PURE MATHEMATICS
A level Practice Papers

PAPER P
MARK SCHEME

1		
	Recognises that two subsequent values will divide to give an equal ratio and sets up an appropriate equation. $\dfrac{2k^2}{4k} = \dfrac{4k}{k+2}$	**M1**
	Makes an attempt to solve the equation. For example, $2k^3 + 4k^2 = 16k^2$ or $2k^3 - 12k^2 = 0$	**M1**
	Factorises to get $2k^2(k-6) = 0$	**M1**
	States the correct solution: $k = 6$. $k \neq 0$ or $k = 0$ is trivial may also be seen, but is not required.	**A1**
	TOTAL: 4 marks	

2		
	Recognises the need to use the chain rule to find $\dfrac{dV}{dt}$ For example $\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dS} \times \dfrac{dS}{dt}$ is seen.	**M1**
	Finds $\dfrac{dV}{dr} = 4\pi r^2$ and $\dfrac{dS}{dr} = 8\pi r$	**M1**
	Makes an attempt to substitute known values. For example, $\dfrac{dV}{dt} = \dfrac{4\pi r^2}{1} \times \dfrac{1}{8\pi r} \times \dfrac{-12}{1}$	**M1**
	Simplifies and states $\dfrac{dV}{dt} = -6r$	**A1**
	TOTAL: 4 marks	

3		
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	**M1**
	States $\sin t = \dfrac{y}{2}$ or $\sin^2 t = \dfrac{y^2}{4}$ Also states $\cos^2 t = \dfrac{1}{x-1}$	**M1**
	Substitutes $\sin^2 t = \dfrac{y^2}{4}$ and $\cos^2 t = \dfrac{1}{x-1}$ into $\sin^2 t + \cos^2 t \equiv 1$ $\dfrac{y^2}{4} + \dfrac{1}{x-1} = 1 \Rightarrow \dfrac{y^2}{4} = \dfrac{x-2}{x-1}$	**M1**
	Solves to find $y = \sqrt{\dfrac{4x-8}{x-1}}$, accept $y = \sqrt{\dfrac{8-4x}{1-x}}$, $x < 1$ or x Ö 2	**A1**
	TOTAL: 4 marks	

4		
States that: $A(2x+5)+B(5x-1)\equiv 6x+42$		**M1**
Equates the various terms. Equating the coefficients of x: $2A+5B=6$ Equating constant terms: $5A-B=42$		**M1***
Multiplies both of the equations in an effort to equate one of the two variables.		**M1***
Finds $A=8$		**A1**
Find $B=-2$		**A1**
TOTAL: 5 marks		

Alternative method

Uses the substitution method, having first obtained this equation: $A(2x+5)+B(5x-1)\equiv 6x+42$

Substitutes $x=-\frac{5}{2}$ to obtain $-\frac{27}{2}B=27$ (**M1**)

Substitutes $x=\frac{1}{5}$ to obtain $\frac{27}{5}A=43.2$ (**M1**)

5		
Differentiates 4^x to obtain $4^x \ln 4$		**M1**
Differentiates $2xy$ to obtain $2x\frac{dy}{dx}+2y$		**M1**
Rearranges $4^x \ln 4 = 2x\frac{dy}{dx}+2y$ to obtain $\frac{dy}{dx}=\frac{4^x \ln 4 - 2y}{2x}$		**A1**
Makes an attempt to substitute (2, 4)		**M1**
States fully correct final answer: $4\ln 4 - 2$	Accept $\ln 256 - 2$	**A1**
TOTAL: 5 marks		

6	Equating the coefficients of x^4: $A = 5$	**A1**
	Equating the coefficients of x^3: $B = -4$	**A1**
	Equating the coefficients of x^2: $2A + C = 17$, $C = 7$	**A1**
	Equating the coefficients of x: $2B + D = -5$, $D = 3$	**A1**
	Equating constant terms: $2C + E = 7$, $E = -7$	**A1**
	TOTAL: 5 marks	

7a	Correctly states that $(1+ax)^{-2} = 1 + (-2)(ax) + \frac{(-2)(-3)(ax)^2}{2} + \frac{(-2)(-3)(-4)(ax)^3}{6} + \ldots$	**M1**
	Simplifies to obtain $(1+ax)^{-2} = 1 - 2ax + 3a^2x^2 - 4a^3x^3 \ldots$	**M1**
	Deduces that $3a^2 = 75$	**M1**
	Solves to find $a = \pm 5$	**A1**
		(4 marks)
7b	$a = 5 \Rightarrow -4(125)x^3 = -500x^3$. Award mark for –500 seen.	**A1**
	$a = -5 \Rightarrow -4(-125)x^3 = 500x^3$. Award mark for 500 seen.	**A1**
		(2 marks)
	TOTAL: 6 marks	

