

2.1 Use Inductive Reasoning

Obj.: Describe patterns and use inductive reasoning.

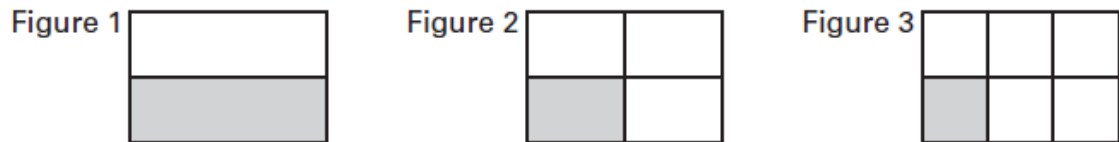
Key Vocabulary

- **Conjecture** - A conjecture is an unproven statement that is based on observations.
- **Inductive reasoning** - You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.
- **Counterexample** - A counterexample is a specific case for which the conjecture is false. You can show that a conjecture is false, however, by simply finding one counterexample.

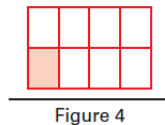
EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Solution:



Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eighths. Shade the section just below the horizontal segment at the left.



EXAMPLE 2 Describe a number pattern

Describe the pattern in the numbers $-1, -4, -16, -64, \dots$ and write the next three numbers in the pattern.

Solution:

Three dots (\dots) tell you that the pattern continues.

Notice that each number in the pattern is four times the previous number.

$$\begin{array}{ccccccc} -1, & -4, & -16, & -64, & \dots \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \times 4 & \times 4 & \times 4 & \times 4 \end{array}$$






The next three numbers are $-256, -1024, \text{ and } -4096$.

EXAMPLE 3 Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution:

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	<u>0</u>	<u>1</u>	<u>3</u>	<u>6</u>	<u>?</u>

$$\begin{array}{ccccccc} & \downarrow & \downarrow & \downarrow & \downarrow \\ & + 1 & + 2 & + 3 & + ? \end{array}$$

Conjecture You can connect five noncollinear points $6 + 4$, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

(2.1 cont.)

Numbers such as 1, 3, and 5 are called *consecutive odd numbers*. Make and test a conjecture about the sum of any three consecutive odd numbers.

Solution:

Step 1 Find a pattern using groups of small numbers.

$$\begin{aligned} 1 + 3 + 5 &= \underline{9} \\ &= 3 \cdot 3 \\ 5 + 7 + 9 &= \underline{21} \\ &= \underline{7} \cdot 3 \end{aligned}$$

$$\begin{aligned} 3 + 5 + 7 &= \underline{15} \\ &= \underline{5} \cdot 3 \\ 7 + 9 + 11 &= \underline{27} \\ &= \underline{9} \cdot 3 \end{aligned}$$

Conjecture The sum of any three consecutive odd numbers is three times the second number.

Step 2 Test your conjecture using other numbers.

$$-1 + 1 + 3 = \underline{3} = \underline{1} \cdot 3 \checkmark$$

$$103 + 105 + 107 = \underline{315} = \underline{105} \cdot 3 \checkmark$$

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Solution:

To find a counterexample, you need to find a difference that is greater than the larger number.

$$8 - (-4) = \underline{12}$$

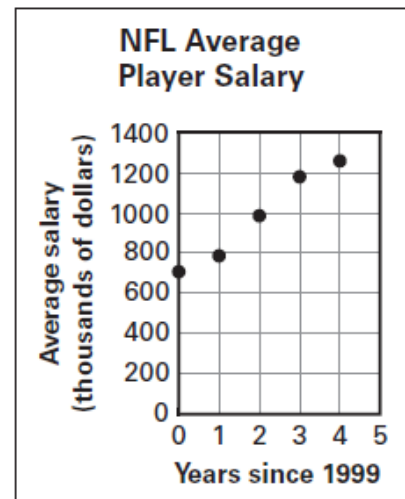
Because 12 \nless 8, a counterexample exists. The conjecture is false.

EXAMPLE 6 Real world application

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.

Solution:

The scatter plot shows that the values increased each year. So, one possible conjecture is that the average player in the NFL is earning more money today than in 1999.



2.1 Cont. (Write these on your paper)

✓ **Checkpoint** Complete the following exercise.

1. Sketch the fifth figure in the pattern in Example 1.

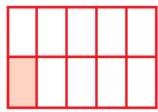


Figure 5

✓ **Checkpoint** Complete the following exercises.

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, ... and write the next three numbers in the pattern.

The numbers are increasing by 1.5; 7, 8.5, 10.

