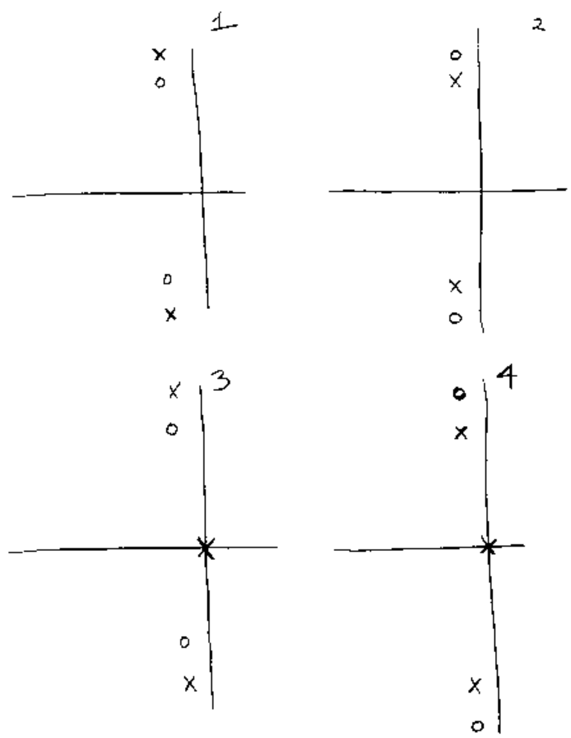
2.14/2.140 Problem Set 4

Assigned: Thurs. March 8, 2007
Due: Thurs. March 15, 2007, in class
Reading: Nise Chapter 8; Notes Chapter 3 on root locus.
Reading for 2.140 students:

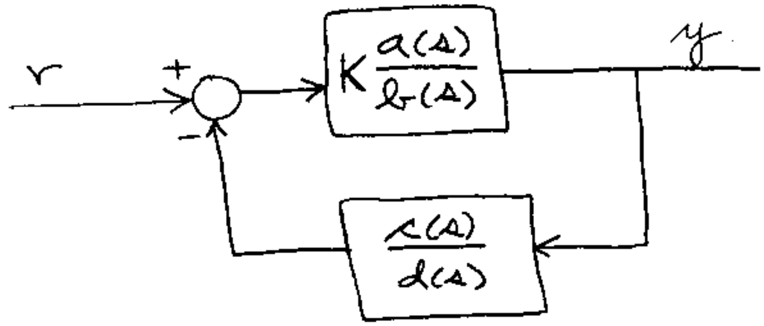
The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 Nise Ch. 8, Problem 2

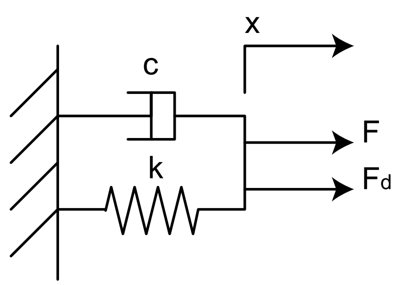
Problem 2 The four plots below show the pole and zero locations of the loop transmission of a feedback system, informally called the 'open-loop' poles and zeros. Each of these loops also have a variable gain $K > 0$, which is used to move the poles along the root locus branches. For the four systems shown below, sketch the approximate shape of the root locus plot for $K > 0$. Note that you will need to pay particular attention to the angle criteria in the vicinity of the complex poles and zeros. If the complex pairs are lightly damped, which of these systems presents a danger of instability as the loop gain is varied? This analysis has practical relevance for the situation where a notch filter is used to help stabilize a system with a lightly-damped pair of poles.



Problem 3 The block diagram for a feedback loop has a forward path transfer function $G(s) = Ka(s)/b(s)$, and a feedback path transfer function $H(s) = c(s)/d(s)$ as shown below. Prove that the closed-loop zeros are located at: 1) the zeros of the forward path and 2) the poles of the feedback path, independent of the loop gain K.

