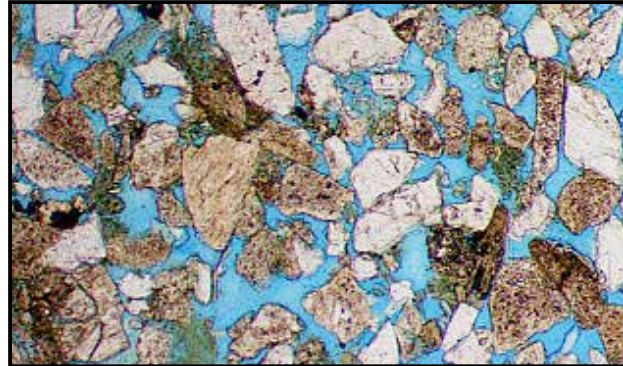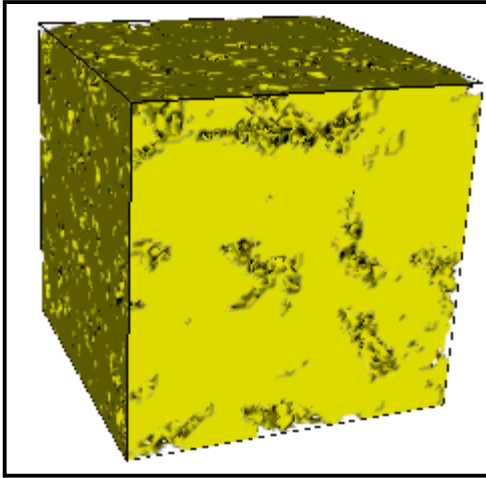# 2. Definitions and Methods

ROCK PHYSICS DEFINITIONS

Volumetrics: Density, Porosity, Saturation

3D volume of sandstone (left) and a sandstone thin section (right). The pore space is blue due to epoxy that impregnated the samples.

Equations for density, porosity, and saturation

Bulk Density

$$\rho_{Bulk} = M_{Total} / V_{Total}$$

Total Porosity

$$\phi_{Total} = V_{Pore} / V_{Total} \equiv \phi$$

Saturation

$$S_{Fluid} = V_{Fluid} / V_{Pore}$$
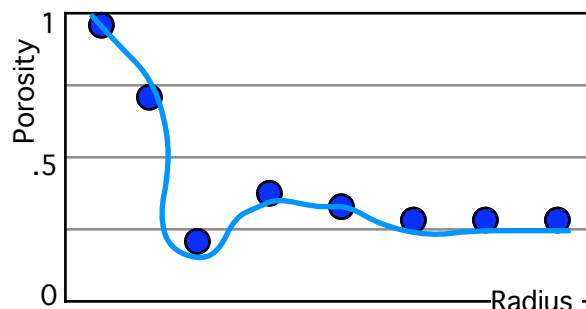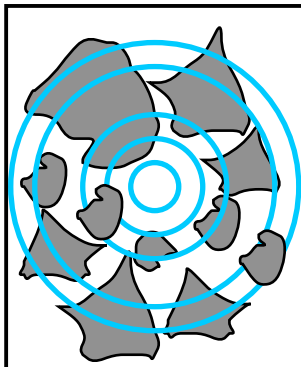
M is mass
V is volume