3. Rework Example 3 if you are given six noncollinear points.

15 different ways

✓ **Checkpoint** Complete the following exercise.

4. Make and test a conjecture about the sign of the product of any four negative numbers.

The result of the product of four negative numbers is a positive number;
 $(-1)(-2)(-5)(-1) = 10$.

✓ **Checkpoint** Complete the following exercises.

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

$$\frac{4}{\frac{1}{2}} = 8$$

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

The average salary of an NFL player in future years will be higher than the previous year; the average salary of an NFL player increased for the 5 years from 1999 to 2003.

2.2 Analyze Conditional Statements

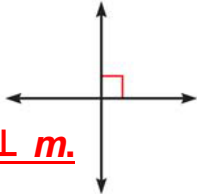
Obj.: Write definitions as conditional statements.

Key Vocabulary

- **Conditional statement** - A conditional statement is a logical statement that has two parts, a hypothesis and a conclusion.
- **Converse** - To write the **converse** of a conditional statement, exchange the hypothesis and conclusion.
- **Inverse** - To write the **inverse** of a conditional statement, negate (not) both the hypothesis and the conclusion.
- **Contrapositive** - To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.
- **If-then form, Hypothesis, Conclusion** - When a conditional statement is written in if-then form, the “if” part contains the **hypothesis** and the “then” part contains the **conclusion**. Here is an example:

If it is raining, then there are clouds in the sky.

hypothesis conclusion

- **Negation** - The **negation** of a statement is the opposite of the original statement.
- **Equivalent statements** - When two statements are both true or both false, they are called **equivalent statements**.
- **Perpendicular lines** - If two lines intersect to form a right angle, then they are **perpendicular lines**. You can write “line l is perpendicular to line m ” as $l \perp m$. 
- **Biconditional statement** – A **biconditional statement** is a statement that contains the converse and the phrase “if and only if.”

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form. All vertebrates have a backbone.

Solution:

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If an animal is a vertebrate, then it has a backbone.

EXAMPLE 2 Write four related conditional statements

(2.2 cont.)

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is *true* or *false*.

Solution:

If-then form

If-then form If you are an Olympian, then you are an athlete. *True*, Olympians are athletes.

Converse

Converse If you are an athlete, then you are an Olympian. *False*, not all athletes are Olympians.

Inverse

Inverse If you are not an Olympian, then you are not an athlete. *False*, even if you are not an Olympian, you can still be an athlete.

Contrapositive

Contrapositive If you are not an athlete, then you are not an Olympian. *True*, a person who is not an athlete cannot be an Olympian.

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

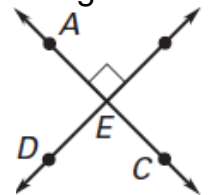
a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AED$ and $\angle BEC$ are a linear pair.

Solution:

a. The statement is *true*. The right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. The statement is *false*. Because $\angle AED$ and $\angle BEC$ are not adjacent angles, $\angle AED$ and $\angle BEC$ are not a linear pair.



EXAMPLE 4 Write a biconditional

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution:

Converse: If two lines are parallel, then they lie in the same plane and do not intersect.

Biconditional: Two lines are parallel if and only if they lie in the same plane and do not intersect.

2.2 Cont. (Write these on your paper)

- ✓ **Checkpoint** Write the conditional statement in if-then form.

1. All triangles have 3 sides. If a figure is a triangle, then it has 3 sides.	2. When $x = 2$, $x^2 = 4$. If $x = 2$, then $x^2 = 4$.
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- ✓ **Checkpoint** Complete the following exercises.

3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Squares are rectangles." Decide whether each statement is *true* or *false*.

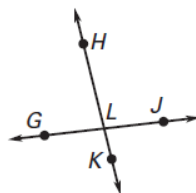
If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.

Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.

Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.

Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.

4. Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.



- a. $\angle GLK$ and $\angle JLK$ are supplementary.

- b. $\overleftrightarrow{GJ} \perp \overleftrightarrow{HK}$

(a) True; linear pairs of angles are supplementary.

(b) False; it is not known that the lines intersect at right angles.

5. Write the statement below as a biconditional.

Statement: If a student is a boy, he will be in group A.
If a student is in group A, the student must be a boy.

A student is in group A if and only if the student is a boy.

2.3 Apply Deductive Reasoning

Obj.: Use deductive reasoning to form a logical argument.

Key Vocabulary

• **Deductive reasoning** - Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

**** This is different from inductive reasoning, which uses specific examples and patterns to form a conjecture.****

Laws of Logic KEY CONCEPT

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis p , then **conclusion q** .
If hypothesis q , then conclusion r .
If these statements are true,

If hypothesis p , then **conclusion r** . then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

Solution:

a. Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true.
So, $\angle A \cong \angle B$.

b. Jesse goes to the gym every weekday. Today is Monday.