8a	States that $x_2 = 4p - 9$	**A1**
	Attempts to substitute x_2 into x_3. $x_3 = p(4p-9)-9$ and simplifies to find $x_3 = 4p^2 - 9p - 9$	**A1**
		(2 marks)
8b	States $4p^2 - 9p - 9 = 46$ or $4p^2 - 9p - 55 = 0$	**M1**
	Factorises to get $(4p+11)(p-5)=0$	**M1**
	States $p = 5$. May also state that $p \neq -\frac{11}{4}$, but mark can be awarded without that being seen.	**A1**
		(3 marks)
8c	$x_4 = 5(46) - 9 = 221$ $x_5 = 5(221) - 9 = 1096$	**A1 ft**
		(1 mark)
	TOTAL: 6 marks	

NOTES: 8c: Award mark for a correct answer using their value of *p* from part **b**.

9a	States $5^2 + 6^2 + (k-10)^2 = (5\sqrt{5})^2$	**M1**
	Makes an attempt to solve the equation. For example, $(k-10)^2 = 64$ is seen.	**M1**
	States $k = 2$ and $k = 18$	**A1**
		(3 marks)
9b	Finds the vector $\overrightarrow{OA} = (-1, 7, 18)$	**M1 ft**
	Finds $\lvert\overrightarrow{OA}\rvert = \sqrt{(-1)^2 + (7)^2 + (18)^2} = \sqrt{374}$	**M1 ft**
	States the unit vector $\frac{1}{\sqrt{374}}(-\mathbf{i} + 7\mathbf{j} + 18\mathbf{k})$	**A1 ft**
		(3 marks)
	TOTAL: 6 marks	

NOTES: 9b

Award ft marks for a correct answer to part **b** using their incorrect answer from part **a.**

10a	Makes an attempt to find $\text{fg}(x)$. For example, writing $\text{fg}(x) = e^{2\ln(x+1)} + 4$	**M1**
	Uses the law of logarithms to write $\text{fg}(x) = e^{\ln(x+1)^2} + 4$	**M1**
	States that $\text{fg}(x) = (x+1)^2 + 4$	**A1**
	States that the range is $y > 4$ or $\text{fg}(x) > 4$	**B1**
		(4 marks)
10b	States that $(x+1)^2 + 4 = 85$	**M1**
	Makes an attempt to solve for x, including attempting to take the square root of both sides of the equation. For example, $x + 1 = \pm 9$	**M1**
	States that $x = 8$. Does not need to state that $x \neq -10$, but do not award the mark if $x = -10$ is stated.	**A1**
		(3 marks)
	TOTAL: 7 marks	

11a	Rearranges $x^4 - 8x^2 + 2 = 0$ to find $x^2 = \frac{x^4+2}{8}$	**M1**
	States $x = \sqrt{\frac{x^4+2}{8}}$ and therefore $a = \frac{1}{8}$ and $b = \frac{1}{4}$ or states $x = \sqrt{\frac{1}{8}x^4 + \frac{1}{4}}$	**A1**
		(2 marks
	Attempts to use iterative procedure to find subsequent values.	**M1**
11b	Correctly finds: $x_1 = 0.9396$ $x_2 = 0.5894$ $x_3 = 0.5149$ $x_4 = 0.5087$	**A1**
		(2 marks)
11c	Demonstrates an understanding that the two values of f(x) to be calculated are for $x = -2.7815$ and $x = -2.7825$.	**M1***
	Finds $\text{f}(-2.7815) = -0.0367...$ and $\text{f}(-2.7825) = (+)0.00485...$	**M1**
	Change of sign and continuous function in the interval $[-2.7825, -2.7815] \Rightarrow$ root	**A1**
		(3 marks)
	TOTAL: 7 marks	

NOTES:

11b

Award M1 if finds at least one correct answer.

11c

Any two numbers that produce a change of sign, where one is greater than –2.782 and one is less than –2.782, and both numbers round to –2.782 to 3 decimal places, are acceptable.

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

12a	Recognises the need to write $\tan^4 x \equiv \tan^2 x \tan^2 x$	**M1**
	Recognises the need to write $\tan^2 x \tan^2 x \equiv \left(\sec^2 x - 1\right)\tan^2 x$	**M1**
	Multiplies out the bracket and makes a further substitution $\left(\sec^2 x - 1\right)\tan^2 x \equiv \sec^2 x \tan^2 x - \tan^2 x \equiv \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$	**M1**
	States the fully correct final answer $\sec^2 x \tan^2 x + 1 - \sec^2 x$	**A1**
		(4 marks)
12b	States or implies that $\int \sec^2 x \, dx = \tan x$	**M1**
	States fully correct integral $\int \tan^4 x \, dx = \frac{1}{3}\tan^3 x + x - \tan x + C$	**M1**
	Makes an attempt to substitute the limits. For example, $\left[\frac{1}{3}\tan^3 x + x - \tan x\right]_0^{\frac{\pi}{4}} = \left(\frac{1}{3}\left(\tan\frac{\pi}{4}\right)^3 + \frac{\pi}{4} - \tan\frac{\pi}{4}\right) - (0)$ is seen.	**M1 ft**
	Begins to simplify the expression $\frac{1}{3} + \frac{\pi}{4} - 1$	**M1 ft**
	States the correct final answer $\frac{3\pi - 8}{12}$	**A1 ft**
		(5 marks)
	TOTAL: 9 marks	

NOTES:

12b

Student does not need to state '+C' to be awarded the second method mark.

