

# 11 FLUID STATICS



**Figure 11.1** The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

## Learning Objectives

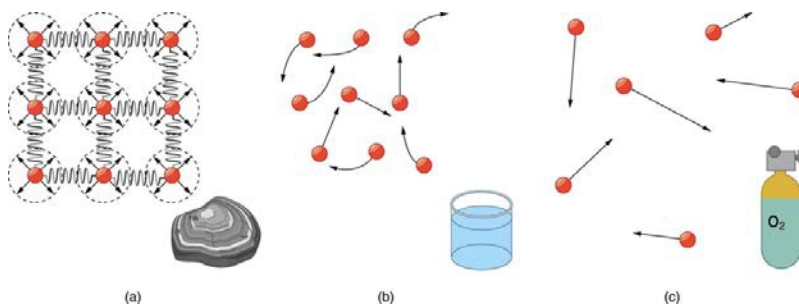
- 11.1. What Is a Fluid?
- 11.2. Density
- 11.3. Pressure
- 11.4. Variation of Pressure with Depth in a Fluid
- 11.5. Pascal's Principle
- 11.6. Gauge Pressure, Absolute Pressure, and Pressure Measurement
- 11.7. Archimedes' Principle
- 11.8. Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action
- 11.9. Pressures in the Body

### Introduction to Fluid Statics

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. **Fluid Dynamics and Its Biological and Medical Applications** explores aspects of fluid flow.

### 11.1 What Is a Fluid?

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See **Figure 11.2**.) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity which is discussed in detail in **Viscosity and Laminar Flow; Poiseuille's Law**. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.



**Figure 11.2** (a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

#### Connections: Submicroscopic Explanation of Solids and Liquids

Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This submicroscopic explanation is one theme of this text and is highlighted in the Things Great and Small features in **Conservation of Momentum**. See, for example, microscopic description of collisions and momentum or microscopic description of pressure in a gas. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

#### PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.



## PhET Interactive Simulation

**Figure 11.3** States of Matter: Basics ([http://cnx.org/content/m42186/1.4/states-of-matter-basics\\_en.jar](http://cnx.org/content/m42186/1.4/states-of-matter-basics_en.jar))

## 11.2 Density

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See **Figure 11.4**.)

**Density**, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}, \quad (11.1)$$

where the Greek letter  $\rho$  (rho) is the symbol for density,  $m$  is the mass, and  $V$  is the volume occupied by the substance.

#### Density

Density is mass per unit volume.

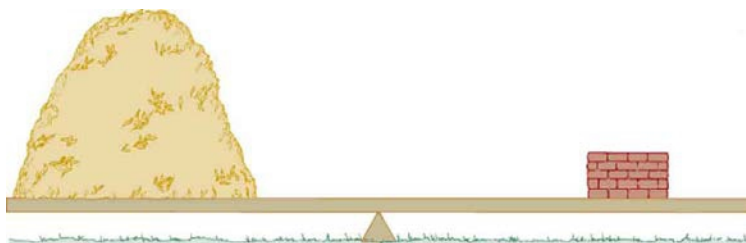
$$\rho = \frac{m}{V}, \quad (11.2)$$

where  $\rho$  is the symbol for density,  $m$  is the mass, and  $V$  is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is  $\text{kg/m}^3$ , representative values are given in **Table 11.1**. The metric system was originally devised so that water would have a density of  $1 \text{ g/cm}^3$ , equivalent to  $10^3 \text{ kg/m}^3$ . Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of  $1000 \text{ cm}^3$ .

**Table 11.1** Densities of Various Substances

Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$
<b>Solids</b>		<b>Liquids</b>		<b>Gases</b>	
Aluminum	2.7	Water (4°C)	1.000	Air	$1.29 \times 10^{-3}$
Brass	8.44	Blood	1.05	Carbon dioxide	$1.98 \times 10^{-3}$
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	$1.25 \times 10^{-3}$
Gold	19.32	Mercury	13.6	Hydrogen	$0.090 \times 10^{-3}$
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	$0.18 \times 10^{-3}$
Lead	11.3	Petrol	0.68	Methane	$0.72 \times 10^{-3}$
Polystyrene	0.10	Glycerin	1.26	Nitrogen	$1.25 \times 10^{-3}$
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	$1.98 \times 10^{-3}$
Uranium	18.70			Oxygen	$1.43 \times 10^{-3}$
Concrete	2.30–3.0			Steam (100° C)	$0.60 \times 10^{-3}$
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth's crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				



**Figure 11.4** A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining **Table 11.1**, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

#### Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

### Example 11.1 Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of  $50.0 \text{ km}^2$  and an average depth of  $40.0 \text{ m}$ . What mass of water is held behind the dam? (See **Figure 11.5** for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

#### Strategy

We can calculate the volume  $V$  of the reservoir from its dimensions, and find the density of water  $\rho$  in **Table 11.1**. Then the mass  $m$  can be found from the definition of density

$$\rho = \frac{m}{V}. \quad (11.3)$$

#### Solution

Solving equation  $\rho = m/V$  for  $m$  gives  $m = \rho V$ .

The volume  $V$  of the reservoir is its surface area  $A$  times its average depth  $h$ :

$$\begin{aligned} V &= Ah = (50.0 \text{ km}^2)(40.0 \text{ m}) \\ &= \left[ (50.0 \text{ km}^2) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right)^2 \right] (40.0 \text{ m}) = 2.00 \times 10^9 \text{ m}^3 \end{aligned} \quad (11.4)$$

The density of water  $\rho$  from **Table 11.1** is  $1.000 \times 10^3 \text{ kg/m}^3$ . Substituting  $V$  and  $\rho$  into the expression for mass gives

$$\begin{aligned} m &= (1.00 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^9 \text{ m}^3) \\ &= 2.00 \times 10^{12} \text{ kg}. \end{aligned} \quad (11.5)$$

#### Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is  $mg = 1.96 \times 10^{13} \text{ N}$ , where  $g$  is the acceleration due to the Earth's gravity (about  $9.80 \text{ m/s}^2$ ). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.



**Figure 11.5** Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is  $181 \text{ m}$  high and  $2.3 \text{ km}$  across. The reservoir made by this dam is  $660 \text{ km}$  long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

## 11.3 Pressure

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure  $P$  is defined as

$$P = \frac{F}{A} \quad (11.6)$$

where  $F$  is a force applied to an area  $A$  that is perpendicular to the force.

#### Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

$$P = \frac{F}{A}. \quad (11.7)$$

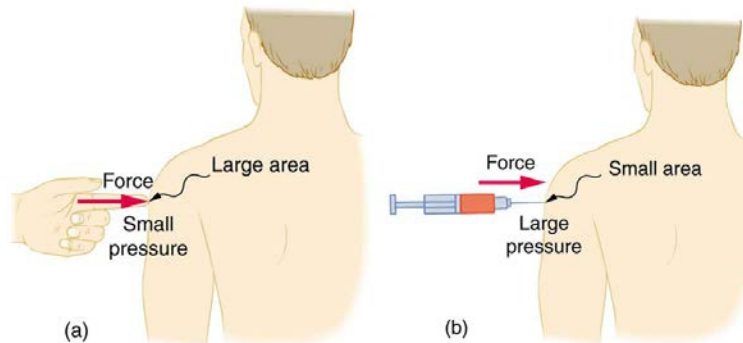
A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in **Figure 11.6**. The SI unit for pressure is the *pascal*, where

$$1 \text{ Pa} = 1 \text{ N/m}^2. \quad (11.8)$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$100 \text{ mb} = 1 \times 10^5 \text{ Pa}. \quad (11.9)$$

Pounds per square inch ( $\text{lb/in}^2$  or psi) is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.



**Figure 11.6** (a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

### Example 11.2 Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads  $6.90 \times 10^6 \text{ Pa}$ . What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk  $0.150 \text{ m}$  in diameter?

#### Strategy

We can find the force exerted from the definition of pressure given in  $P = \frac{F}{A}$ , provided we can find the area  $A$  acted upon.

#### Solution

By rearranging the definition of pressure to solve for force, we see that

$$F = PA. \quad (11.10)$$

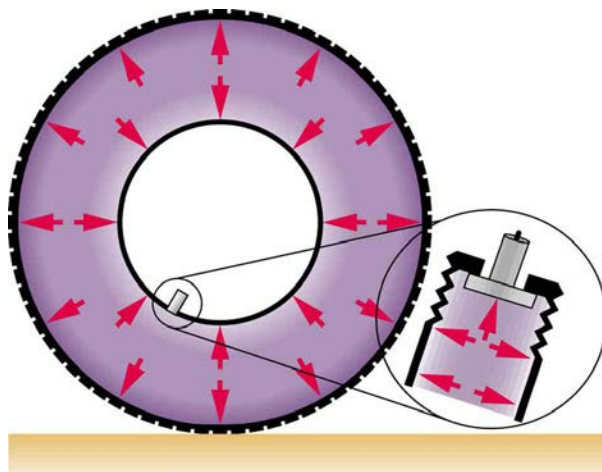
Here, the pressure  $P$  is given, as is the area of the end of the cylinder  $A$ , given by  $A = \pi r^2$ . Thus,

$$\begin{aligned} F &= (6.90 \times 10^6 \text{ N/m}^2)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22 \times 10^5 \text{ N}. \end{aligned} \quad (11.11)$$

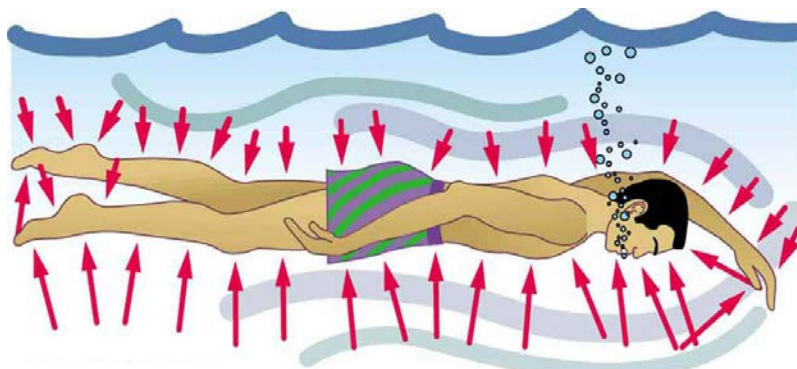
#### Discussion

Wow! No wonder the tank must be strong. Since we found  $F = PA$ , we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in **Figure 11.7**, for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See **Figure 11.8**.)



