

Chords and Arcs

1. Plan

Objectives

- To use congruent chords, arcs, and central angles
- To recognize properties of lines through the center of a circle

Examples

- Using Theorem 12-4
- Using Theorem 12-5
- Using Diameters and Chords



Math Background

Theorem 12-8 can be used to prove the theorem of analytic geometry that states that any three noncollinear points determine a unique circle. It also can be used to justify a method of constructing the circle. Construct the perpendicular bisectors of two of the three possible segments. Construct a circle whose center is the point of intersection of the perpendicular bisectors and whose radius is the distance from the center to any of the three points.

More Math Background: p. 660C

Lesson Planning and Resources

See p. 660E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Using 45°-45°-90° Triangles

Lesson 8-2: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 8

Using 30°-60°-90° Triangles

Lesson 8-2: Example 4
Extra Skills, Word Problems, Proof Practice, Ch. 8

What You'll Learn

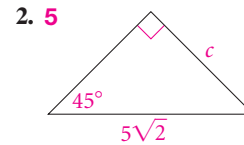
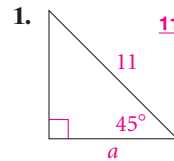
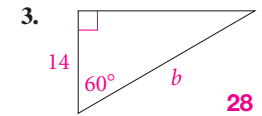
- To use congruent chords, arcs, and central angles
- To recognize properties of lines through the center of a circle

... And Why

To see how an archaeologist finds the center and radius of the rim of a jar, as in Exercise 20

Check Skills You'll Need

Find the value of each variable. Leave your answer in simplest radical form.


GO for Help Lesson 8-2

New Vocabulary • chord

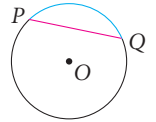
1

Using Congruent Chords, Arcs, and Central Angles

A segment whose endpoints are on a circle is called a **chord**.

The diagram shows the related chord and arc, \overline{PQ} and \widehat{PQ} .

The following theorem is about related central angles, chords, and arcs. It says, for example, that if two central angles in a circle are congruent, then so are the two chords and two arcs that the angles intercept.



Key Concepts



For: Chords and Arcs Activity
Use: Interactive Textbook, 12-2

Theorem 12-4

Within a circle or in congruent circles

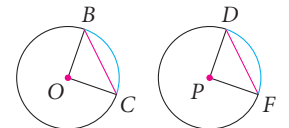
- Congruent central angles have congruent chords.
- Congruent chords have congruent arcs.
- Congruent arcs have congruent central angles.

You will prove Theorem 12-4 in Exercises 23, 24, and 35.

1 EXAMPLE Using Theorem 12-4

In the diagram, $\odot O \cong \odot P$. Given that $\widehat{BC} \cong \widehat{DF}$, what can you conclude?

By Theorem 12-4, $\angle O \cong \angle P$ and $\overline{BC} \cong \overline{DF}$.



Quick Check

- If you are instead given that $\overline{BC} \cong \overline{DF}$, what can you conclude?
 $\angle O \cong \angle P$; $\widehat{BC} \cong \widehat{DF}$

670 Chapter 12 Circles

Differentiated Instruction Solutions for All Learners

Special Needs **L1**

Review with students why *congruent arcs* must be in the same circle or in congruent circles. Also point out that a chord is related to the *minor arc* it intercepts.

Below Level **L2**

Have students draw diagrams for Theorems 12-6, 12-7, and 12-8 that accurately represent the given information.

learning style: verbal

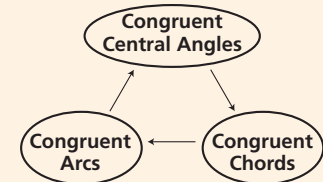
learning style: visual

2. Teach

Guided Instruction

Visual Learners

On the board, copy the diagram below that summarizes Theorem 12-4.



