Comprehensive Curriculum

Algebra I

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Reaching For Results

Algebra I
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Algebra I
Unit 1: Understanding Quantities, Variability, and Change

Time Frame: Approximately three weeks

Unit Description

This unit examines numbers and number sets including basic operations on rational numbers, integer exponents, radicals, and scientific notation. It also includes investigations of situations in which quantities change and the study of the relative nature of the change through tables, graphs, and numerical relationships. The identification of independent and dependent variables is emphasized as well as the comparison of linear and non-linear data.

Student Understandings

Students focus on developing the notion of a variable. They begin to understand inputs and outputs and how they reflect the nature of a given relationship. Students recognize and apply the notions of independent and dependent variables and write expressions modeling simple linear relationships. They should also come to understand the difference between linear and non-linear relationships.

Guiding Questions

1. Can students perform basic operations on rational numbers with and without technology?
2. Can students perform basic operations on radical expressions?
3. Can students evaluate and write expressions using scientific notation and integer exponents?
4. Can students identify independent and dependent variables?
5. Can students recognize patterns in and differentiate between linear and non-linear sequence data?

Unit 1 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
1.	Identify and describe differences among natural numbers, whole numbers, integers, rational numbers, and irrational numbers (N-1-H) (N-2-H) (N-3-H)
2.	Evaluate and write numerical expressions involving integer exponents (N-2-H)
3.	Apply scientific notation to perform computations, solve problems, and write

	representations of numbers (N-2-H)
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
6.	Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3}+5\sqrt{3}=7\sqrt{3}$) (N-5-H)
Algebra	
7.	Use proportional reasoning to model and solve real-life problems involving direct and inverse variation (N-6-H)
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
10.	Identify independent and dependent variables in real-life relationships (A-1-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Data Analysis, Probability, and Discrete Math	
28.	Identify trends in data and support conclusions by using distribution characteristics such as patterns, clusters, and outliers (D-1-H) (D-6-H) (D-7-H)
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)
34.	Follow and interpret processes expressed in flow charts (D-8-H)

Sample Activities

Activity 1: The Numbers (GLEs: 1, 4, 5)

Use a number line to describe the differences and similarities of whole numbers, integers, rational numbers, irrational numbers, and real numbers. Have the students identify types of numbers selected by the teacher from the number line. Have the students select examples of numbers from the number line that can be classified as particular types. Example questions could include:What kind of number is $\frac{9}{2}$? What kind of number is 3.6666? Identify a number from the number line that is a rational number.

Discuss the difference between exact and approximate numbers. Have the students use Venn diagrams and tree diagrams to display the relationships among the sets of numbers.

Help students understand how approximate values affect the accuracy of answers by having them experiment with calculations involving different approximations of a number. For example, have the students compute the circumference and area of a circle using various approximations for π. Use measurements as examples of approximations and show how the precision of tools and accuracy of measurements affect computations of values such as area and volume. Also, use radical numbers that can be written as approximations such as $\sqrt{2}$.

Activity 2: Using a Flow Chart to classify real numbers (GLEs: 1, 34)

A flow chart is a pictorial representation showing all the steps of a process. Guide students to create a flow chart to classify real numbers as rational, irrational, integer, whole and/or natural. A sample flow chart is given at the end of this unit. Tell students that in most flow charts, questions go in diamonds, processes go in rectangles, and *yes* or *no* answers go on the connectors. Have students come up with the questions that they must ask themselves when they are classifying a real number and what the answers to those questions tell them about the number. Many word processing programs have the capability to construct a flow chart. If technology is available, allow students to construct the flow chart using the computer. After the class has constructed the flow chart, give students different real numbers and have the students use the flow chart to classify the numbers. (Flow charts will be revisited in later units to ensure mastery of GLE 34)

Activity 3: Operations on rational numbers (GLE 5)

Have students review basic operations with whole numbers, fractions, decimals, and integers. Include application problems of all types so that students must apply their prior knowledge in order to solve the problems. Discuss with students when it is appropriate to use estimation, mental math, paper and pencil, or technology. Divide students into groups and give examples of problems in which each method is more appropriate; then have students decide which method to use. Have the different groups compare their answers and discuss their choices.

Activity 4: Comparing Radicals (GLE 6)

Have students work with a partner for this activity. Provide the students with centimeter graph paper. Have them draw a right triangle with legs 1 unit long and use the Pythagorean theorem to show that the hypotenuse is $\sqrt{2}$ units long. Then have them repeat with a triangle that has legs that are 2 units long, so they can see that the hypotenuse is $\sqrt{8}$ or $2\sqrt{2}$ units long. Have them continue with triangles that have legs of 3 and 4 units long. For each hypotenuse, have them write the length two different ways and notice any patterns that they see. This activity leads to a discussion of simplifying radicals.

Activity 5: Basic Operations on Radicals (GLEs: 6, 8)

Review the distributive property with students and its relationship to combining like terms. (i.e. $3x+5x=(3+5)x=8x$) Provide students with variable expressions to simplify. Give the following radical expression to students: $3\sqrt{2}+5\sqrt{2}$. Guide students to the conclusion that the distributive property can also be used on radical expressions, thus $3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$. Provide radical expressions for students to simplify.

Activity 6: Scientific Notation (GLEs: 2, 3, 4)

Have students use a calculator to make a chart with powers of 10 from –5 to 5. Discuss the patterns that are observed and the significance of negative exponents. Provide students with real-life situations for which scientific notation may be necessary, such as the distance from the planets to the sun or the mass of a carbon atom. Have students investigate scientific notation using a calculator. Allow students to convert numbers from scientific notation to standard notation and vice versa. Relate the importance of scientific notation in the areas of physical science and chemistry.

Activity 7: Variation (GLEs: 7, 9, 10, 15, 28, 29)

Part 1: Direct variation
Have the students collect from classmates real data that might represent a relationship between two measures (e.g., foot length in centimeters and shoe size for boys and girls) and make charts for boys and girls separately. Discuss independent and dependent variables and have students decide which is the independent and which is the dependent variable in the activity. Instruct the students to write ordered pairs, graph them, and look for relationships from the graphed data. Is there a pattern in the data? (*Yes, as the foot length increases, so does the shoe size.* Does the data appear to be linear? *Data should appear to be linear.)* Help students notice the positive correlation between foot length and shoe size. Have students find the average ratio of foot length to shoe size. This is the constant of variation. Have students write an equation that models the situation (shoe size = ratio x foot length). Following the experiment, discuss direct variation and have the students come up with other examples of direct variation in real life.

Part 2: Inverse variation
Have students work with a partner. Provide each pair with 36 algebra unit tiles. Have students arrange the tiles in a rectangle and record the height and width. Discuss independent and dependent variables. Does it matter in this situation which variable is independent and dependent? (*No, but the class should probably decide together which to use.)* Have students form as many different sized rectangles as possible and record the dimensions. Instruct the students to write ordered pairs, graph them, and look for relationships in the graphed data. Help students understand that the constant of variation in this experiment is a constant product. Have them write an equation to model the situation (*height (or dependent) = 36/width (or independent)*)

Provide students with other data sets that will give them examples of direct variation, inverse variation, and constant of variation. Ask students to write equations that can be used to find one variable in a relationship when given a second variable from the relationship.

Activity 8: Exponential Growth (GLEs: 2, 9, 10, 15, 29)

Give each student a sheet of $8\frac{1}{2}$" by 11" paper. Have them fold the paper in half as many times as they can. After one fold, there will be two regions, after two folds four regions, etc. Ask the students how the area of the new region compares to the area of the original sheet of paper after each fold. Have the students complete a table like the one below and provide the data and variable expressions.

Number of Folds	**Number of Regions**	**Area of Smallest Region**
0	1	1
1	2	$\frac{1}{2}$ or 2^{-1}
2	4	$\frac{1}{4}$ or 2^{-2}
3	8	$\frac{1}{8}$ or 2^{-3}
. . .	. . .	. . .
N	2^n	$\frac{1}{2^n}$ or 2^{-n}

Have the students complete a graph of the number of folds and the number of regions. Have them identify the independent and dependent variables. Is the graph linear? This is called an *exponential growth* pattern. Have the students also graph the number of folds and the area of the smallest region. This is called an *exponential decay* pattern. Include the significance of integer exponents as exponential decay is discussed.

Activity 9: Pay Day! (GLEs: 9, 10, 15, 29)

Which of the following jobs would you choose?

- Job A: Salary of $1 for the first year, $2 for the second year, $4 for the third year, continuing for 25 years
- Job B: Salary of $1 million a year for 25 years

At the end of 25 years, which job would produce the largest amount in total salary?

After some initial discussion of the two options, have the students work to explore the answer. They should organize their thinking using tables and graphs. Have the students represent the yearly salary and the total salary for both job options using algebraic expressions. Have them predict when the salaries would be equal. Return to this problem later in the year and have the students use technology to answer that question. Discuss whether the salaries represent linear or exponential growth.

Activity 10: Linear or Non-linear? (GLEs: 10, 15, 29)

Divide students into groups. Give each group a different set of the sample data at the end of this unit. Have each group identify the independent and dependent variables of the data and graph on a poster board. Let each group investigate their data and decide if it is linear or non-linear and present their findings to the class, displaying each poster in the front of the class. After all posters are displayed, conduct a whole-class discussion on the findings. As an extension, regression equations of the data could be put on cards and have the class try to match the data to the equation. Sample data sets are provided at the end of this unit.

Activity 11: Using Technology (GLEs: 10, 15, 29)

Have students enter data sets used in Activity 5 into lists in a graphing calculator and generate the scatter plots using the calculator.

Activity 12: Understanding Data (GLEs: 5, 10, 28, 29)

The table below gives the box score for game three of the 2003 NBA Championship series.

SAN ANTONIO SPURS						REBOUNDS					
PLAYER	POS	MIN	FGM-A	3GM-A	FTM-A	OFF	DEF	TOT	AST	PF	PTS
TONY PARKER	G	43	9-21	4-6	4-8	1	2	3	6	0	26
STEPHEN JACKSON	G	36	2-7	1-2	2-4	0	6	6	2	3	7
TIM DUNCAN	F	45	6-13	0-0	9-12	3	13	16	7	3	21
BRUCE BOWEN	F	32	0-5	0-2	0-0	1	3	4	0	3	0
DAVID ROBINSON	C	26	1-5	0-0	6-8	1	2	3	0	2	8
Emanuel Ginobili		28	3-6	0-0	2-3	2	0	2	4	2	8
Malik Rose		22	4-7	0-0	0-0	0	2	2	0	2	8
Speedy Claxton		5	2-2	0-0	0-0	0	1	1	0	1	4
Kevin Willis		3	1-1	0-0	0-0	1	0	1	0	1	2
Steve Kerr											
Danny Ferry											
Steve Smith											
TOTAL		240	28-67	5-10	23-35	9	29	38	19	17	84
			41.8%	50.0%	65.7%	Team Rebs: 15					

NEW JERSEY NETS						REBOUNDS					
PLAYER	POS	MIN	FGM-A	3GM-A	FTM-A	OFF	DEF	TOT	AST	PF	PTS
JASON KIDD	G	42	6-19	0-5	0-0	2	1	3	11	3	12
KERRY KITTLES	G	43	8-16	3-5	2-3	1	3	4	1	2	21
KENYON MARTIN	F	42	8-18	0-1	7-8	2	9	11	0	5	23
RICHARD JEFFERSON	F	36	3-11	0-0	0-0	2	7	9	0	2	6
JASON COLLINS	C	25	0-3	0-0	0-0	4	1	5	1	6	0
Lucious Harris		22	1-6	1-2	4-4	1	0	1	3	2	7
Dikembe Mutombo		18	1-1	0-0	0-0	1	2	3	0	3	2
Rodney Rogers		11	0-3	0-0	2-2	0	2	2	0	2	2
Anthony Johnson		6	2-2	0-0	0-0	0	1	1	0	0	4
Aaron Williams		4	1-2	0-0	0-0	1	1	2	1	1	2
Tamar Slay											
Brian Scalabrine											
TOTAL		240	30-81	4-13	15-17	14	27	41	17	26	79
			37.0%	30.8%	88.2%	Team Rebs: 10					

Source: www.nba.com

Key for Table			
Pos	**Position**	**3GM-A**	**3 point goals made – 3 point goals attempted**
Min.	**Minutes Played**	**AST**	**Assists**
FGM–A	**Field goals made–field goals attempted**	**PF**	**Personal Fouls**
FTM–A	**Free throws made–free throws attempted**	**PTS**	**Total Points Scored**
R	**Rebounds**		

Ask detailed questions about the information in the table, such as: Who played the most minutes, who had the most assists, or which team made a larger percentage of free throws? Have students calculate the percentage of field goals made/attempted and the percentage of free throws made/attempted for each player. Which player(s) has the highest percentages? Why do you think this is so?

Ask the students if they think that the players who attempt the most field goals are generally the players who make the most field goals. Is this a linear relationship? Have the students identify the independent and dependent variables and make a scatter plot showing field goals made and field goals attempted. Designate the players from the different teams using team colors or “S” for each Spur player and “N” for each New Jersey Net.

The plot shows a positive correlation. Note that a player who makes every basket will be represented by a point on the line containing the points (0,0), (1,1), (2,2), etc. Have the students identify the points representing the players who were the four perfect shooters.

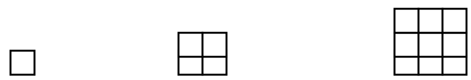
Have the students write a brief description of their interpretations of the scatter plot, noticing that points seem to cluster into two groups. The cluster in the upper right represents players who played more than forty minutes and the cluster in the lower left represents players who played less time.

Ask, Do you think that players who get a lot of rebounds also make a lot of assists (i.e. does the number of rebounds depend on the number of assists)? Have the students construct a scatter plot of rebounds (R) and assists (A). This scatter plot will show that there is no relationship. Have students identify other possible relationships of two-variable data and to investigate whether there is a positive correlation, negative correlation, or if no correlation exists.

Sample Assessments

General Assessments

- The students will explore patterns in the perimeters and areas of figures such as the “trains” described below.

Train 1 

Train number	1	2	3	4	5 …
n	*1*	*2*	*3*	*4*	*5*
Area	*1*	*4*	*9*	*16*	*25*
Perimeter	*4*	*8*	*12*	*16*	*20*

Describe the shape of each train. (*square)*

What is the length of a side of each square? (*n)*

Compare the lengths of the trains with their areas and perimeters. (*length-n, area-* n^2*, perimeter-4n)*

Train 2 

Train Number	1	2	3	4	5 …
n	*1*	*2*	*3*	*4*	*5*
Area	*1*	*3*	*6*	*10*	*15*
Perimeter	*4*	*8*	*12*	*16*	*20*

Formulas: area - $\frac{n(n+1)}{2}$*, perimeter – 4n*

- The students will solve constructed response items, such as:
  1. Cary’s Candy Store sells giant lollipops for \$1.00 each. This price is no longer high enough to create a profit, so Cary decides to raise the price. He doesn’t want to shock his customers by raising the price too suddenly or too dramatically. So, he considers these three plans,
     - ✓ Plan 1: Raise the price by \$0.05 each week until the price reaches \$1.80
     - ✓ Plan 2: Raise the price by 5% each week until the price reaches \$1.80
     - ✓ Plan 3: Raise the price by the same amount each week for 8 weeks, so that in the eighth week the price reaches \$1.80.

a. Make a table for each plan. How many weeks will it take the price to reach $1.80 under each plan? (*Plan 1 – 16 weeks, Plan 2 – 12 weeks, Plan 3 – 8 weeks)*
b. On the same set of axes, graph the data for each plan.
c. Are any of the graphs linear? Explain.
d. Which plan do you think Cary should implement? Give reasons for your choice. (*Answers will vary.)*

2. The table below gives the price that A Plus Car Rentals charges to rent a car including an extra charge for each mile that is driven.

Car Rental prices

Miles	Price
0	$35
1	$35.10
2	$35.20
3	$35.30
4	$35.40
5	$35.50

a. Identify the independent and dependent variables. Explain your choice.
b. Graph the data
c. Write an equation that models the price of the rental car. ($P = 35 + .10m$)
d. How much would it cost to drive the car 60 miles? Justify your answer. (*$41)*
e. If a person only has $40 to spend, how far can they drive the car? Justify your answer. (*50 miles)*

- The students will complete journal writings using such topics as:
  - ✓ Describe the steps used in writing .000062 in scientific notation
  - ✓ How can you tell if two sets of data vary directly?
  - ✓ Explain the error in the following work: $\sqrt{5} + \sqrt{11} = \sqrt{16} = 4$
  - ✓ Explain how one might use a flow chart to help with a process.
  - ✓ Is it true that a person can do many calculations faster using mental math than using a calculator? Give reasons to support your answer.
- The student will complete assessment items that require reflection, writing and explaining why.
- The student will create a portfolio containing samples of their activities.

Activity-Specific Assessments

- Activity 1: Given the set of numbers $A = \left\{\sqrt{3.6},\ 0.36, -\frac{3}{6},\ 0.3\overline{6},\ 0,\ 3^6,\ -3,\ \sqrt{36},\ 3.63363336\ldots\ \right\}$, the student will list the subsets of *A* containing **all** elements of *A* that are also elements of the following sets:

- ✓ natural numbers ($\left\{3^6, \sqrt{36}\right\}$)
- ✓ whole numbers ($\left\{0, 3^6, \sqrt{36}\right\}$)
- ✓ integers ($\left\{0, 3^6, -3, \sqrt{36}\right\}$
- ✓ rational numbers ($\left\{0.36, -3/6, 0.3\overline{6}, 0, 3^6, -3, \sqrt{36}\right\}$)
- ✓ irrational numbers ($\left\{\sqrt{3.6}, 3.633.6333.6...\right\}$)
- ✓ real numbers (*all*)

- Activity 2: The students will use the Internet to find other examples of flow charts. The student will print a flow chart and write a paragraph explaining what process the flow chart is showing and how the different boxes indicate the steps of the process. If Internet access is not available to students, The teacher will provide the student with different examples of flow charts to choose from and write about.

- Activity 7: The students will complete a writing assignment explaining how to tell if an equation is that of an inverse variation or that of a direct variation.

- Activities 8 and 9: The student will graph the following sets of data and write a report comparing the two, including in the report an analysis of the type of data (linear or non-linear).

Average income
Males in the U.S.

