2 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1 Using Inductive and Deductive Reasoning

When you make a conjecture based on a pattern, you use inductive reasoning. You use deductive reasoning to show whether the conjecture is true or false by using facts, definitions, postulates, or proven theorems. If you can find one counterexample to the conjecture, then you know the conjecture is false.

Big Idea 2 Understanding Geometric Relationships in Diagrams

The following can be assumed from the diagram:

A, B, and C are coplanar.

$\angle ABH$ and $\angle HBF$ are a linear pair.

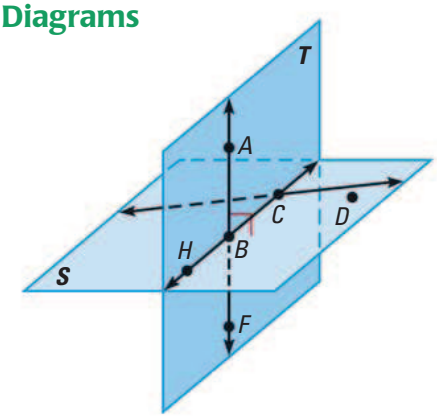
Plane T and plane S intersect in $\overleftrightarrow{BC}$.

$\overleftrightarrow{CD}$ lies in plane S.

$\angle ABC$ and $\angle HBF$ are vertical angles.

$\overleftrightarrow{AB} \perp$ plane S.

Diagram assumptions are reviewed on page 97.



Big Idea 3 Writing Proofs of Geometric Relationships

You can write a logical argument to show a geometric relationship is true. In a two-column proof, you use deductive reasoning to work from GIVEN information to reach a conjecture you want to PROVE.

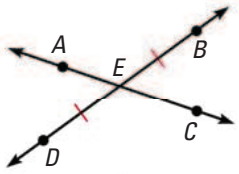


Diagram of geometric relationship with given information labeled to help you write the proof

GIVEN ▶ The hypothesis of an if-then statement

PROVE ▶ The conclusion of an if-then statement

STATEMENTS	REASONS
1. Hypothesis	1. Given
_____	_____
_____	_____
_____	_____
n. Conclusion	*n.* _____

Statements based on facts that you know or conclusions from deductive reasoning

Use postulates, proven theorems, definitions, and properties of numbers and congruence as reasons.

Proof summary is on page 114.

2 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

See pp. 926–931 for a list of postulates and theorems.

- conjecture, *p. 73*
- inductive reasoning, *p. 73*
- counterexample, *p. 74*
- conditional statement, *p. 79*
  converse, inverse, contrapositive
- if-then form, *p. 79*
  hypothesis, conclusion
- negation, *p. 79*
- equivalent statements, *p. 80*
- perpendicular lines, *p. 81*
- biconditional statement, *p. 82*
- deductive reasoning, *p. 87*
- line perpendicular to a plane, *p. 98*
- proof, *p. 112*
- two-column proof, *p. 112*
- theorem, *p. 113*

VOCABULARY EXERCISES

1. Copy and complete: A statement that can be proven is called a(n) __?__.
2. **WRITING** *Compare* the inverse of a conditional statement to the converse of the conditional statement.
3. You know $m\angle A = m\angle B$ and $m\angle B = m\angle C$. What does the Transitive Property of Equality tell you about the measures of the angles?

REVIEW EXAMPLES AND EXERCISES

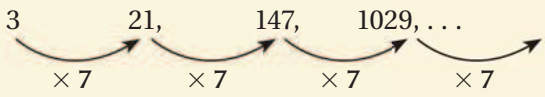
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 2.

2.1 Use Inductive Reasoning *pp. 72–78*

EXAMPLE

Describe the pattern in the numbers 3, 21, 147, 1029, ..., and write the next three numbers in the pattern.

Each number is seven times the previous number.



So, the next three numbers are 7203, 50,421, and 352,947.

