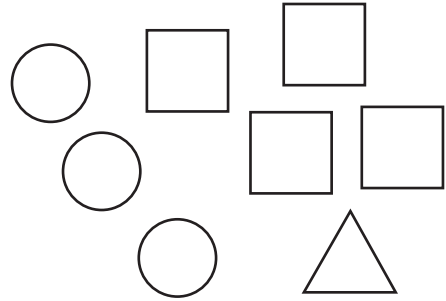


Understanding Ratios

A ratio is a pair of numbers that compares two quantities.

Count to find the ratio of squares to circles.

\downarrow \downarrow
 4 to 3



The ratio 4 to 3 can also be written as 4:3 or $\frac{4}{3}$.

The order of the numbers in a ratio is important.

4:3 is the ratio of squares to circles.

3:4 is the ratio of circles to squares.

Use the picture above for exercises 1 through 6. Write a ratio for each comparison in three ways.

1. The number of triangles to the total number of shapes

\downarrow \downarrow
 1 to 8

1:8; $\frac{1}{8}$

2. The number of squares to the number of triangles

4 to 1; 4:1; $\frac{4}{1}$

3. The number of triangles to the number of squares

1 to 4; 1:4; $\frac{1}{4}$

4. The number of triangles to the number of circles

1 to 3; 1:3; $\frac{1}{3}$

5. The number of circles to the total number of shapes

3 to 8; 3:8; $\frac{3}{8}$

6. The total number of shapes to the number of squares

8 to 4; 8:4; $\frac{8}{4}$

7. There are 14 boys and 16 girls in Mr. Allen's class. What is the ratio of girls to the total number of students in the class? Write the ratio 3 ways.

16 to 30; 16:30; $\frac{16}{30}$

8. **Writing to Explain** At a cat and dog hospital, 9 of the patients were cats, 17 were dogs. Use this fact to write two ratios. Explain what each ratio means.

Sample answer: The number of cats compared to the number of dogs is shown by the ratio 9:17. The number of dogs compared to the total number of animals is shown by the ratio 17:26.

Understanding Ratios

A string quartet consists of 2 violins, 1 viola, and 1 cello. Write a ratio for each comparison in three ways.

- violins to cellos 2:1; 2 to 1; $\frac{2}{1}$
- cellos to violas 1:1; 1 to 1; $\frac{1}{1}$
- violins to all instruments 2:4; 2 to 4; $\frac{2}{4}$

4. **Number Sense** How are the ratios in Exercises 1 and 2 different from the ratio in Exercise 3?

The ratios in Exercises 1 and 2 are comparing parts to parts; the ratio in Exercise 3 compares a part to a whole.

Midland Orchards grows a large variety of apples. The orchard contains 12 rows of Granny Smith trees, 10 rows of Fuji trees, 15 rows of Gala trees, 2 rows of Golden Delicious trees, and 2 rows of Jonathan trees. Write each ratio in three ways.

- rows of Granny Smith trees to rows of Golden Delicious trees 12:2; 12 to 2; $\frac{12}{2}$
- rows of Fuji trees to the total number of rows of trees 10:41; 10 to 41; $\frac{10}{41}$
- A grade school has 45 students who walk to school and 150 students who ride the bus. The other 50 students are driven to school. Which shows the ratio of students who walk to school to the total number of students in the school?

A 45:50

B 45:195

C 45:150

D 45:245

8. **Writing to Explain** Steve said it does not matter which term is first and which term is second in a ratio, since ratios are different than fractions. Is he correct? Explain why or why not.

No; It does matter. If the terms are reversed, then a different comparison is being made.

Equal Ratios and Proportions

You can find equal ratios just like you find equivalent fractions.

Find ratios equal to $\frac{30}{40}$.

Multiply both terms by the same number.

$$\frac{30 \times 2}{40 \times 2} = \frac{60}{80}$$

Divide both terms by the same number. To find the simplest form ratio, divide by the greatest common factor (GCF) of the two numbers.

The GCF of 30 and 40 is 10.

$$\frac{30 \div 10}{40 \div 10} = \frac{3}{4}$$

Two equal ratios form a proportion. The units must be the same in both ratios.

