{425}^3}$ **The ratio of the volumes is $a^3 : b^3$.**

$\dfrac{V_{large}}{1157} = \dfrac{\sqrt{1019}^3}{\sqrt{425}^3}$ **Substitute 1157 for V_{small}.**

$V_{large} = 1157 \cdot \dfrac{\sqrt{1019}^3}{\sqrt{425}^3}$ **Solve for V_{large}.**

$V_{large} \approx 4295$ **Use a calculator.**

● The volume of the large paint can is about 4295 cm^3.

Vocabulary Tip

The <u>subscripts</u> "large" and "small" let you use the letter V for the volumes of both the large can (V_{large}) and small can (V_{small}).

Quick Check **3** The volumes of two similar solids are 128 m^3 and 250 m^3. The surface area of the larger solid is 250 m^2. What is the surface area of the smaller solid? **160 m^2**

Lesson 11-7   Areas and Volumes of Similar Solids   **647**

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2. Teach

Guided Instruction

Teaching Tip
Remind students that when the similarity ratio of two similar plane figures is $a : b$, the ratio of their perimeters is $a : b$ and the ratio of their areas is $a^2 : b^2$.

Tactile Learners
Students can build similar rectangular prisms with cubes to investigate the Areas and Volumes of Similar Solids Theorem.

Auditory Learners
Have students suggest ways to remember whether the ratio of surface areas or volumes is the square of or the cube of the similarity ratio.

2 EXAMPLE **Connection to Algebra**

Students are accustomed to solving an equation for a single variable but not for a ratio. Point out that only the ratio $\frac{a}{b}$ is needed, not a value of a or b.

PowerPoint
Additional Examples

1 Are the two solids similar? If so, give the similarity ratio.

26 in.
8 in.
3 in.
9 in.

no

2 Find the similarity ratio of two similar cylinders with surface areas of 98π ft^2 and 2π ft^2. **7 : 1**

3 Two similar square pyramids have volumes of 48 cm^3 and 162 cm^3. The surface area of the larger pyramid is 135 cm^2. Find the surface area of the smaller pyramid. **60 cm^2**

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The weights of solid objects made of the same material are proportional to their volumes.

4 EXAMPLE **Real-World Connection**

Paperweights A marble paperweight shaped like a pyramid weighs 0.15 lb. How much does a similarly shaped marble paperweight weigh if each dimension is three times as large?

The similarity ratio is 1 : 3. The ratio of the volumes, and hence the ratio of the weights, is $1^3 : 3^3$, or 1 : 27.

$\frac{1}{27} = \frac{0.15}{x}$ **Let x = the weight of the larger paperweight.**

$x = 27(0.15)$ **Use the Cross Product Property.**

$x = 4.05$

● The larger paperweight weighs about 4 lb.

✓ Quick Check **4** Find the weight of a marble bead that is similar to the paperweight in Example 4 but has dimensions half as large. **0.01875 lb**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*
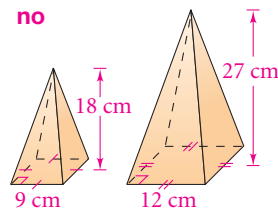
Practice and Problem Solving
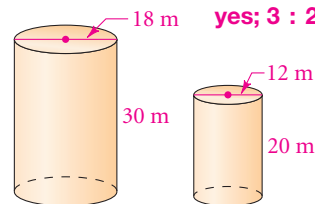
A **Practice by Example**

Example 1 (page 646)

GO for Help

Are the two figures similar? If so, give the similarity ratio.

1. no

18 cm, 27 cm, 9 cm, 12 cm

2. yes; 3 : 2

18 m, 12 m, 30 m, 20 m

3.

8 in., 6 in., 4 in., 6 in., 12 in., 9 in.

yes; 2 : 3

4. no

16 ft, 6 ft, 12 ft, 4 ft
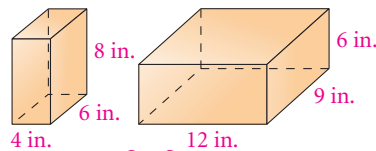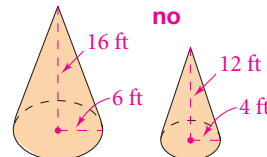
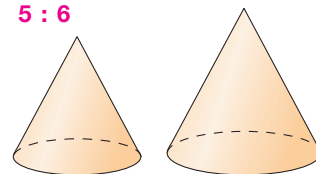5. two cubes, one with 3-cm edges, the other with 4.5-cm edges **yes; 2 : 3**

6. a cylinder and a square prism each with 3-in. radii and 1-in. heights **no**
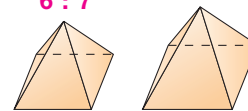
Example 2 (page 647)

Each pair of figures is similar. Use the given information to find the similarity ratio of the smaller figure to the larger figure.

