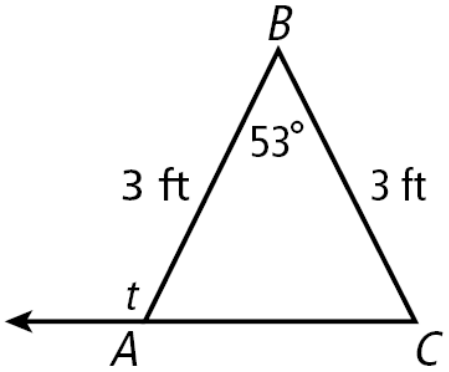
12-4 Inscribed Angles

Warm Up

Find each value.



1. m∠*BCA* 63.5°

2. *t* 116.5°

Solve for *x*.

3. $58 - x = 4(x + 7)$ 6

4. $2(x - 8) = 8$ 12

Objectives

Find the measure of an inscribed angle.

Use inscribed angles and their properties to solve problems.

Vocabulary

inscribed angle

intercepted arc

subtend

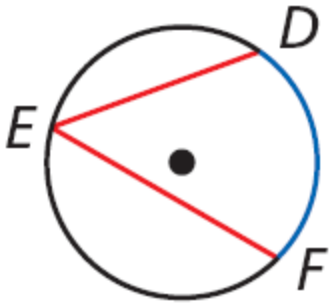
12-4 Inscribed Angles

String art often begins with pins or nails that are placed around the circumference of a circle. A long piece of string is then wound from one nail to another. The resulting pattern may include hundreds of *inscribed angles*.

12-4 Inscribed Angles

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An **intercepted arc** consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them. A chord or arc **subtends** an angle if its endpoints lie on the sides of the angle.

12-4 Inscribed Angles



$\angle DEF$ is an **inscribed angle**.

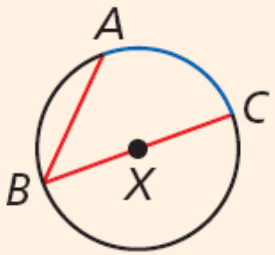
$\overset{\frown}{DF}$ is the **intercepted arc**.

$\overset{\frown}{DF}$ subtends $\angle DEF$.

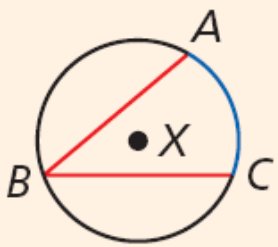
Theorem 11-4-1 **Inscribed Angle Theorem**

The measure of an inscribed angle is half the measure of its intercepted arc.

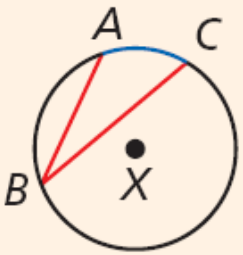
$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$$



Case 1



Case 2

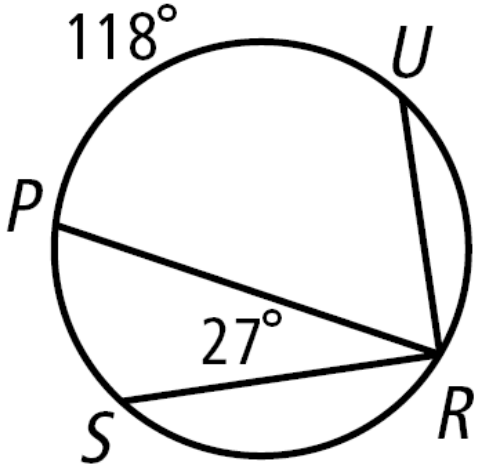


Case 3

Example 1A: Finding Measures of Arcs and Inscribed Angles

Find each measure.