Ask: Can you conclude that congruent chords have congruent central angles? If so, how? **yes; by the Law of Syllogism**

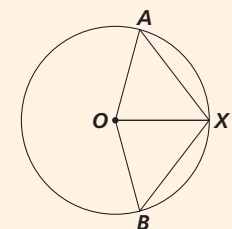
Alternative Method

An alternate proof of part 1 of Theorem 12-5 would use the HL Theorem to prove $\triangle AOE \cong \triangle BOE \cong \triangle COF \cong \triangle DOF$ and then use CPCTC and the Segment Addition Postulate. This method also could be used to prove Theorem 12



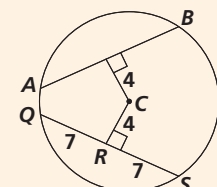
Additional Examples

1 In the diagram, radius \overline{OX} bisects $\angle AOB$. What can you conclude?



$$\angle AOX \cong \angle BOX; \overline{AX} \cong \overline{BX}; \overline{AX} \cong \overline{BX}$$

2 Find AB.



14

Theorem 12-5 shows a relationship between two chords and their distances from the center of a circle. You will prove part (2) in Exercise 38.

Key Concepts

Theorem 12-5

Within a circle or in congruent circles

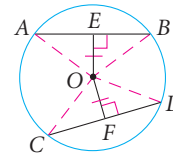
- (1) Chords equidistant from the center are congruent.
- (2) Congruent chords are equidistant from the center.

Proof

Proof of Theorem 12-5, Part (1)

Given: $\odot O, \overline{OE} \cong \overline{OF},$
 $\overline{OE} \perp \overline{AB}, \overline{OF} \perp \overline{CD}$

Prove: $\overline{AB} \cong \overline{CD}$



Statements	Reasons
1. $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$	1. Radii of a circle are congruent.
2. $\overline{OE} \cong \overline{OF}, \overline{OE} \perp \overline{AB}, \overline{OF} \perp \overline{CD}$	2. Given
3. $\angle AEO$ and $\angle CFO$ are right angles.	3. Def. of perpendicular segments
4. $\triangle AEO \cong \triangle CFO$	4. HL Theorem
5. $\angle A \cong \angle C$	5. CPCTC
6. $\angle B \cong \angle A, \angle C \cong \angle D$	6. Isosceles Triangle Theorem
7. $\angle B \cong \angle D$	7. Transitive Property of Congruence
8. $\angle AOB \cong \angle COD$	8. If two \angle s of a \triangle are \cong to two \angle s of another \triangle , then the third \angle s are \cong .
9. $\overline{AB} \cong \overline{CD}$	9. \cong central angles have \cong chords.

Real-World Connection

Steel beams model congruent chords equidistant from the center to give the illusion of a circle.



You can use Theorem 12-5 to find missing lengths in circles.

2 EXAMPLE Using Theorem 12-5

Multiple Choice What is the value of a in the circle at the right?

- (A) 9 (B) 12.5 (C) 18 (D) 25

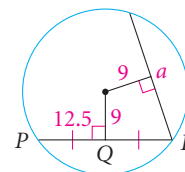
$$PQ = QR = 12.5 \quad \text{Given}$$

$$PQ + QR = PR \quad \text{Segment Addition Postulate}$$

$$25 = PR \quad \text{Substitute.}$$

$$a = PR \quad \text{Chords equidistant from the center of a circle are congruent.}$$

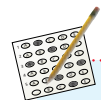
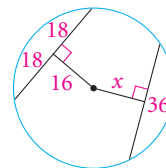
$$a = 25 \quad \text{Substitute.}$$



• The correct answer is D.

2 Find the value of x in the circle at the right.

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Test-Taking Tip

In a circle, the length of the perpendicular segment from the center to a chord is the distance from the center to the chord.

Quick Check

Advanced Learners L4

Have students write a paragraph to explain why the phrase *that is not a diameter* is necessary in Theorem 12-7.

learning style: verbal

English Language Learners ELL

Ask: Is a diameter a chord? Explain. **Yes; it is a segment with two endpoints on the circle.**
Is a radius a chord? Explain. **No; it has only 1 point on the circle.**

learning style: verbal

Guided Instruction

3 EXAMPLE Error Prevention

Because the figures in parts a and b do not show diameters, some students may not understand why Theorems 12-6 and 12-7 apply. Have them reread the section above Theorem 12-6 to reinforce that the theorems apply to lines or segments that contain the center of the circle.