$$S_{Water} + S_{Oil} + S_{Gas} = 1$$

Bulk Density

$$\rho_{Bulk} = (1 - \phi)\rho_{Solid} + \phi\rho_{Fluid} =$$

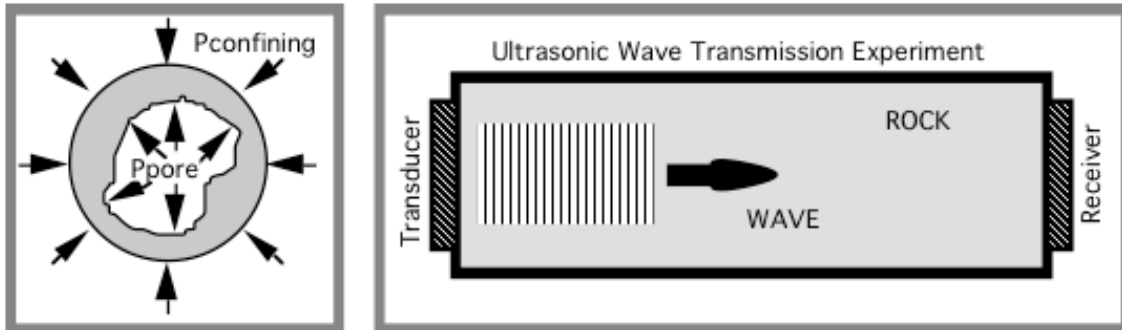$$(1 - \phi)\rho_{Solid} + \phi(S_{Water}\rho_{Water} + S_{Oil}\rho_{Oil} + S_{Gas}\rho_{Gas})$$

Porosity strongly depends on the scale of measurement because rock is heterogeneous at all scales.
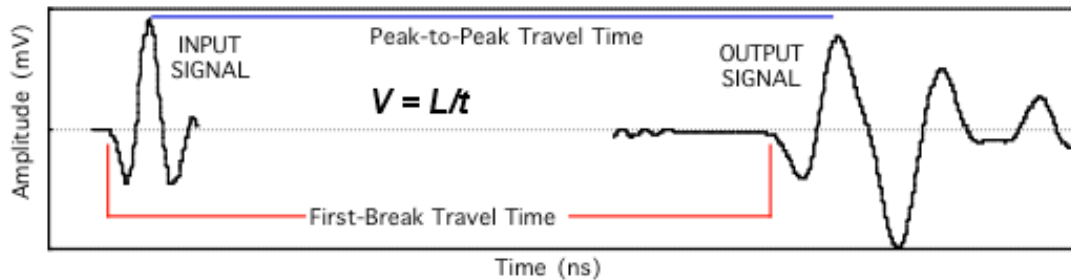
ROCK PHYSICS DEFINITIONS

Velocity

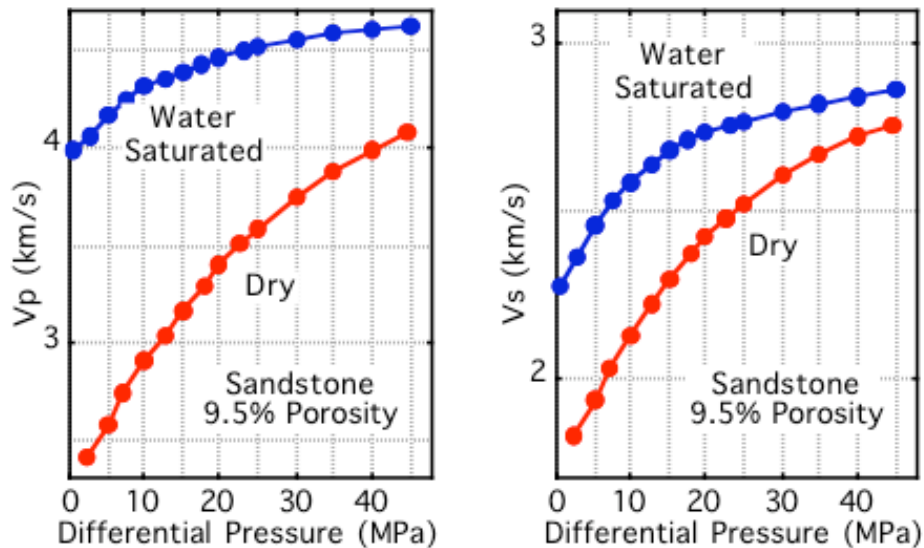**Wave transmission experiment through porous rock**

Pconfining

Ppore

Ultrasonic Wave Transmission Experiment

Transducer

ROCK

WAVE

Receiver

Velocity is defined as the flight time of an elastic signal. There are two types of elastic waves: P-waves and S-waves

Amplitude (mV)

INPUT SIGNAL

Peak-to-Peak Travel Time

OUTPUT SIGNAL

$$V = L/t$$

First-Break Travel Time

Time (ns)

In the laboratory, velocity is measured at changing sample conditions that are typically pressure and saturation. In the field we cannot change conditions. Instead, we measure velocity at many locations (well logging).