Do the ratios 24 ft:16 seconds and 36 ft:24 seconds form a proportion?

First check the units.

Both ratios compare feet to seconds, so the units are the same.

Then write each ratio in simplest form.

$$\frac{24 \text{ ft}}{16 \text{ seconds}} = \frac{3 \text{ ft}}{2 \text{ seconds}}$$

$$\frac{36 \text{ ft}}{24 \text{ seconds}} = \frac{3 \text{ ft}}{2 \text{ seconds}}$$

Compare the simplest form ratios.

They are the same, so the ratios form a proportion.

Write three ratios that are equal to the ratio given.

Sample ratios are given.

1. $\frac{3}{5}$ $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$

2. $\frac{4}{8}$ $\frac{1}{2}, \frac{3}{6}, \frac{50}{100}$

3. $\frac{6}{18}$ $\frac{1}{3}, \frac{4}{12}, \frac{8}{24}$

4. 8:10 4:5, 16:20, 24:30

5. 6:8 3:4, 12:16, 9:12

6. 10:12 5:6, 20:24, 15:18

7. 12 to 18 4 to 6, 2 to 3, 24 to 36

8. 16 to 18 8 to 9, 32 to 36, 48 to 54

9. 5 to 25 1 to 5, 2 to 10, 3 to 15

Write the ratios in simplest form.

10. $\frac{10}{15}$ $\frac{2}{3}$

11. 21 to 14 3 to 2

12. 15:25 3:5

Write = if the ratios form a proportion; if they do not form a proportion, write \neq .

13. $\frac{15}{18} | \frac{10}{12}$ =

14. 20:24 | 24:30 \neq

15. 16 to 20 | 28 to 35 =

16. **Number Sense** Dale says that the ratios 3:5 and 2:10 are equal. Is he correct? Explain. **Sample answer:**

No, 2:10 simplifies 1:5. 3:5 is not equal to 1:5.

Equal Ratios and Proportions

Write three ratios that are equal to the ratio given.

Sample ratios are given.

1. $\frac{8}{10}$ **$\frac{4}{5}, \frac{16}{20}, \frac{24}{30}$** 2. $\frac{2}{3}$ **$\frac{4}{6}, \frac{8}{12}, \frac{6}{9}$** 3. $\frac{3}{4}$ **$\frac{12}{16}, \frac{15}{20}, \frac{6}{8}$**
 4. 21 to 18 **7 to 6, 42 to 36, 63 to 54** 5. 5 to 4 **10 to 8, 15 to 12, 20 to 16** 6. 1 to 3 **2 to 6, 3 to 9, 4 to 12**
 7. 14:16 **7:8, 28:32, 42:48** 8. 2:4 **3:6, 4:8, 1:2** 9. 2:5 **4:10, 6:15, 8:20**

Write = if the ratios form a proportion; if they do not form a proportion, write \neq .

10. 3:12 | 6:24 **=** 11. $\frac{14}{16}$ | $\frac{7}{4}$ **\neq** 12. 4 to 20 | 1 to 4 **\neq**

Find the number that makes the ratios equivalent.

13. $\frac{8}{9} = \frac{32}{\quad}$ /36 14. 15:18 = 5: **6** 15. **3** to 7 = 9 to 21

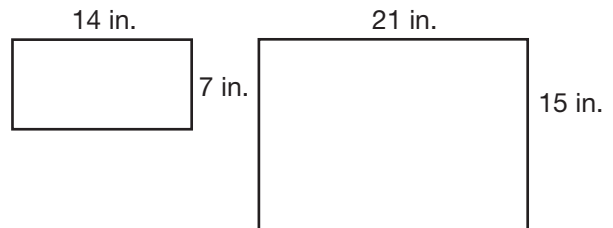
Write the ratios in simplest form.

16. $\frac{42}{28}$ **$\frac{3}{2}$** 17. 21 to 36 **7 to 12** 18. 15:45 **1:3**
 19. $\frac{35}{25}$ **$\frac{7}{5}$** 20. 60 to 30 **2 to 1** 21. 10:40 **1:4**

22. **Writing to Explain** Tell why you cannot multiply or divide by zero to find equal ratios.

Any number multiplied by zero is zero. You cannot divide by zero.