**Figure 11.7** Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



**Figure 11.8** Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.

#### PhET Explorations: Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.



## PhET Interactive Simulation

**Figure 11.9** Gas Properties ([http://cnx.org/content/m42189/1.4/gas-properties\\_en.jar](http://cnx.org/content/m42189/1.4/gas-properties_en.jar))

### 11.4 Variation of Pressure with Depth in a Fluid

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in **Figure 11.10**. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid  $mg$  divided by the area  $A$  supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}. \quad (11.12)$$

We can find the mass of the fluid from its volume and density:

$$m = \rho V. \quad (11.13)$$

The volume of the fluid  $V$  is related to the dimensions of the container. It is

$$V = Ah, \quad (11.14)$$

where  $A$  is the cross-sectional area and  $h$  is the depth. Combining the last two equations gives

$$m = \rho Ah. \quad (11.15)$$

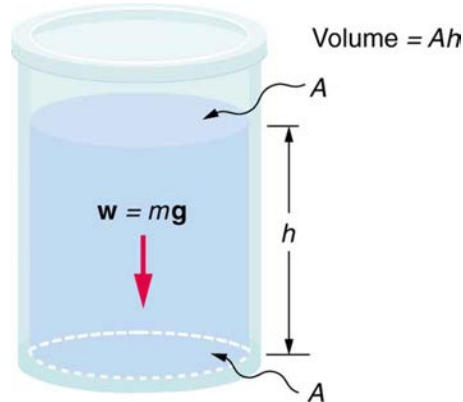
If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho Ah)g}{A}. \quad (11.16)$$

The area cancels, and rearranging the variables yields

$$P = h\rho g. \quad (11.17)$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation  $P = h\rho g$  represents the pressure due to the weight of any fluid of *average density*  $\rho$  at any depth  $h$  below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. **Example 11.4** illustrates this situation.



**Figure 11.10** The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

### Example 11.3 Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In **Example 11.1**, we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See **Figure 11.11**.) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be  $1.96 \times 10^{13}$  N).

#### Strategy for (a)

The average pressure  $\bar{P}$  due to the weight of the water is the pressure at the average depth  $\bar{h}$  of 40.0 m, since pressure increases linearly with depth.

#### Solution for (a)

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g. \quad (11.18)$$

Entering the density of water from **Table 11.1** and taking  $\bar{h}$  to be the average depth of 40.0 m, we obtain

$$\begin{aligned} \bar{P} &= (40.0 \text{ m}) \left( 10^3 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.80 \frac{\text{m}}{\text{s}^2} \right) \\ &= 3.92 \times 10^5 \frac{\text{N}}{\text{m}^2} = 392 \text{ kPa}. \end{aligned} \quad (11.19)$$

#### Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A. \quad (11.20)$$

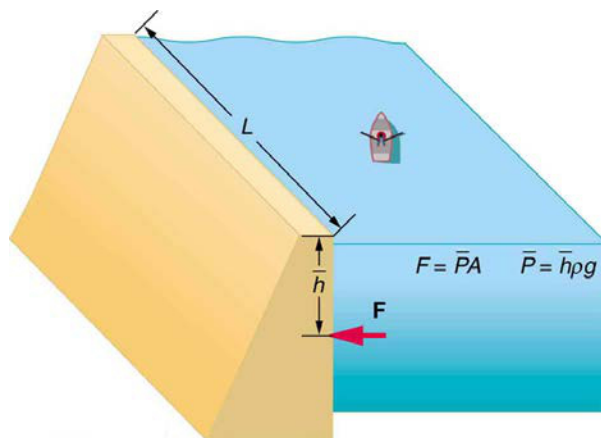
#### Solution for (b)

We have already found the value for  $\bar{P}$ . The area of the dam is  $A = 80.0 \text{ m} \times 500 \text{ m} = 4.00 \times 10^4 \text{ m}^2$ , so that

$$\begin{aligned}
 F &= (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2) \\
 &= 1.57 \times 10^{10} \text{ N.}
 \end{aligned}
 \tag{11.21}$$

**Discussion**

Although this force seems large, it is small compared with the  $1.96 \times 10^{13} \text{ N}$  weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

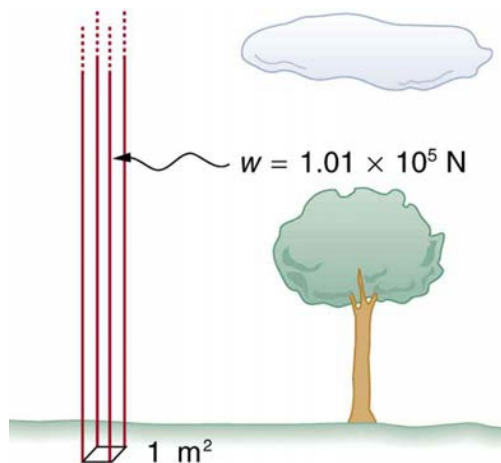


**Figure 11.11** The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

*Atmospheric pressure* is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure*  $P_{\text{atm}}$ , measured to be

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2 = 101 \text{ kPa.} \tag{11.22}$$

This relationship means that, on average, at sea level, a column of air above  $1.00 \text{ m}^2$  of the Earth's surface has a weight of  $1.01 \times 10^5 \text{ N}$ , equivalent to 1 atm. (See **Figure 11.12**.)



**Figure 11.12** Atmospheric pressure at sea level averages  $1.01 \times 10^5 \text{ Pa}$  (equivalent to 1 atm), since the column of air over this  $1 \text{ m}^2$ , extending to the top of the atmosphere, weighs  $1.01 \times 10^5 \text{ N}$ .

### Example 11.4 Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in **Table 11.1**.

**Strategy**

If we solve  $P = h\rho g$  for density, we see that



$$\bar{\rho} = \frac{P}{hg}. \quad (11.23)$$

We then take  $P$  to be atmospheric pressure,  $h$  is given, and  $g$  is known, and so we can use this to calculate  $\bar{\rho}$ .

### Solution

Entering known values into the expression for  $\bar{\rho}$  yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(120 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} = 8.59 \times 10^{-2} \text{ kg/m}^3. \quad (11.24)$$

### Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in **Table 11.1** as  $1.29 \text{ kg/m}^3$ —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

## Example 11.5 Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

### Strategy

We begin by solving the equation  $P = h\rho g$  for depth  $h$ :

$$h = \frac{P}{\rho g}. \quad (11.25)$$

Then we take  $P$  to be 1.00 atm and  $\rho$  to be the density of the water that creates the pressure.

### Solution

Entering the known values into the expression for  $h$  gives

$$h = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m}. \quad (11.26)$$

### Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in **Pascal's Principle** that fluid pressures always add in this way.

## 11.5 Pascal's Principle

**Pressure** is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

### Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

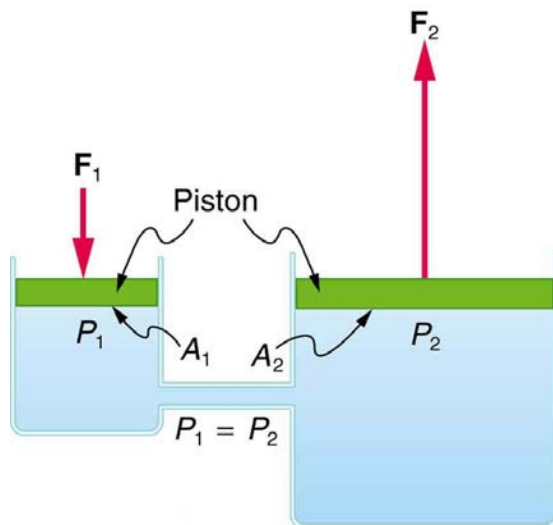
Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory,

and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

### Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in **Figure 11.13**.



**Figure 11.13** A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force  $F_1$  on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force  $F_2$  on the right piston that is larger than  $F_1$  because the right piston has a larger area.

### Relationship Between Forces in a Hydraulic System

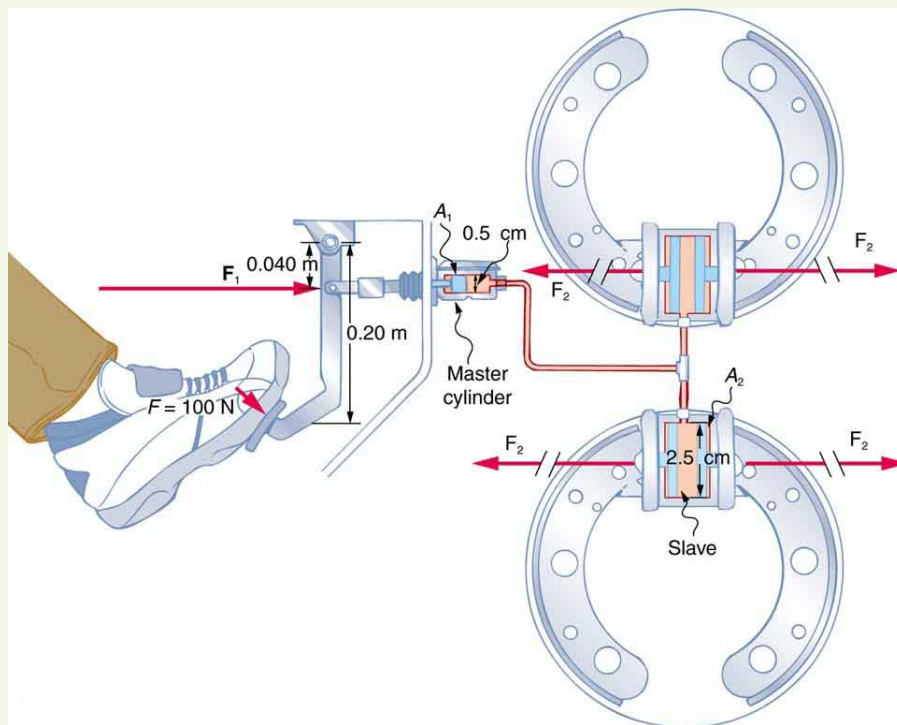
We can derive a relationship between the forces in the simple hydraulic system shown in **Figure 11.13** by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to  $F_1$  acting on area  $A_1$  is simply  $P_1 = \frac{F_1}{A_1}$ , as defined by  $P = \frac{F}{A}$ . According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure  $P_2$  is felt at the other piston that is equal to  $P_1$ . That is  $P_1 = P_2$ .

But since  $P_2 = \frac{F_2}{A_2}$ , we see that  $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ .

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in **Figure 11.13** and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

### Example 11.6 Calculating Force of Slave Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in **Figure 11.14**.



**Figure 11.14** Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical slave cylinders receives the same pressure and, therefore, creates the same force output  $F_2$ . The circular cross-sectional areas of the master and slave cylinders are represented by  $A_1$  and  $A_2$ , respectively

A force of 100 N is applied to the brake pedal, which acts on the cylinder—called the master—through a lever. A force of 500 N is exerted on the master cylinder. (The reader can verify that the force is 500 N using techniques of statics from **Applications of Statics, Including Problem-Solving Strategies**.) Pressure created in the master cylinder is transmitted to four so-called slave cylinders. The master cylinder has a diameter of 0.500 cm, and each slave cylinder has a diameter of 2.50 cm. Calculate the force  $F_2$  created at each of the slave cylinders.