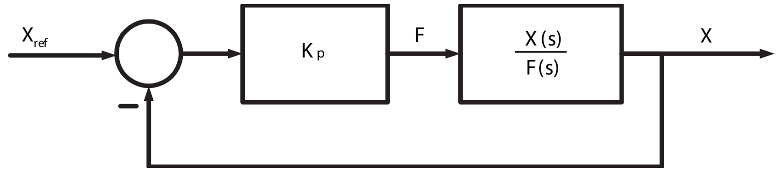


Problem 4 This problem considers the system



a) Derive the transfer function $\frac{X(s)}{F(s)}$ in terms of c and k. For this part of the question, ignore F_d. What is the time constant, τ, of the system?

b) We want to control the position, x, using the force F as the control input and with proportional control K_p, as shown below in the the block diagram for the closed-loop system. Again, ignoring F_d, if c=$10\frac{Ns}{m}$ and $k = 100\frac{N}{m}$, what value of K_p do we need in order to have a closed-loop system time constant $\tau = 10$ ms?



c) With this value of K_p calculate and plot the closed-loop step response $x(t)$ when the commanded position is a step $x_{ref}(t) = 10^{-3}u_s(t)$ m. What is the resulting steady-state position error? Also calculate and plot the closed loop Bode plot from command input x_{ref} to position x.

d) Suppose now there is a disturbance force, F_d. Redraw the block diagram to include this force. Given the value of K_p computed above, find the closed-loop transfer function from the disturbance force, F_d to the output position, x (i.e. $\frac{X(s)}{F_d(s)}$), and sketch its Bode plot. Calculate and plot the closed-loop step response $x(t)$ when the disturbance force is a step $F_d(t) = 1u_s(t)$ N, with the input $x_{ref} = 0$. What is the resulting steady-state position error?

e) Now increase the gain K_p such that the closed-loop time constant is reduced to $\tau = 1$ ms. What gain K_p is required? Repeat parts c) and d) with this new value of K_p.

f) Suppose we now make the model more realistic by including an additional pole in the plant that was not predicted ahead of time. (This is referred to as unmodeled dynamics.) That is, suppose that the transfer function $X(s)/F(s)$ includes an additional term $H(s) = 1/(\tau_u s+1)$. For the controller gain that you calculated in part e) above, investigate the effect of setting $\tau_u = 1$ msec and $\tau_u = 10$ msec. For each of these two values, answer the following questions: What are the closed-loop pole locations? Sketch the closed-loop step responses due to a step input $x_{ref}(t) = 10^{-3}u_s(t)$ m, under the assumption that $F_d = 0$. Also sketch the closed-loop Bode plots for $X(s)/X_{ref}(s)$. How does the unmodeled additional pole affect the closed-loop system?

g) Now suppose that the unmodeled dynamics include a *double* pole $H(s) = 1/(\tau_u s + 1)^2$. Repeat part f) above for this new additional dynamics in the loop, but here use the Matlab `step` and `bode` commands to create the indicated plots. What generalizations can you develop as to the effect of such additional low-pass filtering in the loop? How do the step and Bode plots vary as you reduce K_p to the lower value computed in part b)?

Problem 5 Consider a motor with the following parameters: torque constant $K = 1$ Nm/A, rotor inertia $J = 10^{-2}$ kg m^2, and coil resistance $R = 5\ \Omega$. A torque disturbance T_d acts on the rotor of the motor in opposition to the motor torque $T_m = Ki$, where i is the motor coil current. The voltage at the motor terminals is $V_m = iR + K\omega$, where ω is the motor angular velocity.

a) We consider the motor as a plant with input V_m and output ω. Draw a block diagram for the motor system which shows these signals as well as i, T_m, T_d, and the back emf voltage $v_b = K\omega$.

b) Now we ask you to design a proportional speed controller of the form

$$V_m = K_p(\omega_r - \omega).$$

Here K_p is the proportional gain of the controller, and ω_r is the speed command. Choose K_p such that the closed-loop system has a time constant of $\tau = 1$ msec. For this value of K_p make plots of V_m and ω when the reference is a unit step $\omega_r(t) = 1u_s(t)$ rad/sec.