Below are the results of a typical lab experiment: Vp and Vs versus pressure and saturation

Vp (km/s)

Water Saturated

Dry

Sandstone 9.5% Porosity

Differential Pressure (MPa)

Vs (km/s)

Water Saturated

Dry

Sandstone 9.5% Porosity

Differential Pressure (MPa)

$$P_{differential} = P_{confining} - P_{pore}$$

ROCK PHYSICS BASICS

Normal Reflection
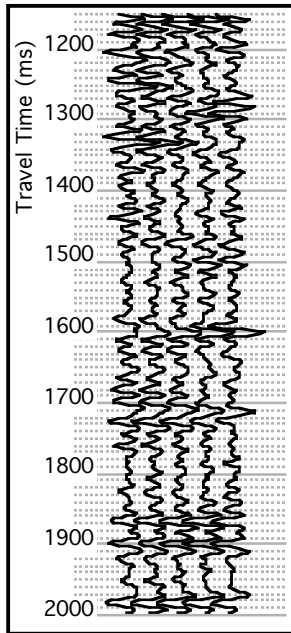
A reflection seismogram is a superposition of signals reflected from interfaces between earth layers of different elastic properties.

It is useful, therefore, to examine reflection at a single interface between two elastic half-spaces.

Two important elastic parameters that affect reflection are derived from velocity and density. They are the acoustic (or P-) impedance I_p and Poisson's ratio ν

$$I_p = \rho V_p$$

$$\nu = \frac{1}{2}\frac{(V_p / V_s)^2 - 2}{(V_p / V_s)^2 - 1}$$

Normal Incidence. The reflection amplitude of a normal-incidence P-wave at the interface between two infinite half-spaces depends on the difference between the impedances of the half-spaces. The same law applies to S-wave reflection.
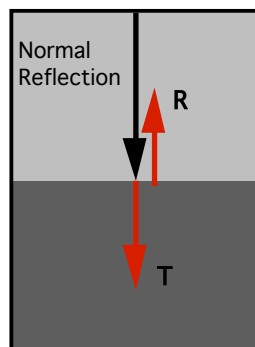
$$R(0) = \frac{I_{p2} - I_{p1}}{I_{p2} + I_{p1}} = \frac{dI_p}{2I_p} = \frac{1}{2}d\ln I_p$$
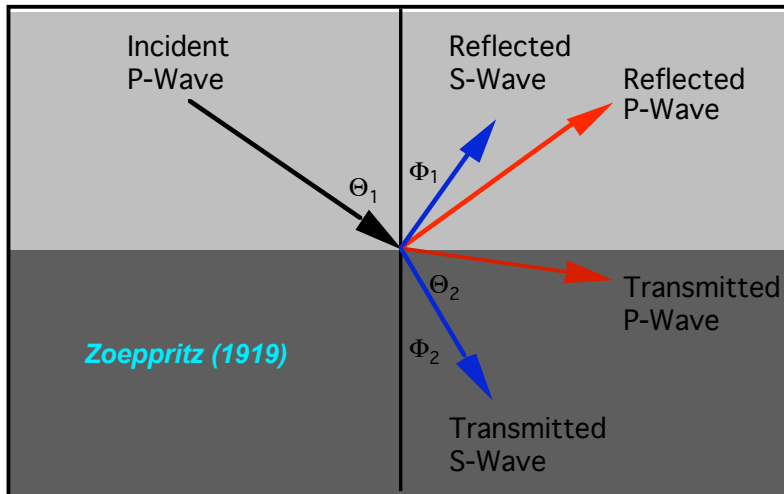
Zoeppritz (1919)

Normal reflection forms a **full differential**. Therefore, it can be integrated to arrive at the absolute values of P-wave impedance. This procedure is called *impedance inversion*.

ROCK PHYSICS BASICS

Reflection at an Angle

The reflection amplitude of a wave propagating at an angle to an interface not only depends on the P-wave elastic properties of the half-spaces but also on the S-wave properties. The rigorous Zoeppritz equations can be linearized to express the reflected amplitude through the P-wave impedance and Poisson's ratio difference.