EXERCISES

EXAMPLES 2 and 5 on pp. 72–74 for Exs. 4–5

4. *Describe* the pattern in the numbers −20,480, −5120, −1280, −320, ... Write the next three numbers.
5. Find a counterexample to disprove the conjecture:

   If the quotient of two numbers is positive, then the two numbers must both be positive.

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Chapter Review Practice

2.2 Analyze Conditional Statements

pp. 79–85

EXAMPLE

Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Black bears live in North America."

a. If-then form: If a bear is a black bear, then it lives in North America.

b. Converse: If a bear lives in North America, then it is a black bear.

c. Inverse: If a bear is not a black bear, then it does not live in North America.

d. Contrapositive: If a bear does not live in North America, then it is not a black bear.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 80–82 for Exs. 6–8

6. Write the if-then form, the converse, the inverse, and the contrapositive of the statement "An angle whose measure is 34° is an acute angle."

7. Is this a valid definition? *Explain* why or why not.

"If the sum of the measures of two angles is 90°, then the angles are complementary."

8. Write the definition of *equiangular* as a biconditional statement.

2.3 Apply Deductive Reasoning

pp. 87–93

EXAMPLE

Use the Law of Detachment to make a valid conclusion in the true situation.

If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

▶ Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 87–89 for Exs. 9–11

9. Use the Law of Detachment to make a valid conclusion.

If an angle is a right angle, then the angle measures 90°. $\angle B$ is a right angle.

10. Use the Law of Syllogism to write the statement that follows from the pair of true statements.

If $x = 3$, then $2x = 6$.

If $4x = 12$, then $x = 3$.

11. What can you say about the sum of any two odd integers? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

2 CHAPTER REVIEW

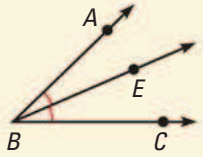
2.4 Use Postulates and Diagrams

pp. 96–102

EXAMPLE

$\angle ABC$, an acute angle, is bisected by $\overrightarrow{BE}$. Sketch a diagram that represents the given information.

1. Draw $\angle ABC$, an acute angle, and label points A, B, and C.
2. Draw angle bisector $\overrightarrow{BE}$. Mark congruent angles.



EXERCISES

EXAMPLES 3 and 4 on p. 98 for Exs. 12–13

12. Straight angle CDE is bisected by $\overrightarrow{DK}$. Sketch a diagram that represents the given information.

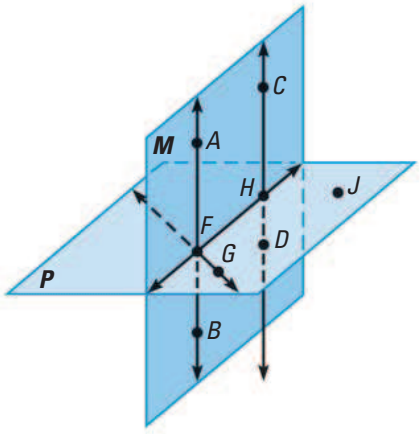
13. Which of the following statements *cannot* be assumed from the diagram?

(A) A, B, and C are coplanar.

(B) $\overleftrightarrow{CD} \perp$ plane P

(C) A, F, and B are collinear.

(D) Plane M intersects plane P in $\overleftrightarrow{FH}$.



2.5 Reason Using Properties from Algebra

pp. 105–111

EXAMPLE

Solve $3x + 2(2x + 9) = -10$. Write a reason for each step.

$3x + 2(2x + 9) = -10$	**Write original equation.**
$3x + 4x + 18 = -10$	**Distributive Property**
$7x + 18 = -10$	**Simplify.**
$7x = -28$	**Subtraction Property of Equality**
$x = -4$	**Division Property of Equality**

EXERCISES

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 14–17

Solve the equation. Write a reason for each step.

14. $-9x - 21 = -20x - 87$

15. $15x + 22 = 7x + 62$

16. $3(2x + 9) = 30$

17. $5x + 2(2x - 23) = -154$

2.6 Prove Statements about Segments and Angles

pp. 112–119

EXAMPLE

Prove the Reflexive Property of Segment Congruence.

