

Conditional Probability



MEDICINE Danielle Jones works in a medical research laboratory where a drug that promotes hair growth in balding men is being tested. The results of the preliminary tests are shown in the table.

	Number of Subjects	
	Using Drug	Using Placebo
Hair growth	1600	1200
No hair growth	800	400

Ms. Jones needs to find the probability that a subject's hair growth was a result of using the experimental drug. *This problem will be solved in Example 1.*

The probability of an event under the condition that some preceding event has occurred is called **conditional probability**. The conditional probability that event A occurs given that event B occurs can be represented by $P(A|B)$. $P(A|B)$ is read "the probability of A given B ."

Conditional Probability

The conditional probability of event A , given event B , is defined as

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \text{ where } P(B) \neq 0.$$

Example



1 MEDICINE Refer to the application above. What is the probability that a test subject's hair grew, given that he used the experimental drug?

Let H represent hair growth and D represent experimental drug usage. We need to find $P(H|D)$.

$$P(H|D) = \frac{P(\text{used experimental drug and had hair growth})}{P(\text{used experimental drug})}$$

$$P(H|D) = \frac{\frac{1600}{4000}}{\frac{2400}{4000}} \leftarrow P(\text{used experimental drug and had hair growth}) = \frac{1600}{4000}$$

$$\leftarrow P(\text{used experimental drug}) = \frac{1600 + 800}{4000}$$

$$P(H|D) = \frac{1600}{2400} \text{ or } \frac{2}{3}$$

The probability that a subject's hair grew, given that they used the experimental drug is $\frac{2}{3}$.

Example 2 Denette tosses two coins. What is the probability that she has tossed 2 heads, given that she has tossed at least 1 head?

Let event A be that the two coins come up heads.

Let event B be that there is at least one head.

$$P(B) = \frac{3}{4} \quad \text{Three of the four outcomes have at least one head.}$$

$$P(A \text{ and } B) = \frac{1}{4} \quad \text{One of the four outcomes has two heads.}$$

$$\begin{aligned} P(A | B) &= \frac{P(A \text{ and } B)}{P(B)} \\ &= \frac{\frac{1}{4}}{\frac{3}{4}} \\ &= \frac{1}{4} \cdot \frac{4}{3} \text{ or } \frac{1}{3} \end{aligned}$$



The probability of tossing two heads, given that at least one toss was a head is $\frac{1}{3}$.

Sample spaces and reduced sample spaces can be used to help determine the outcomes that satisfy a given condition.

Example 3 Alfonso is conducting a survey of families with 3 children. If a family is selected at random, what is the probability that the family will have exactly 2 boys if the second child is a boy?

The sample space is $S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$ and includes all of the possible outcomes for a family with three children.

Determine the reduced sample spaces that satisfy the given conditions that there are exactly 2 boys and that the second child is a boy.

The condition that there are exactly 2 boys reduces the sample space to exclude the outcomes where there are 1, 3, or no boys.

Let X represent the event that there are two boys.

$$\begin{aligned} X &= \{BBG, BGB, GBB\} \\ P(X) &= \frac{3}{8} \end{aligned}$$

The condition that the second child is a boy reduces the sample space to exclude the outcomes where the second child is a girl.

Let Y represent the event that the second child is a boy.

$$\begin{aligned} Y &= \{BBB, BBG, GBB, GBG\} \\ P(Y) &= \frac{4}{8} \text{ or } \frac{1}{2} \end{aligned}$$

$(X \text{ and } Y)$ is the intersection of X and Y . $(X \text{ and } Y) = \{BBG, GBB\}$.

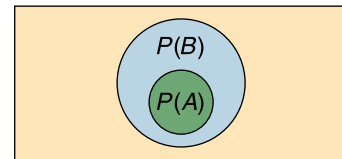
So, $P(X \text{ and } Y) = \frac{2}{8} \text{ or } \frac{1}{4}$.

(continued on the next page)

$$\begin{aligned}
 P(X|Y) &= \frac{P(X \text{ and } Y)}{P(Y)} \\
 &= \frac{\frac{1}{4}}{\frac{1}{2}} \\
 &= \frac{1}{4} \cdot \frac{2}{1} \text{ or } \frac{1}{2}
 \end{aligned}$$

The probability that a family with 3 children selected at random will have exactly 2 boys, given that the second child is a boy, is $\frac{1}{2}$.

In some situations, event A is a subset of event B . When this occurs, the probability that both event A and event B , $P(A \text{ and } B)$, occur is the same as the probability of event A occurring. Thus, in these situations $P(A|B) = \frac{P(A)}{P(B)}$.



Event A is a subset of event B.

Example 4 A 12-sided dodecahedron has the numerals 1 through 12 on its faces. The die is rolled once, and the number on the top face is recorded. What is the probability that the number is a multiple of 4 if it is known that it is even?