Solution:

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is a weekday," and the conclusion is "then Jesse goes to the gym."
"Today is Monday" satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.

EXAMPLE 2 Use the Law of Syllogism

(2.3 cont.)

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

Solution: a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.

If Ron eats lunch today, then he will drink a glass of milk.

b. If $x^2 > 36$, then $x^2 > 30$. If $x > 6$, then $x^2 > 36$.

Solution: b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If $x > 6$, then $x^2 > 30$.

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

Solution: c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

EXAMPLE 3 Use inductive and deductive reasoning

What conclusion can you make about the sum of an odd integer and an odd integer?

Solution:

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$-3 + 5 = \underline{2}, -1 + 5 = \underline{4}, 3 + 5 = \underline{8}$$

$$-3 + (-5) = \underline{-8}, 1 + (-5) = \underline{-4},$$

$$3 + (-5) = \underline{-2}$$

Conjecture: Odd integer + Odd integer = Even integer

Step 2 Let n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ and $2m$ are even integers because any integer multiplied by 2 is even.

$2n - \underline{1}$ and $2m + \underline{1}$ are odd integers because $2n$ and $2m$ are even integers.

$(2n - \underline{1}) + (2m + \underline{1})$ represents the sum of an odd integer $2n - \underline{1}$ and an odd integer $2m + \underline{1}$.

$$(2n - \underline{1}) + (2m + \underline{1}) = \underline{2}(n + m)$$

The result is the product of 2 and an integer $n + m$. So, 2 $(n + m)$ is an even integer.

The sum of an odd integer and an odd integer is an even integer.

EXAMPLE 4 Reasoning from a graph

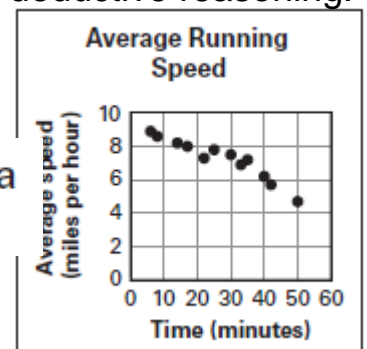
Tell whether the statement is the result of inductive reasoning or deductive reasoning. Explain your choice.

a. The runner's average speed decreases as time spent running increases.

Solution: a. Inductive reasoning, because it is based on a pattern in the data

b. The runner's average speed is slower when running for 40 minutes than when running for 10 minutes.

Solution: b. Deductive reasoning, because you are comparing values that are given on the graph



2.3 Cont.

✓ **Checkpoint** Complete the following exercises.

-
1. If $0^\circ < m\angle A < 90^\circ$, then A is acute. The measure of $\angle A$ is 38° . Using the Law of Detachment, what statement can you make?

$\angle A$ is acute.

2. State the law of logic that is illustrated below.

If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

If you do your homework, then you can watch your favorite show.

Law of Syllogism

✓ **Checkpoint** Complete the following exercise.

-
3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

The sum of a negative integer and itself is twice the integer; $-n + (-n) = -2n = 2(-n)$.

✓ **Checkpoint** Complete the following exercises.

-
4. Use inductive reasoning to write another statement about the graph in Example 4.

Sample answer: The faster the average speed of the runner, the less time he or she is running.

5. Use deductive reasoning to write another statement about the graph in Example 4.

Sample answer: The runner's average speed is faster when running for 10 minutes than when running for 40 minutes.

2.4 Use Postulates and Diagrams

Obj.: Use postulates involving points, lines, and planes.

Key Vocabulary

- **Line perpendicular to a plane** - A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.
- **Postulate** - In geometry, rules that are accepted without proof are called *postulates* or *axioms*.

POSTULATES

Point, Line, and Plane Postulates

POSTULATE 5 - Through any two points there exists exactly one line.

POSTULATE 6 - A line contains at least two points.

POSTULATE 7 - If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 - Through any three noncollinear points there exists exactly one plane.

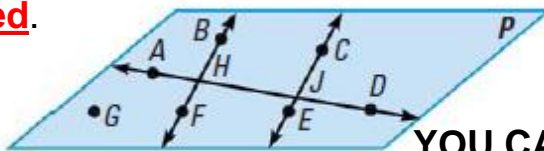
POSTULATE 9 - A plane contains at least three noncollinear points.

POSTULATE 10 - If two points lie in a plane, then the line containing them lies in the plane.

POSTULATE 11 - If two planes intersect, then their intersection is a line.