Year	Annual wages
1970	9521
1973	12088
1976	14732
1979	18711
1985	26365
1987	28313

(Linear)

Average income
Professional baseball players

Year	Annual wages
1970	12000
1973	15000
1976	19000
1979	21000
1985	60000
1991	100000

(Non- linear)

- Activity 12: The teacher will provide the student (or the teacher will assign the teacher to find) similar statistics from the school basketball team, a favorite college team, or another professional basketball team. The student will study the data and develop questions that could be answered using the data. The student will submit the data set, questions, and graphs that must be used to complete the assignment.

Sample Data
Activity 10

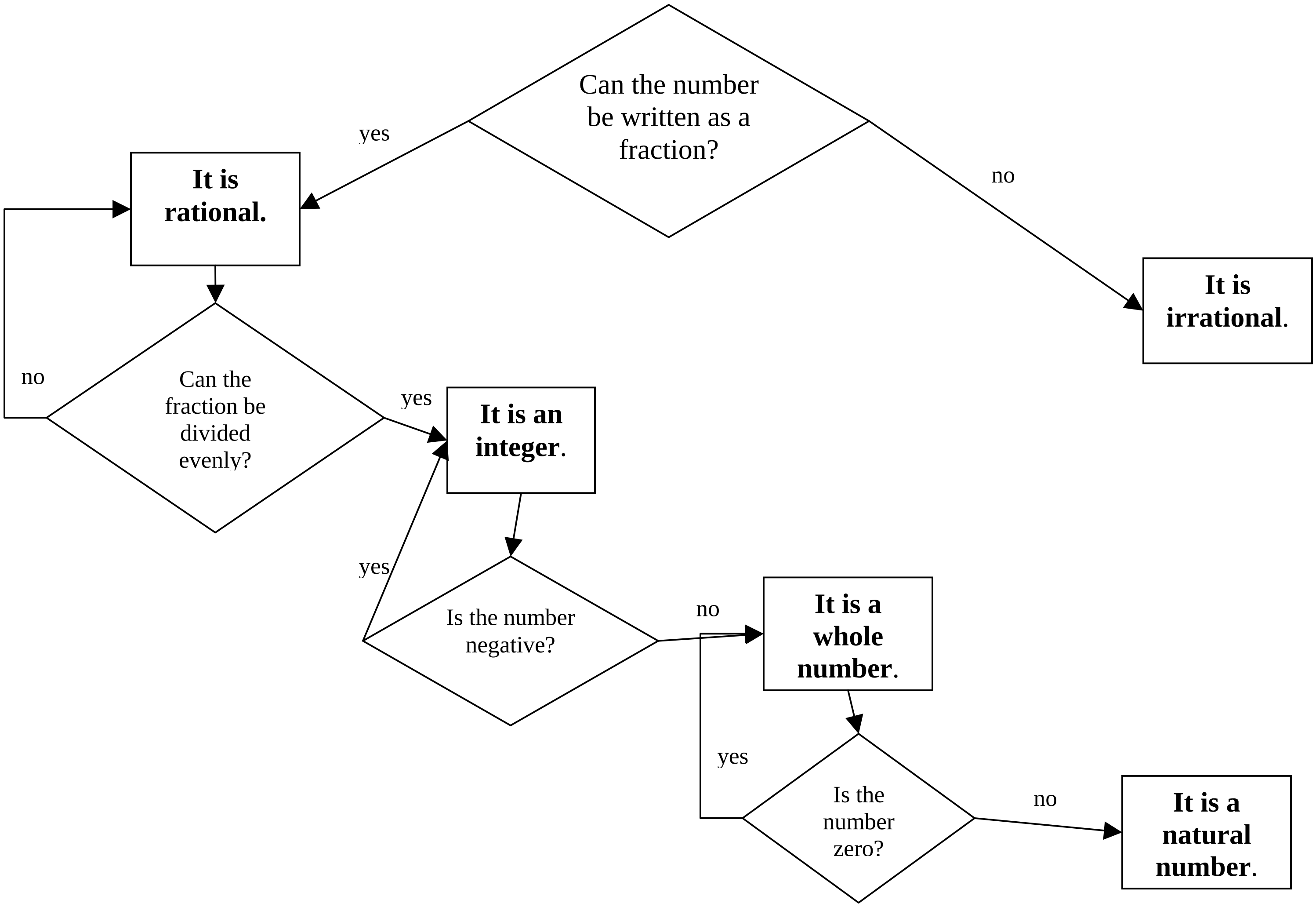
Household with Television Sets (in millions)		Wind Chill	
		Wind Speed	**Wind Chill**
Year	**Television Sets**	**(mph)**	**Fahrenheit**
1986	158	0	35
1987	163	5	32
1988	168	10	22
1989	176	15	16
1990	193	20	11
1991	193	25	8
1992	192	30	6
1993	201	35	4
(linear)		*(Non-linear)*	

Length and Weight of Whales		Median House Prices	
Length	**Weight**	**Year**	**Price**
(feet)	**(long tons)**	1990	85000
40	25	1991	88000
42	29	1992	92000
45	34	1993	100000
46	35	1994	106000
50	43	1995	115500
52	45	1996	125000
55	51	1997	135000
		1998	151000
(Linear)		1999	160000
		(non-linear)	

Presidential Physical Fitness Awards		World Oil Production	
Mile – Run		Year	Barrels
Age	Time (seconds)		(millions)
9	511	1900	149
10	477	1910	328
11	452	1920	689
12	431	1930	1412
13	410	1940	2150
14	386	1950	3803
(linear)		1960	7674
		1970	16690
		1980	21722

Old Faithful geyser eruption		Average Temperature in	
Length of eruption (minutes)	Minutes between eruptions	Month	Temp
		1. Jan	50
		2 Feb	54
2	57	3 Mar	60
2.5	62	4 Apr	67
3	68	5 May	74
3.5	75	6 June	80
4	83	7 July	82
4.5	89	8 Aug	81
5	92	9 Sept	78
		10 Oct	68
(Linear)		11 Nov	59
		(non-linear)	

<u>Sample Flow chart</u>
<u>Classifying Real Numbers</u>



Algebra I
Unit 2: Writing and Solving Proportions and Linear Equations

Time Frame: Approximately three weeks

Unit Description

This unit includes an introduction to the basic forms of linear equations and inequalities and the symbolic transformation rules that lead to their solutions. Topics such as rate of change related to linear data patterns, writing expressions for such patterns, forming equations, and solving them are also included. The relationship between direct variation, direct proportions and linear equations is studied as well as the graphs and equations related to proportional growth patterns.

Student Understandings

Students learn to recognize linear growth patterns and write the related linear expressions and equations for specific contexts. They need to see that linear relationships have graphs that are lines on the coordinate plane when graphed. They also link the relationships in linear equations to direct proportions and their constant differences numerically, graphically, and symbolically. Students can solve and justify the solution graphically and symbolically for single- and multi-step linear equations.

Guiding Questions

1. Can students graph data from input-output tables on a coordinate graph?
2. Can students recognize linear relationships in graphs of input-output relationships?
3. Can students graph the points related to a direct proportion relationship on a coordinate graph?
4. Can students relate the constant of proportionality to the growth rate of the points on its graph?
5. Can students perform simple algebraic manipulations of collecting like terms and simplifying expressions?
6. Can students perform the algebraic manipulations on the symbols involved in a linear equation or inequality to find its solution and relate its meaning graphically?

Unit 2 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Algebra	
7.	Use proportional reasoning to model and solve real-life problems involving direct and inverse variation (N-6-H)
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
Measurement	
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)
22.	Solve problems using indirect measurement (M-4-H)
Data Analysis, Probability, and Discrete Math	
34	Follow and interpret processes expressed in flow charts (D-8-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

Sample Activities

Activity 1: Think of a Number (GLEs: 5, 8, 9)

Number puzzles are an interesting way to review order of operations, properties of a number, and simple algebraic manipulation. Have students answer the following puzzle:

> Think of a number. Add 8 to it. Multiply the result by 2. Subtract 6. Divide by 2. Subtract the number you first thought of. Is your answer five?

Create a table with some numbers from the student results like the table below.

Starting number	6	13	10	24	x
Add 8	14	21	18	32	$x + 8$
Multiply by 2	28	42	36	64	$2(x + 8)$
Subtract 6	22	36	30	58	$2(x + 8) - 6$
Divide by 2	11	18	15	29	$[2(x + 8) – 6] \div 2$
Subtract starting number	5	5	5	5	5

Ask students if they know how the puzzle works. Have students visualize the puzzle by using symbols for the starting number and individual numbers. Then have students use a variable for the beginning number and write algebraic expressions for each step. Have the students develop their own puzzles, using spreadsheets if available. Provide other opportunities for students to review and practice order of operations and algebraic manipulations. Include expressions with various forms of rational numbers so that students can work to demonstrate computational fluency.

Activity 2: Order of Operations and Solving Equations (GLE 5, 8)

Have students work in groups to review solving one-step and multi-step equations. Discuss with students the reason for isolating the variable in an equation and use the comparison of solving an equation to a "balance scale." Then provide students with examples of equations that require simplification using algebraic manipulations and order of operations before they can be solved. Have students cover up one side of the equation and completely simplify the other then repeat with the other side of the equation. Provide students with other opportunities to practice solving different types of linear equations including literal equations. Include equations with various forms of rational numbers so that students can work to demonstrate computational fluency.

Activity 3: Using a flow chart to solve equations (GLE 5, 8, 34)

Review with students the steps to constructing a flow chart from Unit 1 Activity 2. Have the students construct a flow chart for solving equations in one-variable. A sample flow chart is included at the end of this unit. This flow chart is only a sample. Help students come up with other ways to make decisions about solving equations. After the flow charts have been constructed, have the students use the charts to solve different equations.

Activity 4: Linear relationships – Keeping it "real" (GLEs: 7, 9, 13, 37, 39)

Provide students with several input-output tables containing data that depict linear relationships (whose graphs pass through the origin) found in real-world applications. For example, the relationship between the number of gallons of gasoline and the total purchase price or the number of minutes on a cell phone and the total monthly bill both depict a linear function. Have students plot the ordered pairs generated by these data tables on a coordinate graph. See that students recognize that the graph is linear. Revisit direct variation from Unit 1 Activity 7 and discuss with the students that linear data through the origin represents a direct variation. Relate the constant of variation to the rate of change (slope) of the line. Have students write the equation to model the situation. Discuss the real-life meaning of the slope and the *y*-intercept for each table of values. (Although students have not been formally introduced to the terminology of slope and y-intercept, these examples should provide for a good discussion on the real-life meaning of slope and *y*-intercept). Have students state the rate of change in real-life terms. For example: For every gallon of gasoline purchased, the total cost increases by _____. Give students values that provide opportunities for them to solve the linear equations algebraically. For example, if John wants to spend exactly $20 on gasoline, how many gallons can he purchase?

Activity 5: Lines and Direct Proportions (GLEs: 9, 11, 37, 39)

Have students determine some relationships that are direct proportions. For example, they could state that distance traveled is directly proportional to the rate of travel, or the cost of movie tickets is directly proportional to the number purchased, or their total earnings are directly proportional to the hours they work. After some discussion and sharing, divide students into groups and give each group a different proportion. Have students create an input-output table, plot the ordered pairs, and draw the line connecting the ordered pairs. Have students write equations to model each direct proportion. Have students determine the constant of proportionality of each relationship and have each group present their graphs to the entire class. Discuss with the students that the constant of proportionality is the slope (rate of change) for each of the proportions graphed. Have the students state the rate of change in real-life terms. Discuss with students the idea that direct variation and direct proportion are both linear relations passing through the origin. (Other proportional data sets that could be used: The total cost for a bunch of grapes is directly proportional to the number of pounds purchased, the number of miles traveled is directly proportional to the number of kilometers traveled, or if the width of a rectangle is kept constant, then the area of the rectangle is directly proportional to the height.)

Activity 6: Solving Proportions (GLEs: 7, 8, 9, 22)

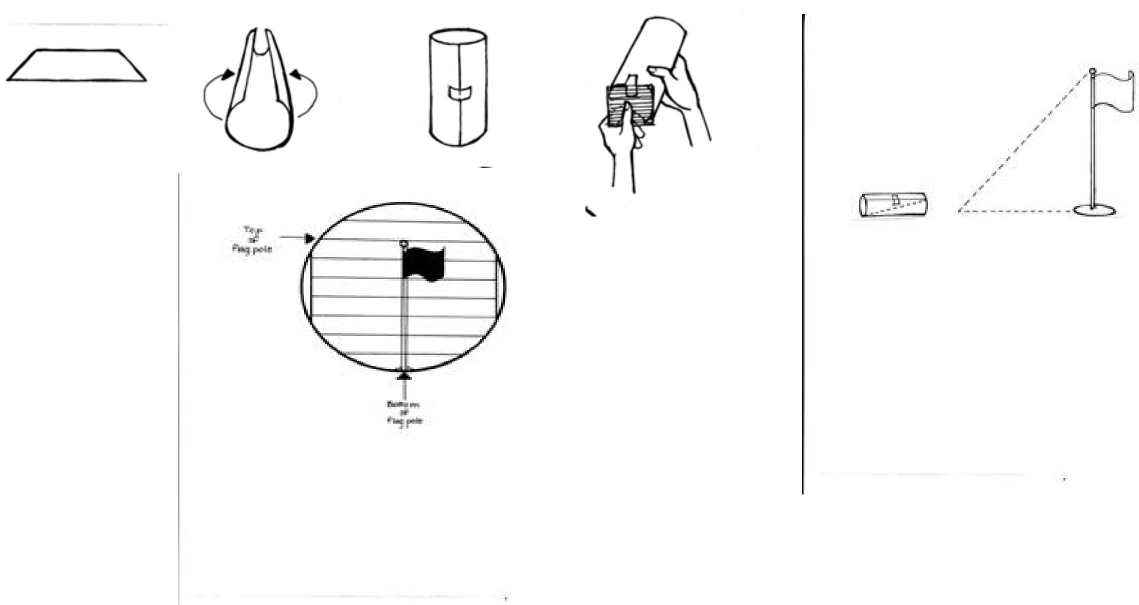
Have students set up and solve proportions that deal with real-life scenarios. For example, many outboard motors require a 50:1 mixture of gasoline and oil to run properly. Have students set up proportions to find the amount of oil to put into various amounts of gasoline. Recipes also provide examples for the application of proportional reasoning. Finding the missing side lengths of similar figures can allow students to set up a proportion as well as find measures by indirect measurement. For example, students can set up and solve a proportion that finds the height of an object by using similar triangles.

Activity 7: Using proportions and direct variation (GLEs: 7, 8)

Review with students the idea that direct variation and direct proportion are both linear relations passing through the origin and that the constant of variation is also called the constant of proportionality. Present students with the following direct variation problem that can be solved using a proportion: The cost of a soft drink varies directly with the number of ounces bought. It cost 75 cents to buy a 12 oz. bottle. How much does it cost to buy a 16 oz. bottle? Have students set up a proportion to solve the problem ($\frac{75}{12} = \frac{c}{16}$). Provide students with other direct variation problems that can be solved using a proportion.

Activity 8: How tall is the flagpole? (GLEs: 21, 22)

In this activity, students build a stadiascope and use it to find the height of the flagpole. A stadiascope is a tool that was used by the ancient Romans to measure the height of very tall objects. Have students work in groups of 3 or 4. Students will need an $8\frac{1}{2}$” x 11” sheet of card stock and a 4-inch square of clear acetate, such as an overhead transparency. Have them mark off equally spaced parallel lines on the acetate about one-half centimeter apart and roll the card stock sideways (not lengthwise) to make a viewing tube (See diagram below). Then tape the acetate to one side of the tube being careful that the bottom parallel line is just at the bottom of the tube. Next, have students measure the distance they are standing from the flagpole and view the entire flagpole through the stadiascope carefully lining up the bottom of the flagpole with the bottom of the tube. Have students decide the appropriate units to use when measuring the stadiascope and the distance to the flagpole. They will then use similar triangles and proportions to find the height of the flagpole. (Similar triangles are formed with the length of the bottom of the stadiascope corresponding with the distance the student is from the flagpole and the height of the top of the sighting of the flagpole in the stadiascope corresponding with the height of the flagpole)



Activity 9: Using inequalities to problem solve (GLE: 11)

Review the basics of solving one-step and multi-step inequalities. Present students with the following problem for class discussion: Trashawn wants to order some DVDs from Yomovies.com. DVDs cost $17 per DVD plus $5.50 for shipping and handling. If Trashawn wants to spend at most $75, how many DVDs can he buy? How much money will he have left over? Have students give more examples of vocabulary that may be used in solving inequalities, such as *at least*, *not more than*, *not to exceed*, etc.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- Performance Task: The student will find something that can be paid for in two different ways, such as admission to an amusement park or museum (e.g., Some museums will charge for each admission or sell a year-round pass, or an amusement park will sell a pay-one-price ticket or a per-ride ticket) and compare the costs. The student will explain the circumstances under which each option is better and justify the answers with a table, graph, and an equation, using inequalities to express their findings.
- The student will find the mistake in the solution of the following equation, explain the mistake, and solve the equation correctly:
$$\begin{aligned} 2x &= 11x + 45 \\ 2x - 11x &= 11x - 11x + 45 \\ 9x &= 45 \\ \frac{9x}{9} &= \frac{45}{9} \\ x &= 5 \end{aligned}$$
- The student will solve constructed response items such as:
  The amount of blood in a person's body varies directly with body weight. Someone weighing 160 lbs. has about 5 qt of blood.
  a. Find the constant of variation and write an equation relating quarts of blood to weight. ($\frac{1}{32}$, $b = \frac{1}{32}w$)
  b. Graph your equation.
  c. Estimate the number of quarts of blood in your body.
- The student will use proportions to solve the missing parts of similar figures.
- The student will determine if the following situations represent direct variation and explain why or why not:
  - ✓ The amount of a gas in a tank in liters and the amount in gallons (*yes*)

- ✓ The temperature in Fahrenheit degrees and in Celsius degrees (*no. Although this relationship is linear, the line does not go through the origin.*)
  - ✓ The price per pound of carrots and the number of pounds (*no*)
  - ✓ The total price of tomatoes and the number of pounds (*yes*)
- The student will submit a portfolio containing artifacts such as
  - ✓ daily student journals
  - ✓ teacher observation checklists or notes
  - ✓ examples of student products
  - ✓ scored tests and quizzes
  - ✓ student work (in-class or homework)
- The student will complete journal writings using such topics as:
  - ✓ Write a letter to a friend explaining order of operations.
  - ✓ Explain how solving an inequality is similar to solving an equation? In what ways is it different?
  - ✓ Describe a situation from your experience in which one variable is:
  - ✓ increasing at a constant rate
  - ✓ decreasing at a constant rate
  - ✓ increasing but not at a constant rate
  - ✓ Explain why the graph of a direct variation $y = kx$ always goes through the origin. Give an example of a graph that shows direct variation and one that does not show direct variation.