### Strategy

We are given the force  $F_1$  that is applied to the master cylinder. The cross-sectional areas  $A_1$  and  $A_2$  can be calculated from their given diameters. Then  $\frac{F_1}{A_1} = \frac{F_2}{A_2}$  can be used to find the force  $F_2$ . Manipulate this algebraically to get  $F_2$  on one side and substitute known values:

### Solution

Pascal's principle applied to hydraulic systems is given by  $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ :

$$F_2 = \frac{A_2}{A_1}F_1 = \frac{\pi r_2^2}{\pi r_1^2}F_1 = \frac{(1.25 \text{ cm})^2}{(0.250 \text{ cm})^2} \times 500 \text{ N} = 1.25 \times 10^4 \text{ N}. \quad (11.27)$$

### Discussion

This value is the force exerted by each of the four slave cylinders. Note that we can add as many slave cylinders as we wish. If each has a 2.50-cm diameter, each will exert  $1.25 \times 10^4$  N.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the slave cylinder moves through a smaller distance than the master cylinder. Furthermore, the more slaves added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

### Making Connections: Conservation of Energy

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

## 11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in **Pascal's Principle**, the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define **gauge pressure** to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

### Gauge Pressure

Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal's principle. The total pressure, or **absolute pressure**, is thus the sum of gauge pressure and atmospheric pressure:  $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$  where  $P_{\text{abs}}$  is absolute pressure,  $P_{\text{g}}$  is gauge pressure, and  $P_{\text{atm}}$  is atmospheric pressure. For example, if your tire gauge reads 34 psi (pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi ( $P_{\text{atm}}$  in psi), or 48.7 psi (equivalent to 336 kPa).

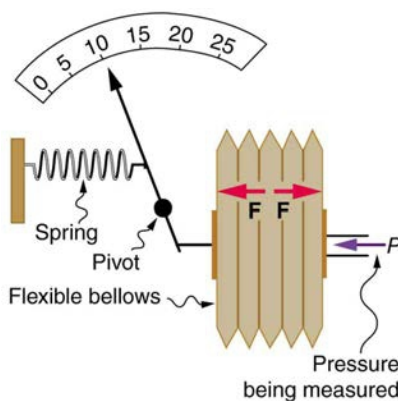
### Absolute Pressure

Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is  $P_{\text{g}} = -P_{\text{atm}}$  (this makes  $P_{\text{abs}}$  zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal's principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person's artery.

**Figure 11.15** shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure gauges, pressure results in a force that is converted (or transduced) into some type of readout.

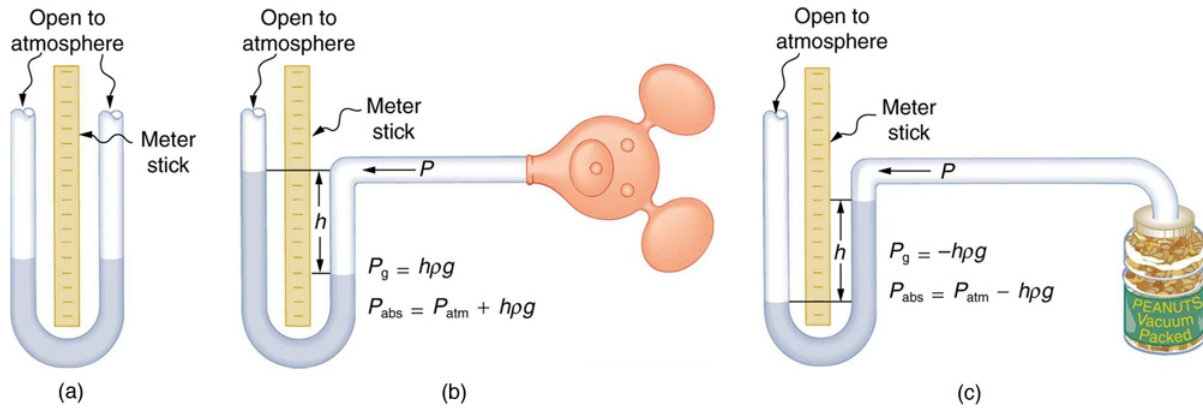


**Figure 11.15** This aneroid gauge utilizes flexible bellows connected to a mechanical indicator to measure pressure.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by  $P = h\rho g$ . Consider the U-shaped tube shown in **Figure 11.16**, for example. This simple tube is called a *manometer*. In **Figure 11.16(a)**, both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure  $P_{\text{abs}}$  such as the toy balloon in **Figure 11.16(b)** or the vacuum-packed peanut jar shown in **Figure 11.16(c)**. Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In **Figure 11.16(b)**,  $P_{\text{abs}}$  is greater than atmospheric pressure, whereas in **Figure 11.16(c)**,  $P_{\text{abs}}$  is less than atmospheric pressure. In both cases,  $P_{\text{abs}}$  differs from atmospheric pressure by an amount  $h\rho g$ , where  $\rho$  is the density of the fluid in the manometer. In **Figure 11.16(b)**,  $P_{\text{abs}}$  can support a column of fluid of height  $h$ , and so it must exert a pressure  $h\rho g$  greater than atmospheric pressure (the gauge pressure  $P_{\text{g}}$  is positive). In **Figure 11.16(c)**, atmospheric pressure can support a column of fluid of height  $h$ , and

so  $P_{\text{abs}}$  is less than atmospheric pressure by an amount  $h\rho g$  (the gauge pressure  $P_g$  is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is  $P_g = h\rho g$  and is found by measuring  $h$ .



**Figure 11.16** An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the deeper side. (b) A positive gauge pressure  $P_g = h\rho g$  transmitted to one side of the manometer can support a column of fluid of height  $h$ . (c) Similarly, atmospheric pressure is greater than a negative gauge pressure  $P_g$  by an amount  $h\rho g$ . The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in **Figure 11.17**. By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called **systolic pressure**, and a minimum, called **diastolic pressure**, with each heartbeat. Systolic pressure is measured by noting the value of  $h$  when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting  $h$  when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be  $1/13.6$  of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that  $1.0 \text{ mm Hg} = 133 \text{ Pa}$ .

#### Systolic Pressure

Systolic pressure is the maximum blood pressure.

#### Diastolic Pressure

Diastolic pressure is the minimum blood pressure.



**Figure 11.17** In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart. Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. (credit: U.S. Army photo by Spc. Micah E. Clare/4TH BCT)

### Example 11.7 Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

#### Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

#### Solution

We first need to convert the pressure into SI units. Since  $1.0 \text{ mm Hg} = 133 \text{ Pa}$ ,

$$P = 18 \text{ mm Hg} \times \frac{133 \text{ Pa}}{1.0 \text{ mm Hg}} = 2400 \text{ Pa}. \quad (11.28)$$

Rearranging  $P_g = h\rho g$  for  $h$  gives  $h = \frac{P_g}{\rho g}$ . Substituting known values into this equation gives

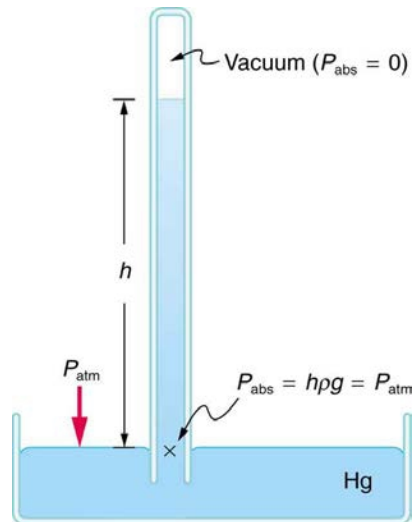
$$\begin{aligned} h &= \frac{2400 \text{ N/m}^2}{(1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} \\ &= 0.24 \text{ m}. \end{aligned} \quad (11.29)$$

#### Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in **Figure 11.18**. This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that  $h\rho g = P_{\text{atm}}$ . When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of

mm Hg are often quoted for atmospheric pressure and blood pressures. **Table 11.2** gives conversion factors for some of the more commonly used units of pressure.



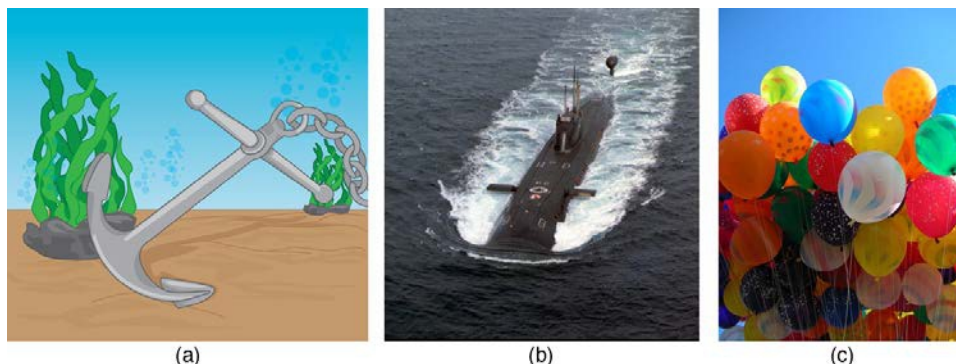
**Figure 11.18** A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight,  $h\rho g$ , equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height  $h$  because the pressure above the mercury is zero.

**Table 11.2** Conversion Factors for Various Pressure Units

Conversion to $\text{N/m}^2$ (Pa)	Conversion from atm
$1.0 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$1.0 \text{ dyne/cm}^2 = 0.10 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1.0 \text{ kg/cm}^2 = 9.8 \times 10^4 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \text{ kg/cm}^2$
$1.0 \text{ lb/in.}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1.0 \text{ atm} = 14.7 \text{ lb/in.}^2$
$1.0 \text{ mm Hg} = 133 \text{ N/m}^2$	$1.0 \text{ atm} = 760 \text{ mm Hg}$
$1.0 \text{ cm Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1.0 \text{ atm} = 76.0 \text{ cm Hg}$
$1.0 \text{ cm water} = 98.1 \text{ N/m}^2$	$1.0 \text{ atm} = 1.03 \times 10^3 \text{ cm water}$
$1.0 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \text{ bar}$
$1.0 \text{ millibar} = 1.000 \times 10^2 \text{ N/m}^2$	$1.0 \text{ atm} = 1013 \text{ millibar}$

## 11.7 Archimedes' Principle

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See **Figure 11.19**.)