m∠*PRU*

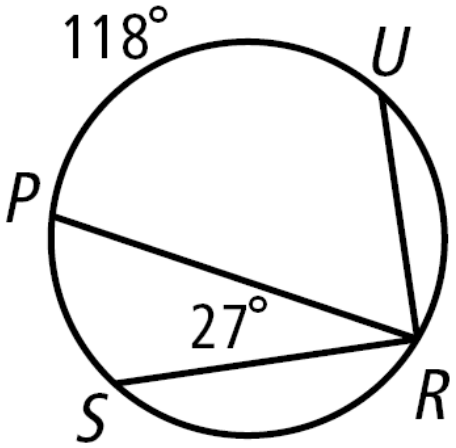


$$m\angle PRU = \frac{1}{2}m\overset{\frown}{PU}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(118°) = 59°$$ *Substitute 118 for $m\overset{\frown}{PU}$.*

Example 1B: Finding Measures of Arcs and Inscribed Angles

Find each measure.



$m\overset{\frown}{SP}$

$m\angle SRP = \frac{1}{2} m\overset{\frown}{SP}$ *Inscribed ∠ Thm.*

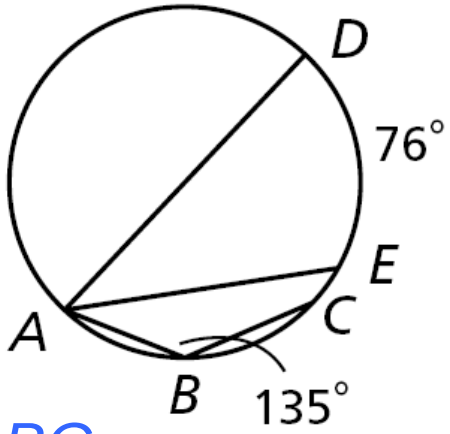
$27° = \frac{1}{2} m\overset{\frown}{SP}$ *Substitute 27 for m∠SRP.*

$m\overset{\frown}{SP} = 54°$ *Multiply both sides by 2.*

Check It Out! Example 1a

Find each measure.

$\overset{\frown}{mADC}$



$m\angle ABC = \frac{1}{2}\overset{\frown}{mADC}$ *Inscribed ∠ Thm.*

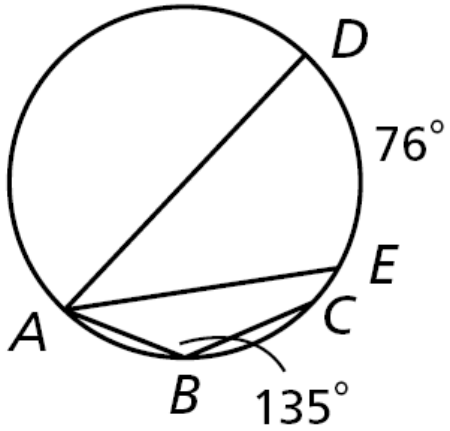
$135° = \frac{1}{2}\overset{\frown}{mADC}$ *Substitute 135 for m∠ABC.*

$270° = \overset{\frown}{mADC}$ *Multiply both sides by 2.*

Check It Out! Example 1b

Find each measure.

m∠*DAE*

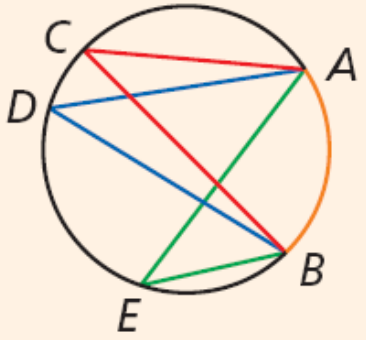


$$m\angle DAE = \frac{1}{2}m\overset{\frown}{DE}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(76°) = 38°$$ *Substitute 76 for $m\overset{\frown}{DE}$.*