Let A represent the event that the number is a multiple of 4. Thus, $A = \{4, 8, 12\}$.

$$P(A) = \frac{3}{12} \text{ or } \frac{1}{4}$$

Let B represent the event that the number is even. So, $B = \{2, 4, 6, 8, 10, 12\}$.

$$P(B) = \frac{6}{12} \text{ or } \frac{1}{2}$$

In this situation, A is a subset of B .

$$P(A \text{ and } B) = P(A) = \frac{1}{4}$$

$$P(B) = \frac{1}{2}$$

$$P(A|B) = \frac{P(A)}{P(B)}$$

$$\begin{aligned}
 &= \frac{\frac{1}{4}}{\frac{1}{2}} \\
 &= \frac{1}{1} \text{ or } \frac{1}{2}
 \end{aligned}$$



The probability that a multiple of 4 is rolled, given that the number is even, is $\frac{1}{2}$.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Explain** the relationship between conditional probability and the probability of two independent events.

2. **Describe** the sample space for $P(\text{face card})$ if the card drawn is black.
3. *Math Journal* Find two real-world examples that use conditional probability. Explain how you know conditional probability is used.

Guided Practice Find each probability.

4. Two number cubes are tossed. Find the probability that the numbers showing on the cubes match given that their sum is greater than five.
5. One card is drawn from a standard deck of cards. What is the probability that it is a queen if it is known to be a face card?

Three coins are tossed. Find the probability that they all land heads up for each known condition.

6. the first coin shows a head
7. at least one coin shows a head
8. at least two coins show heads

A pair of number cubes is thrown. Find each probability given that their sum is greater than or equal to 9.

9. $P(\text{numbers match})$
10. $P(\text{sum is even})$
11. $P(\text{numbers match or sum is even})$

12. **Medicine** To test the effectiveness of a new vaccine, researchers gave 100 volunteers the conventional treatment and gave 100 other volunteers the new vaccine. The results are shown in the table below.

- a. What is the probability that the disease is prevented in a volunteer chosen at random?
- b. What is the probability that the disease is prevented in a volunteer who was given the new vaccine?
- c. What is the probability that the disease is prevented in a volunteer who was not given the new vaccine?

Treatment	Disease Prevented	Disease Not Prevented
New Vaccine	68	32
Conventional Treatment	62	38

13. **Currency** A dollar-bill changer in a snack machine was tested with 100 \$1-bills. Twenty-five of the bills were counterfeit. The results of the test are shown in the chart at the right.

Bill	Accepted	Rejected
Legal	69	6
Counterfeit	1	24

- a. What is the probability that a bill accepted by the changer is legal?
- b. What is the probability that a bill is rejected given that it is legal?
- c. What is the probability that a counterfeit bill is not rejected?



EXERCISES

Practice

Find each probability.

- Two coins are tossed. What is the probability that one coin shows heads if it is known that at least one coin is tails?
- A city council consists of six Democrats, two of whom are women, and six Republicans, four of whom are men. A member is chosen at random. If the member chosen is a man, what is the probability that he is a Democrat?
- A bag contains 4 red chips and 4 blue chips. Another bag contains 2 red chips and 6 blue chips. A chip is randomly selected from one of the bags, and found to be blue. What is the probability that the chip is from the first bag?
- Two boys and two girls are lined up at random. What is the probability that the girls are separated if a girl is at an end?
- A five-digit number is formed from the digits 1, 2, 3, 4, and 5. What is the probability that the number ends in the digits 52, given that it is even?
- Two game tiles, numbered 1 through 9, are selected at random from a box without replacement. If their sum is even, what is the probability that both numbers are odd?

A card is chosen at random from a standard deck of cards. Find each probability given that the card is black.

- | | | |
|---------------------------------|-----------------------------|-----------------------------|
| 20. $P(\text{ace})$ | 21. $P(4)$ | 22. $P(\text{face card})$ |
| 23. $P(\text{queen of hearts})$ | 24. $P(6 \text{ of clubs})$ | 25. $P(\text{jack or ten})$ |

A container holds 3 green marbles and 5 yellow marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find each probability.

- the second marble is green, given that the first marble was green
- the second marble is yellow, given that the first marble was green
- the second marble is yellow, given that the first marble was yellow



Three fish are randomly removed from an aquarium that contains a trout, a bass, a perch, a catfish, a walleye, and a salmon. Find each probability.

- $P(\text{salmon, given bass})$
- $P(\text{not walleye, given trout and perch})$
- $P(\text{bass and perch, given not catfish})$
- $P(\text{perch and trout, given neither bass nor walleye})$

In Mr. Hewson's homeroom, 60% of the students have brown hair, 30% have brown eyes, and 10% have both brown hair and eyes. A student is excused early to go to a doctor's appointment.