23. **Geometry** Is the ratio of length to width for these two rectangles proportional? Tell how you know.



No. $\frac{7}{14} = \frac{1}{2}$; $\frac{15}{21} = \frac{5}{7}$.

Simplest forms are not equal.

24. **Algebra** Which value for x would make the ratios equivalent?

$$\frac{3}{8} = \frac{x}{32}$$

A $x = 4$

B $x = 6$

C $x = 8$

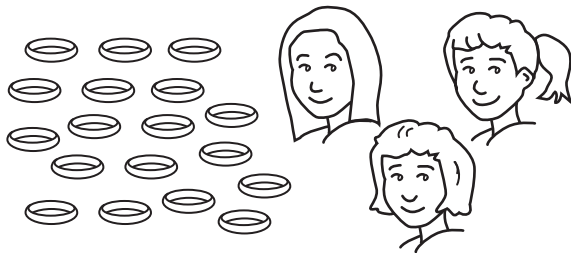
D $x = 12$

Understanding Rates and Unit Rates

A rate is a ratio in which the two terms are measured in different units.

Example: 18 bracelets for 3 girls.

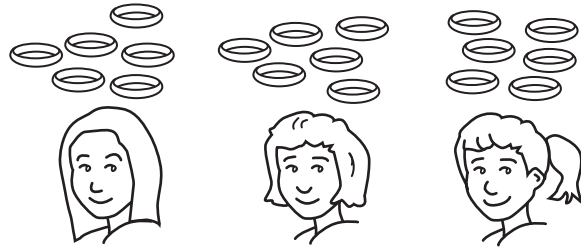
$$\frac{18 \text{ bracelets}}{3 \text{ girls}}$$



In a unit rate, the second number is 1.

Example: 6 bracelets for 1 girl.

$$\frac{6 \text{ bracelets}}{1 \text{ girl}}$$



Remember that the fraction bar shows division. If you know a rate, you can divide to find the unit rate.

Example: 17 goals in 5 games is written as $\frac{17 \text{ goals}}{5 \text{ games}}$.

$$\begin{array}{r} 3.4 \\ 5 \overline{)17.0} \end{array}$$

The unit rate is 3.4 goals per game. (Per means "for each".)

Write the rate and the unit rate.

1. 25 flowers for 5 vases
 $\frac{25 \text{ flowers}}{5 \text{ vases}}$, $\frac{5 \text{ flowers}}{1 \text{ vase}}$

2. 32 games in 8 weeks
 $\frac{32 \text{ games}}{8 \text{ weeks}}$, $\frac{4 \text{ games}}{1 \text{ week}}$

3. 144 pencils in 12 packages
 $\frac{144 \text{ pencils}}{12 \text{ pkgs}}$, $\frac{12 \text{ pencils}}{1 \text{ pkg}}$

4. 252 students in 9 classes
 $\frac{252 \text{ students}}{9 \text{ classes}}$, $\frac{28 \text{ students}}{1 \text{ class}}$

5. \$13.20 for 6 pounds
 $\frac{\$13.20}{6 \text{ lbs}}$, $\frac{\$2.20}{1 \text{ lb}}$

6. 34 minutes for 8 pages
 $\frac{34 \text{ minutes}}{8 \text{ pages}}$, $\frac{4.25 \text{ minutes}}{1 \text{ page}}$

7. **Number Sense** If a car travels 350 miles in 7 hours, what is its rate per hour?

50 miles per hour

8. **Estimation** Bare root plum trees are on sale at 3 for \$40. To the nearest dollar, what is the cost per tree?

\$13 per tree

Understanding Rates and Unit Rates

Write the rate and the unit rate.