Incident P-Wave

Reflected S-Wave

Reflected P-Wave

Θ_1

Φ_1

Θ_2

Transmitted P-Wave

Zoeppritz (1919)

Φ_2

Transmitted S-Wave

A possible simplification for AVO computation. There are many other approximate expressions for reflection at an angle.

$$R(\theta) \approx R(0)\cos^2\theta + d(\frac{1}{1-\nu})\sin^2\theta$$

The exact Zoeppritz equations do not form a full differential. However, their various linearizations do. As a result, these full differentials can be integrated to arrive at a quantity that is called the elastic impedance. It is important to remember that in most cases, the units of the elastic impedance are not the same as of the acoustic impedance. The equation below may serve as a definition of the elastic impedance I_e.

$$R(\theta) \overset{Def}{\equiv} \frac{1}{2}d\ln I_e \approx R(0)\cos^2\theta + d(\frac{1}{1-\nu})\sin^2\theta =$$

$$\frac{1}{2}d\ln I_p + d(\frac{1}{1-\nu} - \frac{1}{2}\ln I_p)\sin^2\theta$$

$$\Rightarrow \ln I_e = \ln I_p + (\frac{2}{1-\nu} - \ln I_p)\sin^2\theta$$

Parameters commonly used to characterize reflection at an angle are intercept and gradient.

$$R(\theta) \approx d(\frac{\ln I_p}{2}) + d(\frac{1}{1-\nu} - \frac{\ln I_p}{2})\sin^2\theta$$

Intercept   Gradient

ROCK PHYSICS BASICS

Reflection at an Angle

A convenient approximation for reflection at an angle belongs to Hilterman (1989). It assumes that, on average, Poisson's ratio is 1/3.

$$R(\theta) \approx R(0)\cos^2\theta + 2.25\Delta\nu\sin^2\theta$$

It follows from this approximation that if the change in Poisson's ratio is negative, the amplitude should decrease with offset. This is what happens between shale and gas sand. Depending on the rock properties, 3 AVO classes have been defined.
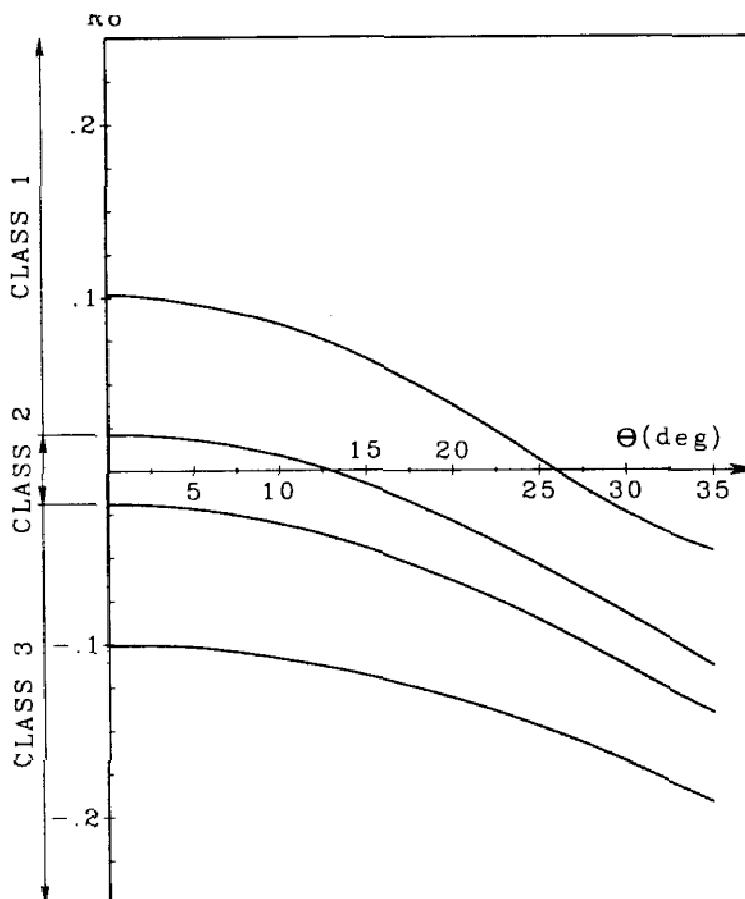
Zoeppritz P-wave reflection coefficients for a shale over gas sand interface for a range of normal reflectivity values. The Poisson's ratio and density were assumed to be 0.38 and 2.4 g/cc for shale and 0.15 and 2.0 g/cc for gas sand (from Rutherford and Williams, 1989).