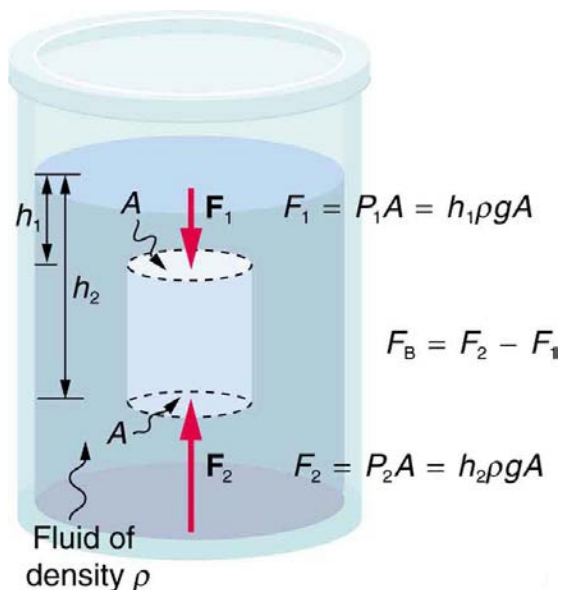


**Figure 11.19** (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See **Figure 11.20**.) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

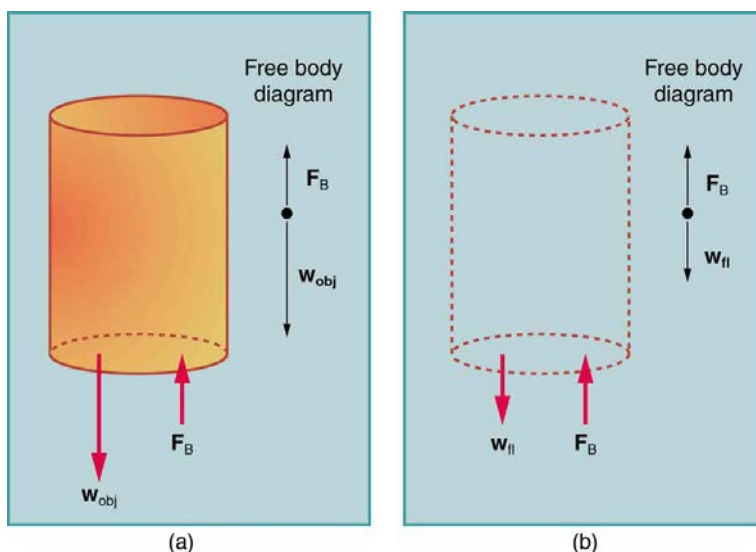
### Buoyant Force

The buoyant force is the net upward force on any object in any fluid.



**Figure 11.20** Pressure due to the weight of a fluid increases with depth since  $P = h\rho g$ . This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force  $F_B$ . (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in **Figure 11.21**.



**Figure 11.21** (a) An object submerged in a fluid experiences a buoyant force  $F_B$ . If  $F_B$  is greater than the weight of the object, the object will rise. If  $F_B$  is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight  $w_{fl}$ . Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is,  $F_B = w_{fl}$ , a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight  $w_{fl}$ . This weight is supported by the surrounding fluid, and so the buoyant force must equal  $w_{fl}$ , the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl}, \quad (11.30)$$



where  $F_B$  is the buoyant force and  $w_{fl}$  is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

### Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl}, \quad (11.31)$$

where  $F_B$  is the buoyant force and  $w_{fl}$  is the weight of the fluid displaced by the object.

*Humm ...* High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

### Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

## Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

### Example 11.8 Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons ( $1.00 \times 10^7$  kg) of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace  $1.00 \times 10^5$  m<sup>3</sup> of water?

#### Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in **Table 11.1**. We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

#### Solution for (a)

First, we use the definition of density  $\rho = \frac{m}{V}$  to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{st} = \frac{m_{st}}{\rho_{st}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3. \quad (11.32)$$

Because the steel is completely submerged, this is also the volume of water displaced,  $V_w$ . We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.28 \times 10^3 \text{ m}^3) \\ &= 1.28 \times 10^6 \text{ kg}. \end{aligned} \quad (11.33)$$

By Archimedes' principle, the weight of water displaced is  $m_w g$ , so the buoyant force is

$$\begin{aligned} F_B &= w_w = m_w g = (1.28 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 1.3 \times 10^7 \text{ N}. \end{aligned} \quad (11.34)$$

The steel's weight is  $m_w g = 9.80 \times 10^7$  N, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

#### Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

#### Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^5 \text{ m}^3) \\ &= 1.00 \times 10^8 \text{ kg}. \end{aligned} \quad (11.35)$$

The maximum buoyant force is the weight of this much water, or

$$\begin{aligned}
 F_B &= w_w = m_w g = (1.00 \times 10^8 \text{ kg})(9.80 \text{ m/s}^2) \\
 &= 9.80 \times 10^8 \text{ N.}
 \end{aligned}
 \tag{11.36}$$

### Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

### Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this “boat,” what shape of the boat would allow it to hold the most “cargo” when placed in water? Test your prediction.

## Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In **Figure 11.22**, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

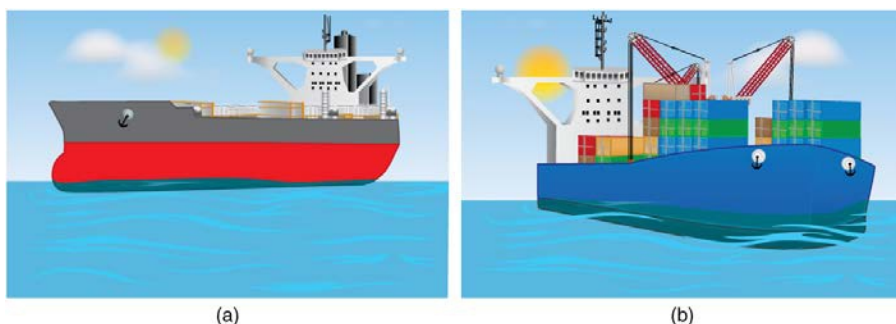
$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.
 \tag{11.37}$$

The volume submerged equals the volume of fluid displaced, which we call  $V_{\text{fl}}$ . Now we can obtain the relationship between the densities by substituting  $\rho = \frac{m}{V}$  into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}} / \rho_{\text{fl}}}{m_{\text{obj}} / \rho_{\text{obj}}},
 \tag{11.38}$$

where  $\rho_{\text{obj}}$  is the average density of the object and  $\rho_{\text{fl}}$  is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.
 \tag{11.39}$$



**Figure 11.22** An unloaded ship (a) floats higher in the water than a loaded ship (b).

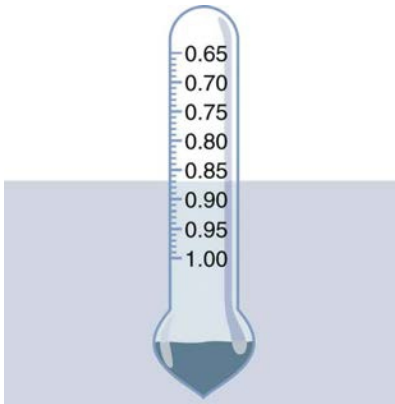
We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\text{specific gravity} = \frac{\rho}{\rho_w},
 \tag{11.40}$$

where  $\rho$  is the average density of the object or substance and  $\rho_w$  is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for  $\rho$ . If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in **Figure 11.23**.

### Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).



**Figure 11.23** This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

### Example 11.9 Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

#### Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}} \quad (11.41)$$

for the density of the object. This yields

$$\bar{\rho}_{\text{obj}} = \bar{\rho}_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}. \quad (11.42)$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

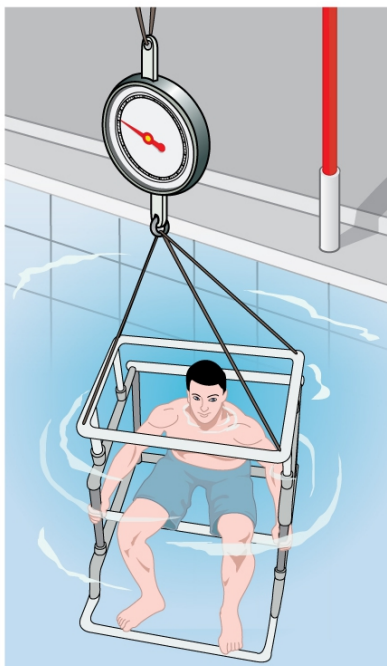
#### Solution

Entering the known values into the expression for her density, we obtain

$$\bar{\rho}_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}. \quad (11.43)$$

#### Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See **Figure 11.24**.)

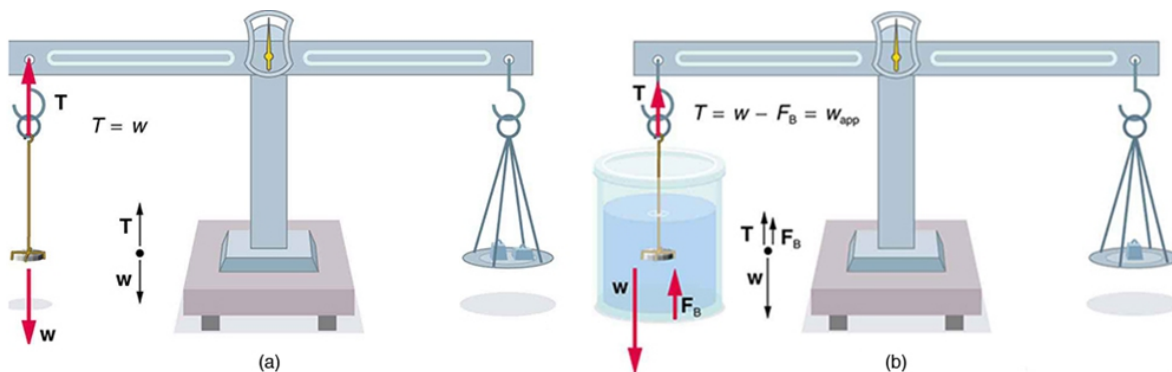


**Figure 11.24** Subject in a “fat tank,” where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air, and pinch tests of strategic fatty areas are used to calculate his percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a “lava lamp,” to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

### More Density Measurements

One of the most common techniques for determining density is shown in **Figure 11.25**.



**Figure 11.25** (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes’ principle.

Archimedes’ principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object’s *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced} \quad (11.44)$$

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.} \quad (11.45)$$

The next example illustrates the use of this technique.

**Example 11.10 Calculating Density: Is the Coin Authentic?**

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in **Figure 11.25**, its apparent mass is 7.800 g. Calculate its density, given that water has a density of  $1.000 \text{ g/cm}^3$  and that effects caused by the wire suspending the coin are negligible.

**Strategy**

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced  $V_w$  can be found by solving the equation for density  $\rho = \frac{m}{V}$  for  $V$ .