1. 42 bricks laid in 2 hours

$$\frac{42 \text{ bricks}}{2 \text{ hours}} \text{ , } \frac{21 \text{ bricks}}{1 \text{ hour}}$$

2. 15 points scored in 4 quarters

$$\frac{15 \text{ points}}{4 \text{ quarters}} \text{ , } \frac{3.75 \text{ points}}{1 \text{ quarter}}$$

3. 225 chairs in 15 rows

$$\frac{225 \text{ chairs}}{15 \text{ rows}} \text{ , } \frac{15 \text{ chairs}}{1 \text{ row}}$$

4. 24 trees pruned in 5 days

$$\frac{24 \text{ trees}}{5 \text{ days}} \text{ , } \frac{4.8 \text{ trees}}{1 \text{ day}}$$

5. 480 miles in 12 hours

$$\frac{480 \text{ miles}}{12 \text{ hours}} \text{ , } \frac{40 \text{ miles}}{1 \text{ hour}}$$

6. \$6.50 for 10 pounds

$$\frac{\$6.50}{10 \text{ lbs}} \text{ , } \frac{\$0.65}{1 \text{ lb}}$$

7. 72 plants in 9 square feet

$$\frac{72 \text{ plants}}{9 \text{ sq ft}} \text{ , } \frac{8 \text{ plants}}{1 \text{ sq ft}}$$

8. 357 miles on 14 gallons

$$\frac{357 \text{ miles}}{14 \text{ gal}} \text{ , } \frac{25.5 \text{ miles}}{1 \text{ gal}}$$

9. **Estimation** Over 5 days, 8,208 people visited an amusement park. About how many people visited the park per day?

About 1,600 per day

10. **Writing to Explain** Explain how you could convert a rate of 18,000 miles per hour to miles per second.

Divide 18,000 by 60 to get miles per minute, $18,000 \div 60 = 300$ miles per minute. Then divide 300 by 60 to get miles per second, $300 \div 60 = 5$ miles per second.

11. **Critical Thinking** Matt makes 5 bookcases in 8 days. What is his unit rate?

0.625 bookcases per day

12. A space shuttle orbits Earth 1 time in 90 minutes. How many times does it orbit Earth in 6 hours?

4 times

13. Which is the unit rate for 39 people in 3 vans?

A 39 people per van

C 13 people per van

B 13 vans per person

D 3 people per van

Comparing Rates

Use unit rates to compare two rates that have the same units of measurement.

Daniel painted 9 planks in 6 minutes. Meredith painted 22 planks in 11 minutes. Who painted at a faster rate?

Write each rate as a unit rate.

$$\text{Daniel's Rate: } \frac{9 \text{ planks}}{6 \text{ min}}$$

$$= \frac{9 \text{ planks} \div 6}{6 \text{ min} \div 6}$$

$$= \frac{1.5 \text{ planks}}{1 \text{ min}}$$

$$\text{Meredith's Rate: } \frac{22 \text{ planks}}{11 \text{ min}}$$

$$= \frac{22 \text{ planks} \div 11}{11 \text{ minutes} \div 11}$$

$$= \frac{2 \text{ planks}}{1 \text{ min}}$$

Since 2 is greater than 1.5, Meredith is the faster painter. The faster rate is 22 planks in 11 min.

Find each unit rate and determine which rate is greater.

1. 51 hits on Jon's website in 3 h or 96 hits on Shana's website in 6 h

Jon's unit rate: $\frac{17 \text{ hits}}{1 \text{ h}}$; Shana's unit rate: $\frac{16 \text{ hits}}{1 \text{ h}}$;

51 hits in 3 h is greater.

2. 330 mi on 15 gal or 240 mi on 10 gal

$\frac{22 \text{ mi}}{1 \text{ gal}}$, $\frac{24 \text{ mi}}{1 \text{ gal}}$; 240 mi on 10 gal is greater.

3. 90 breaths in 6 min or 112 breaths in 8 min

$\frac{15 \text{ breaths}}{1 \text{ min}}$, $\frac{14 \text{ breaths}}{1 \text{ min}}$; 90 breaths in 6 min is greater.

Find each unit price and determine which is a better buy.

4. 20 gallons of gas for \$66.00 or 25 gallons of gas for \$81.25

$\frac{\$3.30}{1 \text{ gal}}$, $\frac{\$3.25}{1 \text{ gal}}$; 25 gallons for \$81.25 is the better buy.

5. **Writing to Explain** Earl and Mia danced in a charity fundraiser.

Earl raised \$275 when he danced for 5 hours. Mia raised \$376 when she danced for 8 hours. Which dancer earned more for each hour danced? Explain how you found your answer.