ROCK PHYSICS BASICS

Relative Impedance Inversion

The Hilterman AVO Approximation. *Hilterman (1989) shows that for not very large angles of incidence and if Poisson's ratio v is about 0.33, the exact Zoeppritz (1919) equation for the P-to-P reflection $R_{pp}(\theta)$ is approximately*

$$R_{pp}(\theta) \approx R_{pp}(0)\cos^2\theta + 2.25\Delta v \sin^2\theta$$
$$\approx R_{pp}(0) + [2.25\Delta v - R_{pp}(0)]\sin^2\theta,$$

where θ is the angle of incidence; $R_{pp}(0)$ is the normal reflection; and Δv is the difference of the Poisson's ratio between the lower and upper half-space.

The normal reflection $R_{pp}(0)$ is

$$R_{pp}(0) = \frac{I_{p2} - I_{p1}}{I_{p2} + I_{p1}} = \frac{dI_p}{2I_p} = \frac{1}{2}d\ln I_p,$$

where I_{p1} is the P-wave impedance in the upper half-space while I_{p2} is that in the lower half-space.

By integrating the last equation we find

$$I_p = \exp[2\int R_{pp}(0)dz].$$

This equation provides a relative impedance because the reflectivity depends on the impedance contrast rather than its absolute value. To convert it to a more realistic impedance map, it has to be normalized by impedance in a well.
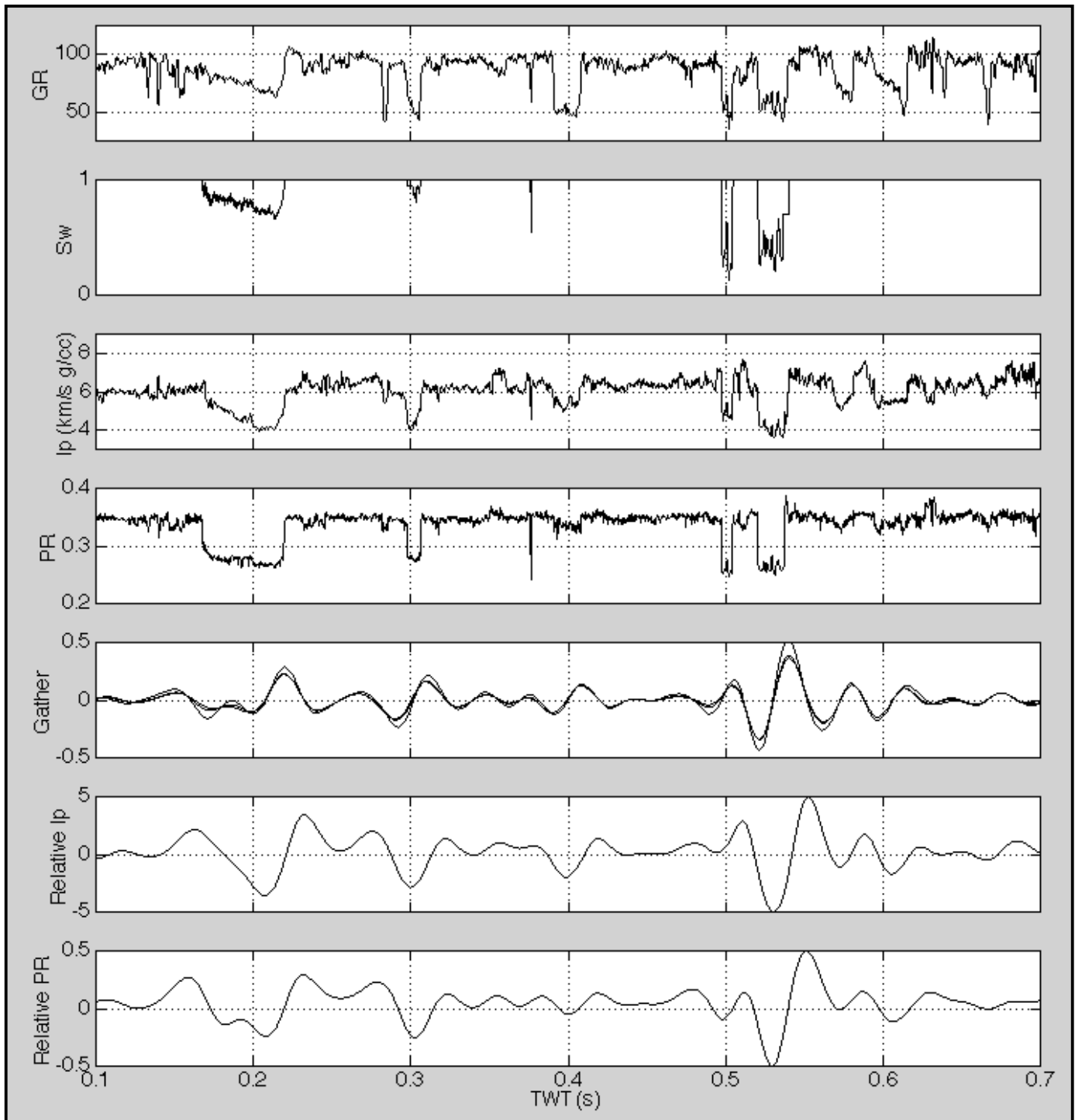
In a similar fashion, we find from Hilterman's approximation that

