14.1 Part I: The Counting Principle

1. **Outcome:** is the result of a single trial

    Ex: Roll a die, an outcome could be a 5

2. **Sample Space:** the set of all outcomes of a trial

    Ex: The sample space of flipping a coin is { heads, tails }

    Ex: The sample space of rolling a die is { 1, 2, 3, 4, 5, 6 }

3. **Event:** consists of one or more outcomes of a trial

4. **Independent Events:** events that do not affect each other

    Ex: Rolling the first die has no effect on the second roll

Ex: A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?

3 choices of hamburger, chicken, or fish and 2 choices of plain or sesame bun

Total Combinations: 3 * 2 = 6

5. **Fundamental Counting Principle:** If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then event *M* followed by event *N* can occur in $m \bullet n$ ways.

Ex: Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of the three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

Total Combinations: 3 * 4 = 12

6. The Fundamental Counting Principle can be used to count the number of outcomes possible for any number of successive events.

Ex: For their vacation, the Murray family is choosing a trip to Atlantic Beach, Charlotte or to the mountains. They can select their transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation?

Total Combinations: 3 * 3 = 9

7. A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected?

Total Combinations: 4 * 5 = 20

8. **Dependent Events**: the outcome of one event **DOES** affect the outcome of another event.

    Ex: picking two cards: one black and the other a king (the first card could have been a black king)

Ex: Carissa wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?

Total Combinations: 6 * 5 * 4 * 3 * 2 * 1 = 720 (Do we recognize 6!)

Ex: Sarah only wants to take 4 different classes. How many different schedules could she have?

Total Combinations: 4 * 3 * 2 * 1 = 24 (Do we recognize 4!)

Ex: For a college application, Marci must select one of five topics on which to write a short essay. He must also select a different topic from the list for a longer essay. How many ways can he choose the topics for the two essays?

Total Combinations: 5 * 4 = 20

Ex: How many 3 letter, 3 digit license plates are there if the letters cannot be repeated but the numbers can?

Total Combinations: 26 * 25 * 24 * 10 * 10 * 10 = 15,600,000

Homework: Counting Principle Worksheet

14.1 Part I Counting Principle Worksheet

I. State whether the events are independent or dependent.

1. Choosing a president, vice president, secretary, and treasurer for Student Council, assuming that a person can hold only one office.

2. Selecting a fiction book and a nonfiction book at the library.

3. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.

4. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them.

II. Solve each problem.

5. Tim wants to buy one of three different albums he sees in a music store. Each is available on tape and CD. How many combinations of album and format does he have to choose?

6. A video store has 8 new releases this week. Each is available on DVD and Blue Ray. How many ways can a customer choose a new release and a format to rent?

7. Carlos has homework to do in Math, Chemistry, and English. How many ways can he choose the order in which to do his homework?

8. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, main course, and dessert be selected to form a meal?

9. A golf club manufacturer makes drivers with 4 different shafts lengths, 3 different lofts, 2 different grips, and 2 different club head materials. How many different combinations are possible?

10. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

11. How many different ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end?

12. In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last?

13. Abby is registering at a Web Site. She must select a password containing 6 numerals to be able to use the site. How many passcodes are allowed if no digits may be used more than once?

14. How many different 5 digit codes are possible if the first digit cannot be 0 and no digit may be used more than once?

15. How many different 3 letter, 4 digit license plates are there if letters can be repeated but the numbers can't?

16. How many numbers between 100 and 999, inclusive, have 7 in the tens place?

17. A coin is tossed four times. How many possible sequences of heads or tails are possible?

18. A coin is tossed 6 times and a die is rolled 3 times. How many possible outcomes are possible?

14.1 Part II Permutations and 14.2 Permutations with Repetitions & Circular Permutations Notes

1. **Permutation:** A permutation of n differenct elements is an ordering of the elements such that one element is first, one is second, one is third, and so on. **ORDER MATTERS**!!

2. Permutation is an ordered arrangement of items that occurs when
   a. No item is used more than once.
   b. The order of arrangement makes a difference

Ex: There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?

Ex: You and 19 friends have decided to form an Internet marketing consulting firm. The group needs to choose three officers-a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

3. **Permutation Formula:** ${}_nP_r = \dfrac{n!}{(n-r)!}$ or $P(n,r) = \dfrac{n!}{(n-r)!}$

4. Eight people enter the Best Pie contest. How many ways can blue, red, and green ribbons be awarded?

5. Suppose you want to rearrange the letters of the word ALGEBRA to see if you can make a different arrangement. If the two A's were not identical, the seven letters in the word could be arranged in P(7,7) or 7! ways. But since the A's are identical, what are we going to do?

6. **Permutations with Repetitions**:

Find the number arrangements for the word:

A. BANANA　　B. BASEBALL　　C. MISSISSIPPI

7. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books have to put together?

8. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books and the English books have to be put together?

9. **Circular Permutations:** If n objects are arranged in a circle, then there are $\frac{n!}{n}$ or $(n-1)!$ permutations of the n objects around the circle; they do not have a beginning or an end.

10. On the buffet there are 7 different appetizers from which to choose. The appetizers are arranged on a revolving tray. How many ways can the appetizers be organized?

11. How many ways can 8 people be seated at a square table?

12. How many ways can 5 men and 5 women be seated at a round table if they have to alternate the men and women?

If *n* objects on a circle are arranged in relation to a fixed point, then there are *n!* Permutations.

If the arrangement can be physically turned over or flipped over, the reflection of the arrangement is possible, divide by 2. (For Example, a key ring can be flipped over but a football team in a huddle cannot)

13. How many ways can 7 beads be placed on a bracelet with no clasp?

14. How many ways can 7 beads be placed on a bracelet that has a clasp?

Homework: Finish Permutation Worksheet from class, pg. 755 #5 – 34, 37 – 40, 46

Permutation Worksheet

I. Evaluate

1. P(8,2)
2. P(9,1)
3. P(7,5)
4. P(12,6)

II. Solve each problem.

5. Find the number of possible ways the winner and first, second, and third runners-up in a contest with 10 finalists can be chosen.

6. Find the number of possible ways the letters in the word "ALGEBRA" can be arranged.

7. Find the number of possible ways an algebra book, a geometry book, a chemistry book, an English book, and a health book can be put on a shelf.