CONCEPT SUMMARY - Interpreting a Diagram

When you interpret a diagram, you can only assume information about size or measure if it is marked.



YOU CAN ASSUME

All points shown are coplanar.

$\angle AHB$ and $\angle BHD$ are a linear pair.

$\angle AHF$ and $\angle BHD$ are vertical angles.

A, H, J, and D are collinear.

\overleftrightarrow{AD} and \overleftrightarrow{BF} intersect at H.

YOU CANNOT ASSUME

G, F, and E are collinear.

\overleftrightarrow{BF} and \overleftrightarrow{CE} intersect.

\overleftrightarrow{BF} and \overleftrightarrow{CE} do not intersect.

$\angle BHA \cong \angle CJA$

$\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90^\circ$

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.



Solution

Postulate 8 Through any three noncollinear points there exists exactly one plane.

EXAMPLE 2 Identify postulates from a diagram

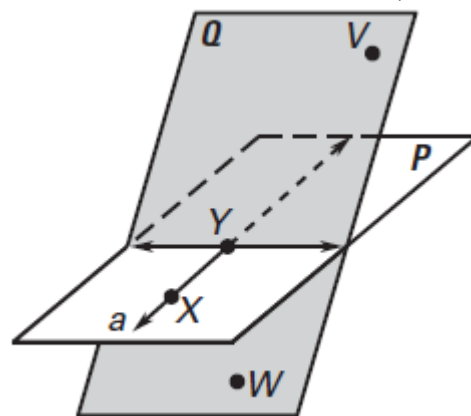
(2.4 cont.)

Use the diagram to write examples of Postulates 9 and 11.

Solution:

Postulate 9 Plane Q contains at least three noncollinear points, W, V, and Y.

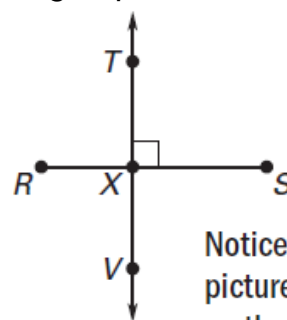
Postulate 11 The intersection of plane P and plane Q is line b.

**EXAMPLE 3 Use given information to sketch a diagram**Sketch a diagram showing \overleftrightarrow{RS} perpendicular to \overleftrightarrow{TV} , intersecting at point X.**Solution:**

Step 1 Draw \overleftrightarrow{RS} and label points R and S.

Step 2 Draw a point X between R and S.

Step 3 Draw \overleftrightarrow{TV} through X so that it is perpendicular to \overleftrightarrow{RS} .



Notice that the picture was drawn so that X does not look like a midpoint of \overleftrightarrow{RS} .

EXAMPLE 4 Interpret a diagram in three dimensions

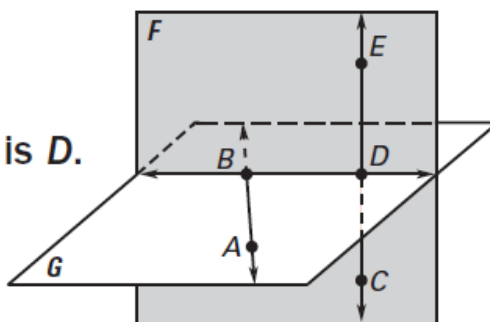
Which of the following statements *cannot* be assumed from the diagram?

$E, D,$ and C are collinear.

The intersection of \overleftrightarrow{BD} and \overleftrightarrow{EC} is D .

$\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$

$\overleftrightarrow{EC} \perp$ plane G



Solution: With no right angles marked, you cannot assume that

$\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$ or $\overleftrightarrow{EC} \perp$ plane G .

2.4 Cont. (Write these on your paper)

✓ **Checkpoint** Use the diagram in Example 2 to complete the following exercises.

-
1. Which postulate allows you to say that the intersection of line a and line b is a point?

Postulate 7

2. Write examples of Postulates 5 and 6.

Line a passes through X and Y ; line a contains points X and Y .

✓ **Checkpoint** Complete the following exercises.

-
3. In Example 3, if the given information indicated that \overline{RX} and \overline{XS} are congruent, how would the diagram change?

Point X would be drawn as the midpoint of \overline{RS} and the congruent segments would be marked.

4. In the diagram for Example 4, can you assume that \overleftrightarrow{BD} is the intersection of plane F and plane G ?

Yes

2.5 Reason Using Properties from Algebra

Obj.: Use algebraic properties in logical arguments.

Key Vocabulary

- **Equation** - An **equation** is a mathematical statement that asserts the equality of two expressions.
- **Solve an equation** - To **solve an equation** is to find what values fulfill a condition stated in the form of an equation. When you solve an equation, you use properties of real numbers.