$$\Delta v = \frac{R_{pp}(\theta) - R_{pp}(0)\cos^2\theta}{2.25\sin^2\theta},$$
$$v = \int \frac{R_{pp}(\theta) - R_{pp}(0)\cos^2\theta}{2.25\sin^2\theta}dz.$$

As with I_p, this Poisson's ratio is not absolute and needs to be normalized by v in a well.

ROCK PHYSICS BASICS

Relative Impedance Example

Relative seismically-derived impedance and Poisson's ratio do not provide the absolute values of these elastic constants. However, they are often useful in obtaining spatial shapes of impedance and Poisson's ratio.
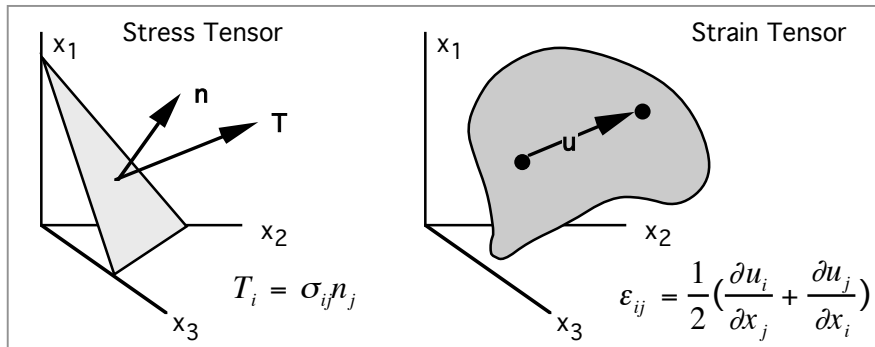
ROCK PHYSICS BASICS

Elasticity

Stress and strain. Forces acting within a mechanical body are mathematically characterized by the stress tensor which is a 3x3 matrix. By using a stress tensor we can find the vector of traction acting on an elemental plane of any orientation within the body (figure on the left).

The deformation within the body is characterized by the strain tensor. This tensor is formed by the derivatives of the components of the displacement of a material point in the body (figure on the right).