8. Find the number of possible ways the letters in the word "PARALLEL" can be arranged.

9. The manager of a four-screen movie theater is deciding which of 12 available movies to show. The screens are in rooms with different seating capacities. How many ways can he show four different movies on the screens?

10. How many ways can 5 players huddle?

11. Find the number of possibilities that four different dishes can be put on a revolving tray in the middle of a table at a Chinese restaurant.

12. Find the number of ways that six quarters with designs from six different states arranged in a circle on top of your desk.

13. A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a line if the bride and groom stand in the middle?

14. A person playing a word game has the following letters in the tray: QUOUNNTAGGRA. How many 12-letter arrangements could she make to check if a single word could be formed from all the letters?

15. How many ways can 3 identical pen sets and 5 identical watches be given to 8 graduates if each person receives one item?

16. Three different hardcover books and five different paperbacks are placed on a shelf. How many ways can they be arranged if all the hardcover books are together?

17. In how many ways can 6 people stand in a ring around the player who is “it”?

18. In how many ways can 5 charms be placed on a bracelet with no clasp?

State whether the events are *independent* or *dependent*.

5. Tossing 3 coins one at a time

6. Choosing 5 numbers in a bingo game

7. Choosing color and size when ordering an item of clothing

8. Choosing a president, secretary, and treasurer for a club

State whether each statement is *true* or *false*.

9. $6! - 3! = 3!$

10. $5 * 4! = 5!$

11. $\frac{8!}{4!} = 2!$

12. $(5 - 3)! = 5! - 3!$

Find each value.

13. P(4,2)

14. P(9,1)

15. P(6,3)

16. P(5,3)

17. P(5,5)

18. P(7,4)

19. P(11,10)

20. $\frac{P(6,4)}{P(5,3)}$

22. How many ways can 7 different books be stacked on a shelf?

23. A penny, a nickel, and a dime are simultaneously. How many different ways can the coins land?

24. There are four roads from Erie to Mead, three from Mead to Titus, and four from Titus to Corry. How many different routes are there from Erie to Corry?

25. Regular license plates in Ohio have three letters followed by three digits. How many possible plates are there?

26. There are 10 students in a class that meets in a room that has 12 chairs arranged in a row. How many ways is it possible for the students to be seated?

Find the number of different ways the letters of the word *pairs* can be arranged given the following. Solve each problem.

31. The first letter must be p.

32. The first letter must be a vowel.

33. The first letter cannot be a vowel.

34. The letter r must be in the middle place.

35. Using the letters from the word equation, how many five letter patterns can be formed in which q is followed immediately by u?

36. How many five-digit whole numbers between and including 56,000 and 59,999 can beformed if no digit is repeated?

Truck license plate numbers in a certain state consist of five digits followed by two letters. Find the number of possible license plates for each situation.

37. The letter O and I cannot be used

38. The letters must be different.

39. The letter O cannot be used .

40. The five digits cannot be 00000

46. ***Sports*** The eight finalists in the men's 100-meter breaststroke at the 1992 Summer Olympics were Nick Gillingham of Great Britain, Vassili Iranov of the Unified Team, Phillip Rogers of Australia, Nelson Diebel of the United States, Norbert Rozsa of Hungary, Akira Hayashi of Japan, Dmitri Volkov of the Unified Team, and Adrian Moorehouse of Great Britain. Medals were given to the first three swimmers to finish the race.

a. How many ways could the medals be awarded in such a race?

b. If all eight swimmers finished the race, how many different orders of finish were possible?

14.3 Combinations

1. **Combination:** An arrangement of objects in which order is **NOT** important

2. **Combination Formula:** The number of combinations of n objects taken r at a time, is written C(n, r).

$$ {}_nC_r = \frac{n!}{(n-r)! \bullet r!} \text{ or } C(n,r) = \frac{n!}{(n-r)! \bullet r!} $$

Ex: There are 4 students on student council that want to be on the recycling committee. How many different groups can be chosen when the recycling committee has 3 openings?

Ex: A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they chose the two students?

Ex: Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people to go?

Ex: From a row of 8 different varieties of soup on a shelf, how many groups of 5 cans can be selected?

Ex: An ice cream shop has eight different toppings from which to choose. This week, if you buy three toppings for your sundae, you get another topping free. How many different ways can the sundae be made (with 4 toppings)?

For each of the following, explain whether the problem is one involving permutations or combinations.

a. Six candidates are running for president, chief technology officer, and director of marketing of an Internet company. The candidate with the greatest number of votes becomes the president, the second biggest vote-getter becomes chief technology officer, and the candidate who gets the third largest number of votes will be the director of marketing. How many different outcomes are possible for these three positions?

b. From the six candidates who desire to hold office in an Internet company, a three-person committee is formed to study ways of finding new investors. How many different committees could be formed?

c. How many ways can you select 6 free videos from a list of 200 videos?

d. In a race in which there are 50 runners and no ties, in how many ways can the first three finishers come in?

3. **In more complicated situations, you may need to multiply combinations and/or permutations.**

4. **Deck of Cards**
   4 suits (spades, hearts, diamonds, clubs)
   Hearts and Diamonds are red
   Spades and Clubs are black
   Each suit has a 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, and ace.

   Ex: In poker, a person is dealt 5 cards from a standard 52-card deck. The order in which you are dealt the 5 cards does not matter. How many different 5 card poker hands are possible?

   Ex: Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

   Ex: 6 cards are drawn from a standard deck of cards. How many hands consist of 2 hearts and 4 spades?

   Ex: Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades?

   Ex: From a group of 6 men and 4 women, how many committees of 2 men and 2 women can be formed?

   Ex: A bag contains 6 green, 5 yellow, and 8 white marbles. How many ways can 2 green, 1 yellow, and 4 white marbles be chosen?

   Ex: A bag contains 4 green, 3 yellow, and 6 white marbles. How many ways can 3 of one color and 3 of another color be chosen?

Homework page 766 #4-21, 26-33 ***Think on # 29

Permutations ~Worksheet 14.2

How many different ways can the letters of each word be arranged?

1. CANADA

2. ILLINI

3. ANNUALLY

4. MEMBERS

Solve.

5. A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a line if the bride and groom stand in the middle?

6. A person playing a word game has the following letters in her tray: QUONUNTAGGRA. How many 12-letter arrangements could she make to check if a single word could be formed from all the letters?