GIVEN ▶ $\overline{AB}$ is a line segment.

PROVE ▶ $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. $\overline{AB}$ is a line segment.	**1.** Given
2. AB is the length of $\overline{AB}$.	**2.** Ruler Postulate
3. $AB = AB$	**3.** Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	**4.** Definition of congruent segments

EXERCISES

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 18–21

Name the property illustrated by the statement.

18. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$.

19. $\angle C \cong \angle C$

20. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

21. Prove the Transitive Property of Angle Congruence.

2.7 Prove Angle Pair Relationships

pp. 124–131

EXAMPLE

GIVEN ▶ $\angle 5 \cong \angle 6$

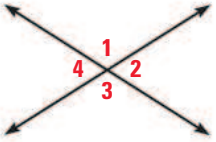
PROVE ▶ $\angle 4 \cong \angle 7$

STATEMENTS	REASONS
1. $\angle 5 \cong \angle 6$	**1.** Given
2. $\angle 4 \cong \angle 5$	**2.** Vertical Angles Congruence Theorem
3. $\angle 4 \cong \angle 6$	**3.** Transitive Property of Congruence
4. $\angle 6 \cong \angle 7$	**4.** Vertical Angles Congruence Theorem
5. $\angle 4 \cong \angle 7$	**5.** Transitive Property of Congruence

EXERCISES

EXAMPLES 2 and 3 on pp. 125–126 for Exs. 22–24

In Exercises 22 and 23, use the diagram at the right.



22. If $m\angle 1 = 114°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

23. If $m\angle 4 = 57°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

24. Write a two-column proof.

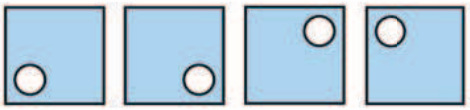
GIVEN ▶ $\angle 3$ and $\angle 2$ are complementary.
$m\angle 1 + m\angle 2 = 90°$

PROVE ▶ $\angle 3 \cong \angle 1$

2 CHAPTER TEST

Sketch the next figure in the pattern.

1.



2.

Describe **the pattern in the numbers. Write the next number.**

3. $-6, -1, 4, 9, \ldots$

4. $100, -50, 25, -12.5, \ldots$

In Exercises 5–8, write the if-then form, the converse, the inverse, and the contrapositive for the given statement.

5. All right angles are congruent.

6. Frogs are amphibians.

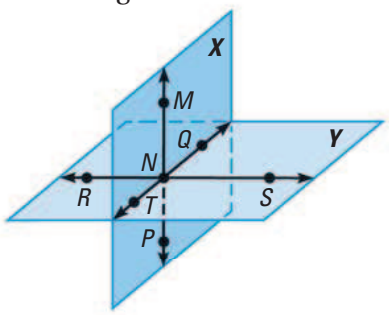
7. $5x + 4 = -6$, because $x = -2$.

8. A regular polygon is equilateral.

9. If you decide to go to the football game, then you will miss band practice. Tonight, you are going the football game. Using the Law of Detachment, what statement can you make?

10. If Margot goes to college, then she will major in Chemistry. If Margot majors in Chemistry, then she will need to buy a lab manual. Using the Law of Syllogism, what statement can you make?

Use the diagram to write examples of the stated postulate.



11. A line contains at least two points.

12. A plane contains at least three noncollinear points.

13. If two planes intersect, then their intersection is a line.

Solve the equation. Write a reason for each step.

14. $9x + 31 = -23$

15. $-7(-x + 2) = 42$

16. $26 + 2(3x + 11) = -18x$

In Exercises 17–19, match the statement with the property that it illustrates.

17. If $\angle RST \cong \angle XYZ$, then $\angle XYZ \cong \angle RST$.

18. $\overline{PQ} \cong \overline{PQ}$

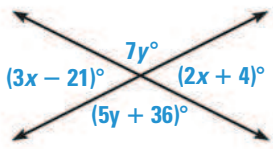
19. If $\overline{FG} \cong \overline{JK}$ and $\overline{JK} \cong \overline{LM}$, then $\overline{FG} \cong \overline{LM}$.

