### 2.3 Conditional Statements

## Objectives

- Analyze statements in if-then form.
- Write the converse, inverse, and contrapositive of if-then statements.


## If-Then Statements

- A conditional statement is a statement that can be written in if then form.

Example: If an animal has hair, then it is a mammal.

- Conditional statements are always written "if $p$, then $q$." The phrase which follows the "if" (p) is called the hypothesis, and the phrase after the "then" (q) is the conclusion.
- We write $p \rightarrow q$, which is read "if $p$, then $q$ " or " $p$ implies $q$."


## Example 1a:

Identify the hypothesis and conclusion of the following statement.
If a polygon has 6 sides, then it is a hexagon.
If a polygon has 6 sides, then it is a hexagon. hypothesis conclusion

Answer: Hypothesis: a polygon has 6 sides
Conclusion: it is a hexagon

## Example 1b:

Identify the hypothesis and conclusion of the following statement.
Tamika will advance to the next level of play if she completes the maze in her computer game.

Answer: Hypothesis: Tamika completes the maze in her computer game Conclusion: she will advance to the next level of play

## Your Turn:

Identify the hypothesis and conclusion of each statement.
a. If you are a baby, then you will cry.

Answer: Hypothesis: you are a baby
Conclusion: you will cry
b. To find the distance between two points, you can use the Distance Formula.

Answer: Hypothesis: you want to find the distance between two points
Conclusion: you can use the Distance Formula

## If-Then Statements

- AS we just witnessed, often conditionals are not written in "if-then" form but in standard form. By identifying the hypothesis and conclusion of a statement, we can translate the statement to "if-then" form for a better understanding.
- When writing a statement in "if-then" form, identify the requirement (condition) to find your hypothesis and the result as your conclusion.


## Example 2a:

Identify the hypothesis and conclusion of the following statement. Then write the statement in the if-then form.

Distance is positive.
Sometimes you must add information to a statement. Here you know that distance is measured or determined.

Answer: Hypothesis: a distance is determined Conclusion: it is positive If a distance is determined, then it is positive.

## Example 2b:

Identify the hypothesis and conclusion of the following statement. Then write the statement in the if-then form.
A five-sided polygon is a pentagon.

Answer: Hypothesis: a polygon has five sides Conclusion: it is a pentagon If a polygon has five sides, then it is a pentagon.

## Your Turn:

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.
a. A polygon with 8 sides is an octagon.

Answer: Hypothesis: a polygon has 8 sides
Conclusion: it is an octagon If a polygon has 8 sides, then it is an octagon.
b. An angle that measures $45^{\circ}$ is an acute angle.

Answer: Hypothesis: an angle measures $45^{\circ}$
Conclusion: it is an acute angle If an angle measures $45^{\circ}$, then it is an acute angle.

## If-Then Statements

- Since a conditional is a statement, it has a truth value. The conditional itself as well as the hypothesis and/or conclusion can be either true or false.


## Example 3a:

Determine the truth value of the following statement for each set of conditions. If Sam rests for 10 days, his ankle will heal.

Sam rests for 10 days, and he still has a hurt ankle.
The hypothesis is true, but the conclusion is false.

Answer: Since the result is not what was expected, the conditional statement is false.

## Example 3b:

Determine the truth value of the following statement for each set of conditions. If Sam rests for 10 days, his ankle will heal.

Sam rests for 3 days, and he still has a hurt ankle.
The hypothesis is false, and the conclusion is false. The statement does not say what happens if Sam only rests for 3 days. His ankle could possibly still heal.

Answer: In this case, we cannot say that the statement is false. Thus, the statement is true.

## Example 3c:

Determine the truth value of the following statement for each set of conditions. If Sam rests for 10 days, his ankle will heal.

Sam rests for 10 days, and he does not have a hurt ankle anymore.

The hypothesis is true since Sam rested for 10 days, and the conclusion is true because he does not have a hurt ankle.

Answer: Since what was stated is true, the conditional statement is true.