Earl earned more for each hour; Earl's unit rate: $\frac{\$55}{1 \text{ h}}$;

Mia's unit rate: $\frac{\$47}{1 \text{ h}}$. Since $\$55 > \47 , Earl earned more for each hour.

Comparing Rates

Find each unit rate and determine which rate is greater.

1. 250 mi per 10 gal or 460 mi per 20 gal

$\frac{25 \text{ mi.}}{1 \text{ gal}}$, $\frac{23 \text{ mi.}}{1 \text{ gal}}$; **250 mi per 10 gal is greater.**

2. 1,000 words in 20 min or 2,475 words in 45 min

$\frac{50 \text{ words.}}{1 \text{ min}}$, $\frac{55 \text{ words.}}{1 \text{ min}}$; **2,475 words in 45 min is greater.**

3. 6 in. of rain in 4 h or 8 in. of rain in 5 h

$\frac{1.5 \text{ in.}}{1 \text{ h}}$, $\frac{1.6 \text{ in.}}{1 \text{ h}}$; **8 in. of rain in 5 h is greater.**

Find each unit price and determine which is a better buy.

4. 1 lb of apples for \$2.15 or 3 lb of apples for \$5.76

$\frac{\$2.15}{1 \text{ lb}}$, $\frac{\$1.92}{1 \text{ lb}}$; **3 lb for \$5.76 is the better buy.**

5. 8 bungee cords for \$10.00 or 20 bungee cords for \$22.00

$\frac{\$1.25}{1 \text{ cord}}$, $\frac{\$1.10}{1 \text{ cord}}$; **20 cords for \$22.00 is the better buy.**

6. 5 oz of insect repellent for \$6.95 or 14 oz of insect repellent for \$19.60

$\frac{\$1.39}{1 \text{ oz}}$, $\frac{\$1.40}{1 \text{ oz}}$; **5 oz for \$6.95 is the better buy.**

7. Fritz earns \$75.60 for each 7-h shift that he works. Which shift pays a higher hourly wage than the wage Fritz earns?

- A \$60.30 for a 6-h shift
- B \$80.00 for an 8-h shift
- C** \$36.30 for a 3-h shift
- D \$40.40 for a 4-h shift

8. **Writing to Explain** Shaunda said that buying 4 towels for \$17 was a better buy than buying 2 towels for \$9. She found her answer by doubling the terms in the ratio $\frac{9}{2}$ and comparing the first terms in the ratios. Is she correct? Use unit prices to support your answer.

Yes, the unit price for 2 towels for \$9 is \$4.50 per towel, and the unit price for 4 towels for \$17 is \$4.25 per towel. Since $\$4.25 < \4.50 , the 4 towels for \$17 are a better buy.

Distance, Rate, and Time

The formula $d = r \times t$ uses symbols to relate the quantities for distance (d), average rate of speed (r), and time (t).

Example 1

How long will it take a car moving at 50 mph to travel 70 mi?

Substitute what you know into the formula $d = r \times t$.

Solve the equation.

$$\begin{aligned} 70 \text{ mi} &= 50 \text{ mph} \times t \\ \frac{70 \text{ mi}}{50 \text{ mph}} &= \frac{50 \text{ mph} \times t}{50 \text{ mph}} \\ 1.4 \text{ h} &= t \end{aligned}$$

It will take 1.4 h to travel 70 mi at 50 mph.

Example 2

A car travels 325 mi in 5 h. What is its rate of speed?

Substitute what you know into the formula $d = r \times t$.

Solve the equation.

$$\begin{aligned} 325 \text{ mi} &= r \times 5 \text{ h} \\ \frac{325 \text{ mi}}{5 \text{ h}} &= \frac{r \times 5 \text{ h}}{5 \text{ h}} \\ 65 \text{ mph} &= r \end{aligned}$$

The rate of speed of a car that travels 325 mi in 5 h is 65 mph.

1. An airplane flies at 250 mph. How far will it travel in 5 h at that rate of speed?

Substitute the information you know into the formula $d = r \times t$:

$$d = \underline{250 \text{ mph} \times 5 \text{ h}}$$

Solve the equation.

$$\underline{d = 1,250 \text{ mi}}$$

Write the answer with the correct units.

$$\underline{1,250 \text{ mi}}$$

Find the missing variable.

