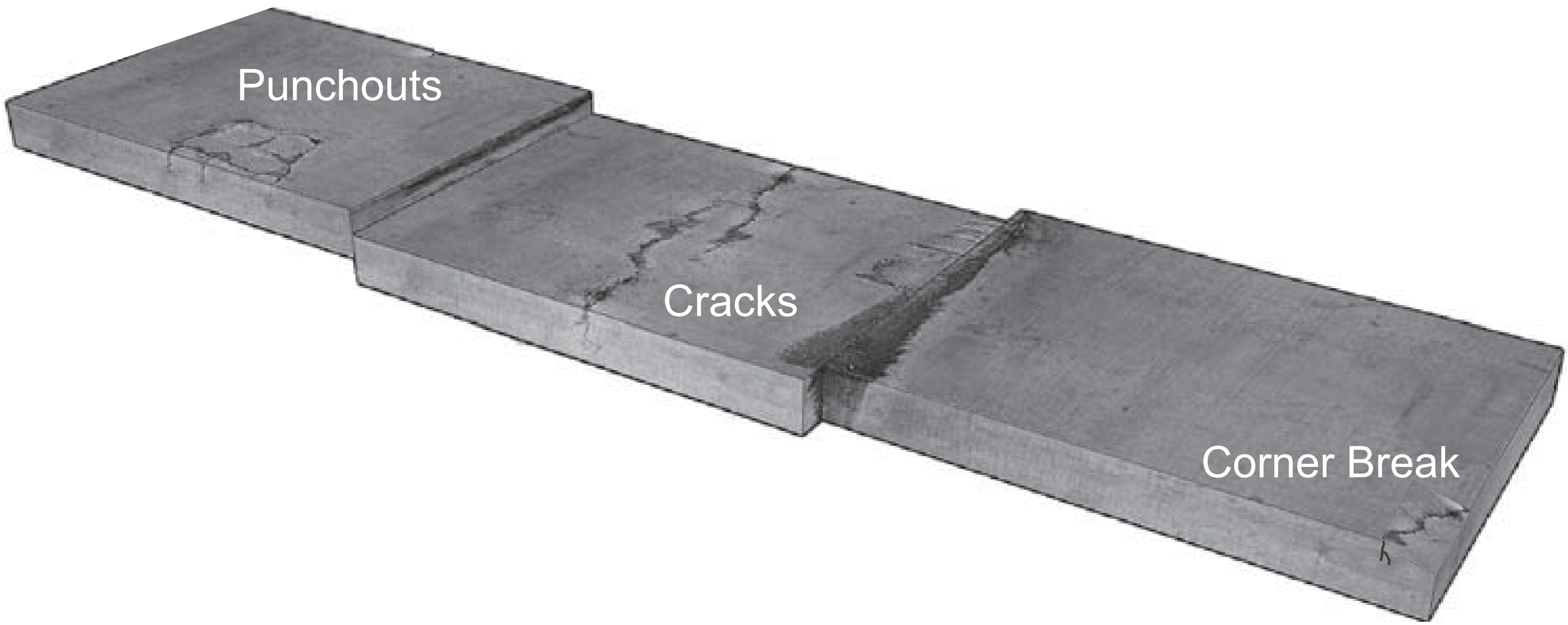
Rigid Pavement Mechanics

Curling Stresses

Major Distress Conditions

- Cracking
  - Bottom-up transverse cracks
  - Top-down transverse cracks
  - Longitudinal cracks
  - Corner breaks
  - Punchouts (CRCP)