PowerPoint

Additional Examples

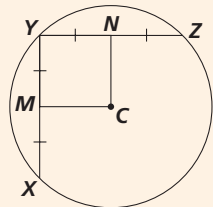
3 P and Q are points on $\odot O$. The distance from O to \overline{PQ} is 15 in., and $PQ = 16$ in. Find the radius of $\odot O$. **17 in.**

Resources

- Daily Notetaking Guide 12-2 **L3**
- Daily Notetaking Guide 12-2—Adapted Instruction **L1**

Closure

\overline{XY} and \overline{YZ} are perpendicular chords within $\odot C$ that are also equidistant from center C. What is the most precise name for quadrilateral MYNC? Explain.



Square; congruent chords are equidistant from the center, and a diameter that bisects a chord is \perp to the chord.

2

Lines Through the Center of a Circle

The Converse of the Perpendicular Bisector Theorem from Lesson 5-2 has special applications to a circle and its diameters, chords, and arcs.



Key Concepts

Theorem 12-6

In a circle, a diameter that is perpendicular to a chord bisects the chord and its arcs.

Theorem 12-7

In a circle, a diameter that bisects a chord (that is not a diameter) is perpendicular to the chord.

Theorem 12-8

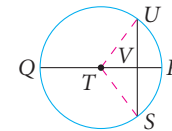
In a circle, the perpendicular bisector of a chord contains the center of the circle.

Proof

Proof of Theorem 12-7

Given: $\odot T$ with diameter \overline{QR} bisecting \overline{SU} at V.

Prove: $\overline{QR} \perp \overline{SU}$



Proof: $TS = TU$ because the radii of a circle are congruent. $VS = VU$ by the definition of bisect. Thus, T and V are equidistant from S and U .

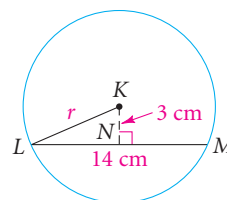
By the Converse of the Perpendicular Bisector Theorem, T and V are on the perpendicular bisector of \overline{SU} . Since two points determine one line, \overline{TV} is the perpendicular bisector of \overline{SU} . Another name for \overline{TV} is \overline{QR} . Thus, $\overline{QR} \perp \overline{SU}$.

You will prove Theorems 12-6 and 12-8 in Exercises 25 and 36, respectively.

3 EXAMPLE Using Diameters and Chords

Algebra Find each missing length to the nearest tenth.

a.



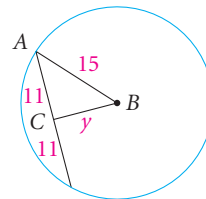
$$LN = \frac{1}{2}(14) = 7$$

$$r^2 = 3^2 + 7^2$$

$$r \approx 7.6$$

A diameter \perp to a chord bisects the chord.
Use the Pythagorean Theorem.
Find the square root of each side.

b.



$$\overline{BC} \perp \overline{AC}$$

$$y^2 + 11^2 = 15^2$$

$$y^2 = 104$$

$$y \approx 10.2$$

A diameter that bisects a chord that is not a diameter is \perp to the chord.
Use the Pythagorean Theorem.
Solve for y^2 .
Find the square root of each side.

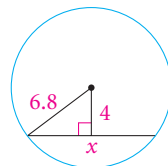


Real-World Connection

The center of the tire is located on the perpendicular bisector of the flat part.



- 3 Use the circle at the right.
- Find the length of the chord. **about 11**
 - Find the distance from the midpoint of the chord to the midpoint of its minor arc. **2.8**



3. Practice

Assignment Guide

- 1** A B 1-8, 17, 23, 24, 27, 29-32, 35
- 2** A B 9-16, 18-22, 25, 26, 28, 33, 34, 36, 37
- C Challenge 38-41
- Test Prep 42-48
- Mixed Review 49-52

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 8, 12, 24, 29, 30.