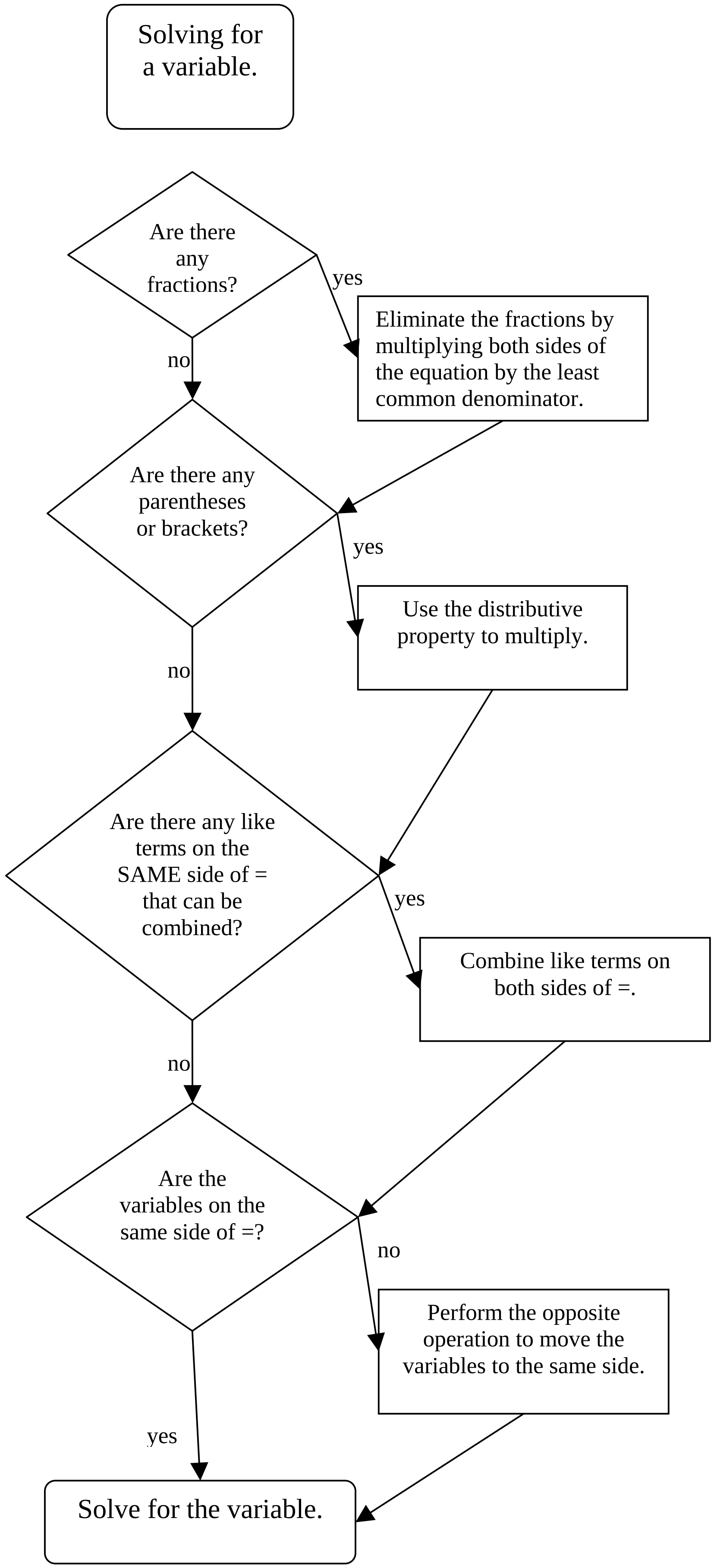
Activity-Specific Assessments

- Activity 4: The student will solve constructed response items such as:
  The drama club is selling tickets to their production of *Grease* for $4 each.
  - ✓ Make a table and a graph showing the amount of money they will make if 0, 5, 10, …, 100 tickets are sold.
  - ✓ Identify the variables and write an equation for the total amount the club will make for each ticket sold. ($y = 4x$)
  - ✓ Use your equation to show how much money the club will make if 250 people attend their production. (*$1000*)
  - ✓ The club spent $500 on their production. How many tickets must they sell to begin to make a profit? Justify your answer. (*125 tickets)*

- Activity 5: The student will choose one of the direct proportion situations and write at least two application problems that can be solved using a linear equation. The student will then write the equation for each application problem and solve it algebraically.
  - ✓ The student will determine the constant of proportionality for a direct proportion by relating it to the slope of the line they obtain from input-output data.

- Activity 8: The students will write a lab report describing the procedure for finding the height of the flagpole. The student will include diagrams and detailed work for justifying the solution as well as the conclusions in the report.

- Activity 9: Given an inequality such as $3x - 15 \geq 45$, the student will write an application problem for the inequality.


Solving for a variable.
Are there any fractions?
yes
Eliminate the fractions by multiplying both sides of the equation by the least common denominator.
no
Are there any parentheses or brackets?
yes
Use the distributive property to multiply.
no
Are there any like terms on the SAME side of = that can be combined?
yes
Combine like terms on both sides of =.
no
Are the variables on the same side of =?
no
Perform the opposite operation to move the variables to the same side.
yes
Solve for the variable.

Algebra I
Unit 3: Linear Functions and Their Graphs, Rates of Change, and Applications

Time Frame: Approximately five weeks

Unit Description

This unit leads to the investigation of the role of functions in the development of algebraic thinking and modeling. Heavy emphasis is given in this unit to understanding rates of change (intuitive slope) and graphing input-output relationships on the coordinate graph. Emphasis is also given to geometric transformations as functions and using their constant difference to relate to slope of linear equations.

Student Understandings

Students need to come to see functions as input-output relationships that have exactly one output for any given input. Central to this unit is the study of rates of change from an intuitive point, noting that the rate of change in graphs and tables is constant for linear relationships (one-differences are constant in tables) and for each change of 1 in *x* (the input), there is a constant amount of growth in *y* (the output). In Unit 2, this relationship for lines through the origin was tied to direct proportion. In this unit, emphasis is given to the formula and rate of change of a direct proportion as $y = kx$ or $\frac{y}{k} = \frac{x}{1}$. That is, as *x* changes 1, *y* changes *k*. Lines that do not run through the origin can be modeled by functions of the form $kx + b$, which are just lines of proportion translated up *b* units. These relationships need to be seen in a wide variety of settings.

Guiding Questions

1. Can students understand and apply the definition of a function in evaluating expressions (output rules) as to whether they are functions or not?
2. Can students apply the vertical line test to a graph to determine if it is a function or not?
3. Can students identify the matched elements in the domain and range for a given function?
4. Can students describe the constant growth rate for a linear function in tables and graphs, as well as connecting it to the coefficient on the *x* term in the expression leading to the linear graph?
5. Can students intuitively relate slope (rate of change) to *m* and the *y*-intercept in graphs to *b* for linear relationships $mx + b$?

Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
10.	Identify independent and dependent variables in real-life relationships (A-1-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
25.	Explain slope as a representation of “rate of change” (G-3-H) (A-1-H)
26.	Perform translations and line reflections on the coordinate plane (G-3-H)
Patterns, Relations, and Functions	
35.	Determine if a relation is a function and use appropriate function notation (P-1-H)
36.	Identify the domain and range of functions (P-1-H)
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
40.	Explain how the graph of a linear function changes as the coefficients or constants are changed in the function’s symbolic representation (P-4-H)

Sample Activities

Activity 1: What’s a Function? (GLEs: 12, 35, 36)

Give students examples of input-output tables that are and are not functions (that are labeled as such) including real-life examples. Pose the question: What is a function? Have students use a Think-Pair-Share process to help them understand what is significant in the tables. Lead them to the discovery of the definition of a function (for every input there is exactly one output). Repeat the activity with graphs that are and are not functions (include a circle and a parabola) and lead them to the discovery of the vertical line test. Ask students to explain why this vertical line test for functions is the same as the definition they used to see if the set of ordered pairs was a function. Discuss domain and range of a function. Introduce function notation *(* $f(x)$ *)*. Give students several input-output rules in the form of two-variable equations. Have students evaluate the expressions for several given inputs, solving for the output in each case. *(i.e If* $f(x) = 2x + 3$ *, find* $f(-2), f(-1), f(0)$ *)*. Next, have students determine if the set of ordered pairs in the input/output tables generated satisfies the definition of a function (i.e., for each element in the domain there is exactly one element in the range). Tell students to plot the ordered pairs and connect them and determine the domain and range. Now have students draw several vertical lines through the input values to illustrate the idea that for a function, a vertical line cuts through the graph at exactly one point.

Activity 2: Identify! (GLEs: 8, 12, 15, 35, and 36)

Give students two handouts, one containing a set of linear equations and the other containing a set of ordered pairs. Have students identify the domain and range of each relation. Have students work in pairs to determine which domain-range pairs match which given equation. The set of linear equations should include some that depict real-world scenarios. These linear equations should also include some that are in unsimplified form (e.g., $3y - 3(4x + 2) = 2y + 3$) so that students can have practice in using order of operations when they plug a value in for one of the variables and solve for the other. Examples of real-world linear equations would include direct proportions such as distance-time functions. Have students determine which relations are also functions. For those relations they determine to be functions, have students identify the independent and dependent variables and rewrite the linear function using function notation. For example, if students determine that $3x + y = 8$ is a linear function, then they could rewrite it as $h(x) = -3x + 8$.

Activity 3: Functions of Time (GLEs: 10, 15, 35, 36)

Have students collect and graph data about something that changes over time. (Ex. The temperature at each hour of the day, the height of a pedal on a bicycle when being ridden, the number of cars in a fast food parking lot at different times of the day, the length of a plastic grow creature as it sits in water, etc.) Have students organize the data in a spreadsheet and make a graph of the data. Have them identify the domain and range of the function. Then have the students construct a *PowerPoint®* presentation and present their findings to the class, perhaps first showing their graphs to the class without labels to see if other students can guess what they observed. (If technology is not available, have students construct the table and graph by hand on a posterboard.)

Activity 4: Patterns and Slope (GLEs: 13, 15, 25)
Divide students into groups and provide them with square algebra tiles. Have the students arrange 3 tiles in a rectangle and record the width (x) and the perimeter (y). Have the students fit 3 more tiles under the previous tiles and continue adding tiles, putting the values in a table. Example:

Width(x)	*Perimeter(y)*
1	*8*
2	*10*
3	*12*
4	*14*

Have students notice that the change in the y-values is the same, graphing the data and deciding if it is linear. Ask students what changed in the pattern *(the widths that keep increasing)* and what remained constant *(the length of the sides added together (3+3))*. Have them write a formula to describe the pattern. $(y = 6 + 2x)$ Guide students to conclude that what remained constant in the pattern will be the constant in the formula and the rate of change in the pattern will be the slope. Guide students to make a connection between the tabular, graphical and algebraic representation of the slope.

Activity 5: Recognizing Linear Relationships (GLEs: 9, 39, 40)

Provide students with several input-output tables (linear) paired with a graph of that same data. Include examples of real-life linear relationships. (Examples of linear data sets can be found in any algebra textbook.) Introduce slope as the concept of $\frac{rise}{run}$. Have students determine the slope of the line and then investigate the change in the x-coordinates and the accompanying change in the y-coordinates. Ask, Was a common difference found? How does this common difference in the y-coordinates compare to the slope (rate of change) found for the line? Using this information, have students conjecture how to determine if an input-output table defines a linear relationship. *(There is a common difference in the change in y over the change in x.)* Have students write a linear equation for each of the graphs. Have students compare the input-output tables, the graphs, and the equations to see how the slope and y-intercepts affect each.

Activity 6: Rate of Change (GLEs: 10, 12, 13, 15, 23, 25, 39)

Introduce the following problem: David owns a farm market. The amount a customer pays for sweet corn depends on the number of ears that are purchased. David sells a dozen ears of corn for $3.00. Place the students in groups and ask each group to make a table reflecting prices for purchases of 6, 12, 18, and 24 ears of corn. Have each group write and graph four ordered pairs that represent the number of ears of corn and the price of the purchase. Have each group write an explanation of how the table was developed, how the ordered pairs were determined, and how the graph was constructed. After ensuring that each group has a valid product, ask the students to use a straightedge to construct the line passing through the points on the graph. Looking at the line constructed, ask each group to find the slope of the line. Review the idea that slope is an expression of a rate of change. Ask students to explain the real-life meaning of the slope. *(For every ear of corn purchased, the price goes up $.25.)* Introduce the slope-intercept form of an equation. Have groups determine the equation of the lines by examining the graph for the slope and *y*-intercept. Point out to the students that the value of *y* (the price of the purchase) is determined by the value of *x* (the number of ears purchased). Therefore, *y* is the dependent variable and *x* is the independent variable. Point out to the student that the value of *y* will always increase as the value of *x* increases. This is indicated by the fact that there is a positive slope. Also, point out that the *y*-intercept is at the origin because no purchase would involve a zero price. Ask the students to use the equation to find the price of a purchase of four ears of corn.

Activity 7: Graph Families (GLEs: 37, 38, 39, 40)

Generate a discussion on families of linear graphs by describing the following situation. Suppose you go to a gourmet coffee shop to buy coffee beans. At the store, you find that one type of beans costs $6.00 per pound and another costs $8.00 per pound. Place the students in groups and ask them to construct a graph for each type of coffee bean where the *x*-axis identifies the number of pounds of beans and the *y*-axis represents the cost of the coffee. Ask the students to write the equations of the lines and to find the slope and *y*-intercept for each of the lines. Ask each group to share its findings, and ensure that each group finds the correct equations, slopes, and *y*-intercepts. Ask the students to describe similarities and differences for the values found for each equation. Tell each group that there was a third type of beans that cost $4.00 per pound. Ask each group to use the same grid to construct the graph of the third type of bean and to write the equation, the slope, and the *y*-intercept for the third line. Ask each group to compare these findings with those for the other beans and to describe differences and similarities. When the class decides that these lines have different slopes and the same *y*-intercept, inform the students that a group of lines that all share at least one common characteristic is called a family of lines. Furthermore, have them recognize that the family of lines shares a common intercept and different slopes.

Write the equations $y=-2x+3$ and $y=-2x-5$ on the board or overhead projector. Ask the student groups to find and compare the slopes and *y*-intercepts for the given lines. Ask each group to write a statement of its findings and have each group report its findings to the entire class. Use a graphing calculator or computer-graphing program to graph both lines on the same grid. Ask each group to look at the graphs and report its observations. The determination should be that lines having the same slope and different *y*-intercepts are parallel. To verify the observation, graph the equation $y=-2x+7$ on the same grid as the other two. The third line should appear to be parallel to the other two. Explain to the students that these three lines are all members of the family of parallel lines, having a slope of –2 and different *y*-intercepts.

Conclude the lesson by clarifying what is meant by the term *family of lines* and discussing similarities and differences of the types of families.

Activity 8: Make that Connection! (GLEs: 10, 12, 13, 15, 25, 36)

Have students generate a table of values for a given linear function expressed as $f(x) = mx + b$. An example would be the cost of renting a car is $25 plus $0.35 per mile. Have students label the input value column of the table "Independent Variable" and the output value column "Dependent Variable." Have students select their own domain values for the independent variable and generate the range values for the dependent variable. Next, have students calculate the differences in successive values of the dependent variable, and find a constant difference. Then have them relate this constant difference to the slope of the linear function. Next, have students graph the ordered pairs and connect them with a straight line. Finally, discuss with the students the connections between the table of values, the constant difference found, the graph, and the function notation. Last, have students do the same activity using a linear function that models a real-world application. For example, students could investigate the connections between the algebraic representation of a cost function, the table of values, and the graph.

Activity 9: Slopes and *y*-Intercepts (GLEs: 38, 40)

Have students use a graphing calculator to graph several linear functions for which the slope is constant and the *y*-intercept changes. Have students explain how the changes in the *y*-intercepts affect the graphs. Next, have students graph several linear functions in which the slope changes and the *y*-intercepts remain constant. Have students explain the effects of the change in the slope on the graphs. Have students make conjectures about positive and negative slopes. Discuss the slopes of horizontal and vertical lines and the lines $y = x$ and $y = -x$. Help students intuitively relate slope (rate of change) to m and the *y*-intercept in graphs to b for each of these linear functions expressed as $f(x) = mx + b$.

Activity 10: Rate of Growth (GLEs: 11, 13, 15, 23, 25, 37, 38)

Provide students with two similar triangles, quadrilaterals, etc. Have students measure the corresponding side lengths in these two similar figures and plot them as ordered pairs (i.e., students would plot the ordered pair [original side length, corresponding side length] for each pair of corresponding sides). Have students first determine the ratio between the side lengths and then compare that ratio to the slope of the line. Have them determine that the graph also indicates that the relationship is proportional since the line passes through the origin. Ask students to write the equation in slope-intercept form to find that $y = kx$, where k is the ratio they found between the corresponding parts of the two similar figures. Have students describe the slopes of these linear functions as they relate to describing the proportional relationship between two similar figures. Next, have the students switch the order of the ordered pairs that were plotted and plot them (i.e., corresponding side length, original side length). Determine the ratio between two corresponding sides and compare to the ratio to the slope of the new line. Ask, How do the ratios compare to one another? (*The ratios are reciprocals.*) What do the equations mean in a real-life setting? (*The two equations indicate how to convert between lengths in the two figures. One says to multiply the values in the smaller figure by some number to get the corresponding values in the larger figure. The other indicates how to find the lengths in the smaller figure from the values in the larger figure.*) Repeat this activity several times until students understand that the equation of the line describes the proportional relationship between the side lengths of the two figures and that the proportional relationship represents a rate of growth from the small figure to the large (or vice versa).

Activity 11: Recognizing Translations (GLEs: 15, 26)

Give students a set of ordered pairs that are the vertices of a triangle, square, or other geometric shape. Also, provide students with a translation rule depicted as an input-output rule. For example, the rule of (x, y) goes in and $(x+2,\ y+3)$ comes out. Have students create a table of ordered pairs and then graph each ordered pair that represents a vertex and the corresponding new ordered pair $(x+2,\ y+3)$. Have them then describe the rule as a translation of each point 2 to the right and up 3. Repeat this activity using several different translation rules.