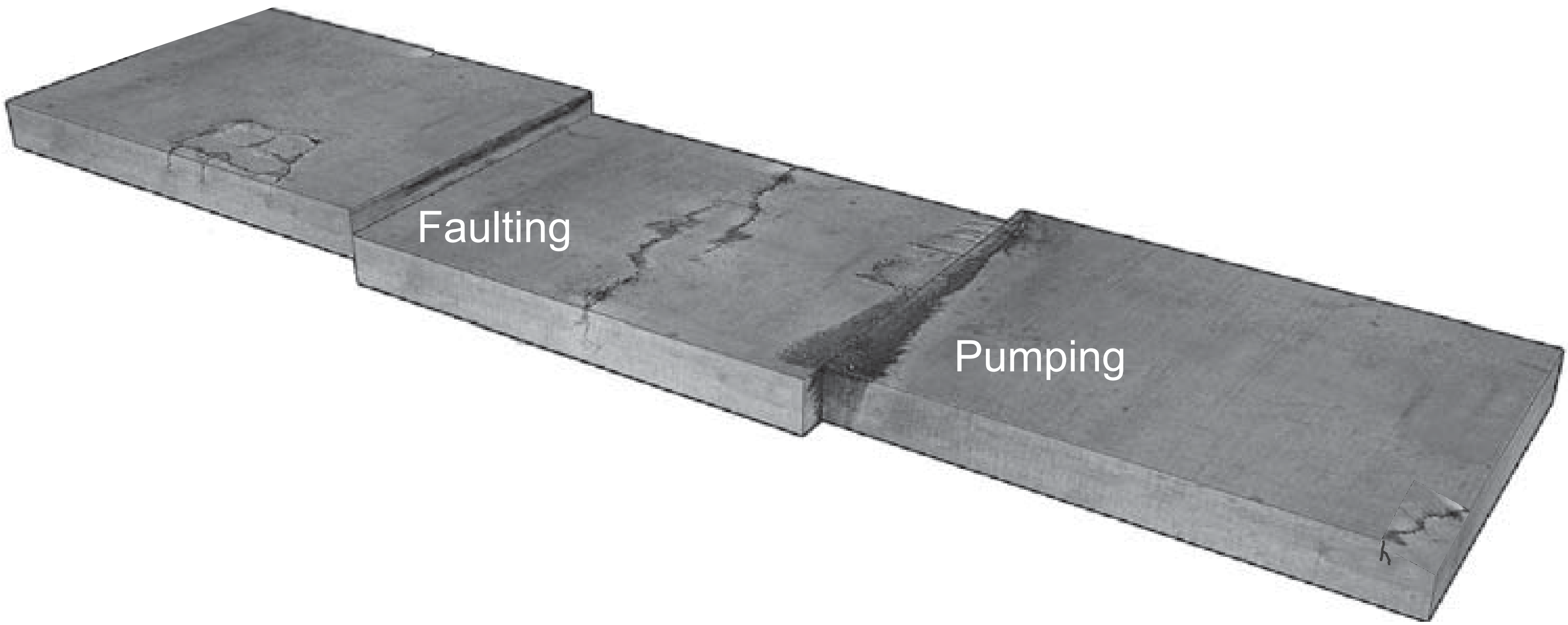
Major Distress Conditions



Major Distress Conditions

- Cracking
  - Bottom-up transverse cracks
  - Top-down transverse cracks
  - Longitudinal cracks
  - Corner breaks
  - Punchouts (CRCP)
- Joint Faulting
- Pumping

Major Distress Conditions



Sources of Stress

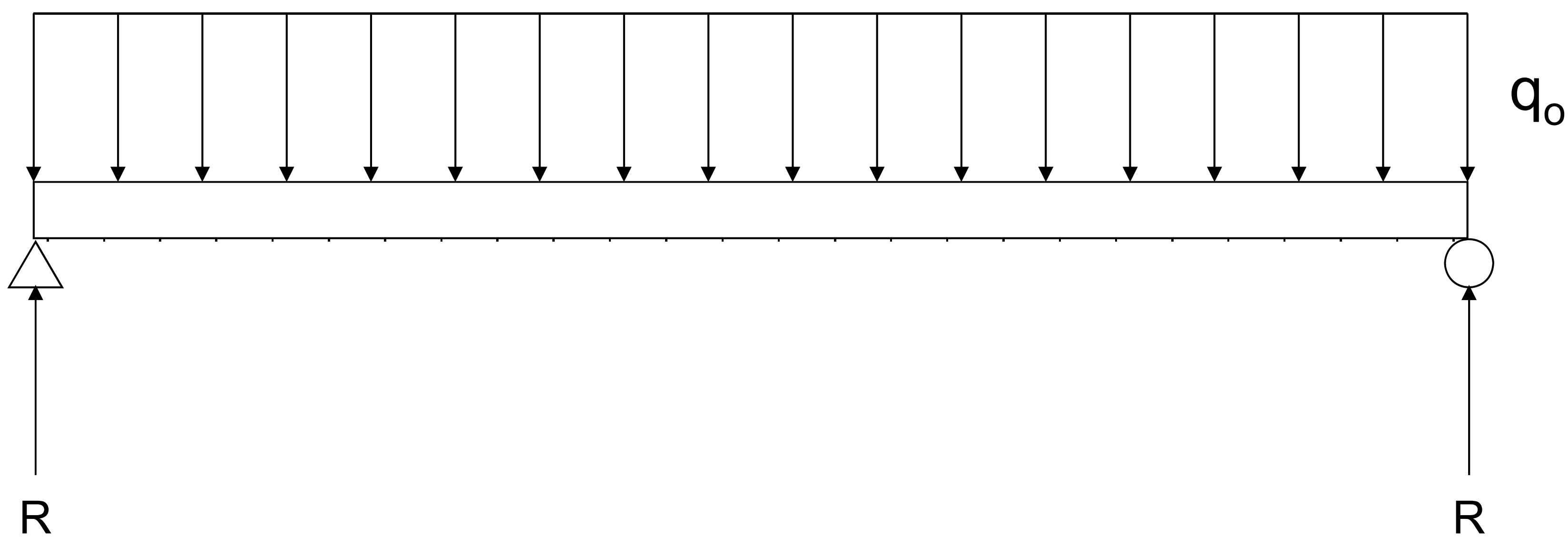
- Wheel Loads
- Curing Shrinkage
- Thermal Contraction/Expansion
- Thermal Curling
- Moisture Warping

Design Considerations

- Slab Thickness
- Base Type and Thickness
- Drainage
- Joint Spacing
- Temperature Steel
- Dowel Bars
- Tie Bars

Beam Bending

$q_o = q \times b$ (applied load per unit length)