7. How many ways can 3 identical pen sets and 5 identical watches be given to 8 graduates if each person receives one item?

8. Three different hardcover books and five different paperbacks are placed on a shelf. How many ways can they be arranged if all the hardcover books are together?

9. In how many ways can 6 people stand in a ring around the player who is “it” ?

10. In how many ways can 5 charms be placed on a bracelet with no clasp?

Combinations ~Worksheet 14.3

Evaluate each expression.

1. ${}_7C_2 = C(7,2)$

2. $C(10,4)$

3. $C(8,8)$

4. $C(10,4) \cdot C(5,3)$

5. $C(12,4) \cdot C(8,3)$

6. $P(4,3) \cdot C(8,6)$

7. $P(12,3) \cdot C(2,1) \cdot C(6,5)$

8. $P(8,3) \cdot C(9,6) \cdot C(8,2)$

Solve.

9. Sally has 7 candles, each a different color. How many ways can she arrange the candles in a candelabra that holds 3 candles?

10. In how many ways can a student choose 4 books from 2 geometry, 4 geography, 5 history, and 2 physics?

11. Eight toppings for pizza are available. In how many ways can Jim choose 3 of the toppings?

12. From a list of 12 books, how many groups of 5 books can be selected?

13. How many committees of 5 students can be selected from a class of 25?

14. Leroy can afford to buy 2 of the 6 CDs he wants. How many possible combinations could he buy?

15. A box contains 12 black and 8 green marbles. In how many ways can 3 black and 2 green marbles be chosen?

16. A box contains 12 black and 8 green marbles. In how many ways can 5 marbles be chosen?

pg. 766 # 4 -21, 26 - 33

State whether each arrangement represents a *permutation* or a *combination*.

4. 10 books on a shelf

5. A subset of 12 elements contained in a set of 26

6. a hand of 7 cards from a deck of 52 cards

7. 8 people seated around a circular table

Find each value.

8. C(4,2)

9. C(12,7)

10. C(6,6)

11. C(3,2) * C(8,3)

12. C(20,15)

13. C(8,5) * C(7,3)

14. C(8,2) * C(5,1) * C(4,2)

15. P(4,2) * C(13,3) * C(13,2)

16. From a list of 10 books, how many groups of 4 books can be selected?

17. There are 85 telephones in the editorial department of Glencoe Publishing Company. How many 2-way connections can be made among the office phones?

18. How many baseball teams of 9 members can be formed from 14 players?

19. The cast of a school play requires four girls and 3 boys. They will be selected from 7 eligible girls and 9 eligible boys. How many ways can the cast be selected?

20. Suppose there are 8 points in a plane., no 3 of which are collinear. How many distinct traingles could be formed with these points as vertices?

21. Consider a deck of 52 cards.

   a. How many different 5-card hands can have 5 cards of the same suit?

   b. How many different 4-card hands can have each card from a different suit?

A bag contains 4 red, 6 white, and 9 blue marbles. How many ways can 5 marbles be selected to meet each condition?

26. All white

27. All blue

28. Exactly 2 are blue

29. 2 one color, 3 another color

From a group of 8 juniors and 10 seniors, a committee of 5 is to be formed to discuss plans for the prom. How many committees can be formed given each condition?

30. All juniors

31. 3 juniors, 2 seniors

32. 1 junior, 4 seniors

33. All seniors

AFM PreQuiz Worksheet Name______________________________________

Write each expression using factorials and then evaluate.

______________1. ${}_{10}C_{3}$ ______________2. ${}_{12}P_{5}$ ______________3. ${}_{7}C_{0}\,{}_{10}C_{9}\,{}_{4}P_{4}$

_________________4. Each question on a ten-question multiple-choice quiz has answer choices labeled A, B, C, D and E. How many different ways can a student answer the ten questions?

_________________5. How many different 3 letter, 4 digit license plates are there without a W and a 2?

_________________6. A coin is tossed eight times. How many possible sequences of heads or tails are possible?

_________________7. Six different hardcover books and four different paperbacks are placed on a shelf. How many ways can they be arranged if all the paperback books are together?

_________________8. In how many ways can 9 people sit around a campfire?

_________________9. In how many ways can 10 charms be placed on a bracelet with no clasp?

_________________10. Caid has 10 holiday candles, each a different color. How many ways can he arrange the candles in a candelabra that holds 5 candles?

_________________11. In how many ways can a student choose 4 books from 20 that are on the library shelf?

_________________12. How many committees of 5 men and 4 women can be selected from a group of 10 men and 12 women?

_________________13. From a group of 10 men and 8 women, how many committees of six contain at least 5 women?

_________________14. Seven cards are drawn from a standard deck of cards. How many hands consist of 4 hearts and 3 clubs?

_________________15. How many five card hands consist of the same suit?

_________________16. How many 9 player baseball teams can be made from a group of 18 players?

_________________17. How many different ways can the letters in “PARALLEL” be arranged?

_________________18. How many ways can seven keys be arranged on a key ring with a VIC card already on the ring?

_________________19. How many ways can 5 people be seated around a table if 2 of the people must be seated next to each other?

_________________20. Five cards are drawn from a standard deck of cards. How many hands consist of 2 diamonds and 3 spades?

Probability: To calculate the probability of an event, count the number of outcomes in the event and in the sample space. The number of outcomes in event E is denoted as n(E) and the number of outcomes in the sample space S is denoted by n(S).

$$P(E) = \frac{n(E)}{n(S)}$$

Ex: Suppose you toss a coin four times.

a. How many different equally likely outcomes are possible?

b. Find the probability of obtaining no heads.

c. Find the probability of obtaining at least one head.

d. Find the probability of obtaining exactly one head.

Ex: A standard deck of cards consist of 52 cards, with 13 cards in each of the four suits (clubs, spades, diamonds, and hearts). Clubs and spades are black cards, and diamonds and hearts are red cards. Also, jacks, queens, and kings are called face cards. What is the probability that the card selected:

a. A red card?

b. A black ace?

c. Not a black ace?

d. A diamond face card?

e. A prime number?

Multiplying Probabilities

1. **Probability of Two Independent Events:**

   If two events, A and B, are independent, then the probability of both events occurring is:

Ex: At a picnic, Julio reaches into an ice-filled cooler containing 8 regular soft drinks and 5 diet soft drinks. He removes a can, then decides he is not really thirty, and puts it back. What is the probability that Julio and the next person to reach into the cooler both randomly select a regular soft drink?