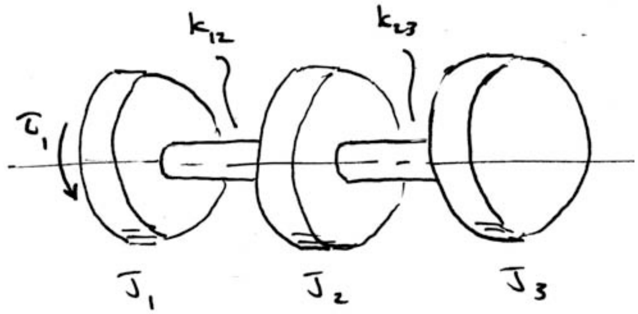
c) Now we ask you to design a proportional plus integral speed controller of the form

$$V_m(s) = K_p(1 + \frac{1}{T_i s})(\omega_r(s) - \omega(s)).$$

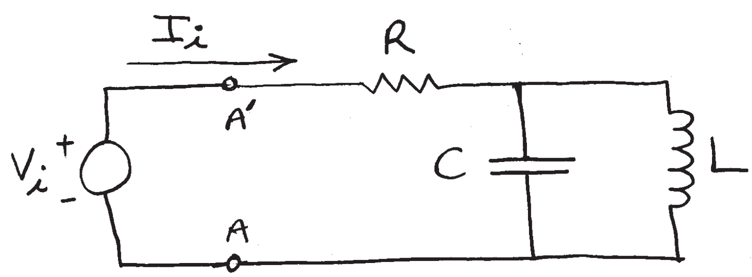
Here K_p is the proportional gain of the controller, and T_i is referred to as the integral time. Choose K_p and T_i such that the closed-loop system has a natural frequency of $\omega_n = 1000$ rad/sec and a damping ratio of $\zeta = 0.4$. For these values make plots of V_m and ω when the reference is a unit step $\omega_r(t) = 1u_s(t)$ rad/sec.

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 Consider a rotor modeled as three lumped inertias J_1, J_2, and J_3 coupled by massless shafts with torsional stiffnesses k_{12} and k_{23} as shown in the figure below. The input to the system is the torque τ_1 applied to J_1, and the outputs are the angles of rotation of the lumped inertias. Suppose $k_{12} = k_{23} = 100$ N·m and $J_1 = J_2 = 10^{-3}$ kg·m^2 and $J_3 = 1.05J_1$. Use Matlab to plot all three Bode plots from the input to the three outputs, respectively. Solve for the mode shapes via the eigenvectors and relate these to the Bode plots. How do the modes and Bode plot change if $J_3 = J_1$ or $J_3 = 0.95J_1$?



Problem G2 For the circuit shown below



a) Solve for the transfer function $Z(s) = V_i(s)/I_i(s)$ in terms of the circuit parameters. It may be helpful to recognize that this transfer function is the impedance seen to the right of the terminals A-A'. Be sure to show your reasoning in your solution.

b) For what complex value(s) s_i does this impedance go to infinity? For what complex values s_j does this impedance go to zero?

c) Sketch a root contour showing the locations of the zeros of $Z(s)$ as a function of resistance R, starting from large values of R and finishing as R approaches zero.

d) Sketch a corresponding set of Bode plots (magnitude and phase) as R varies in several steps from large values to zero. Indicate how the zero locations affect the Bode plot.

Problem G3 The pole-zero plot below shows the singularities of the loop transmission of a feedback loop, which includes a variable gain $K > 0$. The singularities include a zero at the origin, and real-axis poles at $r_1 > 0$ and $r_2 > 0$. Solve for the closed-loop poles of this loop, in terms of the variable gain K. Prove that when the poles are complex-valued, they lie on a circle of radius $\sqrt{r_1 r_2}$ centered on the origin. This is an important pole-zero constellation in root locus sketching: two poles to the right of a real-axis zero will frequently get to the left of the zero by following a circle about the zero. This result does not require that the zero be at the origin; a change of variables should be able to convince you of this.