Activity 12: Recognizing Reflections (GLEs: 15, 26)

Give students a set of ordered pairs that are the vertices of a triangle, square, or other geometric shape. Also, provide students with a reflection rule depicted as an input-output rule. For example, the rule of (x, y) goes in and $(x,\ -y)$ comes out to represent a reflection across the *x*-axis. Have students create a table of ordered pairs and then graph each ordered pair that represents a vertex and the corresponding new ordered pair $(x,\ -y)$. Next have them describe the rule as a reflection of each point across the *x*-axis. Repeat this activity using reflection across the *y*-axis. Be sure to include in the original vertices some points that lie on an axis. As an extension, have students reflect the given vertices across other vertical or horizontal lines.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- The students will submit a portfolio with artifacts such as:
  - daily student journal

- o teacher observation checklists or notes
  - o examples of student products
  - o scored tests and quizzes
  - o teacher observations of group presentations
- The students will use the definition of a function and/or the vertical line test to determine which of several relations are functions.
- The student will generate the functional notation for a linear function expressed in *x* and *y*.
- The student will generate a function's graph from an input-output table.
- The student will make a poster of a function represented in three different ways and describe the domain and range of the function.
- Given a graph that is a function of time, the student will write a story that relates to the graph.
- The students will answer open-ended questions such as:
  Maria is hiking up a mountain. She monitors and records her distance every half hour. Do you think the rates of change for every half hour are constant? Explain your answer.
- The student will solve constructed response items such as:
  Signature Office Supplies is a regional distributor of graphing calculators. When an order is received, a shipping company packs the calculators in a box. They place the box on a scale which automatically finds the shipping cost. The cost *C* depends on the number *N* of the calculators in the box, with rule $C = 4.95 + 1.25N$.
  a. Make a table showing the cost for 0 to 20 calculators.
  b. How much would it cost to ship an empty box? (*4.95)* How is that information shown in the table and the cost rule?
  c. How much does a single calculator add to the cost of shipping a box? (*1.25)* How is that information shown in the table and the cost rule?
  d. Write and solve equations and inequalities to answer the following questions.
     a. If the shipping cost is $17.45, how many calculators are in the box? (*10 calculators)*
     b. How many calculators can be shipped if the cost is to be held below $25? (*16 calculators)*
     c. What is the cost of shipping eight calculators? (*$14.95)*
  e. What questions about shipping costs could be answered using the following equation and inequality?

     $27.45 = 4.95 + 1.25N$

     $4.95 + 1.25N \le 10$
- The students will complete journal writings using such topics as:
  - o Sketch the graph of a relation that is not a function and explain why it is not a function.
  - o Explain algebraically and graphically why $y = 2x^2 - 7$ is a function.
  - o Explain why the vertical line test works.

- A child's height is an example of a variable showing a positive rate of change over time. Give two examples of a variable showing a negative rate of change over time. Explain your answer.
- Explain why the graph of an equation of the form $y = kx$ always goes through the origin. Give an example of a graph that shows direct variation and one that does not show direct variation.
- Explain how you can tell if the relationship between two sets of data is linear.

Activity-Specific Assessments

- <u>Activity 1</u>: The students will decide if the following relations are functions:
  a. number of tickets sold for a benefit play and amount of money made *(yes)*
  b. students' height and grade point averages (*no)*
  c. amount of your monthly loan payment and the number of years you pay back the loan (*no)*
  d. cost of electricity to run an air conditioner during peak usage hours and the number of hours it runs (*yes)*
  e. time it takes to travel 50 miles and the speed of the vehicle (*yes)*

- <u>Activity 3</u>: The student will write a report explaining the procedures and the conclusions of the investigation. The teacher will provide the student a rubric to use when he/she writes the report including questions that must be answered in the report such as: How did you decide on values to use for your axes? And what did you and your partner learn about collecting and graphing data? (A sample rubric is included at the end of this unit.)

- <u>Activity 5</u>: The student will find the rate of change between consecutive pairs of data.

  Example:

x	1	3	4	7
y	3	7	9	15

  Is the relationship shown by the data linear? *(Yes)* Explain your answer. (*There is a common difference between the change in y over the change in x. (2)*)

- <u>Activity 7</u>: The student will sort a set of linear functions into families based on slope and *y*-intercept characteristics.

- <u>Activity 8</u>: The student will solve constructed response items such as:
  Suppose a new refrigerator costs $1000. Electricity to run the refrigerator costs about $68 per year. The total cost of the refrigerator is a function of the number of years it is used.
  a. Identify the independent and dependent variables
  b. State the reasonable domain and range of the function.
  c. Write an equation for the function. ($C = 1000 + 68N$)
  d. Make a table of values for the function.

e. Graph the function.
f. Label the constant difference (slope) on each of the representations of the function.

Functions of Time Project
Rubric

Directions: Write a report explaining the procedures and the conclusion of your function of time investigation. In your report, the following questions must be answered:

1. What did you investigate?
2. How did you and your partner decide what to investigate?
3. What is the domain and range of your function?
4. Did you see any patterns in the relationship you observed?
5. How did you decide what values to use for your axes?
6. How did you divide up the work between you and your partner?
7. Did you have any problems conducting your investigation? If so, explain.

(Teachers may wish to add other questions to ensure understanding of the investigation.)

This rubric must be handed in with your final project.
Name___________________________________

Spreadsheet/Table

Demonstrates mastery of constructing a spreadsheet/table with no errors	Spreadsheet/table is constructed with 1 – 2 errors	Spreadsheet/table is constructed with 3 errors	Spreadsheet/table is constructed with 4 - 5 errors	Spreadsheet/table is constructed with many errors
4 points	3 points	2 points	1 points	0 points

Graph

Graph is exemplary. Title is included, axes are labeled appropriately, all points are plotted correctly.	Graph is sufficient but has 1 - 2 errors in construction.	Graph has 3 errors.	Graph has 4 -5 errors.	Graph is constructed with many errors.
4 points	3 points	2 points	1 point	0 points

Report

Report is exemplary. All questions are answered thoroughly. No grammatical errors.	Report is constructed with few grammatical errors or one question was not answered thoroughly.	Report is constructed with grammatical errors or 2 -3 questions were not answered thoroughly.	Report is constructed with many grammatical errors or 4 questions were not answered thoroughly.	Report is insufficient as explanation of project.
12 points	9 points	6 points	3 points	0 points

Algebra I
Unit 4: Linear Equations, Inequalities, and Their Solutions

Time Frame: Approximately five weeks

Unit Description

This unit focuses on the various forms for writing the equation of a line (point-slope, slope-intercept, two-point, and standard form) and how to interpret slope in each of these settings, as well as interpreting the *y*-intercept as the fixed cost, initial value, or sequence starting-point value. The algorithmic methods for finding slope and the equation of a line are emphasized. This leads to a study of linear data analysis. Linear and absolute value inequalities in one-variable are considered and their solutions graphed as intervals (open and closed) on the line. Linear inequalities in two-variables are also introduced.

Student Understandings

Given information, students can write equations for and graph linear relationships. In addition, they can discuss the nature of slope as a rate of change and the *y*-intercept as a fixed cost, initial value, or beginning point in a sequence of values that differ by the value of the slope. Students learn the basic approaches to writing the equation of a line (two-point, point-slope, slope-intercept, and standard form). They graph linear inequalities in one variable ($2x+3>-x+5$ and $|x|>3$) on the number line and two variables on a coordinate system.

Guiding Questions

1. Can students write the equation of a linear function given appropriate information to determine slope and intercept?
2. Can students use the basic methods for writing the equation of a line (two-point, slope-intercept, point-slope, and standard form)?
3. Can students discuss the meanings of slope and intercepts in the context of an application problem?
4. Can students relate linear inequalities in one variable to real-world settings?
5. Can students perform the symbolic manipulations needed to solve linear and absolute value inequalities and graph their solutions on the number line and the coordinate system?

Unit 4 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Algebra	
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
14.	Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Measurement	
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)
22.	Solve problems using indirect measurement (M-4-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
25.	Explain slope as a representation of “rate of change” (G-3-H) (A-1-H)
Data Analysis, Probability, and Discrete Math	
34.	Follow and interpret processes expressed in flow charts (D-8-H)
Patterns, Relations, and Functions	
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

Sample Activities

Activity 1: Generating Equations (GLEs: 13, 23, 24, 25)

Remind the students that the slope of a line is the ratio of the change in the vertical distance between two points on a line and the change in horizontal distance between the two points. Use a geoboard to model the concept. Ask the students to think of the pegs on the geoboard as points in a coordinate plane and explain that the lower left peg represents the point (1,1). Ask the students to locate the pegs representing the pair (1,1) and the pair (3,5) and place a rubber band around the pegs to model the line segment joining (1,1) and (3,5). Ask them to use a different colored rubber band to show the horizontal from *x* value to *x* value of the two endpoints and use another colored rubber band to show the distance from *y*-value to *y*-value to the endpoints. Ask the students to find the value of the change in *y*-values (3) and the change in *x*-values (2) and show that the defined slope ratio is $\frac{3}{2}$. Ask students to use this procedure to find the slope of the segment from the point (5,2) and (1,4). Lead the students to discover that, because the line moves downward from left to right, the change in *y* would produce a negative value and the slope ratio is negative. Show the class that if the computations above are generalized, the formula $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ where x_2 is not equal to x_1 could determine the slope of the line passing through the two points.

When student understanding of slope is evident, ask them to find the slope between a specific point (x_1, y_1) and a general point (*x*, *y*). Guide them to the conclusion that this slope would be $m = \frac{(y - y_1)}{(x - x_1)}$. Work with the students to algebraically transform this equation into its equivalent form $(y - y_1) = m(x - x_1)$. Explain that this is the point-slope form for the equation of a line and that it may be used to write the equation of a line when a point on the line and the slope of a line are known. Guide the students through the determination of the line with slope 2 and passing through points with coordinates (3, 4).

Ask the students to use a coordinate grid and graph several nonvertical lines. Guide the students to the discovery that all nonvertical lines will intersect the *y*-axis at some point and inform them that this point is called the *y*-intercept. Pick out several points along the *y*-axis and write their coordinates. Through questioning, allow the students to infer that all points on the *y*-axis have *x*-coordinates of 0. Then, establish that a general point of the *y*-intercept of a line could be expressed as (0, *b*). Ask the students to write and simplify the equation of the line with slope *m* and passing through the point (0, *b*). Using the point-slope form for the equation of a line, $(y - b) = m(x - 0)$, have students insert the point (0, *b*) and solve for *y*, producing the slope-intercept form for the equation. Place the students in small groups and have them work collectively to write equations of lines when given the slope and the *y*-intercept.

Activity 2: Points, Slopes, and Lines (GLE: 24)

Provide students with opportunities to plot graphs based on having either a known slope and a point or two points. When given a slope and a point, help students start at the given point and use the slope to move to a second point. Have students label the second point after using the slope to move there. Then have them connect these two points to produce a graph of the line with the given slope which passes through the given point. When given two points, ask students to plot them and then connect them with a line. Next, have students determine the slope of the line by counting vertical and horizontal movement from one of the plotted points to the other plotted point. Repeat this activity with various slopes and points. Then give students an equation in slope-intercept form and provide discussion for graphing a line when the equation is in slope-intercept form.

Activity 3: Applications (GLEs: 4, 5, 11, 13, 21, 22, 23, 24, 25, 38, 39)

This activity includes an investigation that will involve applying the concepts learned in Activities 1 and 2. Have students investigate the linear relationship between a person's foot length and length of the arm from the elbow to fingertip. Also have them collect and organize data, determine line of best fit, investigate slope and *y*-intercept, and use an equation to make predictions. To perform the investigation, they will need tape measures (metric), graph paper, a piece of spaghetti, and a transparency coordinate system. A graphing calculator is optional.

Initially this is done as an in-class activity. Have the students measure their foot length and arm length to the nearest millimeter (a class discussion of measurement techniques and of rounding measurements is appropriate). The foot length should be measured from the heel to the end of the big toe. The arm length should be from the elbow to the tip of the index finger. Have the students agree on measuring technique so that all measures are somewhat standardized. Have students take measurements and compile their data into tables where foot length is the independent variable and arm length is the dependent variable. Have each student graph his/her personal data on the overhead coordinate system. After all points are plotted, discuss what occurs. Ask questions like, Looking at the graph, do you see characteristics? Does there appear to be a relationship? What happens to the *y*-values as the *x*-values increase? Talk about the line of best fit. The piece of spaghetti will be used as a tool to estimate the line of best fit. Allow the students to make suggestions as to where it will be placed on the graph. Once the line is placed, review the ideas of slope of a line, *y*-intercept, point-slope form of a line, dependent and independent variables, etc. Determine two points that are contained in the line of best fit, find the slope of the line, and use the point-slope formula to write the equation. Have students state the real-life meaning of the slope of the line. Explain that this equation could be used as a means of estimating the length of a person's arm when the length of his or her foot is known. Have the students take foot and arm measures of an individual not yet measured (often the teacher is a good candidate for these measures). Place the newly found foot length into the equation to estimate foot length and to compare the actual value with the measured value. Conduct another linear experiment such as timing students in the class as they do the wave where the number of students would be the independent variable and time in seconds would be the dependent variable. Put students in small groups and have each group create the scatter plot, derive the linear equation for the data, state the real-life meaning of the slope, and calculate how long it would take 100 students to do the wave. Compare each group's lines of best fit. Have students identify the characteristics of the different lines that are the same or different. Also have them compare and contrast the linear functions they obtained algebraically in terms of their rates of change and y-intercepts. Many graphing calculators are programmed to use statistical processes to calculate lines of best fit. Students might find it interesting to input class data into the calculator and compare the calculator's estimate with theirs.

Activity 4: Linear Experiments (GLEs: 13, 15, 23, 25, 39)

Have groups of students complete a variety of experiments. For each experiment, have the group collect, record, and graph the data. Have the group discuss the meaning of the *y*-intercept and slope, identify independent and dependent variables, explain why the relationship is linear, write the equation, and extrapolate values. Some sample experiments include:

Bouncing Ball

Goal: to determine how the height of a ball's bounce is related to the height from which it is dropped
Materials: rubber ball, measuring tape
Procedure: Drop a ball and measure the height of the first bounce. To minimize experimental error, you can drop from the same height 3 times, and use the average bounce height as the data value. Repeat using different heights.

Stretched Spring

Goal: to determine the relationship between the distance a spring is stretched and the number of weights used to stretch it
Materials: spring, paper cup, pipe cleaner, weights, measuring tape
Procedure: Suspend a number of weights on a spring and measure the length of the stretch of the spring. A slinky (cut in half) makes a good spring; one end can be stabilized by suspending the spring on a yard stick held between two chair backs. A small paper cup (with a wire or pipe cleaner handle) containing weights, such as peppermints, can be attached to the spring.

Burning Candle

Goal: to determine the relationship between the time a candle burns and the height of the candle.
Materials: birthday candle (secured to a jar lid), matches, ruler, stopwatch
Procedure: Measure the candle; mark the candle in 10 cm or 1/2 in. units. Light the candle while starting the stopwatch. Record time burned and height of candle.

Marbles in Water

Goal: to determine the relationship between the number of marbles in a glass of water and the height of the water.
Materials: glass with water, marbles, ruler or measuring tape
Procedure: Measure the height of water in a glass. Drop one marble at a time into the glass of water, measuring the height of the water after each marble is added.

Marbles and uncooked spaghetti

Goal: to see how many pieces of spaghetti it takes to support a cup of marbles
Materials: paper cup with a hook (paper clip) attached, spaghetti, marbles
Procedure: place the hook on a piece of uncooked spaghetti supported between two chairs, drop in one marble at a time until the spaghetti breaks, repeat with two

pieces of spaghetti, etc. *(number of pieces of spaghetti is ind. and number of marbles is dep.)*

Activity 5: Processes (GLE: 34)

Have students follow the steps in a flow chart for putting a linear equation expressed in standard form into slope-intercept form. A sample flow chart that could be used is included at the end of this unit. Next, have students work in pairs to create a flow chart of steps an "absent classmate" could use to convert a linear equation written in slope-intercept form to standard form. Review the following procedures: questions go in the diamonds; processes go in the rectangles; *yes* or *no* answers go on the connectors. Have a class discussion of the finished flow charts, and then have students construct another flow chart individually to convert a linear equation from point-slope form to standard form. Have them exchange charts with another student and follow them to perform the conversion.

Activity 6: Inequalities (GLEs: 11, 14)

Provide students with real-life scenarios that can be described by an inequality in one variable. Have students graph the inequality and interpret the solution set. Make sure students are given inequalities to interpret that include both weak inequalities (i.e., $\leq$ or $\geq$) and strict inequalities (i.e., < or >), as well as absolute value inequalities. An example follows:

> When Latoya measured Rory's height, she got 172 cm but may have made an error of as much as 1 cm. Letting *x* represent Rory's actual height in cm, write an inequality indicating the numbers that *x* lies between. Write the equivalent inequality using absolute value. *$(171 \leq x \leq 173, |x - 172| \leq 1)$*

Activity 7: Is it Within the Area? Interpreting Absolute Value Inequalities in One Variable (GLEs: 5, 14)

Review with students the idea of being within a certain distance of a location. For example, ask what it means to be within 25 miles of their home. Using only straight-line distances (rather than a circle in two dimensions), have students graph simple absolute value inequalities in one variable on the number line. The location point would always be the number that makes the expression inside the absolute value bars zero. For example, if $|x-3|<5$ is given, then the "location" is 3 because $x-3$ is zero at $x=3$. The area the inequality encompasses is from –2 to 8. This area is found simply by moving 5 units away from the "location" in both directions. Repeat this activity several times. Extend this idea to solving absolute value inequalities like $|ax+b|<c$.