Algebraic Properties of Equality - KEY CONCEPT

Let a , b , and c be real numbers.

Addition Property

If $a = b$, then $a + c = b + c$.

Subtraction Property

If $a = b$, then $a - c = b - c$.

Multiplication Property

If $a = b$, then $ac = bc$.

Division Property

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Substitution Property

If $a = b$, then a can be substituted for b in any equation or expression.

Distributive Property

$a(b + c) = \underline{ab + ac}$, where a , b , and c are real numbers.

KEY CONCEPT

Reflexive Property of Equality

Real Numbers

For any real number a , $a = \underline{a}$.

Segment Length

For any segment \overline{AB} , $\overline{AB} \cong \underline{\overline{AB}}$.

Angle Measure

For any angle $\angle A$, $m\angle A = \underline{m\angle A}$.

Symmetric Property of Equality

Real Numbers

For any real numbers a and b , if $a = b$, then $\underline{b = a}$.

Segment Length

For any segments \overline{AB} and \overline{CD} , if $\overline{AB} \cong \overline{CD}$, then $\underline{\overline{CD} \cong \overline{AB}}$.

Angle Measure

For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $\underline{m\angle B = m\angle A}$.

Transitive Property of Equality

Real Numbers

For any real numbers a , b , and c , if $\underline{a = b}$ and $b = \underline{c}$, then $\underline{a = c}$.

Segment Length

For any segments \overline{AB} , \overline{CD} , and \overline{EF} , if $\underline{\overline{AB} \cong \overline{CD}}$ and $\overline{CD} \cong \underline{\overline{EF}}$, then $\underline{\overline{AB} \cong \overline{EF}}$.

Angle Measure

For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = \underline{m\angle B}$ and $\underline{m\angle B} = m\angle C$, then $\underline{m\angle A = m\angle C}$.

EXAMPLE 1 Write reasons for each stepSolve $2x + 3 = 9 - x$. Write a reason for each step.**Solution:**

(2.5 cont.)

Equation	Explanation	Reason
$2x + 3 = 9 - x$	Write original equation.	Given
$2x + 3 + \underline{x} = 9 - x + \underline{x}$	Add \underline{x} to each side.	<u>Addition Property of Equality</u>
$\underline{3x} + 3 = \underline{9}$	Combine like terms.	<u>Simplify.</u>
$\underline{3x} = \underline{6}$	Subtract $\underline{3}$ from each side.	<u>Subtraction Property of Equality</u>
$x = \underline{2}$	Divide each side by $\underline{3}$.	<u>Division Property of Equality</u>

EXAMPLE 2 Use the Distributive PropertySolve $-4(6x + 2) = 64$. Write a reason for each step.**Solution:**

The value of x is $\underline{2}$.

Equation	Explanation	Reason
$-4(6x + 2) = 64$	Write original equation.	Given
$\underline{-24x - 8} = 64$	Multiply.	<u>Distributive Property</u>
$\underline{-24x} = \underline{72}$	Add $\underline{8}$ to each side.	<u>Addition Property of Equality</u>
$\underline{x} = \underline{-3}$	Divide each side by $\underline{-24}$.	<u>Division Property of Equality</u>

EXAMPLE 3 Use properties in the real world

Speed A motorist travels 5 miles per hour slower than the speed limit s for 3.5 hours. The distance traveled d can be determined by the formula $d = 3.5(s - 5)$. Solve for s .

Solution:

Equation	Explanation	Reason
$d = 3.5(s - 5)$	Write original equation.	Given
$d = \underline{3.5s - 17.5}$	Multiply.	<u>Distributive Property</u>
$d + \underline{17.5} = \underline{3.5s}$	Add $\underline{17.5}$ to each side.	<u>Addition Property of Equality</u>
$\frac{d + \boxed{17.5}}{\boxed{3.5}} = s$	Divide each side by $\underline{3.5}$.	<u>Division Property of Equality</u>

EXAMPLE 4 Use properties of equalityShow that $CF = AD$.**Solution:**

Equation	Reason
$AB = \underline{EF}$	Given
$BC = \underline{DE}$	Given
$AC = AB + BC$	<u>Segment Addition Postulate</u>
$DF = \underline{DE} + \underline{EF}$	Segment Addition Postulate
$DF = BC + AB$	<u>Substitution Property of Equality</u>
$DF = \underline{AC}$	<u>Transitive Property of Equality</u>
$DF + CD = \underline{AC} + CD$	<u>Addition Property of Equality</u>
$\underline{CF} = \underline{AD}$	<u>Substitution Property of Equality</u>



2.5 Cont.

✔ **Checkpoint** Complete the following exercises.