**Solution**

The volume of water is  $V_w = \frac{m_w}{\rho_w}$  where  $m_w$  is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is  $m_w = 8.630 \text{ g} - 7.800 \text{ g} = 0.830 \text{ g}$ . Thus the volume of water is  $V_w = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830 \text{ cm}^3$ . This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_c = \frac{m_c}{V_c} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 \text{ g/cm}^3. \quad (11.46)$$

**Discussion**

You can see from **Table 11.1** that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

**PhET Explorations: Buoyancy**

When will objects float and when will they sink? Learn how buoyancy works with blocks. Arrows show the applied forces, and you can modify the properties of the blocks and the fluid.

**PhET Interactive Simulation**

**Figure 11.26 Buoyancy** ([http://cnx.org/content/m42196/1.8/buoyancy\\_en.jar](http://cnx.org/content/m42196/1.8/buoyancy_en.jar))

**11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action****Cohesion and Adhesion in Liquids**

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See **Figure 11.27**.) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

**Cohesive Forces**

Attractive forces between molecules of the same type are called cohesive forces.

**Adhesive Forces**

Attractive forces between molecules of different types are called adhesive forces.



**Figure 11.27** The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. (credit: Steve Ford Elliott)

## Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

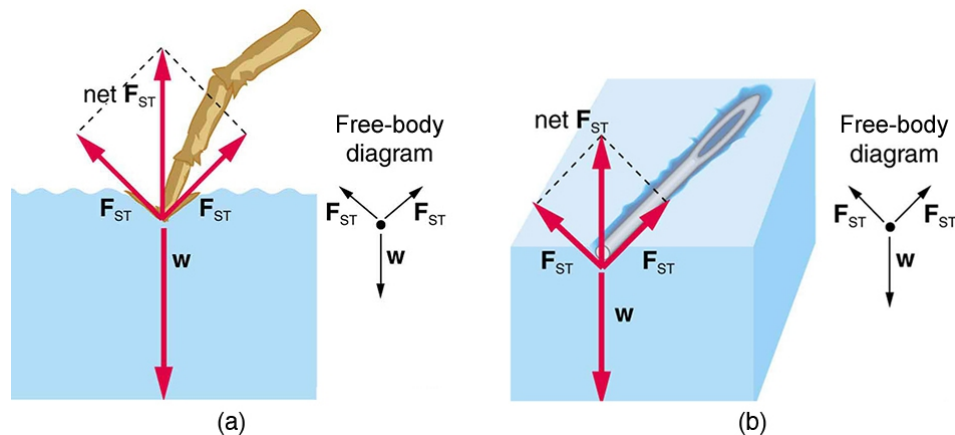
### Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.

### Making Connections: Surface Tension

Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in **Figure 11.28(a)**. **Figure 11.28(b)** shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.



**Figure 11.28** Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface.  $F_{ST}$  is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension  $\gamma$  is defined to be the force  $F$  per unit length  $L$  exerted by a stretched liquid membrane:

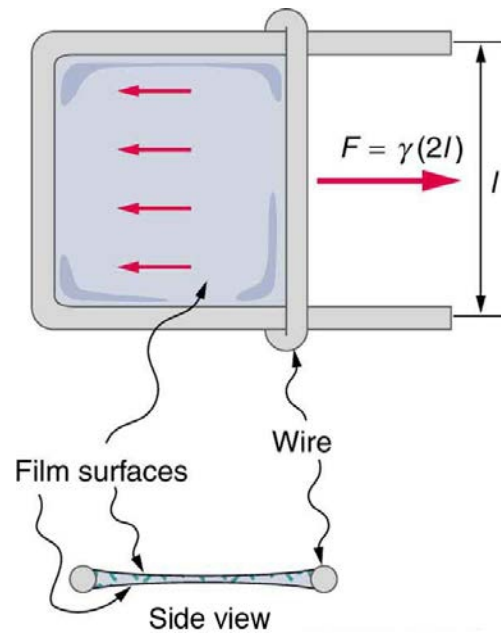
$$\gamma = \frac{F}{L}. \quad (11.47)$$

**Table 11.3** lists values of  $\gamma$  for some liquids. For the insect of **Figure 11.28(a)**, its weight  $w$  is supported by the upward components of the surface tension force:  $w = \gamma L \sin \theta$ , where  $L$  is the circumference of the insect's foot in contact with the water. **Figure 11.29** shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

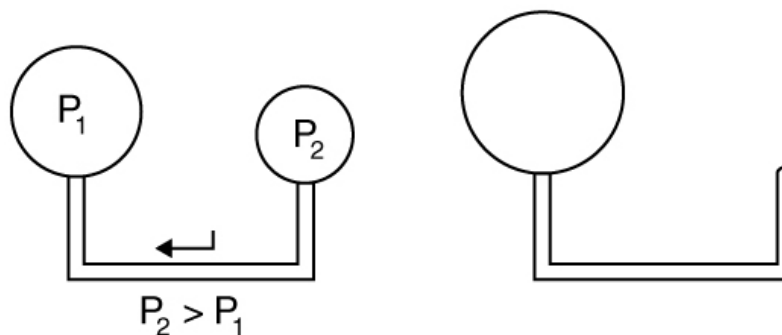
Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure  $P$  inside a spherical bubble is given by

$$P = \frac{4\gamma}{r}, \quad (11.48)$$

where  $r$  is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in **Figure 11.30**. When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.



**Figure 11.29** Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film's surface area. The force needed to hold the wire in place is  $F = \gamma L = \gamma(2l)$ , since there are two liquid surfaces attached to the wire. This force remains nearly constant as the film is stretched, until the film approaches its breaking point.



**Figure 11.30** With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow.

Table 11.3 Surface Tension of Some Liquids<sup>[1]</sup>

Liquid	Surface tension $\gamma$ (N/m)
Water at 0°C	0.0756
Water at 20°C	0.0728
Water at 100°C	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37°C	0.058
Blood plasma at 37°C	0.073
Gold at 1070°C	1.000
Oxygen at -193°C	0.0157
Helium at -269°C	0.00012

### Example 11.11 Surface Tension: Pressure Inside a Bubble

Calculate the gauge pressure inside a soap bubble  $2.00 \times 10^{-4}$  m in radius using the surface tension for soapy water in Table 11.3. Convert this pressure to mm Hg.

#### Strategy

The radius is given and the surface tension can be found in Table 11.3, and so  $P$  can be found directly from the equation  $P = \frac{4\gamma}{r}$ .

#### Solution

Substituting  $r$  and  $\gamma$  into the equation  $P = \frac{4\gamma}{r}$ , we obtain

$$P = \frac{4\gamma}{r} = \frac{4(0.037 \text{ N/m})}{2.00 \times 10^{-4} \text{ m}} = 740 \text{ N/m}^2 = 740 \text{ Pa.} \quad (11.49)$$

We use a conversion factor to get this into units of mm Hg:

$$P = (740 \text{ N/m}^2) \frac{1.00 \text{ mm Hg}}{133 \text{ N/m}^2} = 5.56 \text{ mm Hg.} \quad (11.50)$$

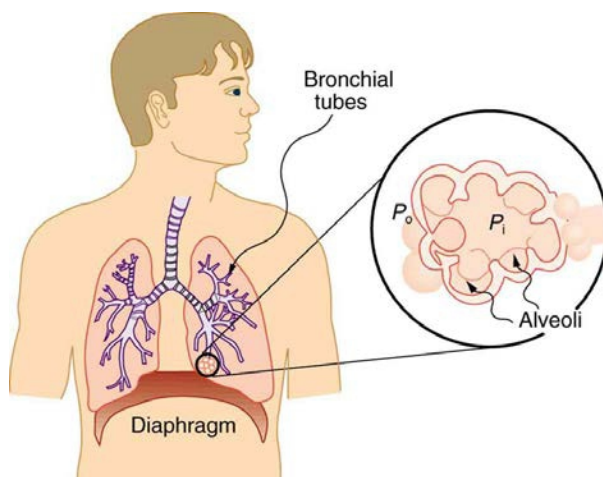
#### Discussion

Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the pressure inside would *increase* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See Figure 11.31.) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.

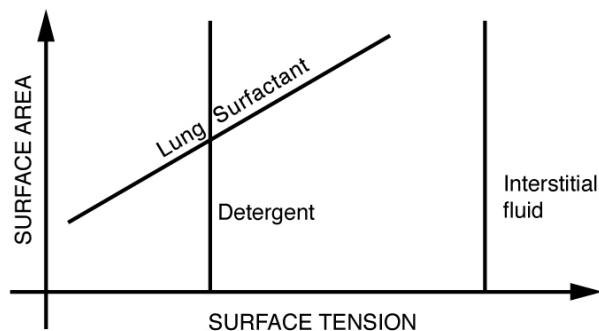
1. At 20°C unless otherwise stated.





**Figure 11.31** Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in **Figure 11.30**). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don't collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See **Figure 11.32**.)



**Figure 11.32** Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don't collapse and large alveoli are not able to over expand.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant's breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

#### Making Connections: Take-Home Investigation

(1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle. (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

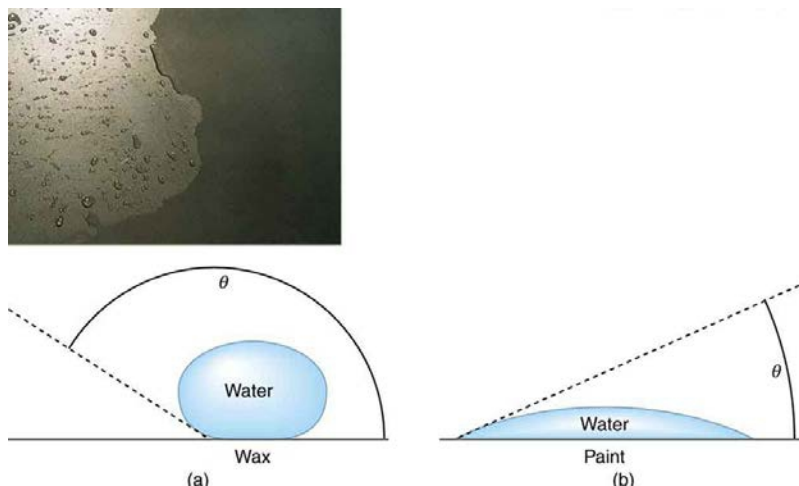
### Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint? The answer is that the adhesive forces between water and wax are much smaller than those between water and paint. Competition between the forces of adhesion and cohesion are important in the macroscopic behavior of liquids. An important factor in studying the roles of these two forces is the angle  $\theta$  between the tangent to the liquid surface and the surface. (See

**Figure 11.33**.) The **contact angle**  $\theta$  is directly related to the relative strength of the cohesive and adhesive forces. The larger the strength of the cohesive force relative to the adhesive force, the larger  $\theta$  is, and the more the liquid tends to form a droplet. The smaller  $\theta$  is, the smaller the relative strength, so that the adhesive force is able to flatten the drop. **Table 11.4** lists contact angles for several combinations of liquids and solids.