Activity 8: Graphing Inequalities in Two Variables (GLE: 14)

Introduce activity by asking students if (5, 3) and (3, 1) are solutions to the inequality $x - y \geq 1$. Ask how many other points are solutions? Have students work with a partner and make a large coordinate grid on poster paper. Both axes should extend from –4 to 4. Have students write the value of $x - y$ on each coordinate point (i.e., on the point (3, 2) the student would write (3 – 2) or 1). Have students circle with a colored pencil several values that satisfy the inequality $x - y \geq 1$. Question students about points that lie between the points (ex. 2.5, 4.5). Have students shade all the solutions to the inequality. Use the students' conclusions about this inequality to guide a discussion on graphing all inequalities in two variables.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- The student will create a portfolio that includes student-selected and teacher-selected work.
- The student will complete constructed response items such as:
  - Each gram of mass stretches a spring .025 cm. Use $m = .025$ and the ordered pair (50, 8.5) to write a linear equation that models the relationship between the length of the spring and the mass.
    $y = .025x + 7.25$
    a) What does the y-intercept mean in this situation? (*When the spring is not stretched at all it is 7.25 cm.*)
    b) What is the length of the spring for a mass of 70 g? (*9 cm)*
  - A taxicab ride that is 2 mi. long costs \$7. One that is 9 mi long costs \$24.50.
    a) Write an equation relating cost to length of ride. ($C = 2.5m + 2$)
    b) What do the slope and y-intercept mean in this situation? (*Slope – the cost goes up \$2.50 for each mile driven, y-intercept – The cost is \$2 for 0 miles driven)*
- The student will complete journal writings using such topics as:
  - Describe two ways to find the slope of the graph of a linear equation. Which do you prefer? Why?
  - Write a few sentences to explain whether a line with a steep slope can have a negative slope?
  - Explain how you would graph the line $y = \frac{3}{4}x + 5$.
  - Explain why absolute value is always a non-negative number.

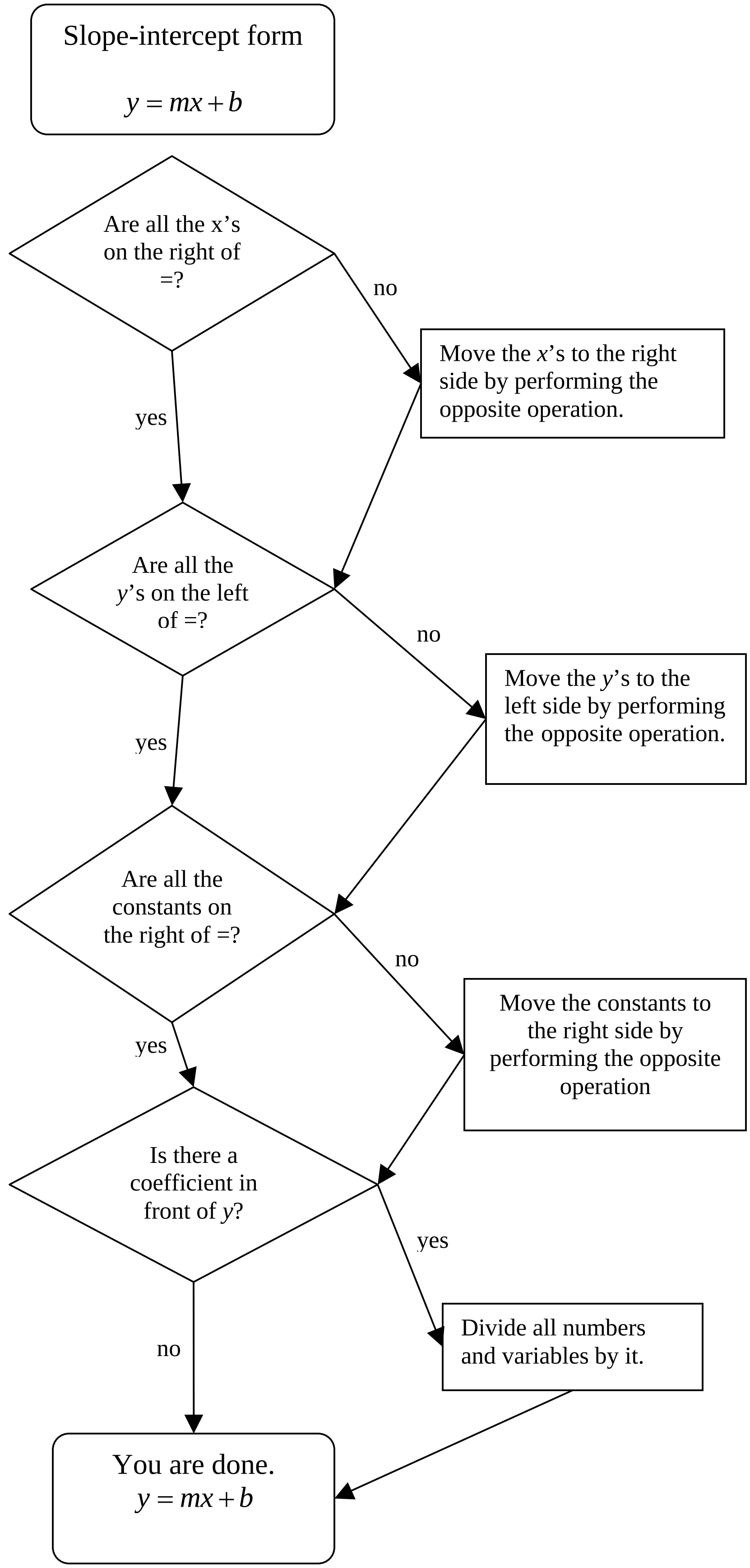
Activity-Specific Assessments

- Activity 1:
  - The student will write the equation of a linear function when given two points or one point and the *y*-intercept.
  - The student will convert one form of a linear equation into another equivalent form.

- Activity 2:
  - Given a linear function and its graph, the student will demonstrate knowledge of the slope and *y*-intercept, found graphically, to the slope and *y*-intercept, found algebraically, as a coefficient of *x* and the constant term, when the equation is in slope-intercept form.
  - The student will interpret the slope and *y*-intercept of a graph that depicts a real-world situation (i.e. state its real-life meaning).

- Activity 3:
  - The student will use any of the linear data sets from Unit 1 and complete the following tasks with and/or without the graphing calculator.
    a. Make a scatter plot of the data
    b. Draw and find the equation of the line of best fit
    c. Give the real-life meaning of the slope and *y*-intercept
    d. Predict for a specific independent variable
    e. Predict for a specific dependent variable

- Activity 4:
  - The student will construct a lab report describing materials, procedures, diagrams, and conclusion of the linear experiment.


Slope-intercept form
$y = mx + b$
Are all the x's on the right of =?
no
Move the x's to the right side by performing the opposite operation.
yes
Are all the y's on the left of =?
no
Move the y's to the left side by performing the opposite operation.
yes
Are all the constants on the right of =?
no
Move the constants to the right side by performing the opposite operation
yes
Is there a coefficient in front of y?
yes
Divide all numbers and variables by it.
no
You are done.
$y = mx + b$

Algebra I
Unit 5: Systems of Equations and Inequalities

Time Frame: Approximately five weeks

Unit Description

In this unit, linear equations are considered in tandem. Solutions to systems of two linear equations are represented using graphical methods, substitution, and elimination. Matrices are introduced and used to solve systems of two and three linear equations with technology. Heavy emphasis is placed on the real-life applications of systems of equations. Graphs of systems of inequalities are considered in the coordinate plane.

Student Understandings

Students need to understand the nature of a solution for a system of equations and a system of inequalities. In the case of linear equations, students need to develop the graphical and symbolic methods of determining the solutions, including matrices. In the case of linear inequalities in two variables, students need to see the role played by graphical analysis.

Guiding Questions

1. Can students explain the meaning of a solution to a system of equations or inequalities?
2. Can students determine the solution to a system of two linear equations by graphing, substitution, elimination, or matrix methods (using technology)?
3. Can students use matrices and matrix methods by calculator to solve systems of two or three linear equations $\boldsymbol{A}x = \boldsymbol{B}$ as $x = \boldsymbol{A}^{-1}\boldsymbol{B}$?
4. Can students solve real-world problems using systems of equations?
5. Can students graph systems of inequalities and recognize the solution set?

Unit 5 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Algebra	
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
14.	Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)

GLE #	GLE Text and Benchmarks
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
16.	Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
Patterns, Relations, and Functions	
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

Sample Activities

Activity 1: Systems of Equations (GLEs: 15, 16, 23)

Ask the students to imagine two people are walking in the same direction at different rates, with the faster walker starting out behind the slower walker. At some point, the faster walker will overtake the slower walker. Suppose that Sam is the slower walker and James is the faster walker. Sam starts his walk and is walking at a rate of 1.5 mph, and one hour later James starts his walk and is walking at a rate of 2.5 miles per hour. Ask the students how to use graphs to determine where and when James will overtake Sam. Review with the students the distance = rate × time relationship and guide them to the establishment of an equation for both Sam and James (*Sam's equation should be* $d = 1.5t$*, and James' equation should be* $d = 2.5(t-1)$). Use a graphing calculator to graph both equations. Find the point of intersection (*2.5, 3.75*). Lead the students to the discovery that two and one-half hours after Sam started, James would overtake him. They both would have walked 3.75 miles. Show the students that the goal of the process is to find a solution that makes each equation true, and that is the solution to the system of equations. Use the same real-life example to show when a system of equations might have no solution or many solutions. Give the students a number of problems involving 2 × 2 systems of equations and have them use a graphing calculator to solve them graphically. Emphasize that the solution of a system is the point(s) where the graphs intersect and that the point(s) is (are) the common solution(s) to both equations. Provide opportunities for students to solve systems of equations by graphing. Include systems with no solutions and an infinite number of solutions.

Activity 2: Battle of the Sexes (GLEs: 11, 15, 16, 23, 39)

Provide students with the following Olympic data of the winning times for men and women's 100-meter freestyle. Have students create scatter plots and find the equation of the line of best fit for each set of data either by hand or with the graphing calculator. Have students find the point of intersection of the two lines and explain the significance of the point of intersection. *(The two lines of best fit intersect leading to the conclusion that eventually women will be faster than men in the 100-Meter Freestyle.)* Also have students compare the two equations in terms of the rates of change. (i.e. How much faster are the women and the men each year?)

Men's 100-Meter Freestyle		Women's 100-Meter Freestyle	
Year	Time (seconds)	Year	Time(seconds)
1920	61.4	1920	73.6
1924	59	1924	72.4
1928	58.6	1928	71
1932	58.2	1932	66.8
1936	57.6	1936	65.9
1948	57.3	1948	66.3
1952	57.4	1952	66.8
1956	55.4	1956	62
1960	55.2	1960	61.2
1964	53.4	1964	59.5
1968	52.2	1968	60
1972	51.2	1972	58.6
1976	50	1976	55.7
1980	50.4	1980	54.8
1984	49.8	1984	55.9
1988	48.6	1988	54.9
1992	49	1992	54.6
1994	48.7	1994	54.5
1996	48.7	1996	54.5

Activity 3: Substitution (GLEs: 11, 12, 15, 16, 23, 39)

Begin by reviewing the process for solving systems of equations graphically. Inform the students that it is not always easy to find a good graphing window that allows the determination of points of intersection from observation. Show them an example of a system that is difficult to solve by graphing. Explain that there are other methods of finding solutions to systems and that one such method is called the substitution method. The following example might prove useful in modeling the substitution method.

Alan Wise runs a red light while driving at 80 kilometers per hour. His action is witnessed by a deputy sheriff, who is 0.6 kilometer behind him when he ran the light. The deputy is traveling at 100 kilometers per hour. If Alan will be out of the deputy's jurisdiction in another 5 kilometers, will he be caught?

Lead the students through the process of determining the system of equations that might assist in finding the solution to the problem. Using the relationship distance = rate × time, where time is given in hours and distance is how far he is from the traffic light in kilometers, show the students that Alan's equation can be described as $d = 80t$. The equation for the deputy then would be $d = 100t - 0.6$. Show the students that the right member of the deputy's equation can be substituted for the left member of Alan's equation to achieve the equation $100t - 0.6 = 80t$. Solve the equation for *t*, and a solution of 0.03 would be determined. Substituting back into either or both of the equations, the value of *d* will be found to be 2.4 kilometers. The point common to both lines is (0.03, 2.4). Because the 2.4 kilometers is less than 5, Alan is within the deputy's jurisdiction and will get a ticket.

Provide additional practice problems where the students can use the substitution method to solve systems. Work with students individually and in small groups to ensure mastery of the process.

Activity 4: Elimination (GLEs: 11, 12, 15, 16, 23, 39)

Begin by reviewing the process for solving systems of equations graphically and by substitution. Inform the students that there is another method of solving systems of equations that is called *elimination*. Write an equation and review the addition property of equality. Show that the same number can be added to both sides of an equation to obtain an equivalent equation. Then introduce the following problem:

A newspaper from Central Florida reported that Charles Alverez is so tall he can pick lemons without climbing a tree. Charles's height plus his father's height is 163 inches, with a difference in their heights of 33 inches. Assuming Charles is taller than his father, how tall is each man? Work with the students to establish a system that could be used to find Charles's height. Let *x* represent Charles's height and *y* represent his father's height and write the two equations $x + y = 163$ and $x - y = 33$. Show the students that the sum of the two equations would yield the equation $2x = 196$, which would indicate that Charles' height is 98 inches (8 ft. 2 in.) tall. Through substitution, the father's height could then be determined.

Continue to show examples that include having to use the multiplication property of equality to establish equivalent equations where like terms in the two equations would add to zero and eliminate a variable. Provide opportunities for students to practice solving systems of equations using elimination including real-world problems.

Activity 5: Supply and Demand (GLEs: 11, 15, 16, 23)

This activity can be found on National Council of Teachers of Mathematics website (http://illuminations.nctm.org/index_d.aspx?id=382). Worksheets can be printed from the website for student use. Students investigate and analyze supply and demand equations using the following data obtained by the BurgerRama restaurant chain as they are deciding to sell a cartoon doll at its restaurants and need to decide how much to charge for the dolls.

Selling Price of Each Doll	Number Supplied per Week per Store	Number Requested per Week per Store
\$1.00	35	530
\$2.00	130	400
\$4.00	320	140

Have students plot points representing selling price and supply and selling price and demand on a graph. Have students estimate when supply and demand will be in equilibrium. Then have students find the equation of each line and solve the system of equations algebraically to find the price in exact equilibrium.
($S = 95p - 60, D = -130p + 66$, price in equilibrium, \$3.20)

Activity 6: Introduction to Matrices (GLE: 16)

This activity provides an introduction to the use of matrices in real-life situations and provides opportunities for students to be familiarized with the operations on matrices before using them to solve systems of equations.

Provide students with the following charts of electronic sales at two different store locations:

Store A Store B

	Jan.	Feb.	Mar.		Jan.	Feb.	Mar.
Computers	55	26	42	Computers	30	22	35
DVD players	28	26	30	DVD players	12	24	15
Camcorders	32	25	20	Camcorders	20	21	15
TVs	34	45	37	TVs	32	33	14

Explain to students that these two charts can be arranged in a rectangular array called a matrix. The advantage of writing the numbers as a matrix is that the entire array can be used as a single mathematical entity. Have the students write the charts as matrix A and matrix B as such:

$$A = \begin{bmatrix} 55 & 26 & 42 \\ 28 & 26 & 30 \\ 32 & 25 & 20 \\ 34 & 45 & 37 \end{bmatrix} \qquad B = \begin{bmatrix} 30 & 22 & 35 \\ 12 & 24 & 15 \\ 20 & 21 & 15 \\ 32 & 33 & 14 \end{bmatrix}$$

Discuss with students the dimensions of the matrices. (Both matrices are 4×3 matrices because they have 4 rows and 3 columns) Tell students that each matrix can be identified using its dimensions. (i.e. $A_{4\times3}$) Provide examples of additional matrices for students to name using the dimensions.

Ask students how they might find the total sales of each category for both stores. Have students come up with suggestions and lead them to the conclusion that when adding matrices together, they should add the corresponding elements. Lead them to discover that two matrices can be added together only if they are the same dimensions. Provide a question for subtraction such as: How many more electronic devices did Store A sell than Store B?

Also provide a question for scalar multiplication such as: Another store, Store C, sold twice the amount of electronics as Store B. How much of each electronic device did they sell? (Scalar multiplication is multiplying every element in Matrix B by 2)
All of the operations in this activity should be shown using paper and pencil and using a graphing calculator. Provide students with other examples of real-life applications of matrices and have them perform addition, subtraction, and scalar multiplication.

Activity 7: Multiplying matrices (GLE: 16)

Provide students with the following charts of T-shirt sales for a school fundraiser and the profit made on each shirt sold.

Number of shirts sold   Profit per shirt

	Small	Medium	Large		Profit
Art Club	52	67	30	Small	$5.00
Science Club	60	77	25	Medium	$4. 25
Math Club	33	59	22	Large	$3.00

Have students write a matrix for each chart. Then have them discuss how to calculate the total profit that each club earned for selling the T-shirts. As students come up with ways to calculate, lead them to the process of multiplying two matrices together. For example:

$$\begin{bmatrix} 52 & 67 & 30 \\ 60 & 77 & 25 \\ 33 & 59 & 22 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ 4.25 \\ 3 \end{bmatrix} = \begin{bmatrix} 52(5)+67(4.25)+30(3) \\ 60(5)+77(4.25)+25(3) \\ 33(5)+59(4.25)+22(3) \end{bmatrix} = \begin{bmatrix} 634.75 \\ 702.25 \\ 481.75 \end{bmatrix}$$

Provide students with one more example for them to try using pencil and paper. Then have them use the graphing calculator to multiply matrices of various dimensions. Provide students with examples that cannot be multiplied and have them discover the rule that in order to multiply two matrices together, their inner dimensions must be equal.

Activity 8: Solving Systems of Equations with Matrices (GLE: 16)

Have students multiply the following two matrices: $\begin{bmatrix} -1 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ The result is $\begin{matrix} -x+2y \\ x+6y \end{matrix}$.

Discuss with students that if they are given $\begin{bmatrix} -1 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$ then the following system of equations would result: $\begin{matrix} -x+2y=12 \\ x+6y=20 \end{matrix}$. Conversely, any system of equations can be written as a matrix multiplication equation. Using technology, matrices provide an efficient way to solve equations, especially multiple equations having many variables. This is true because in any system of equations written as matrix multiplication, **A**x = **B,** the equation can be solved for x as $x = A^{-1}B$, where matrix A is the coefficient matrix, $A = \begin{bmatrix} -1 & 2 \\ 1 & 6 \end{bmatrix}$, and matrix B is the constant matrix, $B = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$. Have students enter matrix A and matrix B into the calculator and type *[A]$^{-1}$[B]* on the home screen. The resulting matrix will be $\begin{bmatrix} -4 \\ 4 \end{bmatrix}$ which means $x = -4$ and $y = 4$. Repeat this activity with 3 x 3 systems of equations.