A. Reflexive Property of Congruence

B. Symmetric Property of Congruence

C. Transitive Property of Congruence

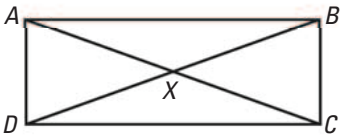
20. Use the Vertical Angles Congruence Theorem to find the measure of each angle in the diagram at the right.



21. Write a two-column proof.

GIVEN ▶ $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

PROVE ▶ $\overline{AC} \cong \overline{BD}$



2 ALGEBRA REVIEW

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SIMPLIFY RATIONAL AND RADICAL EXPRESSIONS

EXAMPLE 1 Simplify rational expressions

a. $\frac{2x^2}{4xy}$ **b.** $\frac{3x^2 + 2x}{9x + 6}$

Solution

To simplify a rational expression, factor the numerator and denominator. Then divide out any common factors.

a. $\frac{2x^2}{4xy} = \frac{2 \cdot x \cdot x}{2 \cdot 2 \cdot x \cdot y} = \frac{x}{2y}$ **b.** $\frac{3x^2 + 2x}{9x + 6} = \frac{x(3x + 2)}{3(3x + 2)} = \frac{x}{3}$

EXAMPLE 2 Simplify radical expressions

a. $\sqrt{54}$ **b.** $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5}$ **c.** $(3\sqrt{2})(-6\sqrt{6})$

Solution

a. $\sqrt{54} = \sqrt{9} \cdot \sqrt{6}$ Use product property of radicals.

$= 3\sqrt{6}$ Simplify.

b. $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5} = -\sqrt{5} - 5\sqrt{2}$ Combine like terms.

c. $(3\sqrt{2})(-6\sqrt{6}) = -18\sqrt{12}$ Use product property and associative property.

$= -18 \cdot 2\sqrt{3}$ Simplify $\sqrt{12}$.

$= -36\sqrt{3}$ Simplify.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Simplify the expression, if possible.

1. $\frac{5x^4}{20x^2}$
2. $\frac{-12ab^3}{9a^2b}$
3. $\frac{5m + 35}{5}$
4. $\frac{36m - 48m}{6m}$
5. $\frac{k + 3}{-2k + 3}$
6. $\frac{m + 4}{m^2 + 4m}$
7. $\frac{12x + 16}{8 + 6x}$
8. $\frac{3x^3}{5x + 8x^2}$
9. $\frac{3x^2 - 6x}{6x^2 - 3x}$

EXAMPLE 2 for Exs. 10–24

Simplify the expression, if possible. All variables are positive.

10. $\sqrt{75}$
11. $-\sqrt{180}$
12. $\pm\sqrt{128}$
13. $\sqrt{2} - \sqrt{18} + \sqrt{6}$
14. $\sqrt{28} - \sqrt{63} - \sqrt{35}$
15. $4\sqrt{8} + 3\sqrt{32}$
16. $(6\sqrt{5})(2\sqrt{2})$
17. $(-4\sqrt{10})(-5\sqrt{5})$
18. $(2\sqrt{6})^2$
19. $\sqrt{(25)^2}$
20. $\sqrt{x^2}$
21. $\sqrt{-(a)^2}$
22. $\sqrt{(3y)^2}$
23. $\sqrt{3^2 + 2^2}$
24. $\sqrt{h^2 + k^2}$

2 ★ Standardized TEST PREPARATION

EXTENDED RESPONSE QUESTIONS

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors, *or*
- solution is without error but incomplete

No Credit
- no solution is given, *or*
- solution makes no sense

PROBLEM

Seven members of the student government (Frank, Gina, Henry, Isabelle, Jack, Katie, and Leah) are posing for a picture for the school yearbook. For the picture, the photographer will arrange the students in a row according to the following restrictions:

Henry must stand in the middle spot.

Katie must stand in the right-most spot.

There must be exactly two spots between Gina and Frank.