## Example 3d:

Determine the truth value of the following statement for each set of conditions. If Sam rests for 10 days, his ankle will heal.

Sam rests for 7 days, and he does not have a hurt ankle anymore.

The hypothesis is false, and the conclusion is true. The statement does not say what happens if Sam only rests for 7 days.

Answer: In this case, we cannot say that the statement is false. Thus, the statement is true.

## Your Turn:

Determine the truth value of the following statements for each set of conditions. If it rains today, then Michael will not go skiing.
a. It does not rain today; Michael does not go skiing. Answer: true
b. It rains today; Michael does not go skiing. Answer: true
c. It snows today; Michael does not go skiing.

Answer: true
d. It rains today; Michael goes skiing.

Answer: false

## If - Then Statements

- From our results in the previous example we can construct a truth table for conditional statements. Notice that a conditional statement is true in all cases except when the conclusion is false.



## Converse, Inverse, and Contrapositive

- From a conditional we can also create additional statements referred to as related conditionals. These include the converse, the inverse, and the contrapositive.


## Converse, Inverse, and Contrapositive

| Statement | Formed by | Symbols | Examples |
| :--- | :--- | :--- | :--- |
| Conditional | Given an <br> hypothesis and <br> conclusion | $\boldsymbol{p \rightarrow \boldsymbol { q }}$ | If 2 angles have the same <br> measure, then they are <br> congruent. |
| Converse | Exchange the <br> hypothesis and <br> conclusion | $\boldsymbol{q} \rightarrow \boldsymbol{p}$ | If 2 angles are congruent, then <br> they have the same measure. |
| Inverse | Negate both the <br> hypothesis and the <br> conclusion | $\sim \boldsymbol{p} \rightarrow \sim \boldsymbol{q}$ | If 2 angles do not have the <br> same measure, then they are <br> not congruent. |
| Contrapositive | Negate both the <br> hypothesis and <br> conclusion of the <br> converse | $\sim \boldsymbol{q} \rightarrow \sim \boldsymbol{p}$ | If 2 angles are not congruent, <br> then they do not have the <br> same measure. |

## Converse, Inverse, and Contrapositive

- Statements that have the same truth value are said to be logically equivalent. We can create a truth table to compare the related conditionals and their relationships.

|  |  | Conditional | Converse | Inverse | Contrapositive |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | $q$ | $p \rightarrow q$ | $q \rightarrow p$ | $\sim p \rightarrow \sim q$ | $\sim q \rightarrow \sim p$ |
| T | T | T | T | T | T |
| T | F | F | T | T | F |
| F | T | T | F | F | T |
| F | F | T | T | T | T |

## Example 4:

Write the converse, inverse, and contrapositive of the statement All squares are rectangles. Determine whether each statement is true or false. If a statement is false, give a counterexample.
First, write the conditional in if-then form.
Conditional: If a shape is a square, then it is a rectangle. The conditional statement is true.

Write the converse by switching the hypothesis and conclusion of the conditional.

Converse: If a shape is a rectangle, then it is a square. The converse is false. A rectangle with $\ell=2$ and $w=4$ is not a square.

## Example 4:

Inverse: If a shape is not a square, then it is not a rectangle. The inverse is false. A 4-sided polygon with side lengths $2,2,4$, and 4 is not a square, but it is a rectangle.

The contrapositive is the negation of the hypothesis and conclusion of the converse.

Contrapositive: If a shape is not a rectangle, then it is not a square. The contrapositive is true.

## Your Turn:

Write the converse, inverse, and contrapositive of the statement The sum of the measures of two complementary angles is 90 . Determine whether each statement is true or false. If a statement is false, give a counterexample.
Answer: Conditional: If two angles are complementary, then the sum of their measures is 90 ; true. Converse: If the sum of the measures of two angles is 90 , then they are complementary; true.
Inverse: If two angles are not complementary, then the sum of their measures is not 90; true. Contrapositive: If the sum of the measures of two angles is not 90 , then they are not complementary; true.

## Assignment

- Geometry:

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