Ex: Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins are dimes?

Ex: In a board game, three dice are rolled to determine the number of moves for the players. What is the probability that the first die shows a 6, the second die shows a 6, and the third die does not?

2. Probability of Dependent Events:

If two events, A and B, are dependent, then the probability of both events occurring is:

Ex: The host of a game show is drawing chips from a bag to determine the prizes from which contestants will play. Of the 10 chips in the bag, 6 show television, 3 show vacation, and 1 shows car. If the host draws the chips at random and does not replace them, find each probability.

a. A vacation, then a car

b. Two televisions

Ex: The next week, the host of the game show draws from a bag of 20 chips, of which 11 say computer, 8 say trip, and 1 says truck. Drawing at random and without replacement, find each probabilities.

a. A computer, then a truck

b. Two trips

Ex: A die is rolled twice. Find each probability.

a. P(5, then 1)

b. P(two even numbers)

Ex: Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a diamond, a club, and another diamond in that order.

Ex: Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart, another heart, and a spade in that order.

Ex: What if the previous question, just asked for the probability of 2 hearts and a spade?

Multiplying Probabilities Worksheet

I. A die is rolled twice. Find each probability.

a. P(2, then 3)

b. P(no 6s)

c. P(two 4s)

d. P(1, then any number)

e. P(two of the same number)

f. P(two different numbers)

II. The tiles A, B, G, I, M, R, and S of a word game are placed face down. If two tiles are chosen at random, find each probability.

a. P(R, then S), if no replacement occurs

b. P(A, then M), if replacement occurs

c. P(2 consonants), if replacement occurs

d. P(2 consonants), if no replacement occurs

e. P(B, then M), if replacement occurs

f. P(selecting the same letter twice), if no replacement

III. Ashley takes her 3-year-old brother Alex into an antique shop. There are 4 statues, 2 picture frames, and 3 vases on a shelf. Alex accidentally knocks 2 items off the shelf and breaks them.
Find each probability.

a. P(breaking 2 vases)

b. P(breaking 2 statues)

c. P(breaking a picture frame, then a vase)

d. P(breaking a statue, then a picture frame)

IV. Determine whether the events are independent or dependent. Then find the probability.

1. There are 3 miniature chocolate bars and 5 peanut butter cups in a candy dish. Judie chooses 2 of them at random. What is the probability that she chooses two miniature chocolate bars?

2. A bowl contains 4 peaches and 5 apricots. Maxine randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were apricots?

3. A bag contains 7 red, 4 blue, and 6 yellow marbles. If 3 marbles are selected in succession, what is the probability of selecting blue, then yellow, then red, if replacement occurs each time?

4. Joe's wallet contains three $1 bills, four $5 bills, and two $10 bills. If he selects three bills in succession, find the probability of selecting a $10 bill, then a $5 bill, and then a $1 bill if the bills are not replaced.

5. What is the probability of getting heads each time if a coin is tossed 5 times?

V. Find each probability if 13 cards are drawn from a standard deck of cards and no replacement occurs.

a. P(all clubs)

b. P(all black)

c. P(all one suit)

d. P(no aces)

pg. 777 # 10 – 12, 13 – 29 odd

There are 5 pennies, 7 nickels, and 9 dimes in an antique coin collection. Suppose two coins are to be selected at random from the collection. Find each probability.

10. *P*(selecting 2 pennies), if no replacement occurs.

11. *P*(selecting 2 pennies), if replacement occurs.

12. *P*(selecting the same coin twice), if no replacement occurs.

Michael is helping his mother do some packing. There are 5 clocks, 5 candles, and 6 picture frames on a table. If Michael accidentally knocks two items off the table and breaks them. Find each probability.

13. *P*(breaking 2 picture frames).

15. *P*(breaking a clock, then a candle)

Two dice are tossed. Find each probability.

17. *P*(no 2's)

19. *P*(two different numbers)

21. A box contains 5 red markers, 4 black markers, and 7 blue markers. Three are selected, one after the other. Find the probability all three are different colors if:

a. no replacement occurs.

b. replacement occurs each time

For a bingo game, wooden balls numbered consecutively from 1 to 75 are placed in a box. Five balls are drawn randomly. Find each probability.

23. *P*(selecting 5 even numbers), if replacement occurs.

25. *P*(selecting 5 consecutive numbers), if no replacement occurs

A standard deck of 52 cards contains 4 suits of 13 cards each. Find each probability if 13 cards are drawn and no replacement occurs.

27. *P*(all one suit)

29. *P*(all face cards)

Adding Probabilities Notes

1. **Simple Event:** Consists of one event

2. **Compound Event:** An event that consists of two or more simple events.

3. **Mutually Exclusive Events:** Events that cannot occur at the same time

4. **Probability of Mutually Exclusive Events:** If two events, A and B, are mutually exclusive, then the probability that A or B happens is the sum of their probabilities.

Ex: Mike has a stack of 8 baseball cards, 5 basketball cards, and 6 football cards. If he selects a card at random from the stack, what is the probability that it is a baseball or football card?

Ex: Sylvia has a stack of playing cards of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a heart or a club?

Ex: There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?

Ex: The Film Club makes a list of 9 comedies and 5 adventure movies they want to see. They plan to select 5 titles at random to show this semester.

a. What is the probability that at least two of the films they select are comedies?

b. What is the probability that at least three of the films they select are adventures?

c. What is the probability that at least five of the films they select are comedies?

d. What is the probability that all of the movies selected will be comedies or all adventures?

5. **Mutually Inclusive Events:** Two events that whose outcomes may be the same (or have parts in common)

6. **Probability of Inclusive Events:** If two events, A and B, are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

Ex: What is the probability of drawing a queen or a diamond?

Ex: What is the probability of drawing a red card or a heart?

Ex: What is the probability of drawing a club or an ace?

Ex: The enrollment at Southburg High School is 1400. Suppose 550 students want to take French, 700 take Algebra, and 400 take both French and Algebra. What is the probability that a student at random takes a French or an Algebra class?

Ex: There are 2400 subscribers to an Internet service provider. Of these, 1200 own Brand A computers, 500 own Brand B, and 100 own both A and B. What is the probability that a subscriber selected at random owns Brand A or Brand B?