Isabelle cannot stand next to Henry.

Frank must stand next to Katie.

a. *Describe* one possible ordering of the students.

b. Which student(s) can stand in the second spot from the left?

c. If the condition that Leah must stand in the left-most spot is added, will there be exactly one ordering of the students? *Justify* your answer.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The method of representation is clearly explained.

a. Using the first letters of the students' names, here is one possible ordering of the students:

I L G H J F K

The conclusion is correct and shows understanding of the problem.

b. The only students without fixed positions are Isabelle, Leah, and Jack. There are no restrictions on placement in the second spot from the left, so any of these three students can occupy that location.

c. Henry, Frank, Katie, and Gina have fixed positions according to the restrictions. If Leah must stand in the left-most spot, the ordering looks like:

L _ G H _ F K

The reasoning behind the answer is explained clearly.

Because Isabelle cannot stand next to Henry, she must occupy the spot next to Leah. Therefore, Jack stands next to Henry and the only possible order would have to be:

L I G H J F K.

Yes, there would be exactly one ordering of the students.

SAMPLE 2: Partial credit solution

a. One possible ordering of the students is:
Jack, Isabelle, Gina, Henry, Leah, Frank, and Katie.

b. There are three students who could stand in the second spot from the left. They are Isabelle, Leah, and Jack.

c. No, there would be two possible orderings of the students. With Leah in the left-most spot, the ordering looks like:

Leah, ____, Gina, Henry, ____, Frank, and Katie

Therefore, the two possible orderings are
Leah, Isabelle, Gina, Henry, Jack, Frank, and Katie
or
Leah, Jack, Gina, Henry, Isabelle, Frank, and Katie.

The answer to part (a) is correct.

Part (b) is correct but not explained.

The student did not recall that Isabelle cannot stand next to Henry; therefore, the conclusion is incorrect.

SAMPLE 3: No credit solution

a. One possible ordering of the students is **L G J H I F K**.

b. There are four students who can stand in the second spot from the left. Those students are Leah, Gina, Isabelle, and Jack.

c. The two possible orderings are **L G J H I F K** and **L J G H I F K**.

The answer to part (a) is incorrect because Isabelle is next to Henry.

Parts (b) and (c) are based on the incorrect conclusion in part (a).

PRACTICE

Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, explain how you would change the solution so that it earns a score of full credit.

a. A possible ordering of the students is I - J - G - H - L - F - K.

b. There are no restrictions on the second spot from the left. Leah, Isabelle, and Jack could all potentially stand in this location.

c. The positions of Gina, Henry, Frank, and Katie are fixed.

_ - _ - G - H - _ - F - K.

Because Isabelle cannot stand next to Henry, she must occupy the left-most spot or the second spot from the left. There are no restrictions on Leah or Jack. That leaves four possible orderings:

I - J - G - H - L - F - K
L - I - G - H - J - F - K
I - L - G - H - J - F - K
J - I - G - H - L - F - K.

If the restriction is added that Leah must occupy the left-most spot, there is exactly one ordering that would satisfy all conditions:

L - I - G - H - J - F - K.

2 ★ Standardized TEST PRACTICE

EXTENDED RESPONSE

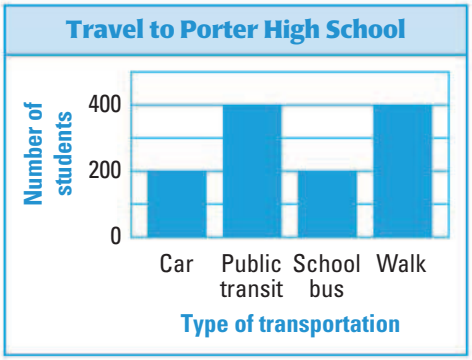
1. In some bowling leagues, the handicap H of a bowler with an average score A is found using the formula $H = \frac{4}{5}(200 - A)$. The handicap is then added to the bowler's score.

a. Solve the formula for A. Write a reason for each step.

b. Use your formula to find a bowler's average score with a handicap of 12.

c. Using this formula, is it possible to calculate a handicap for a bowler with an average score above 200? *Explain* your reasoning.