Beam Bending

$$\frac{dM}{dx} = V$$

$$\Rightarrow \quad \frac{d^2M}{dx^2} = -q_o$$

$$\frac{dV}{dx} = -q_o$$

Shear and Moment Relationships

Beam Bending

$$\kappa = \frac{1}{R} = \frac{d^2 w}{dx^2}$$

$$\Rightarrow \quad M = -EI\,\frac{d^2 w}{dx^2}$$

$$\kappa = -\frac{M}{EI}$$

Moment-Curvature Relationships

Beam Bending

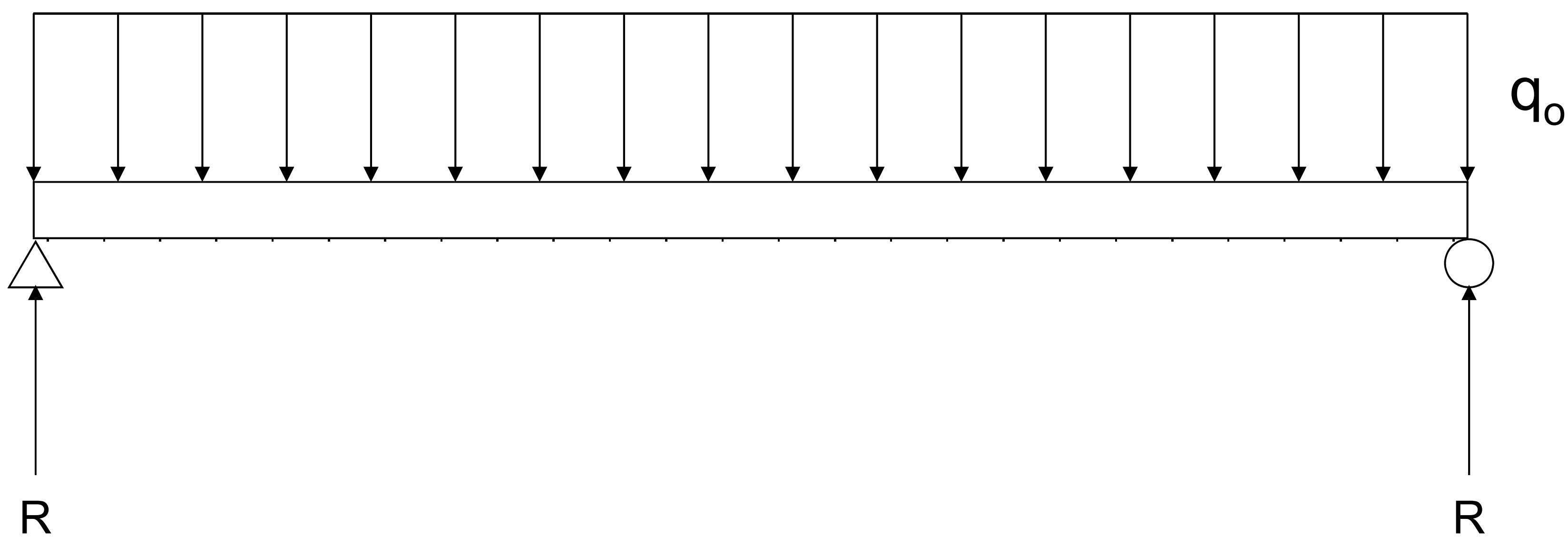
$$\frac{d^2}{dx^2}\left(-EI\,\frac{d^2w}{dx^2}\right) = -q_o$$

$$EI\,\frac{d^4w}{dx^4} = q_o$$

Governing Differential Equation for Beam Bending

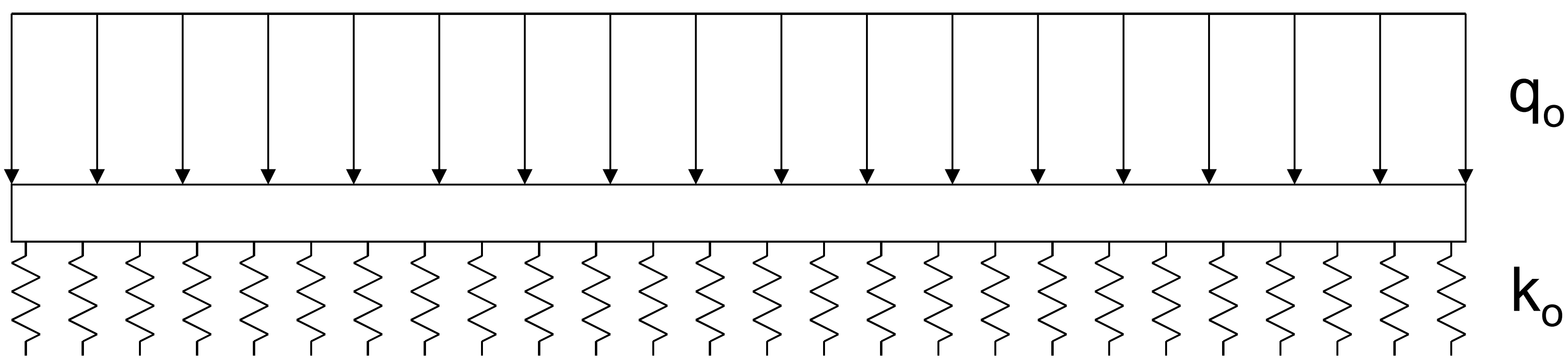
Beams on Elastic Foundations

$q_o = q \times b$ (applied load per unit length)