7. 5 : 6
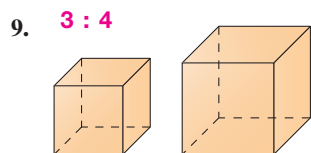

$V = 250\pi$ ft³ $V = 432\pi$ ft³

8. **6 : 7**
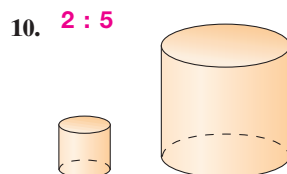

$V = 216$ in.³ $V = 343$ in.³

9. 3 : 4

10. 2 : 5

S.A. = 18 m²   S.A. = 32 m²

S.A. = 20π yd²   S.A. = 125π yd²

Example 3
(page 647)

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

11. S.A. = 18 in.²
S.A. = 98 in.²
$V = 343$ in.³
27 in.³

12. S.A. = 192 m²
S.A. = 1728 m²
$V = 4860$ m³
180 m³

13. S.A. = 52 ft²
S.A. = 208 ft²
$V = 192$ ft³
24 ft³

The volumes of two similar figures are given. The surface area of the smaller figure is given. Find the surface area of the larger figure.

14. $V = 27$ in.³
$V = 125$ in.³
S.A. = 63 in.²
175 in.²

15. $V = 5$ m³
$V = 40$ m³
S.A. = 4 m²
16 m²

16. $V = 54$ yd³
$V = 128$ yd³
S.A. = 18 yd²
32 yd²

Example 4
(page 648)

17. Packaging There are 750 toothpicks in a regular-sized box. If a jumbo box is made by doubling all the dimensions of the regular-sized box, how many toothpicks will the jumbo box hold? **6000 toothpicks**
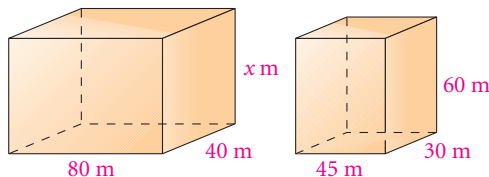
18. Packaging A cylinder 4 in. in diameter and 6 in. high holds 1 lb of oatmeal. To the nearest ounce, how much oatmeal will a similar 10-in.-high cylinder hold? (*Hint:* 1 lb = 16 oz) **74 oz**

19a. It is 64 times the smaller prism.
b. It is 64 times the smaller prism.

19. A regular pentagonal solid prism has 9-cm base edges. A larger, similar solid prism of the same material has 36-cm base edges. How does each indicated measurement for the larger prism compare to the same measurement for the smaller prism? **a–b. See left.**
a. the volume
b. the weight

B   **Apply Your Skills**

20. Two similar prisms have heights 4 cm and 10 cm.
a. What is their similarity ratio? **2 : 5**
b. What is the ratio of their surface areas? **4 : 25**
c. What is the ratio of their volumes? **8 : 125**

21. Atomic Clock A company announced that it had developed the technology to reduce the size of its atomic clock, which is used in electronic devices that transmit data. The company claims that the smaller clock will be similar to the existing clock made of the same material. It will be $\frac{1}{10}$ the size of its existing atomic clocks and $\frac{1}{100}$ the weight. Do these ratios make sense? Explain. **See margin.**

22. Is there a value of x for which the rectangular prisms below are similar? Explain. **Yes; 60;** $\frac{80}{60} = \frac{40}{30} = \frac{60}{45} = \frac{4}{3}$.

x m   60 m
40 m   30 m
80 m   45 m

Real-World **Connection**

The time scale of an atomic clock is based on vibrations of atoms and molecules.

23. The volume of a spherical balloon with radius 3.1 cm is about 125 cm³. Estimate the volume of a similar balloon with radius 6 cm. **about 1000 cm³**

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21. **No; explanations may vary. Sample: If "size" refers to the vol., then the new clock should be at $\frac{1}{10}$ the weight.**

Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 2, 18, 22, 25, 29.