1. Solve $x - 5 = 7 + 2x$. Write a reason for each step.

$x - 5 = 7 + 2x$	Given
$x - 5 - x = 7 + 2x - x$	Subtraction Property of Equality
$-5 = 7 + x$	Simplify.
$-12 = x$	Subtraction Property of Equality

2. Solve $4(5 - x) = -2x$. Write a reason for each step.

$4(5 - x) = -2x$	Given
$20 - 4x = -2x$	Distributive Property
$20 = 2x$	Addition Property of Equality
$10 = x$	Division Property of Equality

✔ **Checkpoint** Complete the following exercises. In Exercises 4–6, name the property of equality that the statement illustrates.

3. Suppose the equation in Example 3 is $d = 5(s + 3)$. Solve for s . Write a reason for each step.

$d = 5(s + 3)$	Given
$d = 5s + 15$	Distributive Property
$d - 15 = 5s$	Subtraction Property of Equality
$\frac{d - 15}{5} = s$	Division Property of Equality

4. If $GH = JK$, then $JK = GH$.

Symmetric Property of Equality for Segment Length

5. If $r = s$, and $s = 44$, then $r = 44$.

Transitive Property of Equality for Real Numbers

6. $m\angle N = m\angle N$

Reflexive Property of Equality for Angle Measure

2.6 Prove Statements about Segments and Angles

Obj.: Write proofs using geometric theorems.

Key Vocabulary

- **Proof** - A **proof** is a logical argument that shows a statement is true. There are several formats for proofs.
- **Two-column proof** - A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.
- **Theorem** - The reasons used in a proof can include definitions, properties, postulates, and theorems. A **theorem** is a statement that can be proven.

THEOREMS

Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB , $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A , $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

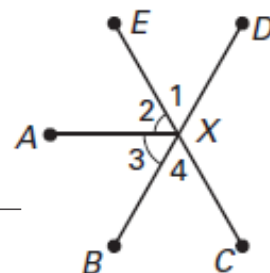
Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

EXAMPLE 1 Write a two-column proof

Use the diagram to prove $m\angle 1 = m\angle 4$.

Given: $m\angle 2 = m\angle 3$, $m\angle AXD = m\angle AXC$

Prove: $m\angle 1 = m\angle 4$



Statements	Reasons
1. $m\angle AXC = m\angle AXD$	1. <u>Given</u>
2. $m\angle AXD = m\angle 1 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle AXC = m\angle 3 + m\angle 4$	3. Angle Addition Postulate
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4. <u>Substitution Property of Equality</u>
5. $m\angle 2 = m\angle 3$	5. <u>Given</u>
6. $m\angle 1 + m\angle 3 = m\angle 3 + m\angle 4$	6. Substitution Property of Equality
7. $m\angle 1 = m\angle 4$	7. <u>Subtraction Property of Equality</u>

Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$

Solution: Symmetric Property of Angle Congruence

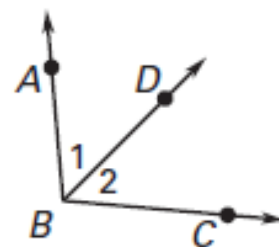
EXAMPLE 3 Use properties of equality

If you know that \overrightarrow{BD} bisects $\angle ABC$, prove that $m\angle ABC$ is two times $m\angle 1$.

Given: \overrightarrow{BD} bisects $\angle ABC$.

Prove: $m\angle ABC = 2 \cdot m\angle 1$

Statements	Reasons
1. \overrightarrow{BD} bisects $\angle ABC$.	1. <u>Given</u>
2. $\angle 1 \cong \angle 2$	2. Definition of angle bisector
3. $m\angle 1 = m\angle 2$	3. Definition of congruent angles
4. $m\angle 1 + m\angle 2 = m\angle ABC$	4. <u>Angle Addition Postulate</u>
5. $m\angle 1 + m\angle 1 = m\angle ABC$	5. Substitution Property of Equality
6. $2 \cdot m\angle 1 = m\angle ABC$	6. Distributive Property

**EXAMPLE 4 Solve a multi-step problem**

Interstate There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and Mason exit. The distance between rest area B and Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

Solution:

Solution

Step 1 Draw a diagram.



Step 2 Draw diagrams showing relationships.



Step 3 Write a proof.

Given R is the midpoint of \overline{AM} , $MB = AR$.

Prove M is the midpoint of \overline{RB} .