### Contact Angle

The angle  $\theta$  between the tangent to the liquid surface and the surface is called the contact angle.



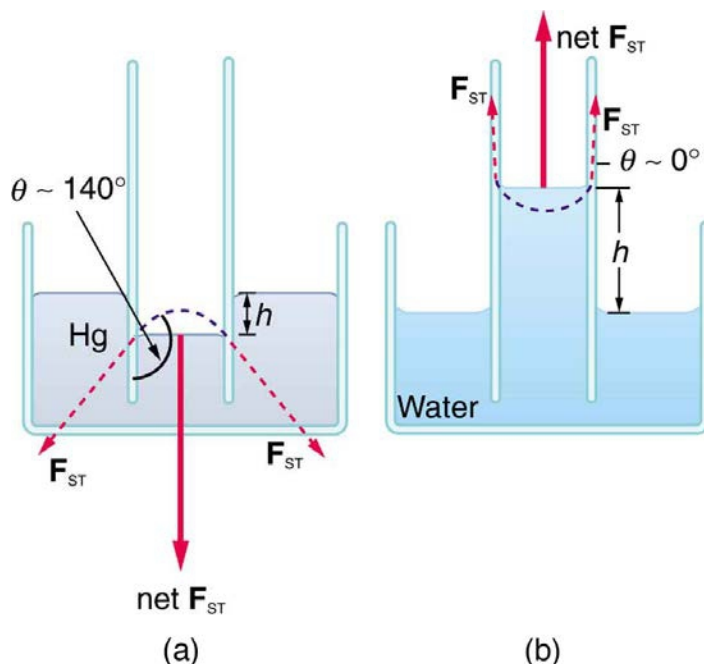
**Figure 11.33** In the photograph, water beads on the waxed car paint and flattens on the unwaxed paint. (a) Water forms beads on the waxed surface because the cohesive forces responsible for surface tension are larger than the adhesive forces, which tend to flatten the drop. (b) Water beads on bare paint are flattened considerably because the adhesive forces between water and paint are strong, overcoming surface tension. The contact angle  $\theta$  is directly related to the relative strengths of the cohesive and adhesive forces. The larger  $\theta$  is, the larger the ratio of cohesive to adhesive forces. (credit: P. P. Urone)

One important phenomenon related to the relative strength of cohesive and adhesive forces is **capillary action**—the tendency of a fluid to be raised or suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

### Capillary Action

The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in **Figure 11.34**, capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle  $\theta$  given in the table. If  $\theta$  is less than  $90^\circ$ , then the fluid will be raised; if  $\theta$  is greater than  $90^\circ$ , it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in **Figure 11.34**.



**Figure 11.34** (a) Mercury is suppressed in a glass tube because its contact angle is greater than  $90^\circ$ . Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension. (b) Water is raised in a glass tube because its contact angle is nearly  $0^\circ$ . Surface tension therefore exerts an upward force when it flattens the surface to reduce its area.

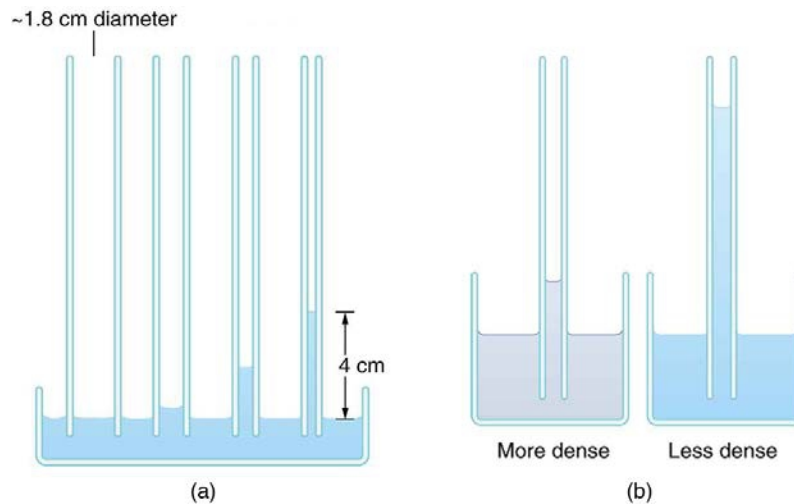
Table 11.4 Contact Angles of Some Substances

Interface	Contact angle $\theta$
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

Capillary action can move liquids horizontally over very large distances, but the height to which it can raise or suppress a liquid in a tube is limited by its weight. It can be shown that this height  $h$  is given by

$$h = \frac{2\gamma \cos \theta}{\rho g r}. \quad (11.51)$$

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly proportional to the surface tension  $\gamma$ , which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius  $r$ , the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density  $\rho$ , since a larger density means a greater mass in the same volume. (See **Figure 11.35**.)



**Figure 11.35** (a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same.

### Example 11.12 Calculating Radius of a Capillary Tube: Capillary Action: Tree Sap

Can capillary action be solely responsible for sap rising in trees? To answer this question, calculate the radius of a capillary tube that would raise sap 100 m to the top of a giant redwood, assuming that sap's density is  $1050 \text{ kg/m}^3$ , its contact angle is zero, and its surface tension is the same as that of water at  $20.0^\circ \text{ C}$ .

#### Strategy

The height to which a liquid will rise as a result of capillary action is given by  $h = \frac{2\gamma \cos \theta}{\rho g r}$ , and every quantity is known except for  $r$ .

#### Solution

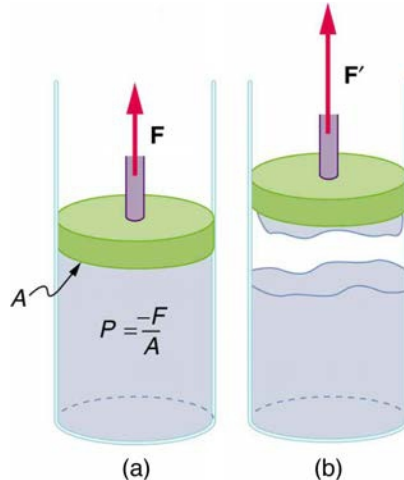
Solving for  $r$  and substituting known values produces

$$\begin{aligned} r &= \frac{2\gamma \cos \theta}{\rho g h} = \frac{2(0.0728 \text{ N/m})\cos(0^\circ)}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(100 \text{ m})} \\ &= 1.41 \times 10^{-7} \text{ m}. \end{aligned} \quad (11.52)$$

#### Discussion

This result is unreasonable. Sap in trees moves through the *xylem*, which forms tubes with radii as small as  $2.5 \times 10^{-5}$  m. This value is about 180 times as large as the radius found necessary here to raise sap 100 m. This means that capillary action alone cannot be solely responsible for sap getting to the tops of trees.

How does sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see **Example 11.5**.) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. **Figure 11.36** shows one device for studying negative pressure. Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.



**Figure 11.36** (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure  $P = -F/A$ . (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

## 11.9 Pressures in the Body

### Pressure in the Body

Next to taking a person's temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

**Table 11.5** lists some of the measured pressures in mm Hg, the units most commonly quoted.

Table 11.5 Typical Pressures in Humans

Body system	Gauge pressure in mm Hg
Blood pressures in large arteries (resting)	
Maximum (systolic)	100–140
Minimum (diastolic)	60–90
Blood pressure in large veins	4–15
Eye	12–24
Brain and spinal fluid (lying down)	5–12
Bladder	
While filling	0–25
When full	100–150
Chest cavity between lungs and ribs	–8 to –4
Inside lungs	–2 to +3
Digestive tract	
Esophagus	–2
Stomach	0–20
Intestines	10–20
Middle ear	<1

## Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem. **Systolic pressure** increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system. **Diastolic pressure** can be an indicator of fluid balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see [Figure 11.37](#)). The pressure differences in the circulation system are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood ( $P = h\rho g$ ). If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg.} \quad (11.53)$$

### Increase in Pressure in the Feet of a Person

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg.} \quad (11.54)$$

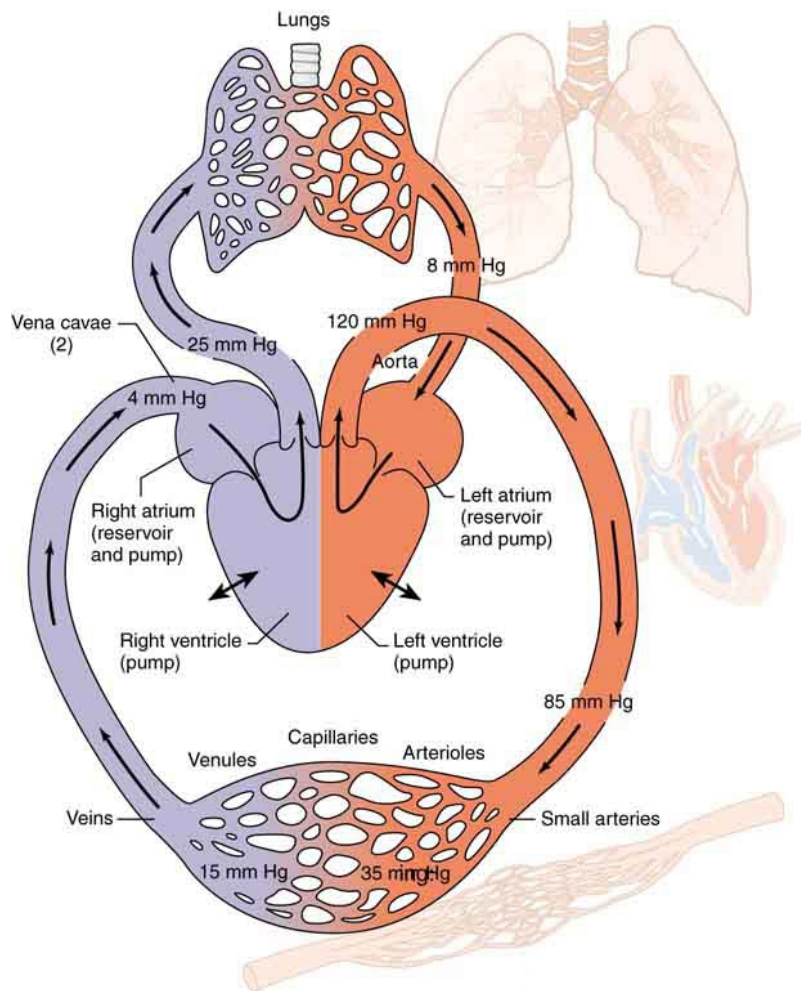
Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs. But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body ([Figure 11.37](#)). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise in the pressure entering the left side of the heart and a drop in aortal pressure. Implications of these and other pressures on flow in the circulatory system will be discussed in more detail in [Fluid Dynamics and Its Biological and Medical Applications](#).

### Two Pumps of the Heart

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.