Activity 9: Systems of Inequalities (GLE: 14)

Review graphing inequalities in two variables. Present the following problem to students: Suppose you receive a $120 gift certificate to a music and book store for your birthday. You want to buy some books and at least 3 CDs. CDs cost $15 and books cost $12. What are the possible ways that you can spend the gift certificate. Have students use a system of inequalities to find the possible solutions and to graph the three inequalities for the problem. *($15x + 12y \le 100, x \ge 3, y \ge 0$)* Have them use different colored pencils or different shading techniques for each inequality. Ask students to explain the significance of the overlapping shaded region. Have them give the possible ways that they can spend the gift certificate. Provide students with other real-world problems that can be solved using systems of linear inequalities.

Activity 10: Name that solution (GLE: 14)

Divide students into groups of 3 or 4. Show students the graph of a system of inequalities on a coordinate grid transparency. Give each group a set of 4 cards, one with the correct system of inequalities, one with each inequality that makes up the system and one with the word none on it. Call out ordered pairs and let each group decide if that ordered pair is a solution to the system, to either inequality, or to none of them. When a group consensus is reached, have one person from each group hold up the card with the correct answer.

Sample Assessments

General Assessments

- Portfolio assessment: On the first day of the new unit, the teacher will give the student an application problem that can be solved using a system of equations. As each new method of solving systems of equations is introduced, the student will solve the problem using the method learned.
- The student will solve constructed response items, such as:

  Prestige Car Rentals charges $44 per day plus $.06 per mile to rent a mid-sized vehicle. Getaway Auto charges $35 per day plus $.09 per mile for the same car.

  a. Write a system of linear equations representing the prices for renting a car for one day at each company. Identify the variables used. (*Prestige:* $C = 44 + .06m$, *Getaway:* $C = 35 + .09m$)
  b. Solve the system of equations graphically and algebraically. ($m = 300$, $C = \$62$)
  c. Suppose you need to rent a car for a day. Which company would you rent from? Justify your answer. (*Prestige, if you were driving more than 300 miles and Getaway, if you were driving less than 300 miles.)*

- The student will solve a 2 × 2 or 3 × 3 system of equations using a graphing calculator and check the solution by hand.
- The student will create a system of inequalities whose solution region is a polygon.
- The student will complete journal writings using such topics as:
  - Describe four methods of solving systems of equations. When would you use each method.
  - What is the purpose of using multiplication as the first step when solving a system using elimination?
  - Describe two ways to tell how many solutions a system of equations has.
  - Describe a linear system that you would prefer to solve by graphing. Describe another linear system that you would prefer to solve using substitution. Provide reasons for your choice.
  - How is solving a system of inequalities like solving a system of equations? How is it different?
- The student will pose and solve problems that require a system of two equations in two unknowns. The student will be able to solve the system using any of the methods learned.

Activity-Specific Assessments

- Activity 2: The student will solve constructed response items such as:
  The table shows the average amounts of red meat and poultry eaten by Americans each year.

Year	1970	1975	1980	1985	1990
Red meat	152 lb	139 lb	146 lb	141 lb	131 lb
Poultry	48 lb	50 lb	60 lb	68 lb	91 lb

  a. Create scatter plots for the amounts of red meat and poultry eaten.
  b. Find the equation of the lines of best fit. (*Red meat* $y = -.8x + 1725.8$ *,* *Poultry:* $y = 2.08x - 4055$ *)*
  c. Does the data show that the average number of pounds of poultry eaten by Americans will ever equal the average number of pounds of red meat eaten? Justify your answer. (*Yes, in the year 2007)*

- Activity 5: The student will solve constructed response items such as:
  The data provided in the table below show the supply and demand for game cartridges at a toy warehouse.

Price	Supply	Demand
$20	150	500
$30	250	400
$50	450	200

  a. Find the supply equation. ($y = 10x - 50$)
  b. Find the demand equation. ($y = -10x + 700$)
  c. Find the price in equilibrium. (*$37.50)*
     Justify each of your answers.

- Activity 10: Given the graph to a system of inequalities the student will list three points that are solutions to the system, to each inequality, and to none of the inequalities.

Algebra 1
Unit 6: Measurement

Time Frame: Approximately three weeks

Unit Description

This unit is an advanced study of measurement. It includes the topics of precision and accuracy and investigates the relationship between the two. The investigation of absolute and relative error and how they each relate to measurement is included. Significant digits are also studied and the computations that can be performed using them.

Student Understandings

Students should be able to find the precision of an instrument and determine the accuracy of a given measurement. They should know the difference between precision and accuracy. Students should see error as the uncertainty approximated by an interval around the true measurement. They should understand significant digits.

Guiding Questions

1. Can students determine the precision of a given measurement instrument?
2. Can students determine the accuracy of a measurement?
3. Can students differentiate between what it means to be precise and what it means to be accurate?
4. Can students discuss the nature of precision and accuracy in measurement and note the differences in final measurement values that may result from error?
5. Can students calculate using significant digits?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Measurement	
17.	Distinguish between precision and accuracy (M-1-H)
GLE #	**GLE Text and Benchmarks**

18.	Demonstrate and explain how the scale of a measuring instrument determines the precision of that instrument (M-1-H)
19.	Use significant digits in computational problems (M-1-H) (N-2-H)
20.	Demonstrate and explain how relative measurement error is compounded when determining absolute error (M-1-H) (M-2-H) (M-3-H)
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)

Sample Activities

Activity 1: What Does it Mean to be Accurate? (GLEs: 4, 17)

Talk with students about the meaning of "accuracy" in measurement. Accuracy indicates how close a measurement is to the accepted "true" value. For example, a scale is expected to read 100 grams if a standard 100 gram weight is placed on it. If the scale does not read 100 grams, then the scale is said to be inaccurate. If possible, obtain a standard weight from one of the science teachers along with several scales. With students, determine which scale is closest to the known value and use this information to determine which scale is most accurate. Next, ask students if they have ever weighed themselves on different scales—if possible, provide different scales for students to weigh themselves. Depending on the scale used, the weight measured for a person might vary according to the accuracy of the instruments being used. Unless "true" weight is known, it cannot be determined which scale is most accurate (unless there is a known standard to judge each scale). Generally, when a scale or any other measuring device is used, the readout is automatically accepted without really thinking about its validity. People do this without knowing if the tool is giving an accurate measurement. Also, modern digital instruments convey such an aura of accuracy and reliability (due to all the digits it might display) that this basic rule is forgotten—there is no such thing as a perfect measurement. Digital equipment does not guarantee 100% accuracy. Have all of the students who have watches to record the time (to the nearest second) at the same moment and hand in their results. Post the results on the board or overhead—there should be a wide range of answers. Ask students, Which watch is the most accurate? Students should see that in order to make this determination, the true time must be known. Official time in the United States is kept by NIST and the United States Naval Observatory, which averages readings from the 60 atomic clocks it owns. Both organizations also contribute to UTC, the world universal time. The website http://www.time.gov has the official U.S. time, but even its time is "accurate to within .7 seconds." Cite this time at the same time the students are determining the time from their watches to see who has the most accurate time. Ultimately, students need to understand that accuracy is really a measure of how close a measurement is to the "true" value. Unless the true value is known, the accuracy of a measurement cannot be determined.

Activity 2: How Precise is Your Measurement Tool? (GLE: 4, 17, 18)

Discuss the term “precision” with the class. Precision is generally referred to in one of two ways. It can refer to the degree to which repeated readings on the same quantity agree with each other. Precision can also be referred to in terms of the unit used to measure an object. Precision depends largely on the way in which the readings are taken—how much care was taken by the person making the measurement, the quality of the instrument, attentiveness of the observer, stability of the environment in which the measurements were taken, etc… Some limitations that hinder the precision of a measurement include the skill of the reader, the way the ruler was placed, whether or not it was viewed at an angle, and so on. Help students to understand that no measurement is perfect. When making a measurement, scientists give their best estimate of the true value of a measurement, along with its uncertainty.

The precision of an instrument reflects the number of digits in a reading taken from it—the degree of refinement of a measurement. Discuss with students the degree of precision with which a measurement can be made using a particular measurement tool. For example, have on hand different types of rulers (some measuring to the nearest inch, nearest $\frac{1}{2}$ inch, nearest $\frac{1}{4}$ inch, nearest $\frac{1}{8}$ inch, nearest $\frac{1}{16}$ inch, nearest centimeter, and nearest millimeter) and discuss with students which tool would give the most precise measurement for the length of a particular item (such as the length of a toothpick). Have students record measurements they obtain with each type of ruler and discuss their findings. Help students understand that the ruler with the smallest markings will provide the most precise measure, but even it has inherent limitations.

Set up measurement stations throughout the class for students to determine the attributes of different items. Include measurements with weight using scales (both in lbs, ounces, and grams), length of items (include diameter of a sphere), and areas of objects which have the shapes of simple 2-D figures (rectangle, circle, parallelogram) and have students measure the appropriate lengths with which to calculate the areas. Discuss the results as a class including sources of error that could account for discrepancies in answers.

Activity 3: Temperature—How Precise Can You Be? (GLEs: 4, 18, 17)

Have students get in groups of three. Provide each team with a thermometer that is calibrated in both Celsius and Fahrenheit. Have each team record the room temperature in both ^{o}C and ^{o}F. Have students note the measurement increments of the thermometer (whether it measures whole degrees, tenths of a degree, etc.) on both scales. Make a class table of the temperatures read by each team. Ask students if it is possible to have an answer in tenths of a degree using their thermometers and why or why not?

Activity 4: Precision vs. Accuracy (GLE: 17)

In this activity, provide students with data tables showing measurements taken, and have students answer questions regarding precision and accuracy, and have them distinguish between the two. For example, provide students with the data tables shown below and have them answer the questions provided.

Example 1: *Using the table below, answer the following questions. Assume that each data set represents 5 measurements taken from the same object.*

- Which of the following sets of data is more precise, based on its range?
  (Solution: Data Set A has a range of .06 while Data Set B has a range of .08, thus the more precise data set is Set A.)
- Do you know which data set is more accurate? Explain
  (Solution: There is no way of knowing which is more accurate since in both cases there is no indication of the true measure of the object being measured.)

Set A	Set B
14.32	36.56
14.37	36.55
14.33	36.48
14.38	36.53
14.35	36.55

Example 2: *The data tables below show measurements that were taken using three different scales. The same standard 100 gram weight was placed on each scale and measured 4 different times by the same reader using the same method each time.*

Trial #	Weight on Scale 1	Weight on Scale 2	Weight on Scale 3
1	101.5	100.00	100.10
2	101.5	100.02	100.00
3	101.5	99.99	99.88
4	101.5	99.99	100.02
Avg. Weight			

- Determine the average weight produced by each scale. Use this average as the actual weight of the 100g mass determined by each scale. Write down the results for each scale used.
  (Solution: Scale 1: 101.5g; Scale 2: 100.00 g; Scale 3: 100.00 g)
- Which scale was the most precise? Explain how you know.
  (Solution: Scale 1 since the range of values is smaller than in the other scales.)
- Which scale was the least precise? Explain how you know.
  (Solution: Scale 3 since the range of values is larger)
- Which scale was the most accurate if we consider the true value of the weight to be 100 grams? Explain your answer.
  (Solution: If we look at the average weights to be the weight given by each scale, then both Scale 2 and Scale 3 are equally accurate.)

Example 3: *Below is a data table produced by 4 groups of students who were measuring the mass of a paper clip which had a known mass of 1.0004 g.*

- Determine the average weight produced by each group's measurements and fill in the results in the table. Use this average as the weight of the paper clip for each group.
  (Solution: Group 1: 1.01 g; Group 2: 3.601267 g; Group 3: 10.13255g; Group 4: 1.01g)
- Which of the group's measurements represents a properly accurate and precise measurement of the mass of the paper clip?
  (Solution: Both Group 1 and Group 4 had an average weight in line with the true weight of the mass; however, Group 4 did not have a precise measurement—the readings have too wide a range . The average just happened to come out to a value close to the true weight; therefore, only Group 1 data represents both an accurate and precise measurement.)
- Which of the group's measurements was the least accurate? Explain why.
  (Solution: Group 3 had the least accurate answer for the weight of the paper clip since its average value is farthest from the actual value of the paper clip.)
- Which of the group's measurements had an accurate answer, but not a precise answer? Explain.
  (Solution: Group 4 had an accurate weight (if the average is used) but was not precise at all.)

Trial #	Group 1 (g)	Group 2 (g)	Group 3 (g)	Group 4 (g)
1	1.01	3.863287	10.13252	2.05
2	1.03	3.754158	10.13258	0.23
3	0.99	3.186357	10.13255	0.75
Average Weight				

Activity 5: Finding a Range of Values for a Measurement—Absolute Error (GLEs: 18, 20)

The uncertainty or error associated with any measurement depends on the measurement tool being used. For example, if the mass of a sample is given as 342 ± 4 mg, the actual value for the mass is somewhere between 338 mg and 346 mg. The reason for the variation may be due to the measurement tool's being unable to sense any changes in mass less than 4 mg (i.e., the pan balance doesn't move for such small changes). This is what is referred to as "absolute error." Discuss with students what absolute error is and how to determine the error of a particular measurement tool. For example, suppose a ruler only measures to the nearest $\frac{1}{2}$ inch, and you take a measurement of an item that lies somewhere between $3\frac{1}{2}$ inches and 4 inches. Since it is closer to $3\frac{1}{2}$ inches, you write this as the length of the item. However, the actual measurement could have been up to $\frac{1}{4}$ inch longer (half of the smallest division associated with the ruler). If someone reads the measurement of $3\frac{1}{2}$ inches, they have no idea how much error was associated with the measurement. A more scientific approach to listing the measure would be to write $3\frac{1}{2} \pm \frac{1}{4}$ inch. Doing so allows the reader to know that the actual length of the item may be anywhere from $3\frac{1}{4}$ inches to $3\frac{3}{4}$ inches in length. Discuss the idea of absolute error and provide students the opportunity to write such error measurements and determine the range associated with different measurement tools. Have students use different rulers (some measuring to the nearest inch, nearest $\frac{1}{2}$ inch, nearest $\frac{1}{4}$ inch, nearest $\frac{1}{8}$ inch, nearest centimeter, and nearest millimeter) to measure various items around the room (i.e., table length, chair height, length of a sheet of paper, height of door, etc.). For each measurement, have the students express their measurement along with the range associated with the error.

Activity 6: What is My Exact Height? Absolute vs. Relative Error (GLEs: 4, 5, 20)

After having discussed absolute error, it is important to talk about a better indication of how accurate a measurement is—a different type of error measurement called "relative error." An accurate measure is one in which the uncertainty is small when compared to the measurement itself. Thus, an uncertainty of ± 4 mg out of a total of 342 mg indicates much more accuracy than ± 4 mg out of a total of 12 mg. For this reason, uncertainty in measurement is often expressed as a percent of uncertainty. This is the relative error associated with the measurement. To determine the relative error, divide the absolute error by the calculated value, and then convert this decimal to a percent by multiplying by 100. For the examples above, $\frac{4}{342} = .011$ which when multiplied by 100 gives a relative error of 1.1%. (Provide students with access to calculators to do this work.) Whereas, in the other measure, $\frac{4}{12} = .333$ which when multiplied by 100 gives a relative error of 33.3%, that is a much higher percentage error, although the absolute errors are the same. Discuss with students how to determine the relative error associated with a measurement, and have students get in groups of 3. Provide two different types of measurement tools for students to make their measurements (a meter stick and a tape measure with English units). Direct two of the students to measure the height of the third, taking turns so that all students in each group are measured in both metric units and customary units. Then, have all three students determine their heights including absolute error and relative error in their measurements.

Activity 7: What's the Cost of Those Bananas? (GLEs: 4, 17, 18)

The following activity can be completed as described below if the activity seems reasonable for the students involved. If not, the same activity can be done if there is access to a pan scale and an electronic balance. If done in the classroom, provide items for students to measure—bunch of bananas, two or three potatoes, or other items that will not deteriorate too fast.

Have the students go to the local supermarket and select one item from the produce department that is paid for by weight. Have them calculate the cost of the object using the hanging pan scale present in the department. Record their data. At the checkout counter, have the students record the weight given on the electronic balance used by the checker. Have students record the cost of the item. How do the two measurements and costs compare? Have students explain the significance of the number of digits (precision) of the scales.

Activity 8: What are Significant Digits? (GLEs: 4, 19)

Discuss with students what significant digits are and how they are used in measurement. Significant digits are those digits of a measurement that represent meaningful data. The more precision there is in the measurement, the more significant digits there will be. Practically speaking, measurements are made to some desired precision that suits the purpose of the person doing the measurement, which normally is determined by the limitations of the measurement instrument available. For example, suppose you measure a room to the nearest millimeter and find its length to be 7.08<u>9</u> meters, the <u>9</u> is the estimated digit in the measurement (remind students the absolute error will be $\frac{1}{2}$ the smallest unit of the measurement tool, which in this case will be ±.5 mm or ± 0.0005 meters). There are four significant digits in all in the measurement. After fully discussing the concept of significant digits with students, provide them with opportunities to determine the number of significant digits given in a particular measurement. Plan with a science teacher if possible.

Activity 9: Measuring the Utilities You Use (GLE: 19)

Have students find the various utility meters (water, electricity) for their households. Have them to record the units and the number of places found on each meter. Have the class get a copy of their family’s last utility bill for each meter they checked. Have students answer the following questions: What units and number of significant digits are shown on the bill? Are they the same? Why or why not? Do your family pay the actual “true value” of the utility used or an estimate? If students do not have access to such information, produce sample drawings of meters used in the community and samples of utility bills so that the remainder of the activity can be completed.

Activity 10: Calculating with Precision (GLEs: 4, 19)

Discuss with students how significant digits are dealt with when making calculations. Students should understand that the precision that results from a calculation cannot be greater than the precision of any of the numbers used in the calculation. For example, consider a rectangle whose sides measure 9.<u>7</u> cm and 4.<u>2</u> cm. Calculating the area of the rectangle using multiplication brings (9.7cm)(4.2cm) = 40.74 sq. cm. Before now, students would probably write the result as 40.74 sq. cm., but a closer look shows the original side length measurements are only precise to the tenth of a centimeter, while the resulting calculation for area is precise to the hundredth of a square centimeter. To correct this, the result should be rounded off so that it has the same precision as the least precise quantity used in the calculation. This rule is the equivalent of saying that making a calculation cannot improve on the precision of the numbers used based on the number least number of significant digits in the factors. Therefore, in the example provided, a more trustworthy answer would be 41 sq. cm. When working with addition and subtraction, the result should be rounded off so that it has the same number of decimal places (to the right of the decimal point) as the quantity in the calculation having the least number of decimal places. After fully discussing calculating with significant figures, have students work computational problems (finding area, perimeter, circumference of 2-D figures) dealing with the topic of calculating with significant digits.