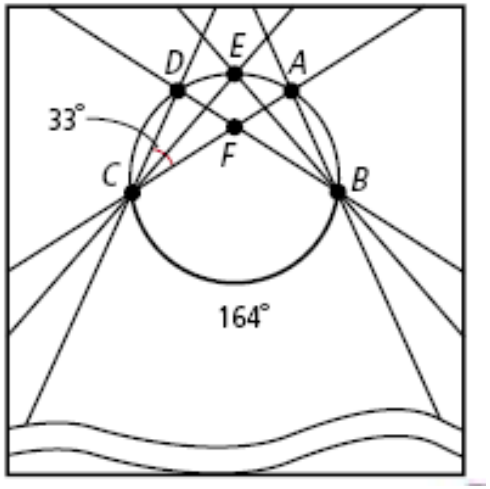
12-4 Inscribed Angles

Corollary 11-4-2

COROLLARY	HYPOTHESIS	CONCLUSION
If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent.	 $\angle ACB$, $\angle ADB$, and $\angle AEB$ intercept $\overset{\frown}{AB}$.	$\angle ACB \cong \angle ADB \cong \angle AEB$ (and $\angle CAE \cong \angle CBE$)

Example 2: Hobby Application

An art student turns in an abstract design for his art project.



Find m∠*DFA*.

$m\angle DFA = m\angle DCF + m\angle CDF$ *Ext ∠ Thm.*

$= m\angle DCF + \frac{1}{2}m\overset{\frown}{BC}$ *Inscribed ∠ Thm.*

$= 33° + \frac{1}{2}(164°)$ *Substitute.*

$= 115°$ *Simplify.*

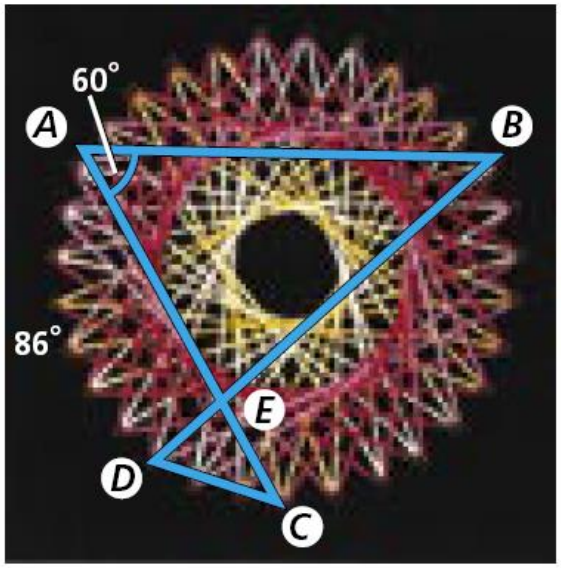
Check It Out! Example 2

Find $m\angle ABD$ and $m\overset{\frown}{BC}$ in the string art.

$$m\angle ABD = \frac{1}{2}m\overset{\frown}{DA}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(86°)$$ *Substitute.*

$$= 43°$$



$$m\angle CAB = \frac{1}{2}m\overset{\frown}{BC}$$ *Inscribed ∠ Thm.*

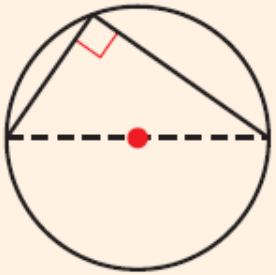
$$60° = \frac{1}{2}m\overset{\frown}{BC}$$ *Substitute.*

$$m\overset{\frown}{BC} = 120°$$

12-4 Inscribed Angles

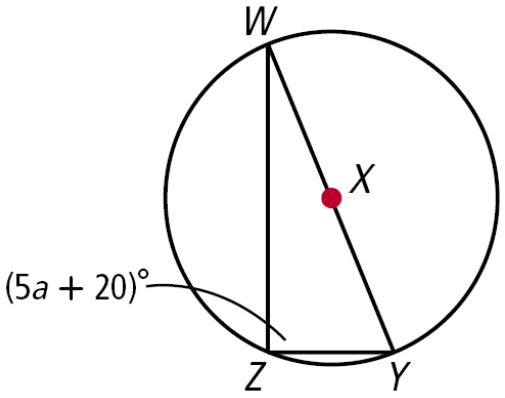
Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