- If the student has brown hair, what is the probability that the student also has brown eyes?
- If the student has brown eyes, what is the probability that the student does not have brown hair?
- If the student does not have brown hair, what is the probability that the student does not have brown eyes?

In a game played with a standard deck of cards, each face card has a value of 10 points, each ace has a value of 1 point, and each number card has a value equal to its number. Two cards are drawn at random.

36. At least one card is an ace. What is the probability that the sum of the cards is 7 or less?
37. One card is the queen of diamonds. What is the probability that the sum of the cards is greater than 18?

**Applications
and Problem
Solving**



38. **Health Care** At Park Medical Center, in a sample group, there are 40 patients diagnosed with lung cancer, and 30 patients who are chronic smokers. Of these, there are 25 patients who have lung cancer and smoke.
- Draw a Venn diagram to represent the situation.
 - If the medical center currently has 200 patients, and one of them is randomly selected for a medical study, what is the probability that the patient has lung cancer, given that the patient smokes?

39. **Business** The manager of a computer software store wants to know whether people who come in and ask questions are more likely to make a purchase than the average person. A survey of 500 people exiting the store found that 250 people bought something, 120 asked questions and bought something, and 30 people asked questions but did not buy anything. Based on the survey, determine whether a person who asks questions is more likely to buy something than the average person.

40. **Critical Thinking** In a game using two number cubes, a sum of 10 has not turned up in the past few rolls. A player believes that a roll of 10 is “due” to come up. Analyze the player’s thinking.

41. **Testing** Winona’s chances of passing a precalculus exam are $\frac{4}{5}$ if she studies, and only $\frac{1}{5}$ if she decides to take it easy. She knows that $\frac{2}{3}$ of her class studied for and passed the exam. What is the probability that Winona studied for it?

42. **Manufacturing** Three computer chip companies manufacture a product that enhances the 3-D graphic capacities of computer displays. The table below shows the number of functioning and defective chips produced by each company during one day’s manufacturing cycle.

Company	Number of functioning chips	Number of defective chips
CyberChip Corp.	475	25
3-D Images, Inc.	279	21
MegaView Designs	180	20

- What is the probability that a randomly selected chip is defective?
- What is the probability that a defective chip came from 3-D Images, Inc.?
- What is the probability that a randomly selected chip is functioning?
- If you were a computer manufacturer, which company would you select to produce the most reliable graphic chip? Why?

43. **Critical Thinking** The probability of an event A is equal to the probability of the same event, given that event B has already occurred. Prove that A and B are independent events.

Mixed Review

44. **City Planning** There are 6 women and 7 men on the committee for city park enhancement. A subcommittee of five members is being selected at random to study the feasibility of redoing the landscaping in one of the parks. What is the probability that the committee will have at least three women? (Lesson 13-4)

45. Suppose there are 9 points on a circle. How many 4-sided closed figures can be formed by joining any 4 of these points? (Lesson 13-1)

46. Write $\sum_{b=1}^{\infty} 3(0.5)^b$ in expanded form. Then find the sum. (Lesson 12-5)

47. Compare and contrast the graphs of $y = 3^x$ and $y = -3^x$ (Lesson 11-2)

48. Graph the system of inequalities. (Lesson 10-8)

$$x^2 + y^2 \leq 81$$

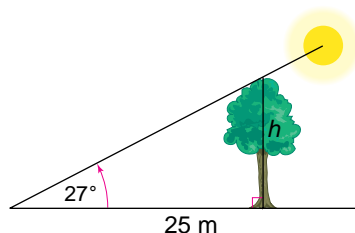
$$x^2 + y^2 \geq 64$$

49. **Navigation** A submarine sonar is tracking a ship. The path of the ship is recorded as $r \cos\left(\theta - \frac{\pi}{2}\right) + 5 = 0$. Find the linear equation of the path of the ship. (Lesson 9-4)

50. Graph the line whose parametric equations are $x = 4t$, and $y = 3 + 2t$. (Lesson 8-6)

51. Find the area of the sector of a circle of radius 8 feet, given its central angle is 98° . Round your answer to the nearest tenth. (Lesson 6-1)

52. When the angle of elevation of the sun is 27° , the shadow of a tree is 25 meters long. How tall is the tree? Round your answer to the nearest tenth. (Lesson 5-4)



53. **Photography** A photographer has a frame that is 3 feet by 4 feet. She wants to mat a group photo such that there is a uniform width of mat surrounding the photo. If the area of the photo is 6 square feet, find the width of the mat. (Lesson 4-2)

54. Find the value(s) of x at which $f(x) = \frac{5}{x^2 - 4}$ is discontinuous. Use the continuity test to justify your answer. (Lesson 3-5)

55. **SAT/ACT Practice** In parallelogram $ABCD$, the ratio of the shaded area to the unshaded area is

- A 1:2 B 1:1
 C 4:3 D 2:1
 E It cannot be determined from the information given.