Activity 11: Which Unit of Measurement? (GLEs: 5, 21)

Divide students into groups. Provide students with a centimeter ruler and have them measure the classroom and calculate the area of the room in centimeters. Then provide them with a meter stick and have them calculate the area of the room in meters. Discuss with students which unit of measure was most appropriate to use in their calculations. Ask students if they were asked to find the area of the school parking lot, which unit would they definitely want to use. What about their entire town? In that case, kilometers would probably be better to use. Provide opportunities for discussion and/or examples of measurements of weight (weigh a quarter on a bathroom scale or a food scale) and mass (fill a large bucket with water using a cup or a gallon jug) similar to the linear example of the area of the room. Use concrete examples for students to visually explore the most appropriate units and scales to use when solving measurement problems.

Sample Assessments

General Assessments

- Portfolio Assessment: The student will create a portfolio divided into the following sections:
    1. Accuracy
    2. Precision
    3. Precision vs. Accuracy
    4. Absolute error
    5. Relative error
    6. Significant digits

  In each section of the portfolio, the student will include an explanation of each, examples of each, artifacts that were used during the activity, sample questions given during class, etc. The portfolio will be used as an opportunity for students to demonstrate a true conceptual understanding of each concept.

- The student will complete journal writings using such topics as:
    - Darla measured the length of a book to be $11\frac{1}{4}$ inches with her ruler and $11\frac{1}{2}$ inches with her teacher's ruler. Can Darla tell which measurement is more accurate? Why or why not? *(She cannot tell unless she knows which ruler is closer to the actual standard measure)*
    - What does it mean to be precise? Give examples to support your explanation.
    - What is the difference between being precise and being accurate? Explain your answer.
    - Explain the following statement: The more significant digits there are in a measurement, the more precise the measurement is.
    - When would it be important to measure something to three or more significant digits? Explain your answer.

Activity-Specific Assessments

- <u>Activity 1</u>: The student will write a paragraph explaining in his/her own words what it means to be accurate. He/she will give an example of a real-life situation in which a measurement taken may not be accurate.

- <u>Activity 2</u>: The student will keep a log of the various measurements that are taken at different measurement stations. The student will record each measurement of each item and then decide which measurement would be more precise. The student will be required to justify each answer with a written explanation.

- <u>Activity 4</u>: The student will be quizzed on the difference between being precise and being accurate. Given examples similar to the ones in the activity, the student will answer questions about the measurements.

- <u>Activity 6</u>: The student will solve sample test questions, such as:
  Raoul measured the length of a wooden board that he wants to use to build a ramp. He measured the length to be 4.2 m. The absolute error of his measurement is $\pm.1$ m. His friend, Cassandra, measured a piece of molding to decorate the ramp. Her measurement was .25 m with an absolute error of $\pm.1$ m. Find the relative error of each of their measurements. Whose measurement was better? Explain your answer. *(Raoul – 2%, Cassandra – 40%, Raoul because his percentage of relative error was smaller.)*

- <u>Activity 11</u>: The student will be able to determine the most appropriate unit and/or instrument to use in both English and Metric units when given examples such as:
  How much water a pan holds
  Weight of a crate of apples
  Distance from New Orleans to Baton Rouge

How long it takes to run a mile
Length of a room
Weight of a Boeing 727
Weight of a t-bone steak
Thickness of a pencil
Weight of a slice of bread

Algebra I
Unit 7: Exponents, Exponential Functions, and Nonlinear Graphs

Time Frame: Approximately four weeks

Unit Description

This unit is an introduction to exponential functions and their graphs. Special emphasis is given to examining their rate of change relative to that of linear equations. Focus is on the real-life applications of exponential growth and decay. Laws of exponents are introduced as well as the simplification of polynomial expressions. Radicals and scientific notation are re-introduced.

Student Understandings

Students need to develop the understanding of exponential growth and its relationship to repeated multiplications, rather than additions, and its relationship to exponents and radicals. Students should be able to understand, recognize, graph, and write symbolic representations for simple exponential relationships of the form $a \cdot b^x$. They should be able to evaluate and describe exponential changes in a sequence by citing the rules involved.

Guiding Questions

1. Can students recognize the presence of an exponential rate of change from data, equations, or graphs?
2. Can students develop an expression or equation to represent a straightforward exponential relation of the form $y - a \cdot b^x$.
3. Can students differentiate between the rates of growth for exponential and linear relationships?
4. Can students use exponential growth and decay to model real-world relationships?
5. Can students use laws of exponents to simplify polynomial expressions?

Unit 7 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
2.	Evaluate and write numerical expressions involving integer exponents (N-2-H)
3.	Apply scientific notation to perform computations, solve problems, and write representations of numbers (N-2-H)

GLE #	GLE Text and Benchmarks
6.	Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3}+5\sqrt{3}=7\sqrt{3}$) (N-5-H)
Algebra	
7.	Use proportional reasoning to model and solve real-life problems involving direct and inverse variation (N-6-H)
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
10.	Identify independent and dependent variables in real-life relationships (A-1-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Patterns, Relations, and Functions	
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)

Sample Activities

Activity 1: Evaluation (GLEs: 2, 10, 12, 15)

Give students the two functions, $f(x)=3x$ and $f(x)=3^x$, and have them generate an input-output table using the same domain for both functions. Have students plot the ordered pairs for each function and connect them. Next, have students calculate the difference between successive *y*-coordinates in each function and compare them. Discuss with students the fact that the rate of change varies for a nonlinear function as opposed to the constant rate of change found in linear functions. (This is called the method of finite differences. It will be studied in depth in Algebra II.) Relate this varying rate of change to the shape of the graph and the degree of the function. Repeat this activity with other exponential functions. The following is an example.

Atoms of radioactive elements break down very slowly into atoms of other elements. The amount of a radioactive element remaining after a given amount of time is an exponential relationship. Given an 80-gram sample of an isotope of mercury, the number of grams (*y*) remaining after *x* days can be represented by the formula $y=80\left(0.5^x\right)$.

- Create a table for this function to show the number of grams remaining for 0, 1, 2, 3, 4, 5, 6, and 7 days. Identify the dependent and independent variables.

0	80
1	40
2	20
3	10
4	5
5	2.5
6	1.25
7	.625

- If half-life is defined as the time it takes for half the atoms to disintegrate, what is the half-life of this isotope? (*1 year)*
- Use a graphing calculator to display the graph.

Activity 2: The King's Chessboard – Modeling exponential growth (GLEs: 9, 10, 15, 29)

Present students with the following folktale from India (the children's book *The King's Chessboard* by David Birch could also be used to set the activity):

> A man named Sissa Ben Dahir invented the game of chess. The king liked the game so much that he wanted to reward Sissa with 64 gold pieces, one for each square on the chessboard. Instead, Sissa asked for 1 grain of wheat for the first square on the chessboard, 2 grains for the second, 4 grains for the third, 8 grains for the second, etc.
> How many grains of wheat will Sissa receive for the 64th square? (2^{63})

Have groups of three students model the problem using grains of rice and a chessboard. Have them construct a table for the square number and the number of grains of wheat and graph the data on graph paper. The graphing calculator can also be used to graph a scatter plot. Have students write the exponential equation that models the situation and answer the question in the problem. Revisit the paper folding activity and the Pay Day activity from Unit 1 and have students compare and contrast the two activities and their demonstration of exponential growth.

Activity 3: What's with my M&Ms®? Modeling exponential decay (GLEs: 9, 10, 15, 29)

Give each student a bag of M&Ms®. Have them empty their bag onto a paper plate and count the M&Ms®. They should create a table with the trial number and number of M&Ms® remaining. Record the first trial as 0 and the total number of M&Ms® as total. The students should pour out the M&Ms® again and count the number that show an "m" and place these back in the bag. Discard or eat the others. Repeat this process until the number of M&Ms® remaining is less than 5 but greater than 0. Have students graph the data by hand and with the graphing calculator. Have them use the calculator to find the equation of the exponential regression. Discuss with students exponential decay and the significance of the values of *a* and *b* in the exponential regression. Revisit the paper folding activity in Unit 1 and compare and contrast the two examples of exponential decay.

Activity 4: Vampire simulation (GLEs: 10, 11, 15, 29)

Explore the common vampire folklore with students: When a vampire bites another person, that person becomes a vampire. If three vampires come into (their town) and each vampire will bite another person each hour, how long will it take for the entire town to become vampires?

Have one student at the board make a table of the following experiment using hour as the independent variable and number of vampires as the dependent variable. Begin with three students (vampires) in front of the classroom. Have each student pick (bite) another student to bring in front of the classroom. Now there are six vampires. Have those two students each bring a student to the front of the classroom. Continue until all of the students have become vampires. Have the students return to their desks and copy the table, graph the data by hand, and find the equation to model the situation. Discuss with students the development of the equation of the form $y = a \cdot b^x$ ($y = 3 \cdot 2^x$). They should then use the equation to predict how long it would take for the entire town to become vampires. Students can then use the graphing calculator to check their answers.

Activity 5: Exponential Decay in Medicine (GLEs: 10, 11, 15, 29)

Pose the following problem: In medicine, it is important for doctors to know how long medications are present in a person's bloodstream. For example, if a person is given 300 mg of a pain medication and every four hours the kidneys eliminate 25% of the drug from the bloodstream, is it safe to give another dose after four hours? When will the drug be completely eliminated from the body?

The following activity could be done in groups or conducted as a demonstration by the teacher. Students will need clear glass bowls, measuring cup, 4 cups of water, 5 drops of food coloring. Have students pour 4 cups of water into the bowl and add the food coloring to it. Have students simulate the elimination of 25% of the drug by removing one cup of the colored water and adding one cup of clear water to the bowl. Have students repeat the steps and investigate how many times the steps need to be repeated until the water is clear. Have students make a table of values using end of time period (every four hours) as the independent variable and amount of medicine left in the body as dependent variable. Help students to develop the equation to model the situation ($y = 300 \cdot 0.75^x$). Have them graph the equation by hand or with the graphing calculator to investigate when the medicine will be completely eliminated from the body. Question students about whether the function will ever reach zero.

Activity 6: Exploring Exponents (GLEs: 2, 8)

Have students work with a partner to discover the laws of exponents. Provide them with charts similar to the ones below. Have them complete the chart and develop a formula for each situation.

Product of powers	Expanded product	Product as a single power
$x^2 \cdot x^3$	$(x \cdot x) \cdot (x \cdot x \cdot x)$	x^5
$x^5 \cdot x^4$		
$x \cdot x^3$		
$x^2 \cdot x^4$		
$x^m \cdot x^n$		

Quotient of powers	Expanded quotient	Quotient as a single power
$\frac{x^5}{x^2}$	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$	x^3
$\frac{x^4}{x^3}$		
$\frac{x^9}{x^4}$		
$\frac{x^2}{x^5}$		
$\frac{x^m}{x^n}$		

Power of a power	Expanded power	Power as a single power
$(x^2)^3$	$x^2 \cdot x^2 \cdot x^2$	x^6
$(x^3)^4$		
$(x^5)^2$		
$(x^6)^4$		
$(x^m)^n$		

Discuss with students the formulas that they discovered placing emphasis on negative exponents as they were introduced in Unit 1. Provide example and practice problems for students to simplify that include using order of operations.

Activity 7: Operations on Polynomials using Algebra Tiles (GLEs: 2, 8)

Give students examples of expressions that are and are not polynomials and help them to develop the definition of polynomial. Also include an introduction on monomials, binomials, trinomials, etc. Divide students into groups and provide each group with a set of algebra tiles. Algebra tiles are manipulatives that help students visualize polynomial expressions. They can be made using card stock and the template at the end of this unit. Use two different colors of card stock to represent positive and negative values. Introduce algebra tiles to students and help them to understand the representation of each ($x^2, -x^2, x, -x, 1, -1$). Give students different polynomials such as $2x^2 + 3x - 4$ and have the students model each polynomial with their algebra tiles. Discuss adding polynomials giving examples and have the students model each. Include a discussion of positive and negative tiles "canceling" out or adding up to zero.

Subtraction can be demonstrated by adding the opposite or changing the sign to addition and flipping the tiles in the expression being subtracted. Multiplication of polynomials can be shown with algebra tiles where the two expressions being multiplied are the dimensions of a rectangle and the simplified expression is the area of the rectangle. Include examples of multiplying a monomial times a binomial and multiplying two binomials together. Provide examples for groups to practice. Help students make the connection from concrete examples to abstract examples. Provide opportunities for students to practice simplifying polynomial expressions.

Activity 8: Scientific Notation (GLE: 3)

Review scientific notation with students. Discuss using laws of exponents to multiply and divide using scientific notation. Provide opportunities for students to apply these laws in real-life situations, such as the following:

There are approximately 50,000 genes in each human cell and about 50 trillion cells in the human body.

- Write these numbers in scientific notation. ($50{,}000 = 5 \times 10^4$, $50 \text{ trillion} = 5 \times 10^{13}$)
- Find an approximate number of genes in the human body. (2.5×10^{18})

The sun contains about 1×10^{57} atoms. The volume of the sun is approximately 8.5×10^{31} cubic inches. Approximately how many atoms are contained in each cubic inch? 1.2×10^{25}

Activity 9: Combining Radicals (GLEs: 2, 6, 11)

Review simplifying and performing basic operations on radicals. Have students create and solve riddles that can be solved by finding a root of an integer or by combining like radicals. For example, "I am positive. Four times my cube is 32. What am I?" Students would first write the equation $4x^3 = 32$ and then solve by dividing by 4 and then taking the cube root of 8 to find $x = 2$. Riddles that require students to add or subtract like radicals could be created; for example, three times a certain radical added to the square root of two gives four square roots of two. What is the radical?

Activity 10: Revisiting Inverse Variation (GLE: 7)

In Unit 1, students observed the difference between direct and inverse variation. Have students revisit that experiment possibly having them redo the investigation in its entirety. Have students note the difference in the graphs of the functions $y = kx$ and $y = \frac{k}{x}$ noting specifically that inverse variation is a non-linear function. Provide students with real-life examples of inverse variation and have them solve the problems using proportional reasoning.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- In Unit 1, students compared two data sets of salaries as examples of linear and non-linear data. The students will revisit that report and find the regression equations for each set of data. The student will also make predictions using the equations.
- The student will obtain population data for Louisiana as far back as possible. The student will graph the data and find the regression equation. The student will then predict the population in the state for the year 2010. The student will write a report summarizing his/her findings and include why it would be important to be able to estimate the future population of the state.
- The student will solve constructed response items such as:
  - Over a one-year time period, an insect population is known to quadruple. The starting population is fifteen insects.
    a. Make a table and a graph to show the growth of the population from 0 through 6 years.
    b. How many insects would there be at the end of 10 years? (*15,728,640)*
    c. Write an exponential equation that describes the growth. ($y = 15 \cdot 4^x$)
    d. Would your equation correctly describe the insect population after 50 years? Justify your answer.
- The student will solve open response items such as:
  - Decide if the following situations are linear or exponential. Use examples to justify your answer.
    a. A constant change in the independent variable produces a constant change in the dependent variable. (*linear)*

b. A constant change in the independent variable produces a constant percentage change in the dependent variable. (*Non-linear)*

- The student will create and solve a radical riddle.
- The student will use scientific notation to describe a very large quantity.
- The student will complete journal writings using such topics as:
  - o Compare the graphs of $y = 4^x$ and $y = \left(\frac{1}{4}\right)^x$. How are they alike? How are they different?
  - o Explain what is meant by exponential growth and exponential decay.
  - o How many ways are there to write x^{12} as a product of two powers. Explain your reasoning.
  - o Raul and Luther used different methods to simplify $\left(\frac{m^8}{m^2}\right)^3$. Are both methods correct? Explain your answer

    Raul: $\left(\frac{m^8}{m^2}\right)^3 = \frac{m^{24}}{m^6} = m^{18}$

    Luther: $\left(\frac{m^8}{m^2}\right)^3 = \left(m^6\right)^3 = m^{18}$

    - o Describe some real-life examples of exponential growth and decay. Sketch the graph of one of these examples and describe what it shows.

Activity-Specific Assessments

- Activity 1:
  - o Given an algebraic representation and a table of values of an exponential function, the student will verify the correctness of the values.
  - o The student will demonstrate the connection between
    - o a constant rate of change and a linear graph
    - o a varying rate of change and a nonlinear graph

- Activity 2: The student will decide which job offer they would take given the following two scenarios.
  Job A: A starting salary of $24,000 with a 4% raise each year for ten years.
  Job B: A starting salary of $24,000 with a $1000 raise each year for ten years.
  The student will justify their answer with tables, graphs and formulas.

- Activity 3: The student will solve constructed response items such as:
  Use the following data:

African Black Rhino Population

Year	Population (in 1000s)
1960	100
1980	15
1991	3.5
1992	2.4

a. Using your calculator and graphing paper, make a scatter plot of the data
b. Find the regression equation for the data. ($y = 1.74 \cdot 0.89^x$)
c. Use your model to predict the rhino population for the years 1998 and 2004. (*1,500, 770)*
d. Use your model to determine the rhino population in 1950. (*342,000)*
e. Should scientists be concerned about this decrease in population.
f. Compare your equation for M&M data to your equation for the rhino data. How are they alike? How are they different?