Beams on Elastic Foundations

$q_o = q \times b$ (applied load per unit length)



$p = k_o\, w = k\, b\, w$ (resistance per unit length)

Modulus of Subgrade Reaction

Beams on Elastic Foundations

$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = k_o w - q_o$$

$$\Rightarrow \quad \frac{d^2 M}{dx^2} = k_o w - q_o$$

Shear and Moment Relationships

Beam Bending

$$\kappa = \frac{1}{R} = \frac{d^2w}{dx^2}$$

$$\Rightarrow \quad M = -EI\,\frac{d^2w}{dx^2}$$

$$\kappa = -\frac{M}{EI}$$

Moment-Curvature Relationships

Beams on Elastic Foundations

$$\frac{d^2}{dx^2}\left(-EI\,\frac{d^2 w}{dx^2}\right) = k_o w - q_o$$

$$EI\,\frac{d^4 w}{dx^4} + k_o w = q_o$$

Governing Differential Equation for Beam Bending

Beams on Elastic Foundations

$$EI\frac{d^4w}{dx^4}+k_o w=0$$

$$w=e^{\beta x}\left(C_1\sin\beta x+C_2\cos\beta x\right)+e^{-\beta x}\left(C_3\sin\beta x+C_4\cos\beta x\right)$$

$$\beta=\sqrt[4]{\frac{k_o}{4EI}}$$ (flexibility)