Example 3A: Finding Angle Measures in Inscribed Triangles

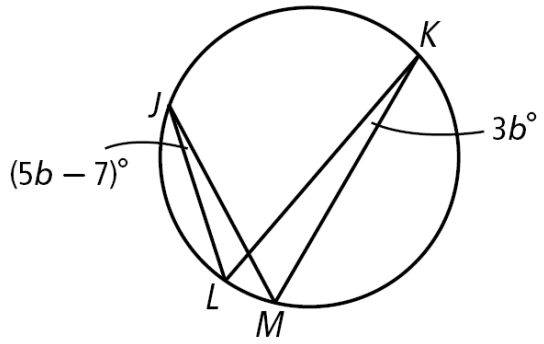
Find *a*.



$\angle WZY$ is a right angle	*$\angle WZY$ is inscribed in a semicircle.*
$m\angle WZY = 90°$	*Def of rt. $\angle$*
$5a + 20 = 90$	*Substitute 5a + 20 for $m\angle WZY$.*
$5a = 70$	*Subtract 20 from both sides.*
$a = 14$	*Divide both sides by 5.*

Example 3B: Finding Angle Measures in Inscribed Triangles

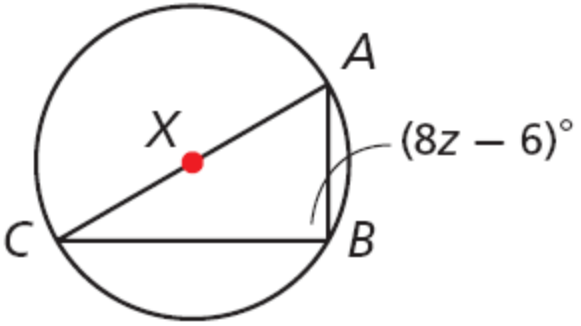
Find m∠*LJM*.



$m\angle LJM = m\angle LKM$	*m∠LJM and m∠LKM both intercept $\overset{\frown}{LM}$.*
$5b - 7 = 3b$	*Substitute the given values.*
$2b - 7 = 0$	*Subtract 3b from both sides.*
$2b = 7$	*Add 7 to both sides.*
$b = 3.5$	*Divide both sides by 2.*
$m\angle LJM = 5(3.5) - 7 = 10.5°$	*Substitute 3.5 for b.*

Check It Out! Example 3a

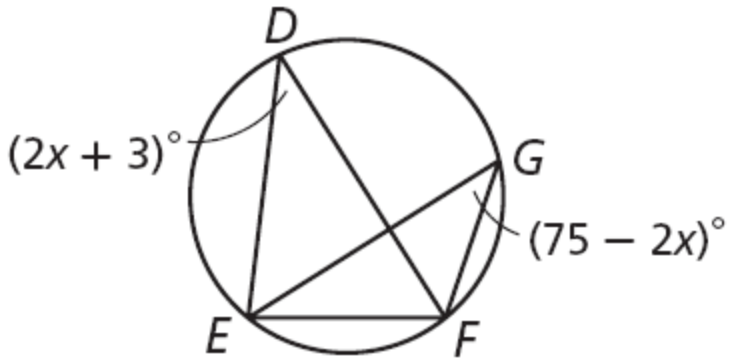
Find *z*.



$\angle ABC$ is a right angle	*$\angle ABC$ is inscribed in a semicircle.*
$m\angle ABC = 90°$	*Def of rt. $\angle$*
$8z - 6 = 90$	*Substitute.*
$8z = 96$	*Add 6 to both sides.*
$z = 12$	*Divide both sides by 8.*

Check It Out! Example 3b

Find m∠*EDF*.