Exercises 12, 14 Students may find it helpful to draw and label the third side of the triangle, using the fact that all radii are congruent.

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 670)



Example 2
(page 671)

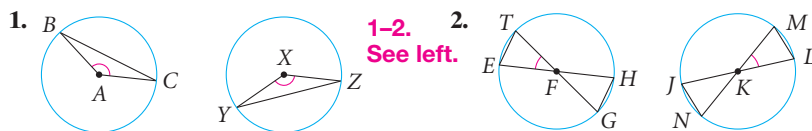
- $\overline{BC} \cong \overline{YZ}; \overline{BC} \cong \overline{YZ}$
- $\overline{ET} \cong \overline{GH} \cong \overline{JN} \cong \overline{ML}; \angle TFE \cong \angle HFG; \angle JKN \cong \angle MKL$

Example 3
(page 672)

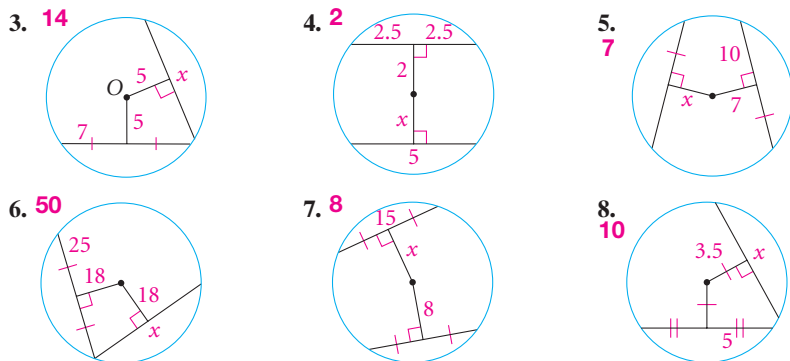
9. Answers may vary. Samples are given.

- \overline{CE}
- \overline{DE}
- $\angle CEB$
- $\angle DEA$

In Exercises 1 and 2, the circles are congruent. What can you conclude?

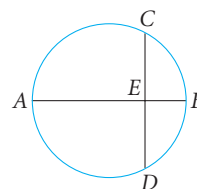


Find the value of x .

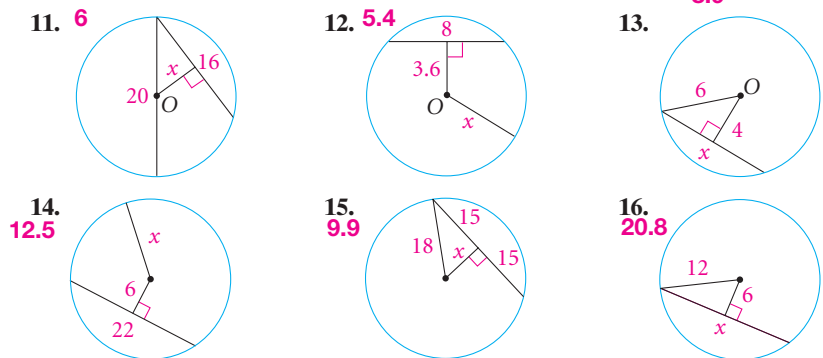


Use the diagram at the right to complete Exercises 9 and 10.

- Given that \overline{AB} is a diameter of the circle and $\overline{AB} \perp \overline{CD}$, then a. $\underline{\quad} \cong \underline{\quad}$ and c. $\underline{\quad} \cong \underline{\quad}$. **See left.**
- Given that \overline{AB} is the perpendicular bisector of \overline{CD} , then \overline{AB} contains $\underline{\quad}$. **the center of the circle**



Algebra Find the value of x to the nearest tenth.



Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3

Practice 12-2 Translations

What is the image of Z under each translation?

- $(2, -2)$
- $(5, -1)$
- $(2, -6)$
- $(4, -6)$
- $(0, 0)$
- $(-2, -4)$

Find the vector that describes the given translation.