Homework: Adding Probabilities Worksheet

Adding Probabilities Worksheet

I. Lisa has 9 rings in her jewelry box. Five are gold and 4 are silver. If she randomly selects 3 rings to wear to a party, find each probability.

1. P (2 silver or 2 gold)
2. P (all gold or all silver)
3. P (at least 2 gold)
4. P (at least 1 silver)

II. Seven girls and six boys walk into a video store at the same time. There are five salespeople available to help them. Find the probability that the salespeople will help the given numbers of girls and boys.

5. P (4 girls or 4 boys)
6. P (3 girls or 3 boys)
7. P (all girls or all boys)
8. P (at least 3 girls)
9. P (at least 4 girls or at least 4 boys)
10. P (at least 2 boys)

III. Determine whether the events are mutually exclusive or inclusive. Then find the probability.

11. There are 3 literature books, 4 Algebra books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature book or an Algebra book?

12. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?

13. In the Math Club, 7 of the 20 girls are seniors, and 4 of 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest?

14. A card is drawn form a standard deck of cards. What is the probability of drawing an ace or a face card?

15. One tile with each letter of the alphabet is placed in a bag, and one is drawn at random. What is the probability of selecting a vowel or a letter from the word equation?

16. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3?

17. Two cards are drawn from a standard deck of cards. Find each probability.

a. P (both kings or both black)

b. P (both kings or both face cards)

c. P (both face cards or both red)

d. P (both either red or a king)

Expected Value and Fairness Notes

Expected Value:

To calculate the expected value, you multiply the probability of each outcome by the possible outcomes.

$$E(x)=x_1 p_1+x_2 p_2+.....+x_n p_n$$

1. The daily earnings of an employee who works on a commission basis are given by the following probability distribution. Find the employee's expected earnings.

x (in $)	0	25	50	75	100	125	150
P(x)	0.07	.12	.17	.14	.28	.18	.04

2. Consider the experiment of rolling two dice and adding the numbers on top of the faces. Calculate the expected value of the probability distribution.

3. If the sum of two rolled dice is 8 or more, player A wins $2; if not, you lose $1. Find the expected value of the game for you.

4. Consider a family with 3 children. Assume that births of boys and girls are equally likely. Find the expected value for the number of girls.

5. Consider a family with 5 children. Assume that births of boys and girls are equally likely. Find the expected value for the number of boys.

In a lottery, the value of a ticket is a random variable, defined to be the amount of money you win less the cost of playing.

6. Suppose that in a lottery for charity with 225 tickets, each ticket costs $1. First prize is $50, second prize $30, and third prize is $20. Then the possible values of the random variable are $49, $29, $19, and $-1.
   a. Why is one of the values negative?

   b. The probability of winning first prize is $\frac{1}{225}$. The same probability holds for second and third prizes. Find the probability of winning nothing.

   c. Find the expected value of a ticket.

7. The Island Club is holding a fund-raising raffle. Ten thousand tickets have been sold for $2 each. There will be a first prize of $3000, 3 second prizes of $1000 each, 5 third prizes of $500 each, and 20 consolation prizes of $100 each.. Letting X denote the net winnings associated with the tickets, find E(x). Interpret your result. (Note: net winnings is the amount won after the cost of the ticket)

8. In a lottery, 120 tickets are sold at $1 each. First prize is $50 and second is $20. Find the expected value of a ticket.

9. In a certain state's lottery, six numbers are randomly chosen without repetition from the numbers 1 to 40. If you correctly pick all 6 numbers, only 5 of the 6, or only 4 of the 6, then you will $1 million, $1000 or $100, respectively. What is the value of a $1 lottery ticket?

10. In the game of roulette as played in Las Vegas casinos, the wheel is divided into 38 compartments numbered 1-36, 0, and 00. One half of the numbers 1-36 are red, the other half are black, and 0 and 00 are green. Of the many types of bets that may be placed, one type involves betting on the outcome of the color of the winning number. For example, one may place a certain sum of money on red. If the winning number is red, one wins an amount equal to the bet placed and losses the bet otherwise. Find the expected value of the winnings on a $1 best placed on red.

Fairness in a Game:

For a game to be fair, no one has advantage of another player. This translates to having an expected value of 0. So to determine if something is fair, calculate the expected value.

11. Most game of chance are not fair but people participate anyway.

12. If the sum of two rolled dice is 8 or more, you win \$2; if not, you lost \$1.
    a. Show that this is not a fair game.

    b. To have a fair game, the \$2 winnings should instead be what amount?

13. Two coins are tossed. If both land heads up, then player A wins \$4 from player B. If exactly one coin lands heads up, then B wins \$1 from A. If both land tails up, then B wins \$2 from A. Is this a fair game?

14. Two dice are rolled. If the sum of the numbers showing on the dice is odd, player A wins \$1 from player B. If both dice show the same number, A wins \$3 from B. Otherwise B wins \$3 from A. Is this a fair game?

15. Mike and Bill play a card game with a standard deck of 52 cards. Mike selects a card from a well-shuffled deck and receives A dollars from Bill if the card selected is a diamond; otherwise, Mike pays Bill a dollar. Determine the value of A if the game is to be fair.

16. Latoya and Richard play a dice game with two die. Latoya rolls the die and will receive \$0.12 each time the sum of 3,4 or 5 occur, but will lose B when any other sum shows. Determine the value of B if the game is to be fair.

Homework: Worksheet on Expected Value and Fairness

Worksheet on Expected Value and Fairness

1. The number of accidents that occur at a certain intersection know as "Five Corners" on a Friday afternoon between the hours of 3 PM and 6 PM, along with the corresponding probabilities are in the table. Find the expected number of accidents during the period in question.

Accidents	0	1	2	3	4
Probabilities	.935	.03	.02	.01	.005

2. A bank has two automatic tellers at its main office and two at each of its three branches. The number of machines that break down on a given day, along with the corresponding probabilities, are in the table. Find the expected value of machines that will break down on a given day.

Machines that Break Down	0	1	2	3	4	5	6	7	8
Probability	.43	.19	.12	.09	.04	.03	.03	.02	.05

3. A box contains 3 red balls and 2 green balls. Two balls are randomly chosen without replacement. If both are green, you win \$2. If just one is green, you win \$1. Otherwise you lose \$1. What is your expected gain or loss?