12b

Award ft marks for a correct answer using an incorrect initial answer.

13a	Begins the proof by assuming the opposite is true. 'Assumption: there exists a number n such that n^2 is even and n is odd.'	**B1**
	Defines an odd number (choice of variable is not important) and successfully calculates n^2 Let $2k + 1$ be an odd number. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$	**M1**
	Factors the expression and concludes that this number must be odd. $4k^2 + 4k + 1 = 2\left(2k^2 + 2k\right) + 1$, so n^2 is odd.	**M1**
	Makes a valid conclusion. This contradicts the assumption n^2 is even. Therefore if n^2 is even, n must be even.	**B1**
		(4 marks)
13b	Begins the proof by assuming the opposite is true. 'Assumption: $\sqrt{2}$ is a rational number.'	**B1**
	Defines the rational number: $\sqrt{2} = \frac{a}{b}$ for some integers a and b, where a and b have no common factors.	**M1**
	Squares both sides and concludes that a is even: $\sqrt{2} = \frac{a}{b} \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$ From part **a**: a^2 is even implies that a is even.	**M1**
	Further states that if a is even, then $a = 2c$. Choice of variable is not important.	**M1**
	Makes a substitution and works through to find $b^2 = 2c^2$, concluding that b is also even. $a^2 = 2b^2 \Rightarrow (2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2$ From part **a**: b^2 is even implies that b is even.	**M1**
	Makes a valid conclusion. If a and b are even, then they have a common factor of 2, which contradicts the statement that a and b have no common factors. Therefore $\sqrt{2}$ is an irrational number.	**B1**
		(6 marks)
	TOTAL: 10 marks	

14a	States $\frac{dV}{dt} = -kV$	**M1**
	Separates the variables $\int \frac{1}{V}\,dV = \int -k\,dt$	**M1**
	Finds $\ln V = -kt + C$	**A1**
	Shows clearly progression to state $V = V_0 e^{-kt}$ For example, $V = e^{-kt+C} = e^{-kt}e^{C}$ is seen. May also explain the $V_0 = e^{C}$ where e^{C} is a constant.	**A1**
		(4 marks)
14b	States $\frac{1}{5}V_0 = V_0 e^{-kt}$	**M1**
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln\frac{1}{5} = -kt$	**M1**
	States that $k = -\frac{1}{10}\ln\frac{1}{5}$	**A1**
		(3 marks)
14c	States $\frac{1}{20}V_0 = V_0 e^{-kt}$	**M1**
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln\frac{1}{20} = t\left(\frac{1}{10}\ln\frac{1}{5}\right)$	**M1**
	Finds $t = 18.613\ldots$ years. Accept 18.6 years.	**A1**
		(3 marks)
	TOTAL: 10 marks	

15a	States: $R\cos(\theta+\alpha)\equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ Or: $5\cos\theta - 8\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	**M1**
	Deduces that: $5 = R\cos\alpha$ $8 = R\sin\alpha$	**M1**
	States that $R=\sqrt{89}$ Use of $\sin^2\theta+\cos^2\theta=1$ might be seen, but is not necessary to award the mark.	**A1**
	Finds that $\alpha = 1.0122$ $\tan\alpha = \frac{8}{5}$ might be seen, but is not necessary to award the mark.	**A1**
		(4 marks)
15b	Uses the maths from part **a** to deduce that $T_{max} = 1100+\sqrt{89} = 1109.43°C$	**A1**
	Recognises that the maximum temperature occurs when $\cos\left(\frac{x}{3}+1.0122\right)=1$	**M1**
	Solves this equation to find $\frac{x}{3} = 2\pi - 1.0122$	**M1**
	Finds x = 15.81 hours	**A1**
		(4 marks)
15c	Deduces that $1097 = 1100+\sqrt{89}\cos\left(\frac{x}{3}+1.0122\right)$	**M1**
	Begins to solve the equation. For example, $\cos\left(\frac{x}{3}+1.0122\right) = -\frac{3}{\sqrt{89}}$ is seen.	**M1**
	States that $\frac{x}{3}+1.0122 = 1.8944,\ 2\pi-1.8944,\ 2\pi+1.8944$ Further values may be seen, but are not necessary in order to award the mark.	**M1**
	Finds that x = 2.65 hours, 10.13 hours, 21.50 hours	**A1**
		(4 marks)
	TOTAL: 12 marks	

(TOTAL: 100 MARKS)