Solution of the Homogeneous Differential Equation

Slabs on Elastic Foundations

$$M = -EI\frac{d^2w}{dx^2}$$ Beams

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$ Slabs

Moment-Curvature Relationships

Slabs on Elastic Foundations

$$D = \frac{Eh^3}{12\left(1 - \nu^2\right)}$$

Slab Stiffness

Slabs on Elastic Foundations

$$EI\frac{d^4w}{dx^4}+k_o w=q_o$$ Beams

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+kw=q$$ Slabs

Modulus of Subgrade Reaction

Governing Differential Equation for Slab Bending

Slabs on Elastic Foundations

$$\beta = \sqrt[4]{\frac{k_o}{4EI}}$$ Beam Flexibility

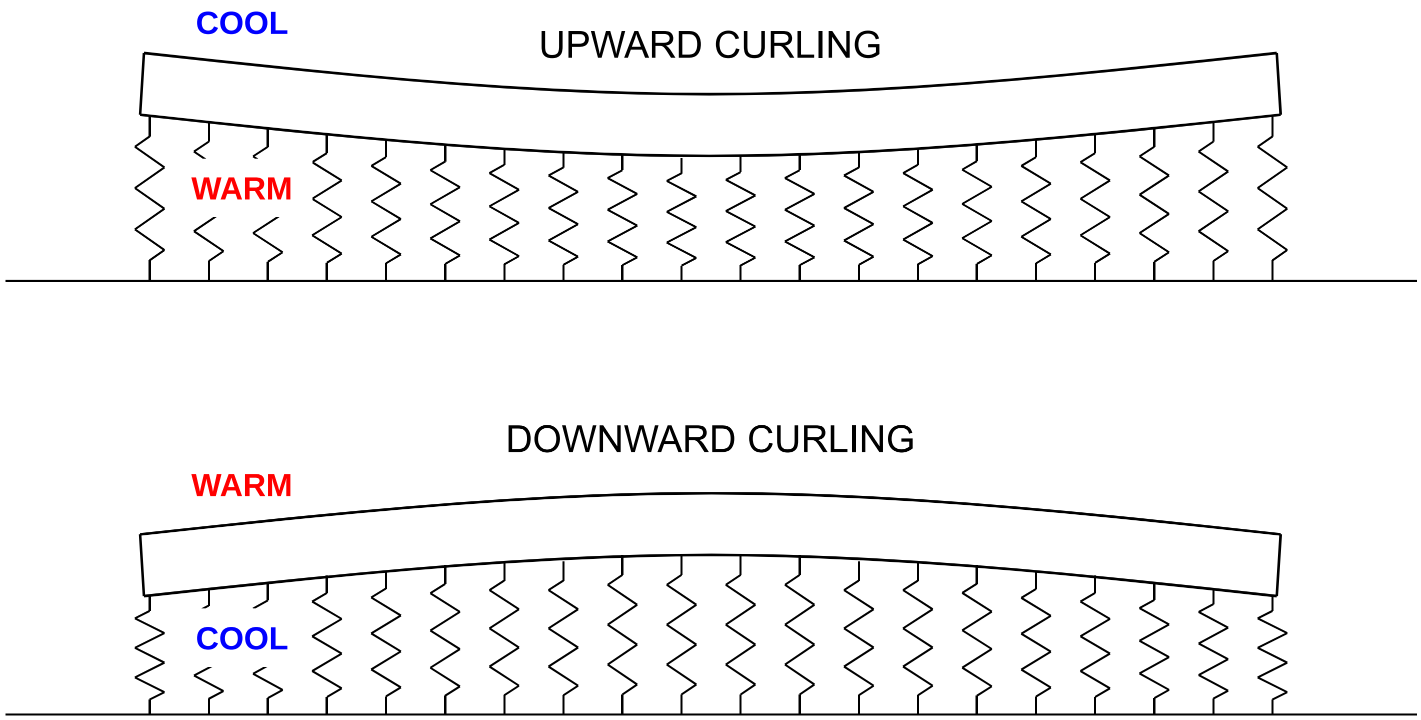
Radius of Relative Stiffness

$$\ell = \sqrt[4]{\frac{D}{k}} = \sqrt[4]{\frac{Eh^3}{12(1-\nu^2)k}}$$ Slab Stiffness

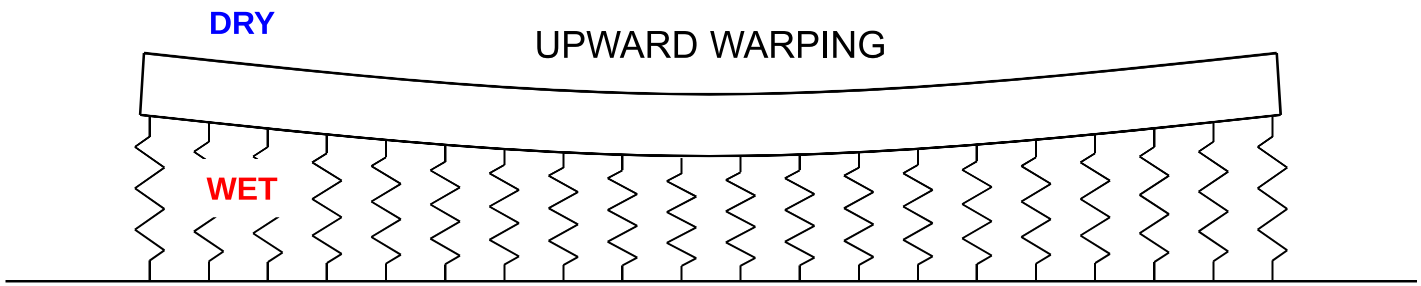
Solution of the Homogeneous Differential Equation