4. A lottery has one \$1000 prize, five \$100 prizes, and twenty \$10 prizes. What is the expected value from buying one of the 2000 tickets sold for \$1 each?

5. Two coins are tossed. If both coins are heads or tails, player A gets 4 points. If the coins are different, player B gets 6 points.
   a. What is the expected value for player A? for player B?

   b. Is this game fair? Why or why not? If the game is not fair, how could you change the point values to make it fair?

6. Players A and B play a game in which a die is rolled and A wins 2 points from B if a 5 or 6 appears. Otherwise, B wins 1 point from A. Decide if this is a fair game.

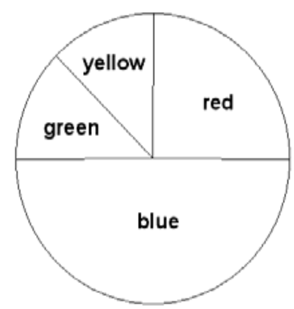
7. A box contains 2 red balls, and 1 white ball. Two balls are randomly chosen without replacement. If both are red, player A wins $5 from player B. Otherwise B wins $2 from A. Is this game fair?

8. Two dice are rolled. If the sum is 6, 7, or 8, player A wins $5 from player B. Otherwise B wins $4 from A. Is this game fair.

9. A spinner is used that is 3 red slices and 2 blue slices, where the each slice is the same size. Player A wins if the spinner lands on red. Player B wins if the spinner lands on blue. If Player B gets 6 points each time he lands on blue, how many points would Player A need to get each time he lands on red in order for this game to be fair?

10. A carnival game is designed as follows. A player spins a four color spinner like the one at the right and then rolls a dice. Prizes are awards as follows as shown in chart below. It costs $1 to play.

Event	Dollars Won
Yellow and 6	$6
Green and 1	$4
Red and even number	$2
Everything else	$0



    a. Calculate the Expected Value of the game.

    b. Is the game fair?

11. At many airports, a person can pay only $1 for a $100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays $100,000 if the insured person dies from a possible flight crash; otherwise the company gains $1 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers. How much can the company expect to gain on one policy?

Geometric Probability Notes

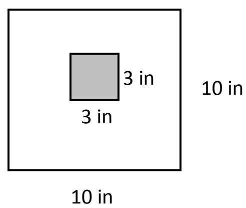
1. Instead of using algebraic formulas to solve probability problems, sometimes you can use geometric figures. The geometric figures allow you to picture the possibilities of a situation before solving the problem. You can then use geometric relationships to find the solution.

To Find Geometric Probability:

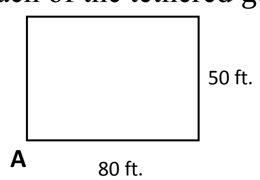
Find the area of success then divide it by the total area.

$$Probability = \frac{Area\ of\ Success}{Total\ Area}$$

Ex: Sue is throwing darts at a square game board as shown in the figure. What is the chance that she will throw a dart in the shaded area in the center of the board? (Assume that every dart thrown lands somewhere on the board.)



Ex: Moe Mentum owns a goat that will eat anything especially the tennis balls that are hit into Moe's yard from a neighboring tennis court. To keep his goat from eating a lot of tennis balls, Moe decides to tether it in a corner of a yard at Point A as shown. The tether is 20 feet long. What is the probability that a tennis ball hit into Moe's yard will be within reach of the tethered goat?



Ex: Washington, D.C. was originally laid out as a square with sides ten miles in length. On a visit to Washington, Myles Away plans to visit all the important sites such as the White House, the Smithsonian Museum, the Capitol, and the National Zoo. What is the probability that any one of these sites in within a mile of the center of Washington?

Ex: A rectangular field measures 27 feet by 15 feet. A small shed is on the field. Its dimensions are 6 feet by 5 feet. There is also an oak tree in the field whose branches form a circular canopy with a diameter of 10 feet. (Assume the shed is not under the tree.)

a. What is the probability that a single drop of rain that lands in the field would hit the shed?

b. What is the probability that a single drop of rain that lands in the field would *not* hit the shed?

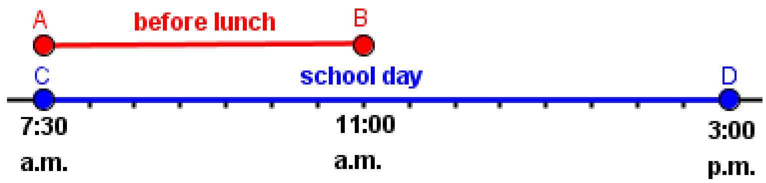
c. What is the probability that a single drop of rain that lands in the field would miss both the shed and the tree?

2. Some problems involve linear information. The best way to approach these types of problems is to draw the picuture with the given information then use

$$Probability = \frac{Length\ of\ line\ that\ represents\ success}{Total\ Length}$$

Ex: Suppose that your school day begins at 7:30 a.m. and ends at 3:00 p.m. You eat lunch at 11:00 A.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch?

You can use line segments to model the probability.



Ex: Holly Mackerel and Patty Cake are driving from New York City to Washington, D.C., a distance of about 300 miles. Their car has a broken gas gauge, but Holly knows her car's gas tank holds exactly enough gas to make the trip without having to stop for gas. Unfortunately, they hit bad weather, which causes traffic delays, and they run out of gas. What is the probability that they will be within 50 miles of Washington when they run out of gas?

Ex: Buses arrive at a resort hotel every 15 minutes. They wait for three minutes while passengers get on and get off, and then the buses depart. What is the probability that there is a bus waiting when a hotel guest walks out of the door at a randomly chosen time?

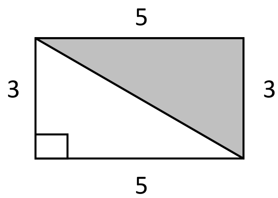
Ex: You are visiting San Francisco and are taking a trolley ride to a store on Market Street. You are supposed to meet a friend at the store at 3:00 p.m. The trolleys run every 10 minutes and the trip to the store is 8 minutes. You arrive at the trolley stop at 2:48 p.m. What is the probability that you will arrive at the store by 3:00 p.m.?