$m\angle EDF = m\angle EGF$ — *m∠EGF and m∠EDF both intercept $\overset{\frown}{EF}$.*

$2x + 3 = 75 - 2x$ — *Substitute the given values.*

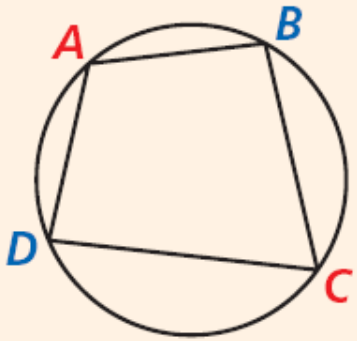
$4x = 72$ — *Add 2x and subtract 3 from both sides.*

$x = 18$ — *Divide both sides by 4.*

$m\angle EDF = 2(18) + 3 = 39°$

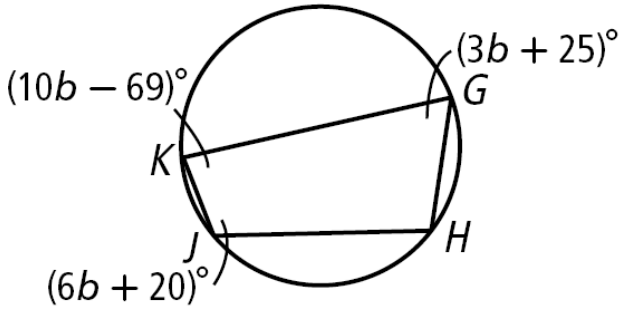
12-4 Inscribed Angles

Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 *ABCD* is inscribed in ⊙*E*.	∠*A* and ∠*C* are supplementary. ∠*B* and ∠*D* are supplementary.

Example 4: Finding Angle Measures in Inscribed Quadrilaterals

Find the angle measures of *GHJK*.



Step 1 Find the value of *b*.

$m\angle G + m\angle J = 180°$ *GHJK is inscribed in a ⊙.*

$3b + 25 + 6b + 20 = 180$ *Substitute the given values.*

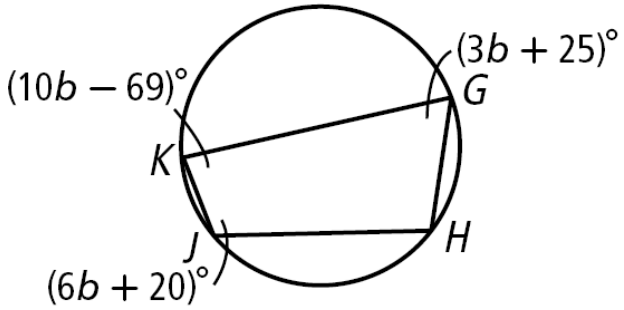
$9b + 45 = 180$ *Simplify.*

$9b = 135$ *Subtract 45 from both sides.*

$b = 15$ *Divide both sides by 9.*

Example 4 Continued

Step 2 Find the measure of each angle.



$m\angle G = 3(15) + 25 = 70°$ *Substitute 15 for b*

$m\angle J = 6(15) + 20 = 110°$ *in each expression.*

$m\angle K = 10(15) - 69 = 81°$

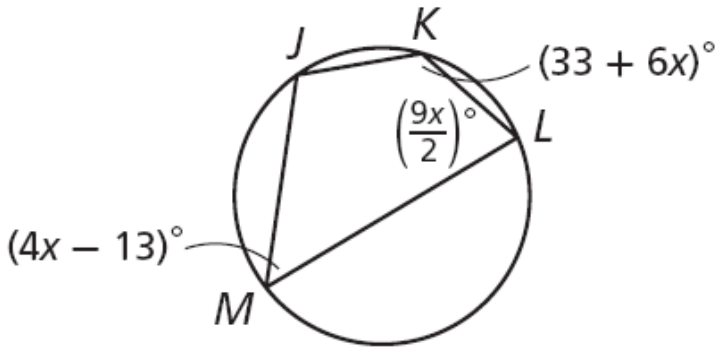
$m\angle H + m\angle K = 180°$ *$\angle H$ and $\angle K$ are supp.*

$m\angle H + 81° = 180°$ *Substitute 81 for $m\angle K$.*

$m\angle H = 99°$ *Subtract 81 from both sides*

Check It Out! Example 4

Find the angle measures of *JKLM*.