- $Z \rightarrow Y$
- $Y \rightarrow W$
- $U \rightarrow X$
- $Y \rightarrow W$
- $U \rightarrow Z$
- $W \rightarrow V$

Use matrices to find the image of each figure under the given translation.

- translation $(2, 4)$
- translation $(-2, 1)$
- translation $(5, -3)$

Write a rule to describe each translation.

-
-
-

Find a single translation that has the same effect as each composition of translations.

- $(3, 5.2)$ followed by $(1, 2.6)$
- $(4, -8)$ followed by $(8, -5)$
- $(7, 11)$ followed by $(-7, -11)$
- $(1, 2)$ followed by $(2, 1)$

23. $\triangle PNQ$ has vertices $P(2, 5)$, $N(-3, -1)$, and $Q(4, 0)$.

- Determine the image of P under the translation $(-5, -6)$.
- Use matrices to find the image of $\triangle PNQ$ under the translation $(-2, 3)$.

Error Prevention!

Exercises 17–19 Students may confuse the measure of an arc with arc length. Remind them that the letter m signals the measure of the arc.

Technology Tip

Exercise 19 Review with students how to use the \sin^{-1} calculator function key.

Tactile Learners

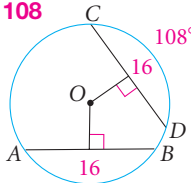
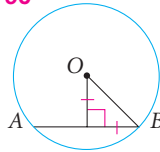
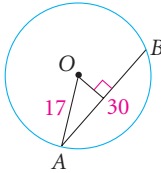
Exercise 20 Have students use a compass and straightedge and the concepts in this exercise to find the center of a circle circumscribed about three noncollinear points. This method also can be used in Exercise 30.

Exercises 30–32 Discuss these exercises as a class. Have students suggest different solution methods, such as using the Pythagorean Theorem or showing that $ACBD$ is a rhombus.

Exercise 33 Show students how they can substitute $x = 2$ into $x^2 + y^2 = 25$ to find the positive and negative y -coordinates for the chord.

B Apply Your Skills

Find $m\widehat{AB}$. (Hint: You will need to use trigonometry in Exercise 19.)

17. **108**  18. **90**  19. **about 123.9** 



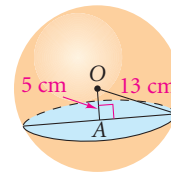
Real-World Connection

Careers Field archaeologists analyze artifacts to provide glimpses of life in the past.

- 20. Archaeology** An archaeologist found several jar fragments including a large piece of the circular rim. How can she find the center and radius of the rim to help her reconstruct the jar? **See margin.**

- 21. Geometry in 3 Dimensions** In the figure at the right, sphere O with radius 13 cm is intersected by a plane 5 cm from center O . Find the radius of cross section $\odot A$. **12 cm**

- 22. Geometry in 3 Dimensions** A plane intersects a sphere that has radius 10 in. forming cross section $\odot B$ with radius 8 in. How far is the plane from the center of the sphere? **6 in.**

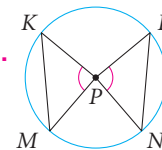


- 23.** Complete this proof of Theorem 12-4, Part (1).

Given: $\odot P$ with $\angle KPM \cong \angle LPN$

Prove: $\overline{KM} \cong \overline{LN}$ a-c. \overline{PL} ; \overline{PM} ; All radii of a circle are \cong .

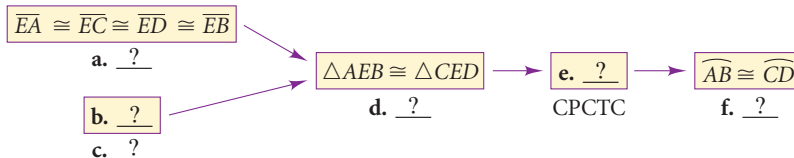
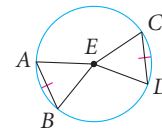
Proof: $\overline{KP} \cong$ a. $\underline{\quad}$ \cong b. $\underline{\quad}$ $\cong \overline{NP}$ because c. $\underline{\quad}$.
 $\triangle KPM \cong$ d. $\underline{\quad}$ by e. $\underline{\quad}$. $\overline{KM} \cong \overline{LN}$ by f. $\underline{\quad}$.
d-f. $\triangle LPN$; SAS; CPCTC



- 24.** Complete this proof of Theorem 12-4, Part (2).