Ex: Dwayne Pipe is driving one car in a line of cars, with about 150 feet between successive cars. Each car is 13 feet long. At the next overpass, there is a large icicle. The icicle is about to crash down onto the highway. If the icicle lands on or within 30 feet of the front of a car, it will cause an accident. What is the chance that the icicle will cause an accident?

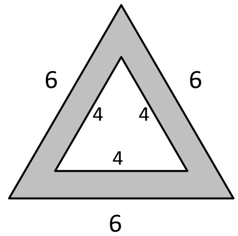
AFM – Geometric Probability Worksheet

Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

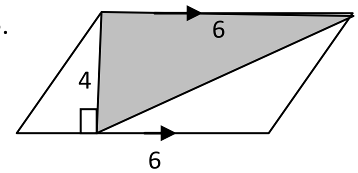
1.



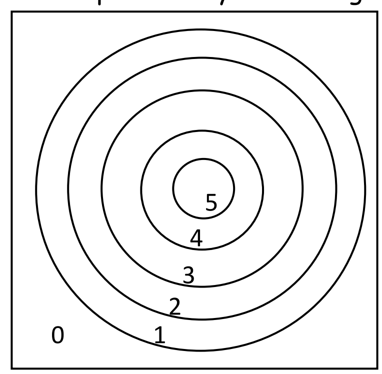
2.



3.



The dart board shown has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm , and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points.



4. 0 points

5. 1 point

6. 2 points

7. 3 points

8. 4 points

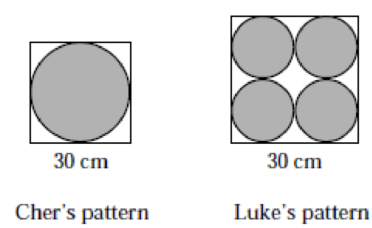
9. 5 points

10. Moe Mentum's goat is at it again. This time Moe decided to tether the goat midway along the longer side of the yard. (The yard is 80 ft. x 50 ft. and the tether is 20 ft.)

a) What is the probability that any tennis balls which are hit into Moe's yard will be within the goat's reach?

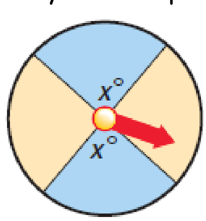
b) What is the probability of the goat eating the ball if Moe shortens the length of the goat's tether to only 10 ft.?

11. Lauren Order and Luke Warm are designing a dart board for their school
The diagrams show the pattern each student prefers for the dart board design. Which design has the greater probability for a contestant to thr
dart in the shaded area?



12. Find the value of x so that the probability of the spinner landing on a blue sector is the value given.

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$



13. That state of Connecticut is approximated by a rectangle 100 mi by 50 mi. Hartford is approximately at the center of the state. If a meteor hit earth within 200 mi of Hartford, find the probability that the meteor landed in Connecticut.

14. A stop light at an intersection stays red for 60 second, changes to green for 45 seconds, and then yellow for 15 seconds. If Joel arrives at the intersection at a random time, what is the probability that he will have to wait at a red light for more than 15 seconds?

15. A washing machine has the following cycle: soak for 20 minutes, wash for 30minutes, and rinse for 15 minutes. If you approached the machine at a random time when it is running,what is the probability that the machine was rinsing when you arrived?

16. You are expecting a call from a friend anytime between 6:00 P.M. and 7:00 P.M. Unexpectedly, you have to run an errand for a relative and are gone from 5:45 P.M. until 6:10 P.M. What is the probability that you missed your friend's call?

17. The midpoint of $\overline{JK}$ is M. What is the probability that a randomly selected point on $\overline{JK}$ is closer to M than to J or K?

18. Belle Tower brought her camera film from a recent trip to Washington ,D.C., to the Someday My Prints Will Come Photography Shop. Unfortunately, the shop ruined 4 photos in a row from Belle's 24-exposure roll. What is the probability that the ruined photos included the eighth, ninth, or tenth photos on the roll (the photos of the White House)?

19. Otto Mation has bought a copy of the new tape "Face Burn" by Cement. The A side of the tape is 30 minutes long and contains his favorite song "Homework Blues." The song "Homework Blues" lasts 4 minutes. Alas, Otto's sister uses the A side of the tape to record her favorite song "Lunch Time Laughs" at the beginning of the tape. "Lunch Time Laughs" lasts 8 minutes. What is the probability that all of Otto's favorite song is still on side A of the tape? (Hint: What is the feasible region for the start of "Homework Blues"?)

14.8 Binomial Probability

1. Conditions of a Binomial Experiment
   A binomial experiment exists if and only if these conditions occur:
   - The experiment consists of n identical trials.
   - Each trial results in one of two possible outcomes.
   - The trials are independent.

2. Determine if each situation is binomial or not.

   a. A fair coin is tossed 10 times and “heads” or “tails” is recorded each time.

   b. A pair of fair dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

   c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded.

   d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded.

3. Binomial Theorem for Probability Formula:

$$\boxed{P(k\ successes) = {}_nC_k\, p^k\, q^{n-k}}$$

where p is the probability of success in one trial and q is the probability of failure.

4. While pitching for the Toronto Blue Jays, 4 of every 7 pitches Fireball Roberts threw in the first 5 innings were strikes. What is the probability that in the next inning, Fireball Roberts will throw exactly one strike out of his first five pitches?

5. If there are 10 true/false questions on a quiz, what is the probability that exactly 8 answers are correct?

6. If 6 coins are tossed, what is the probability of each?

   a. 3 heads and 3 tails   b. at least 4 heads   c. 2 heads or 3 tails   d. all heads or all tails

7. The probability of Chris making a free throw is 60%. If she shoots five times, what is the probability of each?

   a. all missed   b. all made   c. exactly 4 made   d. at least 3 made

8. If you have 6 blue marbles, 4 red marbles, and 5 yellow marbles. Five marbles are being selected. Tell whether each situation is binomial, and if so, find the probability.

   a. P(2 blue); with replacement   b. P(2 red); without replacement   c. P(1 red, 1 yellow); w/ replacement

9. The recovery rate for a certain cancer patient is 76%. If 12 men were afflicted by the disease, what is the probability that exactly 9 will recover?

10. A manufacturing company has a machine that averages one faulty unit for every 1000 it produces. What is the probability that an order of 200 units will have one or more faulty units?