Step 1 Find the value of *b*.

$m\angle M + m\angle K = 180°$ *JKLM is inscribed in a ⊙.*

$4x - 13 + 33 + 6x = 180$ *Substitute the given values.*

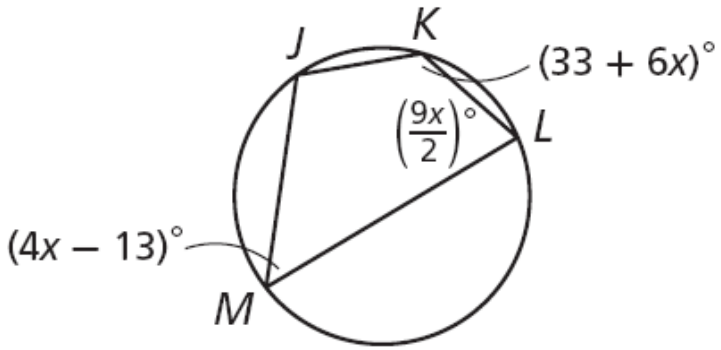
$10x + 20 = 180$ *Simplify.*

$10x = 160$ *Subtract 20 from both sides.*

$x = 16$ *Divide both sides by 10.*

Check It Out! Example 4 Continued

Find the angle measures of *JKLM*.



Step 2 Find the measure of each angle.

$m\angle M = 4(16) - 13 = 51°$

$m\angle K = 33 + 6(16) = 129°$

$$m\angle L = \frac{9(16)}{2} = 72°$$

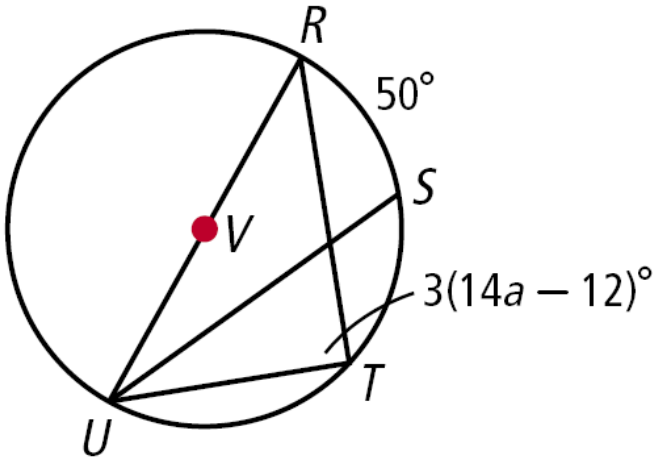
$m\angle J = 360° - 252° = 108°$

Lesson Quiz: Part I

Find each measure.

1. $\angle RUS$ 25°

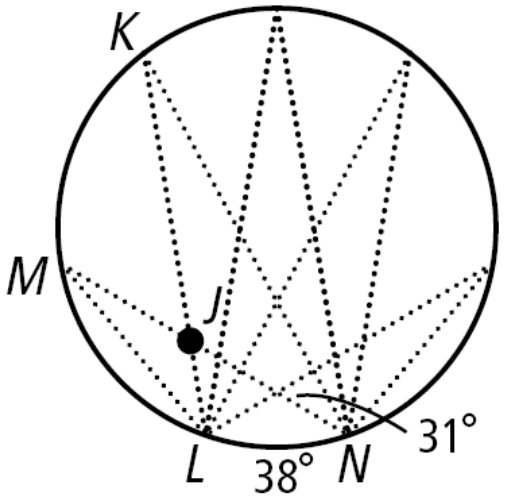
2. a 3



Lesson Quiz: Part II

3. A manufacturer designs a circular ornament with lines of glitter as shown. Find m∠*KJN*.

130°



4. Find the angle measures of *ABCD*.

m∠ *A* = 95°
m∠ *B* = 85°
m∠ *C* = 85°
m∠ *D* = 95°