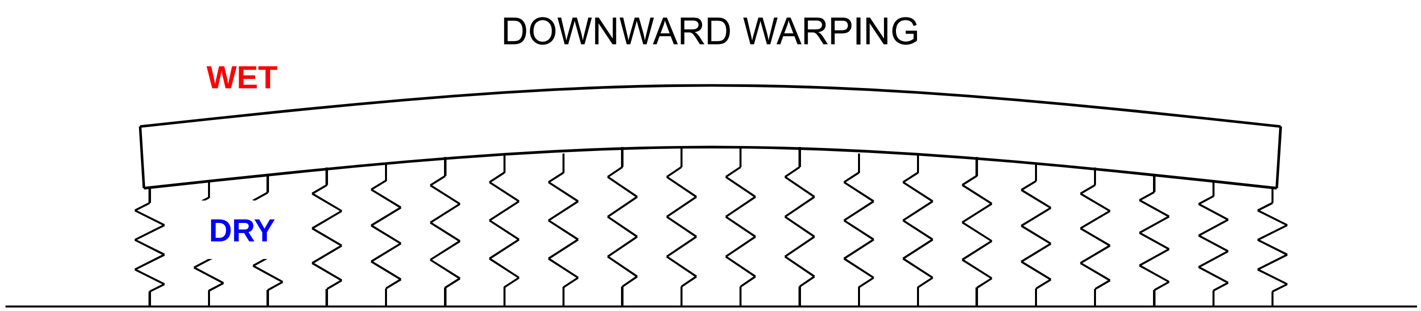
Stresses Due to Curling

Temperature Curling

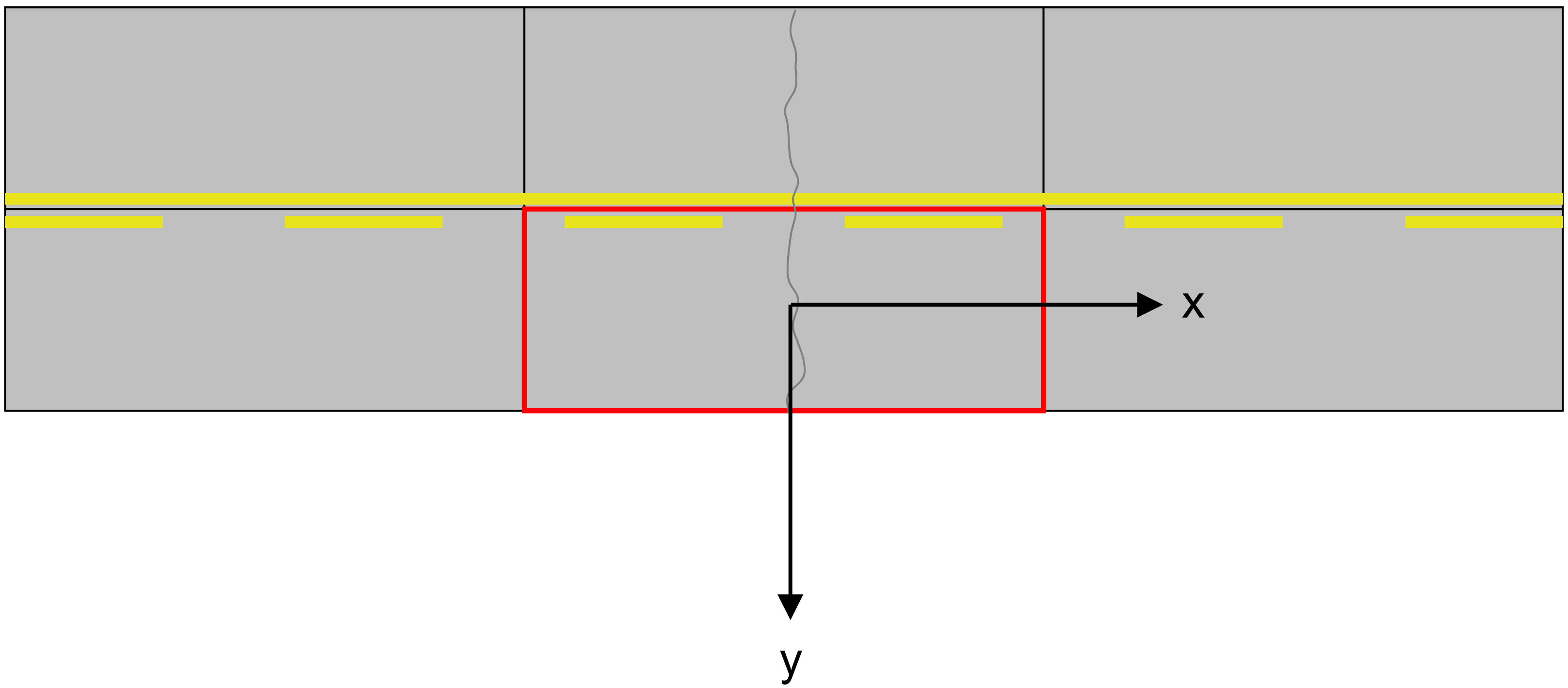


Moisture Warping





Reference Axes



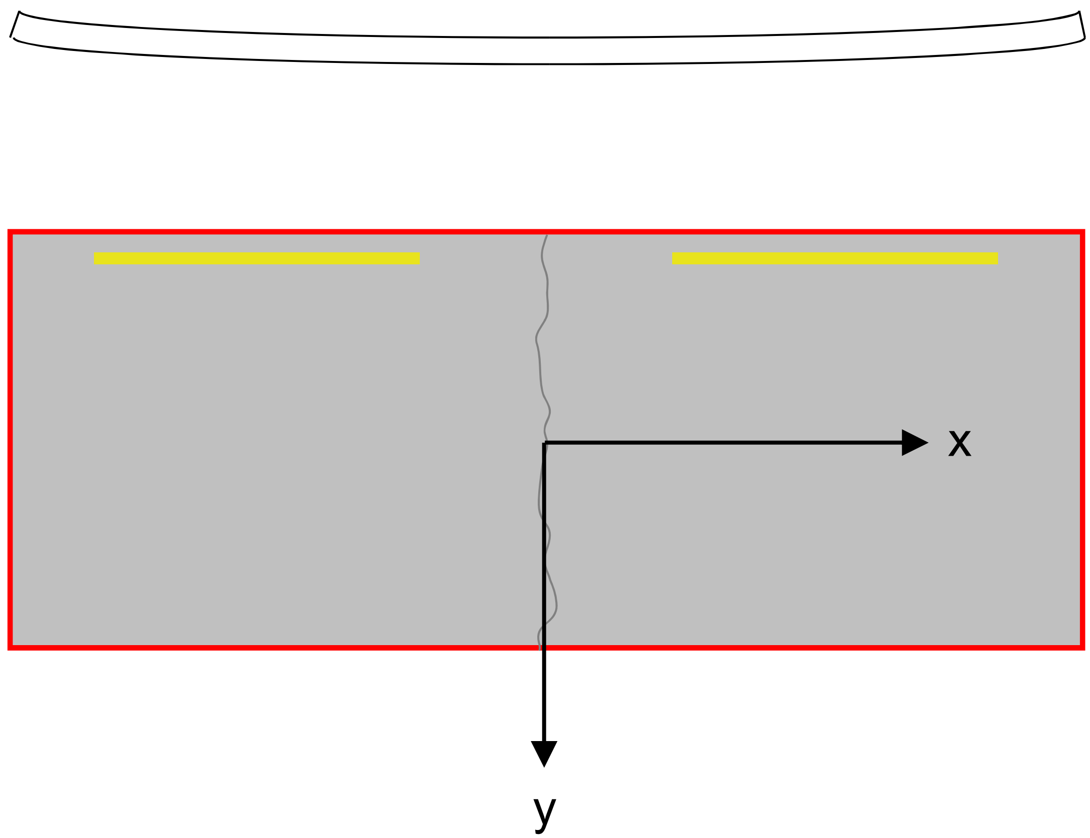
Infinite Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