Given: $\odot E$ with congruent chords \overline{AB} and \overline{CD}

Prove: $\widehat{AB} \cong \widehat{CD}$ **See margin.**



Problem Solving Hint

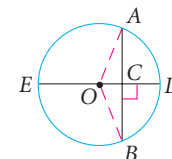
Recall that in a circle congruent central angles intercept congruent arcs.

- Proof 25.** Prove Theorem 12-6. **See back of book.**

Given: $\odot O$ with diameter $\overline{ED} \perp \overline{AB}$ at C

Prove: $\overline{AC} \cong \overline{BC}$ and $\widehat{AD} \cong \widehat{BD}$

(Hint: Begin by drawing \overline{OA} and \overline{OB} .)

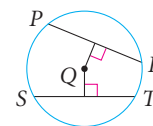


GO for Help

For a guide to solving Exercise 26, see p. 677.

- 26.** Two concentric circles have radii of 4 cm and 8 cm. A segment tangent to the smaller circle is a chord of the larger circle. What is the length of the segment? **about 13.9 cm**

- 27. Error Analysis** Scott looks at this figure and concludes that $\overline{ST} \cong \overline{PR}$. What is wrong with Scott's conclusion? **See left.**



- 28. Open-Ended** Use a circular object such as a can or a saucer to draw a circle. Construct the center of the circle.

- 29. Writing** Theorems 12-4 and 12-5 both begin with the phrase "Within a circle or in congruent circles." Explain why "congruent" is essential for both theorems. **Circles can have \cong chords or \cong central \triangle without having both.**

- 20.** She can draw 2 chords, and their \perp bisectors, of the partial circle. The intersection pt. of the \perp bisectors will be the center and she can then measure the radius.

- 24.** a. All radii of a circle are \cong .

b. $\overline{AB} \cong \overline{CD}$

c. Given

d. SSS

e. $\angle AEB \cong \angle CED$

f. \cong central \triangle have \cong arcs.

- 36.** X is equidist. from W and Y , since \overline{XW} and \overline{XY} are radii. So X is on the \perp bis. of \overline{WY} by the Conv. of the \perp Bis. Thm. But ℓ is the \perp bis. of \overline{WY} , so ℓ contains X .

- 38.** All radii of $\odot O$ are \cong , so $\triangle AOB \cong \triangle COD$ by SSS. $\angle A \cong \angle C$ by CPCTC. Also, $\angle OEA \cong \angle OFC$ since both are rt. \triangle . Thus, $\triangle OEA \cong \triangle OFC$ by AAS, and $\overline{OE} \cong \overline{OF}$ by CPCTC.

- 39.** 1. $\odot A$ with $\overline{CE} \perp \overline{BD}$ (Given) 2. $\overline{CF} \cong \overline{CF}$ (Refl. Prop. of \cong) 3. $\overline{BF} \cong \overline{FD}$ (diameter \perp to a chord bisects the chord.) 4. $\angle CFB$ and

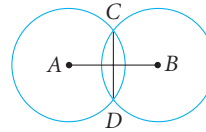
$\angle CFD$ are rt. \triangle (Def. of \perp). 5. $\triangle CFB \cong \triangle CFD$ (SAS) 6. $\overline{BC} \cong \overline{CD}$ (CPCTC) 7. $\widehat{BC} \cong \widehat{DC}$ (\cong chords have \cong arcs.)

⊙A and ⊙B are congruent. \overline{CD} is a chord of both circles.