Using the Calculator to do Binomial Theorem:

Find the probability of 2 successes out of 5 tries if the probability of a success is .3.
Go to 2^{nd} VARS Scroll down to A (binompdf). Enter (5,.3,2). The syntax is binompdf(n, p, x-value)

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Determine whether each situation represents a binomial experiment. Solve the binomial experiments.

3. What is the probability of 2 heads and 2 tails if Jud tosses a coin 4 times?

4. What is the probability of Buddy drawing 4 kings from a deck of cards for each condition?

   a) He replaces the card each time   b) He does not replace the card

5. There are 8 pennies, 4 nickels, and 6 dimes in an antique coin collection. Two coins are selected with replacement after the first selection. Find each probability.

   a) P(both pennies)   b) P(both nickels)   c) P(both dimes)

   d) P(1 penny, 1 dime)   e) P(1 penny, 1 nickel)   f) P(1 nickel, 1 dime)

Find each probability if a coin is tossed three times.

6. P(all heads)   7. P(exactly 2 tails)   8. P(at least 2 heads)

Find each probability if a die is tossed five times.

9. P(only one 4)   10. P(at least three 4s)   11. P(no more than two 4s)   12. P(exactly five 4s)

Reena carries tubes of lipstick in a bag in her purse. The probability of pulling out the color she wants is $\frac{2}{3}$. Suppose she uses her lipstick 4 times a day. Find each probability.

13. P(never the correct color)   14. P(correct at least 3 times)

15. P(no more than 3 times correct)   16. P(correct exactly 2 times)

Megan guesses at all 10 true/false questions on her psychology test. Find each probability.

17. P(7 correct)   18. P(all incorrect)   19. P(at least 6 correct)   20. P(at least half correct)

Tori plays for the Worthington Wolves softball team. She is now batting .200. Find each probability for the next five times she goes to bat.

21. P(exactly 1 hit)   22. P(exactly 3 hits)   23. P(at least 4 hits)

Find each probability if three coins are tossed.

24. P(3 heads)   25. P(3 tails)   26. P(at least 2 heads)   27. P(exactly 2 tails)

The probability of a tack landing point up is $\frac{2}{5}$. Find each probability if 10 tacks have been dropped.

28. P(all point up)   29. P(exactly 3 point up)   30. P(exactly 5 point up)   31. P(at least 6 point up)

AFM Unit 7 Probability Review

1. How many subcommittees of 2 Democrats and 3 Republicans can be formed from a committee whose membership is 6 Democrats and 8 Republicans?

2. In how many ways can 5 people be seated in a room containing 2 chairs?

3. A quality control engineer must inspect a sample of 3 fuses from a box of 100. How many different samples can he choose?

4. How many numbers of three or fewer digits can be formed from the digits 2, 3, 4, 5, and 6? Assume there is no repetition of digits.

5. How many three-digit numbers can be formed from the digits 2, 3, 4, 5, & 6 if repetitions are allowed?

6. A witness to a holdup reports that the license of the getaway car consisted of 6 different digits. He remembers the first three but has forgotten the rest. How many licenses do the police have to check?

7. In how many ways can the letters from the word ***television*** be arranged?

8. How many ways can 8 people be seated at a round table?

9. A clown has 8 balloons, each a different color. There are 6 children. How many ways can the clown give each child a balloon?

10. How many 9-member baseball teams can be formed from 15 players if only 3 pitch while the others play the remaining 8 positions?

11. A photographer is taking a picture of a bride and a groom together with 6 attendants. How many ways can he arranged the 8 people in a line if the bride and groom stand in the middle?

12. Two dice are rolled. What is the probability that their sum is 6 or 8?

13. A bag contains 4 red balls and 2 white balls. If two different balls are selected are random, what is the probability of getting: (a) both red; (b) one of each color?

14. A committee of 5 is selected from a group of 9 people (6 women and 3 men). What is the probability that it will have exactly 3 women and 2 men?

15. Find the probability of drawing two aces from a deck of cards if the first card is not replaced before the second is drawn.

16. A box contains 10 red, 8 green, and 12 blue tickets. Two successive tickets are drawn without replacement. Find the probability of drawing (without regard to order):
    a) one blue and one green ticket
    b) two red tickets
    c) no blue ticket

17. Solve using the binomial probability theorem: What is the probability of getting exactly 2 “fives” in 4 rolls of a die?

18. A coin is tossed 4 times. Find the probability of getting the same number of heads and tails.

19. A coin is flipped eight times. Find the probability of getting exactly six heads.

20. Assume that ¾ of all drivers use seat belts on long trips. If 4 cars are checked on the highway, find the probability that seat belts are used by **at most** two drivers.

21. There are 6 women and 7 men on the committee for city park enhancement. A subcommittee of 5 members is being selected at random to study the feasibility of redoing the landscaping in one of the parks. What is the probability that the committee will have at least 3 women?

22. In his pocket, Ben has 5 dimes, 6 nickels, and 4 pennies. He selects 3 coins. What is the probability that he selects exactly 2 dimes and 1 penny?

23. How many ways can 8 charms be arranged on a bracelet **with no clasp** if 3 of the charms are identical?

24. A college library has three math books, 4 social science books and 3 biology books displayed on a shelf. In how many ways can the 10 books be arranged on the shelf if books on the same subject matter are together?

25. One card is drawn at random from a standard deck. What is the probability of drawing an ace or a red card?

26. Determine if the following is a fair game: Two dice are rolled. If the sum is less than 7, then player A wins \$5 from player B; otherwise, B wins \$4 from A.

27. Kay Paso, who is 3 years old, tears the labels off all 10 of the soup cans on her mother's shelf. Her mother knows that there were 2 cans of tomato soup and 8 cans of vegetable soup. She selects 4 cans at random. What is the probability that exactly one of the cans is tomato?

28. A coin is tossed three times. What is the probability that not all three tosses are the same?
    (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

29. Five cards are dealt from a deck of 52 cards. Which of the following shows the probability that 4 aces will be dealt?
    (a) $\frac{{}_4C_4}{{}_{12}C_5}$ (b) $\frac{{}_{52}C_4}{{}_{52}C_5}$ (c) $\frac{({}_4C_4)({}_{48}C_1)}{{}_{52}C_5}$ (d) $\frac{({}_4C_4)({}_{52}C_1)}{{}_{52}C_5}$