Hooke's Law for an Infinite Elastic Plate

Slab Bent About y-axis



Infinite Slab Curling

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} = 0$$

$$\sigma_y - \nu\sigma_x = 0$$

$$\sigma_y = \nu\sigma_x$$

If slab is bent about the y axis, $\varepsilon_y = 0$

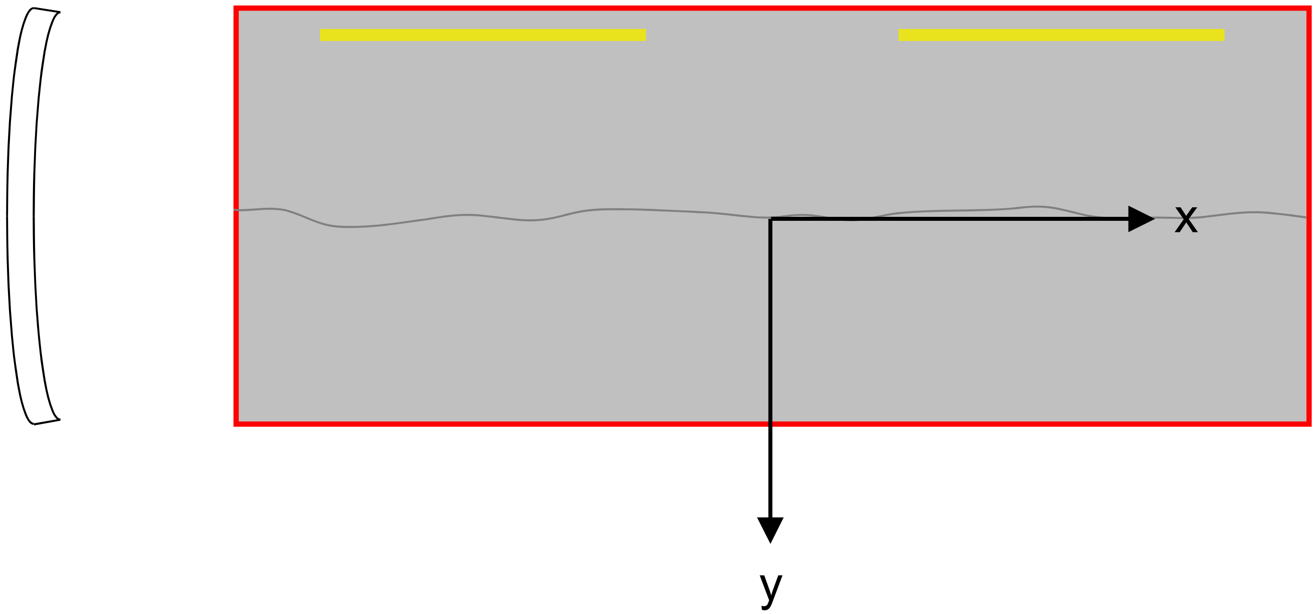
Infinite Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\nu \sigma_x}{E} = \frac{\sigma_x}{E}\left(1 - \nu^2\right)$$

$$\sigma_x = \frac{E\varepsilon_x}{1 - \nu^2}$$

Slab bent about y axis

Slab Bent About x-axis



Infinite Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = 0$$

$$\sigma_x - \nu \sigma_y = 0$$

$$\sigma_x = \nu \sigma_y$$

If slab is bent about the x axis, $\varepsilon_x = 0$

Infinite Slab Curling

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\nu \sigma_y}{E} = \frac{\sigma_y}{E}\left(1 - \nu^2\right)$$

$$\sigma_y = \frac{E\varepsilon_y}{1 - \nu^2}$$

Slab bent about x axis

Infinite Slab Curling

Coefficient of
Thermal Expansion

$$\varepsilon_x = \varepsilon_y = \frac{\alpha \Delta t}{2}$$

Strains due to temperature difference Δt through slab depth

Coefficient of Thermal Expansion

Aggregate Type	Coefficient (10^{-6} in/in/^{o}F)
Quartz	6.6
Sandstone	6.5
Gravel	6.0
Granite	5.3
Basalt	4.8
Limestone	3.8

Average 5.5

Infinite Slab Curling

$$\sigma_x = \frac{E\varepsilon_x}{1-\nu^2} = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

$$\sigma_y = \nu\sigma_x = \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Stresses due to curling about y axis due to temperature difference

Infinite Slab Curling

$$\sigma_y = \frac{E\varepsilon_y}{1-\nu^2} = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

$$\sigma_x = \nu\sigma_y = \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Stresses due to curling about x axis due to temperature difference

Infinite Slab Curling

$$\sigma_{x,y} = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)} + \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