Statements	Reasons
1. R is the midpoint of \overline{AM} , $MB = AR$.	1. <u>Given</u>
2. $\overline{AR} \cong \overline{RM}$	2. Definition of midpoint
3. $AR = RM$	3. Definition of congruent segments
4. $MB = RM$	4. <u>Transitive Property of Congruence</u>
5. $\overline{MB} \cong \overline{RM}$	5. Definition of congruent segments
6. M is the midpoint of \overline{RB} .	6. <u>Definition of midpoint</u>

CONCEPT SUMMARY - Writing a Two-Column Proof

In a proof, you make **one** statement at a time, until you reach the **conclusion**. Because you make statements based on **facts**, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

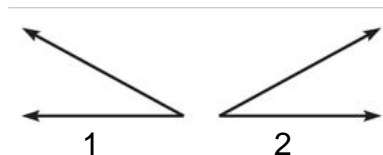
Proof of the Symmetric Property of Angle Congruence

GIVEN $\angle 1 \cong \angle 2$

PROVE $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. <u>Given</u>
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Copy or **draw diagrams** and **label** given information to help develop proofs.



The **number** of statements will **vary**

Remember to give a **reason** for the **last** statement.

Definitions, postulates

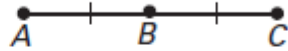
Theorems Statements based on **facts** that you know or on conclusions from deductive **reasoning**

2.6 Cont.

✓ **Checkpoint** Complete the following exercises.

1. Three steps of a proof are shown. Give the reasons for the last two steps.

Given $BC = AB$



Prove $AC = AB + AB$

Statements

Reasons

1. $BC = AB$

1. Given

2. $AC = AB + BC$

2. Segment Addition Postulate

3. $AC = AB + AB$

3. Substitution Property of Equality

2. Name the property illustrated by the statement.
If $\angle H \cong \angle T$ and $\angle T \cong \angle B$, then $\angle H \cong \angle B$.

Transitive Property of Angle Congruence

✓ **Checkpoint** Complete the following exercise.

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

Rest area A and rest area B

2.7 Prove Angle Pair Relationships

Obj.: Use properties of special pairs of angles.

Key Vocabulary

- **Complementary angles** - Two angles whose measures have the sum 90° .
- **Supplementary angles** - Two angles whose measures have the sum 180° .
- **Linear pair** - Two adjacent angles whose noncommon sides are opposite rays.
- **Vertical angles** – Two angles whose sides form two pairs of opposite rays.

Right Angles Congruence Theorem

All right angles are congruent.

Abbreviation

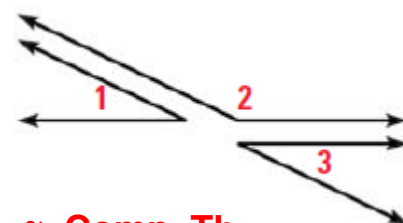
All Rt. \angle 's \cong

Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

\cong Supp. Th

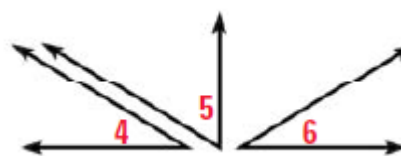


Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

\cong Comp. Th



Linear Pair Postulate

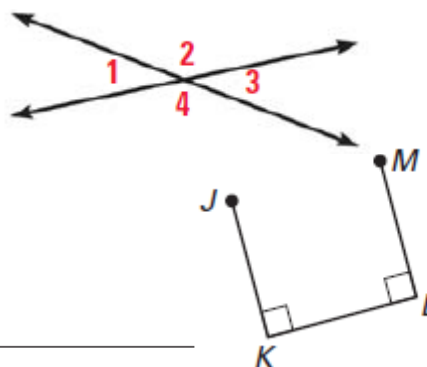
If two angles form a linear pair, then they are supplementary. $\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.



Vertical Angles Congruence Theorem

Vertical angles are congruent.

$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$



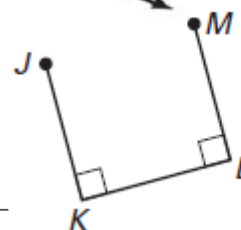
EXAMPLE 1 Use right angle congruence

Write a proof.

GIVEN: $\overline{JK} \perp \overline{KL}, \overline{ML} \perp \overline{KL}$

PROVE: $\angle K \cong \angle L$

Statements	Reasons
1. $\overline{JK} \perp \overline{KL}, \overline{ML} \perp \overline{KL}$	1. <u>Given</u>
2. <u>$\angle K$ and $\angle L$ are right angles.</u>	2. Definition of perpendicular lines
3. $\angle K \cong \angle L$	3. <u>Right Angles Congruence Theorem</u>



EXAMPLE 2 Use Congruent Supplements Theorem

(2.7 cont.)