Error Prevention!

Exercises 7–10 Students may forget to take the square or cube root to find the similarity ratio. Suggest that they write the measurement units as part of the ratio.

Exercise 22 Do this exercise as a class to help correct the common misconception that similar prisms must be shown in the same orientation.

Connection to Biology

Exercise 35 The Areas and Volumes of Similar Solids Theorem has dramatic implications for the scale and size of animals. Recommend that interested students read J.B.S. Haldane's classic essay *On Being the Right Size.*

Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3

Lesson Quiz

For Exercises 1 and 2, are the two solids similar? If so, give the similarity ratio of the smaller figure to the larger figure.

1.

10 in.

5 in.

5 in.   10 in.

no

2.

4 cm   6 cm
8 cm

6 cm
12 cm   9 cm
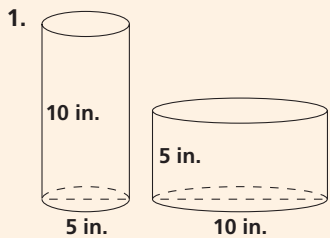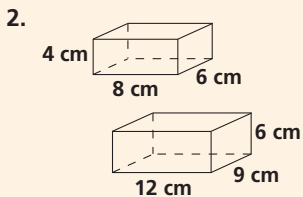
yes; 2 : 3

3. Find the similarity ratio of two spheres with volumes of 20π m^3 and 160π m^3. **1 : 2**

4. The volumes of two similar solids are 54 ft^3 and 250 ft^3. The surface area of the smaller solid is 45 ft^2. Find the surface area of the larger solid. **125 ft^2**

5. A solid chocolate rabbit is 6 in. high and weighs 0.25 lb. A similar chocolate rabbit is 12 in. high. How much does it weigh? **2 lb**

Alternative Assessment

Assign students to groups of three or four. Have each student find one set of possible dimensions for two similar rectangular prisms whose volumes are 288 cm^3 and 972 cm^3, draw the prisms, and list the similarity ratio, ratio of surface areas, and ratio of volumes. Students then should compare their drawings to see the number of different figures that are represented.

24. No; an increase in the lengths of sides does not create prop. ratios unless the box is a cube.

25. Answers may vary. Sample: A sphere has only one measure, **r,** so there's only one possible ratio.

C   Challenge

35c. His weight is 1000 times the weight of an average person, but his bones can only support 600 times the weight.

24. Critical Thinking A carpenter is making a blanket chest based on an antique chest. Both chests have the shape of a rectangular prism. The length, width, and height of the new chest will all be 4 in. greater than the respective dimensions of the antique. Will the chests be similar? Explain. **See left.**

25. Writing Explain why all spheres are similar. **See left.**

26. Two similar pyramids have lateral area 8 ft^2 and 18 ft^2. The volume of the smaller pyramid is 32 ft^3. Find the volume of the larger pyramid. **108 ft^3**

27. The volumes of two spheres are 729 in.^{3} and 27 in.^{3}.
  a. Find the ratio of their radii.   **3 : 1**
  b. Find the ratio of their surface areas.   **9 : 1**

28. The volumes of two similar pyramids are 1331 cm^3 and 2744 cm^3.
  a. Find the ratio of their heights.   **11 : 14**
  b. Find the ratio of their surface areas.   **121 : 196**

29. A clown's face on a balloon is 4 in. high when the balloon holds 108 in.^{3} of air.
GPS How much air must the balloon hold for the face to be 8 in. high?   **864 in.^{3}**

Copy and complete the table for the similar solids.

	Similarity Ratio	Ratio of Surface Areas	Ratio of Volumes	
30.	1 : 2	■ : ■	■ : ■	**1 : 4; 1 : 8**
31.	3 : 5	■ : ■	■ : ■	**9 : 25; 27 : 125**
32.	■ : ■	49 : 81	■ : ■	**7 : 9; 343 : 729**
33.	■ : ■	■ : ■	125 : 512	**5 : 8; 25 : 64**

34. Literature In *Gulliver's Travels* by Jonathan Swift, Gulliver first traveled to Lilliput. The Lilliputian average height was one twelfth of Gulliver's height.
  a. How many Lilliputian coats could be made from the material in Gulliver's coat? *(Hint: Use the ratio of surface areas.)* **144 coats**
  b. How many Lilliputian meals would be needed to make a meal for Gulliver? *(Hint: Use the ratio of volumes.)* **1728 meals**