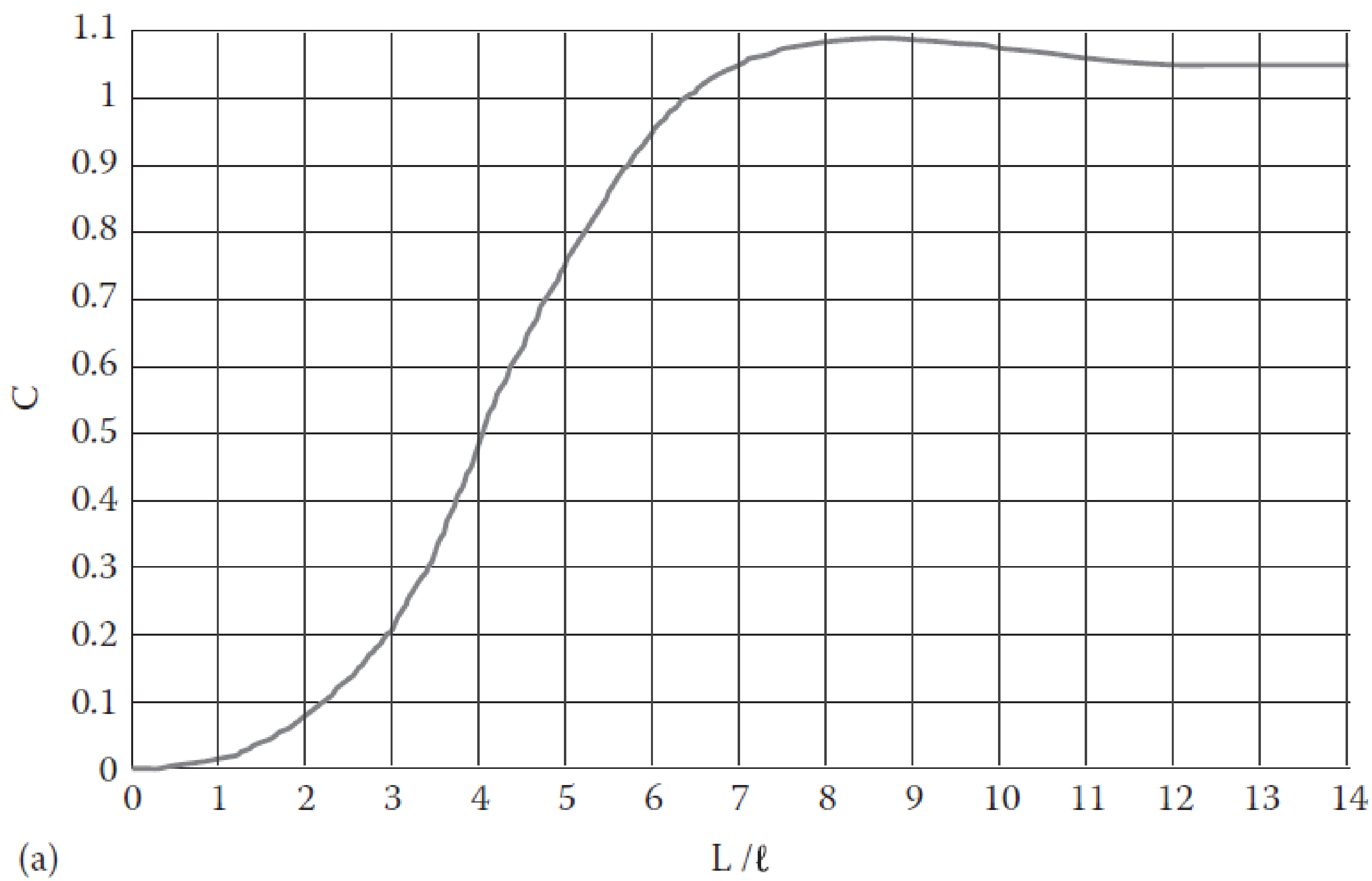
Maximum curling stresses in an infinite slab

Finite Slab Curling

$$\sigma_{x,y} = \overset{\parallel}{C_1} \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)} + \overset{\perp}{C_2} \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Curling stresses in the Interior of a Finite Slab

Finite Slab Curling



Finite Slab Curling

$$\sigma_y = \nu\sigma_x = 0 \quad \Rightarrow \quad \nu = 0$$

$$\parallel$$

$$\sigma_{x,y} = C_1 \frac{E\alpha\Delta t}{2}$$

Curling stresses at the Edges of a Finite Slab

Temperature Gradients

Source	Slab Thickness	Temperature Gradient
AASHO Road Test	6.5"	3.2°F/in ∩
		1.5°F/in ∪
Arlington Road Test	6"	3.7°F/in ∩
	9"	3.4°F/in ∩
Typical Assumption	6“ – 9"	3.0°F/in ∩
		1.5°F/in ∪

Example

Calculate the curling stresses in a concrete slab 25′×12′×8″ thick subject to a daytime temperature difference of 24°F (i.e., a temperature gradient of 3°F/in). Assume the slab is resting on a foundation with a 200-psi/in modulus of subgrade reaction.

Standard Assumptions

- Assume ν = 0.15 (use for all PCC)
- Assume α = 5.5×10^{-6} in/in/°F (typical)
- Assume f_c = 5000 psi (typical)
- Estimate $E = 57{,}000\sqrt{f_c} = 4 \times 10^{-6}$ psi
- Estimate $MOR = 8.4\sqrt{f_c} = 593$ psi*
- Estimate $f_t = 6.7\sqrt{f_c} = 474$ psi*

* Not needed for this problem