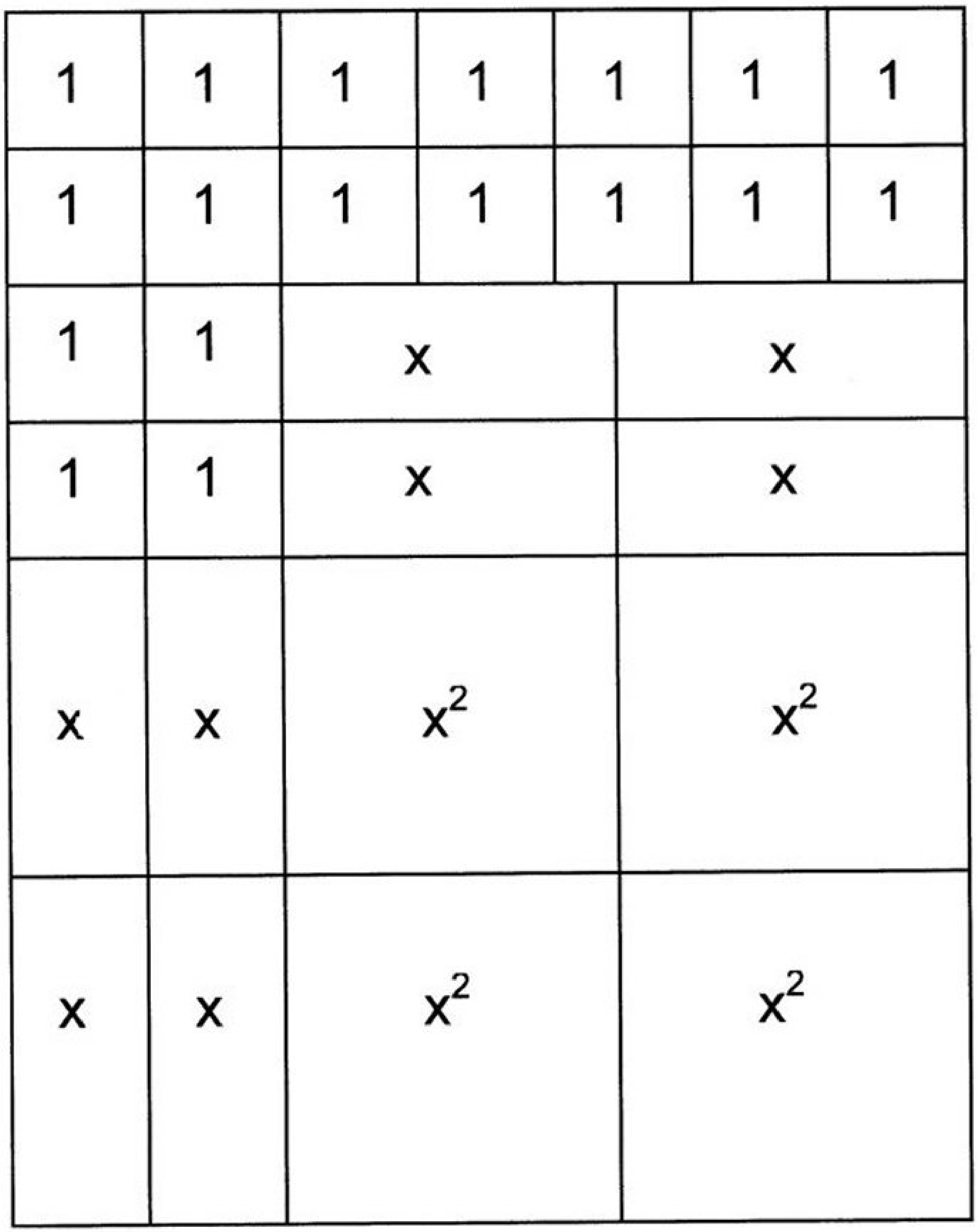
- Activity 4: The student will solve constructed response items such as:

The following data represents the number of people at South High who have heard a rumor:

# of hours after the rumor began	# of people who have heard it
0	5
1	10
2	20
3	40
4	80

a. Graph the data.
b. Find the exponential equation that models the data ($y = 5 \cdot 2^x$)
c. Use your equation to determine the number of people who have heard the rumor in 10 hours. (*5,120)*

Algebra Tile Template



Algebra I
Unit 8: Data, Chance, and Algebra

Time Frame: Approximately four weeks

Unit Description

This unit is a study of probability and statistics. The focus is on examining probability through simulations and the use of odds. Probability concepts are extended to include geometric models, permutations, and combinations. Measures of central tendency are also studied to investigate which measure best represents a set of data.

Student Understandings

Students study the relationships between experimental (especially simulation-based) and theoretical probabilities. There is more emphasis on counting and grouping methods from permutations and combinations. In the former, more emphasis is placed on with and without replacement contexts. Students also look at measures of central tendency and which measure best represents a set of data.

Guiding Questions

1. Can students create simulations to approximate the probabilities of simple and conditional events?
2. Can students relate the probabilities associated with experimental and theoretical probability analyses?
3. Can students find probabilities using combinations and permutations?
4. Can students relate probabilities of events to the odds associated with those events?
5. Can students determine the most appropriate measure of central tendency for a set of data?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Data Analysis, Probability, and Discrete Math	
27.	Determine the most appropriate measure of central tendency for a set of data based on its distribution (D-1-H)
30.	Use simulations to estimate probabilities (D-3-H) (D-5-H)
31.	Define probability in terms of sample spaces, outcomes, and events (D-4-H)

GLE #	GLE Text and Benchmarks
32.	Compute probabilities using geometric models and basic counting techniques such as combinations and permutations (D-4-H)
GLE #	**GLE Text and Benchmarks**
33.	Explain the relationship between the probability of an event occurring, and the odds of an event occurring and compute one given the other (D-4-H)

Sample Activities

Activity 1: Measures of Central Tendency (GLE: 27)

Provide students with the following scenario: The basketball coach wants to compare the attendance at basketball games with other schools in the area. She collected the following numbers for attendance at games: 100, 107, 98, 110, 115, 90, 62, 50, 97, 101, 100. What measure of central tendency would be the best measure to use when comparing with other schools? Have students graph the data on a line plot and mark and label each measure of central tendency on the graph. Discuss with students the significance of outliers and how they affect the measures of central tendency. Have students decide which measure best represents the attendance data. *In this set, the median best represents the data because 50 and 62 are outliers.* Divide students into groups and give them different sets of specific data, such as salaries for baseball players, test scores of students in a certain class, temperature in a certain city on a given day and have them construct a line plot and find the measure of central tendency that best represents the data.

Activity 2: Mean, Median, or Mode? (GLE: 27)

This activity continues to help students develop a better understanding of finding the most appropriate measure of central tendency for a given data set. Have students work with a partner. Provide students with the different characteristics of a data set and have them develop sets of data that meet the criteria. For example: The data set has seven numbers, the mode is 1, the median is 3, and the mean is 9. And/or the data set has 10 numbers, the median is 6, the mean is 8, all numbers in the data set are modes, and the number 6 is not in the data set. After students have been given time to find the data sets, have them discuss their strategies for developing their data sets. Have one student from each pair write their data sets on the board. Compare the sets and have students decide which measure of central tendency is most appropriate for each set. (Have some additional examples available that show cases in which each measure is more appropriate should the student examples not provide opportunities for comparison.) Have the students work with a different partner. Provide students with characteristics specific to most appropriate measure of central tendency to use to develop additional data sets. For example: The set contains five numbers and the mean is the most appropriate measure of central tendency, the set contains 8 numbers and the median best represents the data, and/or the set contains 15 numbers and the mode is the measure of central tendency that best represents the data. Have students share their answers and discuss how they developed their data sets with the class.

Activity 3: Probability Experiments (GLEs: 30, 31)

Review theoretical probability with students. Divide the class into five groups and have each group conduct a different probability experiment. Example experiments could be: place 10 blue chips, 10 white chips, and 10 red chips in a bag and draw 100 times with replacement; roll 1 die 100 times; spin a spinner 100 times; flip a coin 100 times, and flip a coin and roll a die 100 times. Have students list the sample space of their experiment. Have them make a tally chart of the experiment. Explain to students that experimental probability is probability based on an experiment. Have students discuss the difference between theoretical and experimental probability for each of their experiments. Have each group give an oral presentation on their experiment including the sample space of the experiment and the comparison of the experimental and theoretical probability.

Activity 4: Remove One (GLEs: 30, 31)

This activity begins with a game that the teacher plays with the students. Have students write the numbers 2 through 12 down the left side of a sheet of paper. Distribute 15 chips or counters to students. Tell them to place their 15 chips next to any of the numbers on the sheet with the understanding that a chip will be removed when that sum is rolled on two dice. They may place more than one chip by a number. Roll the dice and call out the sums. Have the students remove a chip when that number is called. The first person to remove all of their chips wins. As the sums are called out, have students make a tally chart of the numbers that are called. Lead students to create the sample space for the game. Analyze the sample space and lead students to conclude that some sums have a higher probability than others. Compare the theoretical and experimental probability. Play the game again to determine if there are fewer rolls of the dice since the students have this new information.

Activity 5: What's the Probability? (GLEs: 31, 32)

Have students write the numbers 1 through 10 on their paper. Then have them write true or false next to each of the numbers before asking the questions. Read a set of easy questions and have the students check how many were right or wrong. Sample questions that could be used: Today is Monday; Prince Charles is your principal; school is closed tomorrow. After the students write the percent correct on top of their papers, ask them what they think the typical score was. Graph the results of the scores. Use the results to discuss sample space, theoretical and experimental probability.

Activity 6: Geometric Probability (GLEs: 31, 32)

In this activity, students will conduct an experiment on geometric probability. Have students work with a partner. Have them divide a regular sheet of paper into four equal regions and shade one of the regions. Students will drop a 1-inch square piece of paper onto the paper from about 4 inches above. Have them predict the probability that the paper will land on the shaded region. Students will drop the paper 30 times recording each outcome. Landing on the shaded region is considered a win and landing on the other regions is a loss. Students will calculate the experimental probability and discuss its comparison to the theoretical probability. Lead students to a discussion of geometric probability as $\frac{\text{area of feasible region}}{\text{area of sample space}}$.

Activity 7: What are the odds? (GLE: 33)

Inform the students that in addition to probability, another method may be used to describe the likelihood of an event's occurring. Explain to them that the *odds* in favor of an event is the ratio that compares the number of ways an event can occur to the ways the event cannot occur. Ask the students to create the sample space describing the outcomes of tossing two coins (heads-heads, heads-tails, tails-heads, tails-tails). Ask the class to decide how many ways two heads can be obtained from the experiment (1). Ask the class to decide how many ways something other that two heads can be a result (3). Explain to the class that this would mean that the *odds* of getting two heads when flipping two coins would be $\frac{1}{3}$ or 1:3. Ask the class to determine the probability of getting two heads ($\frac{1}{4}$) and compare that number to the odds of getting two heads. Provide additional practice by using the experiment of rolling one number cube. Ask the students to find the odds of a 3 (1:5); a 3 and a 6 (2:4 or 1:2), or a 2, 3, 5, or 6 (4:2 or 2:1). Place the students into groups and ask each group to write a paragraph that compares and contrasts the meanings of the terms *probability* and *odds*. Have each group share its paragraph with the rest of the class. Use the outputs of the groups to discuss the relationship between the probability of an event's occurring and the odds of an event's occurring.

Activity 8: It's Conditional! (GLEs: 30, 31, 32)

Have students calculate the probability of rolling a 7 on two dice. Then have them find the probability of rolling a 9. Now ask students to determine the $P(7 \text{ or } 9)$. Because these events are independent, the probability is found by adding $P(7)$ and $P(9)$, giving $P\left(7 \text{ or } 9\right)=\frac{6}{36}+\frac{4}{36}=\frac{10}{36}=\frac{5}{18}$. Tell students to suppose they have been told that the first die has been rolled and the number is either a 2 or 3. Ask them to determine the probability of getting a sum of 7 or 9, knowing that the first die is a 2 or 3. Students should be able to count to find this probability to be $\frac{3}{12}=\frac{1}{4}$ because there are 12 possibilities, of which only 3 are sums of 7 or 9. Reiterate that they have found the probability of getting a sum of 7 or 9, given that the first die rolled was a 2 or 3. Discuss with students how the condition

of knowing what the first die was helped to reduce the sample space for this conditional experiment. Repeat this activity using other conditions, such as knowing the first die was a 1, 2, or 3. Students could also perform the following experiment. Have them use three containers, one with two red balls, one with two blue balls, and one with a red and a blue ball. Have them first determine the experimental probability of drawing a red ball after a red ball has been drawn and not replaced by performing several repetitions of the experiment. Be sure to combine all the data from the class to get a better approximation of the theoretical probability. Next, have students calculate $P(A|B)$ (this notation is read the probability of B given A), where event A is “the second ball in the container is red” and event B is “the first ball in the container is red.” That is, students will determine the conditional probability of drawing a red ball on the second draw, knowing that the first draw was a red ball. Students should find the probability to be $\frac{2}{3}$. Provide students with different types of number cubes (e.g., 8-sided, 10-sided, or 12-sided cubes) and have them repeat the activity on computing a conditional probability.
The Web site http://www.shodor.org/interactivate/discussions/pd12.html provides some useful information about conditional probability.

Activity 9: Permutations, combinations, and probability (GLE: 32)

This activity could be done in groups as a discovery activity or as a teacher-led whole-class discussion. Give students four index cards and have them write the letters of a four-letter word on the index cards. Have students find all possible four-letter arrangements. They do not have to form real words. Have them construct a tree diagram of the experiment. Have students observe how many choices there are for the first letter, second letter, etc. Lead students to the definition of the multiplication counting principle, *n!*, and permutations. Ask the question: If a word is formed at random using the letters they wrote on the cards, what is the probability that it will be the original word they wrote? $\frac{1}{24}$ Discuss with students what would happen if only 3 letters of the four were used to form words and lead them to the discovery of the permutation formula of n items arranged r at a time ($_nP_r = \frac{n!}{(n-r)!}$). Next, provide students with construction paper circles and samples to demonstrate 8 pizza toppings. Have students find how many ways they can create a 2-topping pizza from the 8 original toppings. Have them list the possible outcomes of 2-topping pizzas. Use the permutation formula to lead students to the discovery of the combination formula ($_nC_r = \frac{n!}{(n-r)!r!}$). Ask the question: What is the probability that a pizza chosen at random will be a beef and pepperoni pizza? ($\frac{1}{28}$) Demonstrate to students how to find combinations and permutations using a calculator since most calculators can perform the operations without using the formula. Have students discuss the difference between combinations and permutations and have them devise rules for deciding whether a situation is a permutation or combination. Present various situations and have students decide whether it is a permutation or combination.

Activity 10: Dependent vs. Independent Events (GLE: 33)

Use Activity 8, "The Gambler's Fantasy," from *Facing the Odds—The Mathematics of Gambling,* to demonstrate the difference between dependent and independent events and how to compute the probability of a group of events. The *Facing the Odds* document is available from the Louisiana Department of Education. The website address is http://www.louisianaschools.net/lde/curriculum/home.html. Click on *Facing the Odds* from the *Mathematics* pull-down menu.

Activity 11: The Probability of Possible Combinations (GLE: 32)

Use Activity 9, "Winning and Losing the Lottery," from *Facing the Odd—The Mathematics of Gambling*. This activity shows how to use basic counting processes to find permutations and combinations in a given situation and how to determine the probability of possible combinations. The *Facing the Odds* document is available from the Louisiana Department of Education. The website address is http://www.louisianaschools.net/lde/curriculum/home.html. Click on *Facing the Odds* from the *Mathematics* pull-down menu.

Sample Assessments

General Assessments

- The student will find a graph in a newspaper or magazine and write two probability problems that can be answered using the graph.
- The student will design a dartboard with 25, 50, and 100 point sections using the following guidelines:
  a. the probability of getting 25 points should be 60%
  b. the probability of getting 50 points should be 30%
  c. the probability of getting 100 points should be 10%
  The student will write a report describing the design and how it was constructed.
- The student will construct a probability scale that is similar to a number line from 0 to 1 and divide it into fourths and label with low probability and high probability in the appropriate places. The student will place the following situations on the probability scale.
  a. It will snow in July in Shreveport, LA.
  b. It will rain in August in Lafayette, LA.
  c. My bicycle will have a flat tire today.
  d. A coin will land heads up.
  e. The color of an apple will be blue.
  f. You will make an A on your next math test.

- The student will play a game of chance and then determine the probability of winning.
- The student will convert probabilities into odds.
- The student will determine the measures of central tendency for use in reporting the “average” of different types of data (e.g., average grade, average salary for a given profession, average height of adult males or females) and then select the measure that is best suited for that data set.
- The student will develop simulations to help determine an experimental probability for a complicated set of events.
- The student will research the square miles of land, water, and the United States on the Earth and determine the probability that a meteor hitting the earth would hit land, water, or the United States.
- The student will solve constructed response items, such as:
    - The bull’s eye of a standard dart board has a radius of 1 inch. The inner circle has a radius of 5 inches, and the outer circle has a radius of 9 inches. Assume that when a dart is thrown at the board, the dart is equally likely to hit any point inside the outer circle
        a. What is the probability that a dart that hits the dart board lands on the bull’s eye? Justify your answer.
        b. What is the probability that a dart that hits the dart board lands between the inner and outer rings? Justify your answer.
- The student will complete journal writings using such topics as:
    - Suppose that 50% is a passing score on a test. Do you think a true/false test is a good way to see if a student understands a topic? Why or why not?
    - Would you use theoretical or experimental probability to find the probability that a particular player will hit the bull’s eye on a dart board? Explain why and how.
    - Give an example of something that has a probability of 0 and a probability of 1. Explain why you chose each.
    - When tossing a coin five times, explain why the probability of getting one head and five tails is the same as getting one tail and five heads.
    - Explain to a student who was absent how to find the measure of central tendency that best represents a set of data. Include an example in your explanation

Activity-Specific Assessments

- Activity 1: The student will solve constructed response items, such as:
  A class of 25 students is asked to determine approximately how much time the average student spends on homework during a one-week period. Each student is to ask one of his/her friends for information, making sure that no one student is asked more than once. The number of hours spent on homework per week are as follows: 8, 0, 25, 9, 4, 19, 25, 9, 9, 8, 0, 8, 25, 9, 8, 7, 8, 3, 7, 8, 5, 3, 25, 8, 10.

a. Find the mean, median, and mode for the data. Explain or show how you found each answer. (*Mean – 10, median – 8, mode – 8)*
b. Based on this sample, which measure (or measures) best describes the typical student? Explain your answer. (*The median and/or mode. The four answers of 25 skewed the mean so that it is not representative of those surveyed.)*

- Activity 3: The student will write a paragraph comparing and contrasting experimental and theoretical probability, including examples of each in the paragraph, and explain why he/she chose the examples

- Activity 4: The student will write a paragraph telling how he/she determined placement of the chips for the first game and what the result was. Did he/she win? How many chips were left on the board when someone won? Then the student will write a second paragraph explaining what changes weremade to play the game the second time, why the changes were made, and what the results were.

- Activity 6: The student will solve constructed response items, such as:
  Ann E. Flyer is competing in a parachuting competition. She must land on a foam pad in the middle of a field. The foam pad has a diameter of 30 ft. and it is in the middle of a field that is 200 ft by 350 ft.
  a. Draw and label a diagram of the field.
  b. If she only controls her flight enough to land in the field, what is the probability that Ann will land on the pad? (*About 1%)*