2. A survey was conducted at Porter High School asking students what form of transportation they use to go to school. All students in the high school were surveyed. The results are shown in the bar graph.



a. Does the statement "About 1500 students attend Porter High School" follow from the data? *Explain*.

b. Does the statement "About one third of all students at Porter take public transit to school" follow from the data? *Explain*.

c. John makes the conclusion that Porter High School is located in a city or a city suburb. *Explain* his reasoning and tell if his conclusion is the result of *inductive reasoning* or *deductive reasoning*.

d. Betty makes the conclusion that there are twice as many students who walk as take a car to school. *Explain* her reasoning and tell if her conclusion is the result of *inductive reasoning* or *deductive reasoning*.

3. The senior class officers are planning a meeting with the principal and some class officers from the other grades. The senior class president, vice president, treasurer, and secretary will all be present. The junior class president and treasurer will attend. The sophomore class president and vice president, and freshmen treasurer will attend. The secretary makes a seating chart for the meeting using the following conditions.

The principal will sit in chair 10. The senior class treasurer will sit at the other end.

The senior class president will sit to the left of the principal, next to the junior class president, and across from the sophomore class president.

All three treasurers will sit together. The two sophomores will sit next to each other.

The two vice presidents and the freshman treasurer will sit on the same side of the table.

a. Draw a diagram to show where everyone will sit.

b. *Explain* why the senior class secretary must sit between the junior class president and junior class treasurer.

c. Can the senior class vice-president sit across from the junior class president? *Justify* your answer.

STATE TEST PRACTICE
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MULTIPLE CHOICE

4. If d represents an odd integer, which of the expressions represents an even integer?

Ⓐ $d + 2$

Ⓑ $2d - 1$

Ⓒ $3d + 1$

Ⓓ $3d + 2$

5. In the repeating decimal 0.23142314. . . , where the digits 2314 repeat, which digit is in the 300th place to the right of the decimal point?

Ⓐ 1

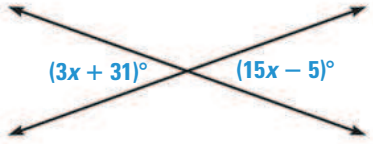
Ⓑ 2

Ⓒ 3

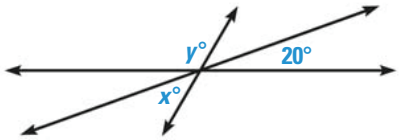
Ⓓ 4

GRIDDED ANSWER

6. Use the diagram to find the value of x.



7. Three lines intersect in the figure shown. What is the value of $x + y$?



8. R is the midpoint of $\overline{PQ}$, and S and T are the midpoints of $\overline{PR}$ and $\overline{RQ}$, respectively. If $ST = 20$, what is PT?

SHORT RESPONSE

9. Is this a correct conclusion from the given information? If so, *explain* why. If not, *explain* the error in the reasoning.

If you are a soccer player, then you wear shin guards. Your friend is wearing shin guards. Therefore, she is a soccer player.

10. *Describe* the pattern in the numbers. Write the next number in the pattern.

192, −48, 12, −3, . . .

11. Points A, B, C, D, E, and F are coplanar. Points A, B, and F are collinear. The line through A and B is perpendicular to the line through C and D, and the line through C and D is perpendicular to the line through E and F. Which four points must lie on the same line? *Justify* your answer.

12. Westville High School offers after-school tutoring with five student volunteer tutors for this program: Jen, Kim, Lou, Mike, and Nina. On any given weekday, three tutors are scheduled to work. Due to the students' other commitments after school, the tutoring work schedule must meet the following conditions.

Jen can work any day except every other Monday and Wednesday.

Kim can only work on Thursdays and Fridays.

Lou can work on Tuesdays and Wednesdays.

Mike cannot work on Fridays.

Nina cannot work on Tuesdays.

Name three tutors who can work on *any* Wednesday. *Justify* your answer.