GPS 30. $AB = 8$ in., $CD = 6$ in. How long is a radius? **5 in.**

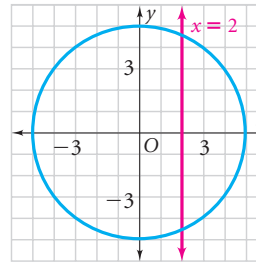
31. $AB = 24$ cm, radius = 13 cm. How long is \overline{CD} ? **10 cm**

32. radius = 13 ft, $CD = 24$ ft. How long is \overline{AB} ? **10 ft**



33. **Multiple Choice** In the diagram at the right, the endpoints of the chord are the points where the line $x = 2$ intersects the circle $x^2 + y^2 = 25$. What is the length of the chord? **C**

- (A) 4.6
 (B) 8.0
 (C) 9.2
 (D) 10.0



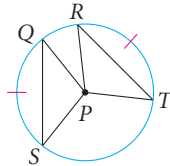
34. **Critical Thinking** The diameter of a circle is 20 cm. Two chords parallel to the diameter are 6 cm and 16 cm long. What are the possible distances between the chords to the nearest tenth of a centimeter? **3.5 cm, 15.5 cm**

35.1. ⊙P with $\overline{QS} \cong \overline{RT}$
 (Given) 2. $m\overline{QS} = m\angle QPS$ and $m\overline{RT} = m\angle RPT$ (Arc measure = central \angle measure.)
 3. $m\overline{QS} = m\overline{RT}$ (Def. of \cong) 4. $\angle QPS \cong \angle RPT$ (Subst.)

Proof 35. Prove Theorem 12-4, Part (3).

Given: ⊙P with $\overline{QS} \cong \overline{RT}$

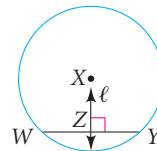
Prove: $\angle QPS \cong \angle RPT$ **See left.**



36. Prove Theorem 12-8.

Given: ℓ is the \perp bisector of \overline{WY} .

Prove: ℓ contains the center of ⊙X. **See margin p. 674.**

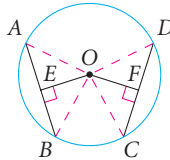


Proof 37. \overline{PQ} and \overline{PR} are chords of ⊙C. Prove that if C is on the bisector of $\angle QPR$, then $PQ = PR$. **See back of book.**

Challenge **Proof** 38. Prove Theorem 12-5, Part (2).

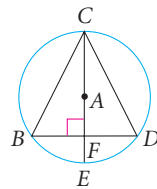
Given: ⊙O with $\overline{AB} \cong \overline{CD}$

Prove: $\overline{OE} \cong \overline{OF}$ **38-39. See margin.**



39. **Given:** ⊙A with $\overline{CE} \perp \overline{BD}$

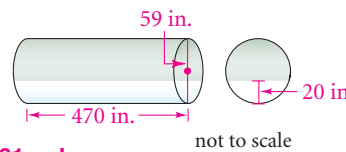
Prove: $\overline{BC} \cong \overline{DC}$



Real-World Connection

The cylinders used on milk tank trucks lie on the lateral surface and have a vertical base at each end.

40. Dairy The diameter of the base of a cylindrical milk tank is 59 in. The length of the tank is 470 in. You estimate that the depth of the milk in the tank is 20 in. Find the number of gallons of milk in the tank to the nearest gallon. (1 gal = 231 in.³) **1661 gal**



Proof 41. If two circles are concentric and a chord of the larger circle is tangent to the smaller circle, prove that the point of tangency is the midpoint of the chord. **See margin.**

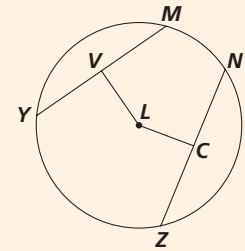
41. Let O be the center of the circles, and P be the pt. of tangency of the larger circle's chord to the smaller circle. Then \overline{OP} is \perp to the chord, and therefore bisects it.

So P is the midpt. of the chord.

4. Assess & Reteach

Lesson Quiz

For Exercises 1–5, use the diagram of ⊙L below.