35. Indirect Reasoning Some stories say that Paul Bunyan was ten times as tall as the average human. Assume that Paul Bunyan's bone structure was proportional to that of ordinary people.
  a. Strength of bones is proportional to the area of their cross section. How many times as strong as the average person's bones would Paul Bunyan's bones be? **100 times**
  b. Weights of objects made of like material are proportional to their volumes. How many times the average person's weight would Paul Bunyan's weight be? **1000 times**
  c. Human leg bones can support about 6 times the average person's weight. Use your answers to parts (a) and (b) to explain why Paul Bunyan could not exist with a bone structure that was proportional to that of ordinary people.

36. Square pyramids A and B are similar. In pyramid A, each base edge is 12 cm. In pyramid B, each base edge is 3 cm and the volume is 6 cm^3.
  a. Find the volume of pyramid A. **384 cm^3**
  b. Find the ratio of the surface area of A to the surface area of B. **16 : 1**
  c. Find the surface area of each pyramid.
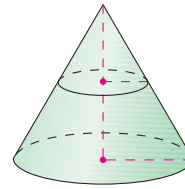   **pyramid A: 384 cm^2; pyramid B: 24 cm^2**

GO **for Help**

A frustum of a cone is defined on page 636.

37. The cone is cut by a plane parallel to its base. The small cone on top is similar to the large cone. The ratio of the slant heights of the cones is 1 : 2. Find the ratio indicated.
  a. the surface area of the large cone to that of the small cone; the volume of the large cone to that of the small cone **4 : 1; 8 : 1**
  b. the surface area of the frustum to that of the large cone; to that of the small cone **See left.**
  c. the volume of the frustum to that of the large cone; to that of the small cone **7 : 8; 7 : 1**

Test Prep

Gridded Response

38. The slant heights of two similar pyramids are in the ratio 1 : 5. The volume of the smaller pyramid is 60 m^3. What is the volume in cubic meters of the larger pyramid? **7500**

39. A lawn chair weighs 8 lb. A child's lawn chair has dimensions exactly one half those of the larger chair. How many pounds does the child's chair weigh? **1**

40. A model of a historical home has dimensions that are one fifteenth the dimensions of the actual home. The area of a window in the model is 2 cm^2. What is the area in square centimeters of the corresponding window in the actual home? **450**

41. The volumes of two similar rectangular prisms are 64 cm^3 and 1000 cm^3. The surface area of the smaller figure is 112 cm^2. What is the surface area in square centimeters of the larger figure? **700**

42. The surface areas of two similar cylinders are 54 ft^2 and 96 ft^2. The volume of the smaller cylinder is 216 ft^3. What is the volume in cubic feet of the larger cylinder? **512**

Mixed Review

GO **for Help**

Lesson 11-6

43. Sports Equipment The circumference of a regulation basketball is between 75 cm and 78 cm. What are the smallest and the largest surface areas that a basketball can have? Give your answers to the nearest whole unit.
  1790 cm^2 and 1937 cm^2

Find the volume and surface area of each sphere to the nearest tenth.

44. diameter = 6 in.
113.1 in.^{3}; 113.1 in.^{2}

45. circumference = 2.5π m
8.2 m^3; 19.6 m^2

46. radius = 6 in.
904.8 in.^{3}; 452.4 in.^{2}

Lesson 7-4

47. The altitude to the hypotenuse of a right triangle ABC divides the hypotenuse into 12-mm and 16-mm segments. Find the length of each of the following.
  a. the altitude to the hypotenuse **$8\sqrt{3}$ mm**
  b. the shorter leg of $\triangle ABC$ **$4\sqrt{21}$ mm**
  c. the longer leg of $\triangle ABC$ **$8\sqrt{7}$ mm**

Lesson 7-1 x^2 **Algebra** Solve each proportion.

48. $\frac{25}{16} = \frac{x}{16}$ **25**

49. $\frac{21}{x} = \frac{8}{5}$ **13.125**

50. $\frac{3}{8} = \frac{n}{n+4}$ **2.4**

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 657
- Test-Taking Strategies, p. 652
- Test-Taking Strategies with Transparencies