**Figure 11.37** Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body. Long-term deviations from these pressures have medical implications discussed in some detail in the **Fluid Dynamics and Its Biological and Medical Applications**. Only aortal or arterial blood pressure can be measured noninvasively.

## Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called **intraocular pressure**, which is normally in the range of 12.0 to 24.0 mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called **glaucoma**. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of  $6.0 \text{ cm}^2$ , and the net pressure is 85.0 mm Hg. Force is given by  $F = PA$ . To get  $F$  in newtons, we convert the area to  $\text{m}^2$  ( $1 \text{ m}^2 = 10^4 \text{ cm}^2$ ). Then we calculate as follows:

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m})(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N}. \quad (11.55)$$

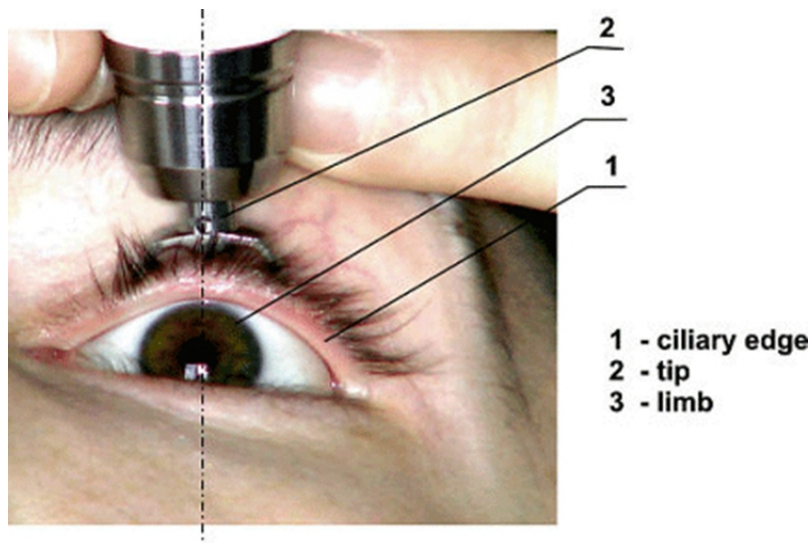
### Eye Pressure

The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m})(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N}. \quad (11.56)$$

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye's response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye (**Figure 11.38**). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.



**Figure 11.38** The intraocular eye pressure can be read with a tonometer. (credit: DevelopAll at the Wikipedia Project.)

### Example 11.13 Calculating Gauge Pressure and Depth: Damage to the Eardrum

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of  $1.00 \text{ cm}^2$ , calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

#### Strategy for (a)

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

#### Solution for (a)

$$P_g = F/A = 3.00 \text{ N}/(1.00 \times 10^{-4} \text{ m}^2) = 3.00 \times 10^4 \text{ N/m}^2. \quad (11.57)$$

We now need to convert this to units of mm Hg:

$$P_g = 3.0 \times 10^4 \text{ N/m}^2 \left( \frac{1.0 \text{ mm Hg}}{133 \text{ N/m}^2} \right) = 226 \text{ mm Hg}. \quad (11.58)$$

#### Strategy for (b)

Here we will use the fact that the water pressure varies linearly with depth  $h$  below the surface.

#### Solution for (b)

$P = h\rho g$  and therefore  $h = P/\rho g$ . Using the value above for  $P$ , we have

$$h = \frac{3.0 \times 10^4 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.06 \text{ m}. \quad (11.59)$$

#### Discussion

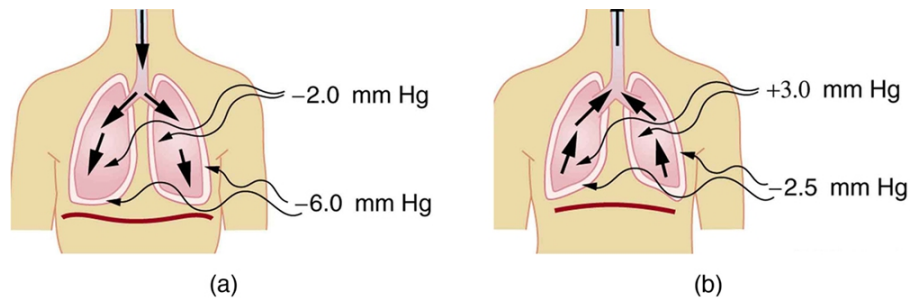
Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

## Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them **Figure 11.39**. Surface tension in the alveoli creates a positive pressure opposing inhalation. (See **Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action**.) You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from  $-4$  to  $-8$  mm Hg during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.



**Figure 11.39** (a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation.

## Other Pressures in the Body

### Spinal Column and Skull

Normally, there is a 5- to 12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

### Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of  $500 \text{ cm}^3$ . This pressure triggers the **micturition reflex**, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

### Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high values of initial force, and the small areas to which this force is applied, such as in the joints. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as  $10 \text{ cm}^2$ . The pressure created is  $P = F/A = (5000 \text{ N}) / (10^{-3} \text{ m}^2) = 5.0 \times 10^6 \text{ N/m}^2$  or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity. (See **Forces and Torques in Muscles and Joints**.)

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

Many pressures in the human body are associated with the flow of fluids. Fluid flow will be discussed in detail in the **Fluid Dynamics and Its Biological and Medical Applications**.

## Glossary

**Archimedes' principle:** the buoyant force on an object equals the weight of the fluid it displaces

**absolute pressure:** the sum of gauge pressure and atmospheric pressure

**adhesive forces:** the attractive forces between molecules of different types

**buoyant force:** the net upward force on any object in any fluid

**capillary action:** the tendency of a fluid to be raised or lowered in a narrow tube

**cohesive forces:** the attractive forces between molecules of the same type

**contact angle:** the angle  $\theta$  between the tangent to the liquid surface and the surface

**density:** the mass per unit volume of a substance or object



**diastolic pressure:** the minimum blood pressure in the artery

**diastolic pressure:** minimum arterial blood pressure; indicator for the fluid balance

**fluids:** liquids and gases; a fluid is a state of matter that yields to shearing forces

**gauge pressure:** the pressure relative to atmospheric pressure

**glaucoma:** condition caused by the buildup of fluid pressure in the eye

**intraocular pressure:** fluid pressure in the eye

**micturition reflex:** stimulates the feeling of needing to urinate, triggered by bladder pressure

**Pascal's Principle:** a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

**pressure:** the force per unit area perpendicular to the force, over which the force acts

**pressure:** the weight of the fluid divided by the area supporting it

**specific gravity:** the ratio of the density of an object to a fluid (usually water)

**surface tension:** the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

**systolic pressure:** the maximum blood pressure in the artery

**systolic pressure:** maximum arterial blood pressure; indicator for the blood flow

## Section Summary

### 11.1 What Is a Fluid?

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

### 11.2 Density

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}.$$

- The SI unit of density is  $\text{kg/m}^3$ .

### 11.3 Pressure

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as

$$P = \frac{F}{A}.$$

- The SI unit of pressure is pascal and  $1 \text{ Pa} = 1 \text{ N/m}^2$ .

### 11.4 Variation of Pressure with Depth in a Fluid

- Pressure is the weight of the fluid  $mg$  divided by the area  $A$  supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}.$$

- Pressure due to the weight of a liquid is given by

$$P = h\rho g,$$

where  $P$  is the pressure,  $h$  is the height of the liquid,  $\rho$  is the density of the liquid, and  $g$  is the acceleration due to gravity.

### 11.5 Pascal's Principle

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

### 11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and one end is always open. It is used to measure pressure.
- A mercury barometer is a device that measures atmospheric pressure.

### 11.7 Archimedes' Principle

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

### 11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

### 11.9 Pressures in the Body

- Measuring blood pressure is among the most common of all medical examinations.
- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

## Conceptual Questions

### 11.1 What Is a Fluid?

1. What physical characteristic distinguishes a fluid from a solid?
2. Which of the following substances are fluids at room temperature: air, mercury, water, glass?
3. Why are gases easier to compress than liquids and solids?
4. How do gases differ from liquids?

### 11.2 Density

5. Approximately how does the density of air vary with altitude?
6. Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?
7. **Figure 11.40** shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.



Figure 11.40

### 11.3 Pressure

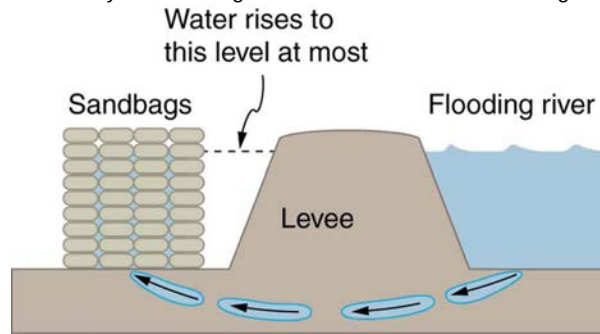
8. How is pressure related to the sharpness of a knife and its ability to cut?
9. Why does a dull hypodermic needle hurt more than a sharp one?
10. The outward force on one end of an air tank was calculated in **Example 11.2**. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)
11. Why is force exerted by static fluids always perpendicular to a surface?
12. In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?
13. How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?
14. Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.
15. How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

### 11.4 Variation of Pressure with Depth in a Fluid

16. Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?
17. Why does atmospheric pressure decrease more rapidly than linearly with altitude?

18. What are two reasons why mercury rather than water is used in barometers?

19. **Figure 11.41** shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.



**Figure 11.41** Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

20. Why is it difficult to swim under water in the Great Salt Lake?

21. Is there a net force on a dam due to atmospheric pressure? Explain your answer.

22. Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

23. You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

### 11.5 Pascal's Principle

24. Suppose the master cylinder in a hydraulic system is at a greater height than the slave cylinder. Explain how this will affect the force produced at the slave cylinder.

### 11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

25. Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.

26. **Figure 11.17** shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.

27. Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

### 11.7 Archimedes' Principle

28. More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

29. Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

30. Will the same ship float higher in salt water than in freshwater? Explain your answer.

31. Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

### 11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

32. The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?

33. Is surface tension due to cohesive or adhesive forces, or both?

34. Is capillary action due to cohesive or adhesive forces, or both?

35. Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.

36. Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.

37. Could capillary action be used to move fluids in a "weightless" environment, such as in an orbiting space probe?

38. What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.

39. Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

## Problems & Exercises

### 11.2 Density

40. Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?
41. Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?
42. (a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.
43. A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 cm<sup>3</sup> of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)
44. Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.
45. (a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.
46. A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?
47. A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?
48. What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.
49. There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10<sup>3</sup> kg/m<sup>3</sup>. The nucleus of an atom has a radius about 10<sup>-5</sup> that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is 7 × 10<sup>8</sup> m)?