2. Distance = 60 km

time = 4 h

rate = 15 km/h

3. Distance = 24 cm

time = 12 sec

rate = 2 cm/sec

4. Distance = 56 yd

time = 7 min

rate = 8 yd/min

5. Distance = 4,000 m

time = 25 d

rate = 160 m/d

6. **Writing to Explain** A storm is 15 mi from Lodi. If the storm travels at 6 mph towards the city, how many hours will it take for the storm to get to Lodi? Show your work.

$$\underline{2\frac{1}{2} \text{ h}; d = r \times t; 15 = 6 \times t; 15 \div 6 = t; t = 2\frac{1}{2}}$$

Distance, Rate, and Time

Find the missing variable.

1. Distance = 15 mi time = 2 h rate = 7.5 mph

2. Distance = 56 km time = 4 h rate = $\frac{14 \text{ km}}{\text{h}}$

3. Distance = 72 yd time = 6 min rate = $\frac{12 \text{ yd}}{\text{min}}$

4. Distance = 27 cm time = 9 sec rate = $\frac{3 \text{ cm}}{\text{sec}}$

5. Distance = 10,000 m time = 2 d rate = $\frac{5,000 \text{ m}}{\text{d}}$

6. Distance = 480 ft time = 6 wk rate = $\frac{80 \text{ ft}}{\text{wk}}$

7. The California Speedway hosts automobile races. Which rate of speed is higher: a car completing a 500-mi race in about $3\frac{1}{3}$ h or a car completing a 300-mi race in about $2\frac{1}{2}$ h? 500 mi race in about $3\frac{1}{3}$ h8. A train traveled 250 mi in 2 h. If it traveled at the same rate of speed, how long would it take the train to travel 600 mi? 4.8 h

9. The space shuttle travels 4,375 mi in 15 min as it orbits the earth. Estimate its average rate of speed during that time to the nearest hundred.

A About 400 mi per min

B About 300 mi per min

C About 60,000 mi per min

D About 70,000 mi per min

10. **Writing to Explain** Kevin drove his scooter 62 km in 2 h. Explain how to find how far he drives if he drives at the same rate for 3 h.

93 km; Substitute known values in formula $d = r \times t$ to get $62 = r \times 2$. Then solve the equation to find $r = \frac{31 \text{ km}}{\text{hr}}$. Multiply the rate of $\frac{31 \text{ km}}{\text{h}}$ and the time of 3 h to find the distance, 93 km.

Problem Solving: Draw a Picture

Veronica is celebrating her birthday by having a skating party. As part of a birthday special, Veronica paid for 10 tickets and 2 guests received free admission. What fraction of the people at Veronica's party were not charged for admission?

Read and Understand

What do you know? There were 10 paid admissions and 2 free admissions.

What are you trying to find? The fraction of people attending Veronica's party that were admitted at no charge.

Plan and Solve

What strategy will you use? Draw a picture to show the 10 paid admissions and the 2 free admissions.

Count the boxes. There were 12 people admitted. Since 2 of the 12 people were admitted at no charge, the fraction is $\frac{2}{12}$, or $\frac{1}{6}$ in simplest form.



Look Back and Check

Is your answer reasonable? Yes. The picture shows 2 out of 12 boxes, which is $\frac{2}{12}$, or $\frac{1}{6}$.

Draw or use a picture to solve each problem.

One afternoon, the ratio of black shirts sold to white shirts sold at The Clothes Horse was 2:1. Complete the picture to show the ratio.



- How many boxes are shaded in all?
- What fraction of the shirts sold were black?
- The Clothes Horse sold 12 shirts that afternoon. How many black shirts were sold? HINT: YOU CAN ADD TO THE PICTURE UNTIL THERE ARE 12 SHADED BOXES TO REPRESENT THE PROBLEM.
- Ilene earns \$20. She saves \$2 for every \$8 that she spends. How much of the \$20 will she save?

3 boxes

$\frac{2}{3}$

8 black shirts

\$4

Dollars Saved
Dollars Spent

X X X X
X X X X X X X X X X X X X X