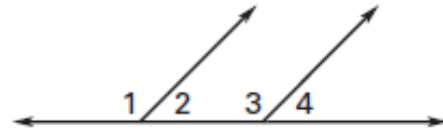
Write a proof.

GIVEN: $\angle 1$ and $\angle 2$ are supplements.

$\angle 1$ and $\angle 4$ are supplements.

$$m\angle 2 = 45^\circ$$

PROVE: $m\angle 4 = 45^\circ$



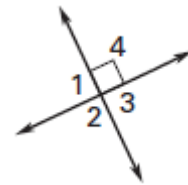
Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 1$ and $\angle 4$ are supplements.	1. <u>Given</u>
2. <u>$\angle 2 \cong \angle 4$</u>	2. Congruent Supplements Theorem
3. $m\angle 2 = m\angle 4$	3. <u>Definition of congruent angles</u>
4. $m\angle 2 = 45^\circ$	4. <u>Given</u>
5. <u>$m\angle 4 = 45^\circ$</u>	5. Substitution Property of Equality

EXAMPLE 3 Use the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN: $\angle 4$ is a right angle.

PROVE: $\angle 2$ and $\angle 4$ are supplementary.



Statements	Reasons
1. $\angle 4$ is a right angle.	1. <u>Given</u>
2. <u>$m\angle 4 = 90^\circ$</u>	2. Definition of a right angle
3. $\angle 2 \cong \angle 4$	3. <u>Vertical Angles Congruence Theorem</u>
4. <u>$m\angle 2 = m\angle 4$</u>	4. Definition of congruent angles
5. $m\angle 2 = 90^\circ$	5. <u>Substitution Property of Equality</u>
6. <u>$\angle 2$ and $\angle 4$ are supplementary.</u>	6. $m\angle 2 + m\angle 4 = 180^\circ$

EXAMPLE 4 Find angle measures

Write and solve an equation to find x . Use x to find the $m\angle FKG$.

Solution:

Because $m\angle FKG$ and $m\angle GKH$ form a linear pair, the sum of their measures is 180° .

$$(4x - 1)^\circ + 113^\circ = \underline{180^\circ}$$

Write equation.

$$4x + \underline{112} = \underline{180}$$

Simplify.

$$4x = \underline{68}$$

Subtract 112 from each side.

$$x = \underline{17}$$

Divide each side by 4.

Use $x = \underline{17}$ to find $m\angle FKG$.

$$m\angle FKG = (4x - 1)^\circ$$

$$= [4(\underline{17}) - 1]^\circ$$

$$= [\underline{68} - 1]^\circ$$

$$= \underline{67^\circ}$$

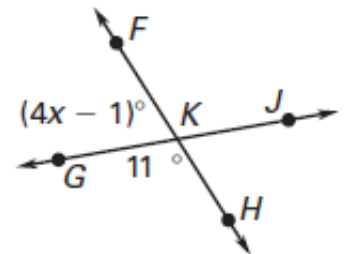
Write equation.

Substitute 17 for x .

Multiply.

Simplify.

The measure of $\angle FKG$ is 67° .



2.7 Cont.

✓ **Checkpoint** Complete the following exercises.

1. In Example 1, suppose you are given that $\angle K \cong \angle L$. Can you use the Right Angles Congruence Theorem to prove that $\angle K$ and $\angle L$ are right angles? *Explain.*

No, you cannot prove that $\angle K$ and $\angle L$ are right angles, because the converse of the Right Angles Congruence Theorem is not always true.

2. Suppose $\angle A$ and $\angle B$ are complements, and $\angle A$ and $\angle C$ are complements. Can $\angle B$ and $\angle C$ be supplements? *Explain.*

No, $\angle B$ and $\angle C$ are complements by the Congruent Complements Theorem, so they cannot be supplements.

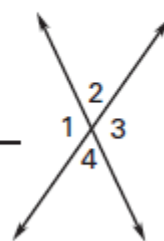
✓ **Checkpoint** In Exercises 3 and 4, use the diagram.

3. If $m\angle 4 = 63^\circ$, find $m\angle 1$ and $m\angle 2$.

$$m\angle 1 = 117^\circ, m\angle 2 = 63^\circ$$

4. If $m\angle 3 = 121^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

$$m\angle 1 = 121^\circ, m\angle 2 = 59^\circ, m\angle 4 = 59^\circ$$



✓ **Checkpoint** Complete the following exercise.

5. Find $m\angle AEB$.

$$m\angle AEB = 70^\circ$$