1. If \overline{YM} and \overline{ZN} are congruent chords, what can you conclude?

$\overline{YM} \cong \overline{ZN}$; $\angle YLM \cong \angle ZLN$

2. If \overline{YM} and \overline{ZN} are congruent chords, explain why you cannot conclude that $LV = LC$. **You do not know whether LV and LC are perpendicular to the chords.**

3. Suppose that \overline{YM} has length 12 in., and its distance from point L is 5 in. Find the radius of ⊙L to the nearest tenth. **7.8 in.**

For Exercises 4 and 5, suppose that $\overline{LV} \perp \overline{YM}$, $YV = 11$ cm, and ⊙L has a diameter of 26 cm.

4. Find YM. **22 cm**

5. Find LV to the nearest tenth. **6.9 cm**

Alternative Assessment

Have students use only pictures and mathematical symbols to express each theorem in this lesson. Then have each student exchange their work with a partner who writes a paragraph below the picture evaluating the presentation for accuracy and clarity. Use the drawings and paragraphs to assess students' understanding.



Resources

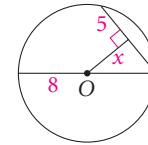
For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 711
- Test-Taking Strategies, p. 706
- Test-Taking Strategies with Transparencies

Multiple Choice

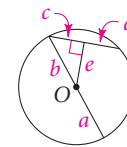
42. The diameter of a circle is 25 cm and a chord of the same circle is 16 cm. To the nearest tenth, what is the distance of the chord from the center of the circle? **B**
- A. 9.0 cm B. 9.6 cm C. 18.0 cm D. 19.2 cm

43. In the figure at the right, what is the value of x to the nearest tenth? **G**
- F. 3.0 G. 6.2
H. 6.8 J. 9.0



44. A 9-cm chord is 11 cm from the center of a circle. What is the radius of the circle? **B**
- A. 9.0 cm B. 11.9 cm C. 13.0 cm D. 14.2 cm
45. The radius of a circle is 10.8 ft. The length of a chord is 12 ft. What is the approximate distance of the chord from the center of the circle? **H**
- F. 1.2 ft G. 4.7 ft H. 9.0 ft J. 12.4 ft

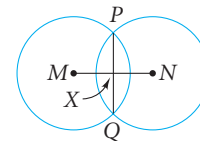
46. What can you NOT conclude from the diagram at the right? **D**
- A. $c = d$ B. $a = b$
C. $c^2 + e^2 = b^2$ D. $e = d$



47. $\odot A$ and $\odot B$ intersect at points C and D . Each circle has radius 6 in. and $AB = 8$ in. What is CD ? **J**
- F. 4.5 in. G. 6 in. H. 8 in. J. 8.9 in.

Short Response

48. Circles M and N are congruent with radii measuring 13 cm. \overline{PQ} is a chord of both circles and $PQ = 18$ cm. To the nearest tenth, find MN . Justify your answer. **See margin.**

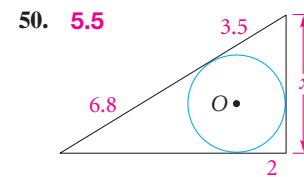
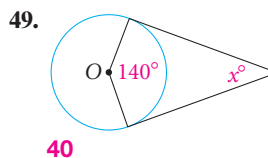


Mixed Review



Lesson 12-1

Assume that lines that appear to be tangent are tangent. O is the center of each circle. Find the value of x to the nearest tenth.



Lesson 8-5

51. From the top of a building you look down at an object on the ground. If your eyes are 50 feet above the ground and the angle of depression is 50° , how far is the object on the ground from the base of the building? **about 42 ft**

Lesson 7-5

52. The legs of a right triangle are 10 in. and 24 in. long. Find the lengths, to the nearest tenth, of the segments into which the bisector of the right angle divides the hypotenuse. **18.4 in. and 7.6 in.**

48. [2] \overline{PN} is a radius of $\odot N$. Thus, $PN = 13$ cm. $PX = \frac{1}{2}PQ = 9$, $\triangle PNX$ is a rt. \triangle , so

$NX \approx 9.38$ cm. Thus, $MN = 18.8$ cm, to the nearest tenth.

[1] incorrect length OR incorrect explanation