### 11.3 Pressure

50. As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50 cm<sup>2</sup> and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)
51. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m<sup>2</sup>?
52. Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of 3.00 × 10<sup>9</sup> N/m<sup>2</sup>? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

### 11.4 Variation of Pressure with Depth in a Fluid

53. What depth of mercury creates a pressure of 1.00 atm?
54. The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.
55. Verify that the SI unit of  $h\rho g$  is N/m<sup>2</sup>.
56. Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of 3.00 × 10<sup>5</sup> N/m<sup>2</sup>?
57. The aqueous humor in a person's eye is exerting a force of 0.300 N on the 1.10-cm<sup>2</sup> area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?
58. How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?
59. What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?
60. Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is 50.0 cm<sup>2</sup> and he exerts a force of 800 N on it. Express the pressure in N/m<sup>2</sup> and compare it with the 1.00 × 10<sup>6</sup> Pa pressures sometimes encountered in the skeletal system.
61. The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of 15.0 cm<sup>2</sup>. What force does it exert to accomplish this?
62. Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by  $F = \rho g h^2 L / 2$ , where  $\rho$  is the density of water,  $h$  is its depth at the dam, and  $L$  is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See Figure 11.42.)

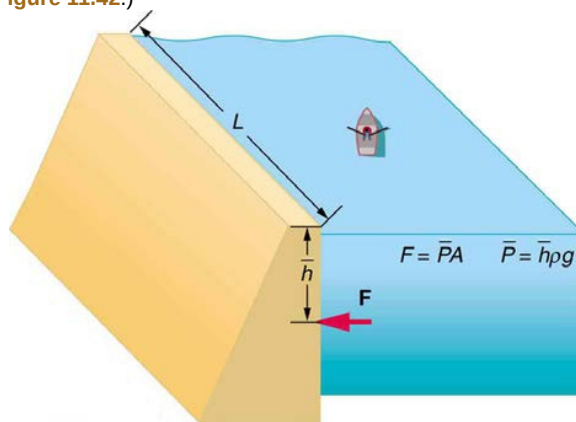


Figure 11.42

### 11.5 Pascal's Principle

63. How much pressure is transmitted in the hydraulic system considered in Example 11.6? Express your answer in pascals and in atmospheres.
64. What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00-cm diameter and the slave has a 24.0-cm diameter.
65. A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he

pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

**66.** A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

**67.** (a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

## 11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

**68.** Find the gauge and absolute pressures in the balloon and peanut jar shown in **Figure 11.16**, assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking  $h = 0.0500$  m for each.

**69.** (a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid ( $P = h\rho g$ ) rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.

**70.** How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?

**71.** Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is 25.0 cm in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

**72.** Suppose you measure a standing person's blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).

**73.** A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.

**74.** Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is  $3.50 \times 10^5$  Pa.

## 11.7 Archimedes' Principle

**75.** What fraction of ice is submerged when it floats in freshwater, given the density of water at 0°C is very close to  $1000 \text{ kg/m}^3$ ?

**76.** Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

**77.** Find the density of a fluid in which a hydrometer having a density of 0.750 g/mL floats with 92.0% of its volume submerged.

**78.** If your body has a density of  $995 \text{ kg/m}^3$ , what fraction of you will be submerged when floating gently in: (a) Freshwater? (b) Salt water, which has a density of  $1027 \text{ kg/m}^3$ ?

**79.** Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

**80.** A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

**81.** Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in **Table 11.1** (c) Calculate the fluid's density and identify it.

**82.** In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

**83.** Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is  $1015 \text{ kg/m}^3$ ?

**84.** (a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.

**85.** (a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?

**86.** A certain man has a mass of 80 kg and a density of  $955 \text{ kg/m}^3$  (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

**87.** A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

**88.** What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?

**89.** Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

**90.** A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?

**91.** Referring to **Figure 11.21**, prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes'

principle). You may assume that the buoyant force is  $F_2 - F_1$  and that the ends of the cylinder have equal areas  $A$ . Note that the volume of the cylinder (and that of the fluid it displaces) equals  $(h_2 - h_1)A$ .

**92.** (a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

### 11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

**93.** What is the pressure inside an alveolus having a radius of  $2.50 \times 10^{-4}$  m if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.

**94.** (a) The pressure inside an alveolus with a  $2.00 \times 10^{-4}$ -m radius is  $1.40 \times 10^3$  Pa, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in **Table 11.3**.)

**95.** What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?

**96.** Calculate the force on the slide wire in **Figure 11.29** if it is 3.50 cm long and the fluid is ethyl alcohol.

**97.** **Figure 11.35(a)** shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height  $h$  for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?

**98.** We stated in **Example 11.12** that a xylem tube is of radius  $2.50 \times 10^{-5}$  m. Verify that such a tube raises sap less than a meter by finding  $h$  for it, making the same assumptions that sap's density is  $1050 \text{ kg/m}^3$ , its contact angle is zero, and its surface tension is the same as that of water at  $20.0^\circ \text{C}$ .

**99.** What fluid is in the device shown in **Figure 11.29** if the force is  $3.16 \times 10^{-3}$  N and the length of the wire is 2.50 cm? Calculate the surface tension  $\gamma$  and find a likely match from **Table 11.3**.

**100.** If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?

**101.** Calculate the gauge pressures inside 2.00-cm-radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any effects of evaporation?

**102.** Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?

**103.** Calculate the contact angle  $\theta$  for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?

**104.** When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?

**105.** Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.

**106.** What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

### 11.9 Pressures in the Body

**107.** During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the  $600 \text{ cm}^2$  surface area of the diaphragm?

**108.** You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of 500 N with your tooth on an area of  $1.00 \text{ mm}^2$ ?

**109.** One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of  $50.0 \text{ cm}^2$ ?

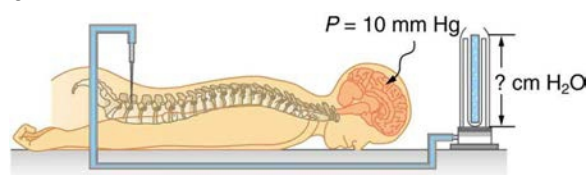
**110.** Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve  $-3.00$  cm water pressure with your lungs 60.0 cm below the surface?

**111.** Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is  $70.0 \text{ cm}^2$ . (b) What is the net force acting on the skull?

**112.** A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of  $90.0 \text{ cm}^2$ ? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).

**113.** If the pressure in the esophagus is  $-2.00$  mm Hg while that in the stomach is  $+20.0$  mm Hg, to what height could stomach fluid rise in the esophagus, assuming a density of  $1.10 \text{ g/mL}$ ? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

**114.** Pressure in the spinal fluid is measured as shown in **Figure 11.43**. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is  $1.05 \text{ g/mL}$ .



**Figure 11.43** A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere. The measured pressure will be considerably greater if the person sits up.

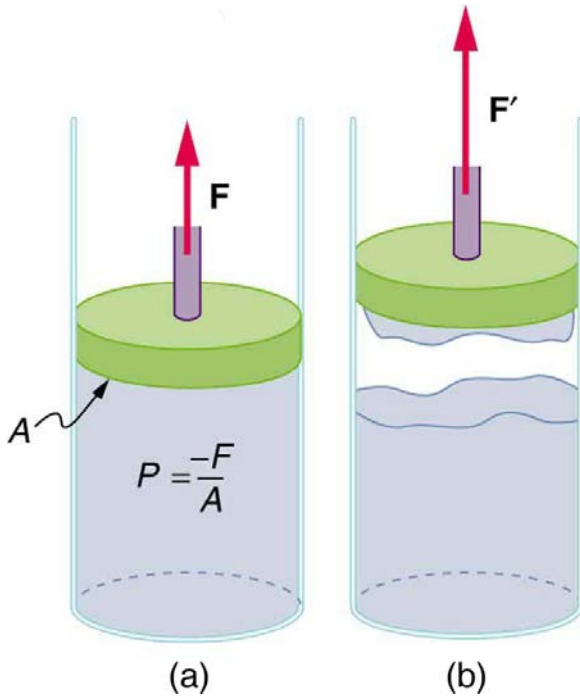
**115.** Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is  $20.0 \text{ cm}^2$ . Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.

**116.** During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. (a) What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius? (b) What deformation is produced if the disk is 0.800 cm thick and has a Young's modulus of  $1.5 \times 10^9 \text{ N/m}^2$ ?

**117.** When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.

**118.** (a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.

**119.** A negative pressure of 25.0 atm can sometimes be achieved with the device in **Figure 11.44** before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?

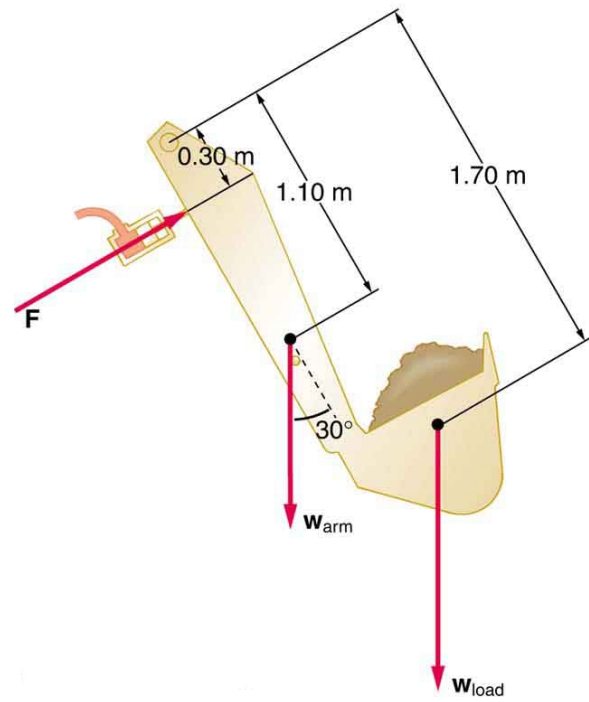


**Figure 11.44** (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure  $P = -F/A$  (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

**120.** Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at 15.0 m/s and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?

**121.** Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is 11.0 km and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?

**122.** The hydraulic system of a backhoe is used to lift a load as shown in **Figure 11.45**. (a) Calculate the force  $F$  the slave cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the slave cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a master cylinder 0.800 cm in diameter to create this pressure?



**Figure 11.45** Hydraulic and mechanical lever systems are used in heavy machinery such as this back hoe.

**123.** Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?

**124.** You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius.

(a) What force in newtons must you exert to create a pressure of  $6.90 \times 10^5$  Pa (b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?

**125.** Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person's head and arms above water without swimming or treading water.

**126.** The alveoli in emphysema victims are damaged and effectively form larger sacs. Construct a problem in which you calculate the loss of pressure due to surface tension in the alveoli because of their larger average diameters. (Part of the lung's ability to expel air results from pressure created by surface tension in the alveoli.) Among the things to consider are the normal surface tension of the fluid lining the alveoli, the average alveolar radius in normal individuals and its average in emphysema sufferers.