Stress Tensor

Strain Tensor

$$T_i = \sigma_{ij} n_j$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Both stress and strain tensors are symmetrical matrices.

$$\sigma_{ij} = \sigma_{ji} \ i \neq j; \ \varepsilon_{ij} = \varepsilon_{ji} \ i \neq j.$$

Hooke's law relates stress to strain. It postulates that this relation is linear. In general, there are 21 independent elastic constants that linearly relate stress to strain.

$$\sigma_{ij} = c_{ijkl} e_{kl}; \ c_{ijkl} = c_{jikl} = c_{ijlk} = c_{jilk}, \ c_{ijkl} = c_{klij}.$$

Fortunately, if a body is *isotropic*, only two independent elastic constants are required. These constants are called *elastic moduli*.

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{\alpha\alpha} + 2\mu \varepsilon_{ij}; \ \varepsilon_{ij} = \left[(1+\nu)\sigma_{ij} - \nu \delta_{ij} \sigma_{\alpha\alpha}\right]/E.$$

Commonly used constants are: λ and μ -- Lame's constants; ν -- Poisson's ratio; E -- Young's modulus. The elastic moduli are determined by the experiment performed. For example, the bulk modulus is measured in the hydrostatic compression experiment. The shear modulus is measured in the shear deformation experiment.

Bulk Modulus	Shear Modulus	Compressional Modulus
Hydrostatic Loading	*Distortion, no volume change*	*No lateral deformation*

$$K = \lambda + 2\mu / 3$$

Shear

$$\sigma_{xz} = 2\mu \varepsilon_{xz}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = 0$$

$$M = K + (4/3)\mu$$

Elastic moduli derived from loading experiments are called static moduli

ROCK PHYSICS BASICS

Elastic-Wave Velocity

An elastic stress wave can propagate through an elastic body and generate stress and strain disturbance. The magnitude of deformation generated by a propagating wave is usually very small, on the order of 10^{-7}. As a result, the stress perturbation is also very small, much smaller than the ambient state of stress.

The speed of an elastic wave is related to the elastic moduli via the wave equation (below) where u is displacement, t is time, z is the spatial coordinate, M is the elastic modulus, and ρ is the density.

$$\frac{\partial^2 u}{\partial t^2} = \frac{M}{\rho}\frac{\partial^2 u}{\partial z^2}$$
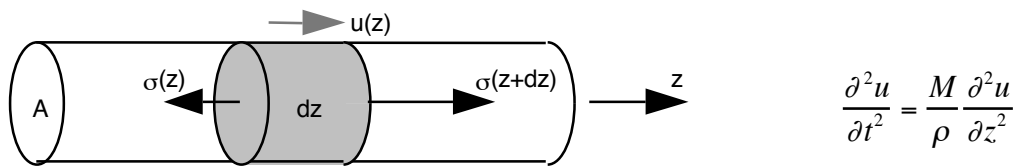
It follows from the wave equation that the speed of wave propagation is

$$\sqrt{M/\rho}$$

$$V_p = \sqrt{M/\rho} = \sqrt{(K + 4G/3)/\rho}$$

$$V_s = \sqrt{G/\rho}$$

$$M = \rho V_p^2; \quad G = \rho V_s^2; \quad K = \rho(V_p^2 - 4V_s^2/3); \quad \lambda = \rho(V_p^2 - 2V_s^2).$$

Poisson's Ratio

$$\nu = \frac{1}{2}\frac{(V_p/V_s)^2 - 2}{(V_p/V_s)^2 - 1}$$

M is the compressional modulus or M-modulus
G (or μ) is the shear modulus
K is the bulk modulus
E is Young's modulus
ν is Poisson's ratio
λ is Lame's constant

Elastic moduli derived from velocity data are called dynamic moduli

ROCK PHYSICS BASICS

Relations between Elastic Constants

> **Poisson's ratio relates to the ratios of various elastic moduli and elastic-wave velocities.**

$$\nu = \frac{1}{2}\frac{(V_p/V_s)^2 - 2}{(V_p/V_s)^2 - 1} = \frac{1}{2}\frac{M/G - 2}{M/G - 1} = \frac{1}{2}\frac{K/G - 2/3}{K/G + 1/3} = \frac{1}{2}\frac{\lambda/G}{\lambda/G + 1}$$

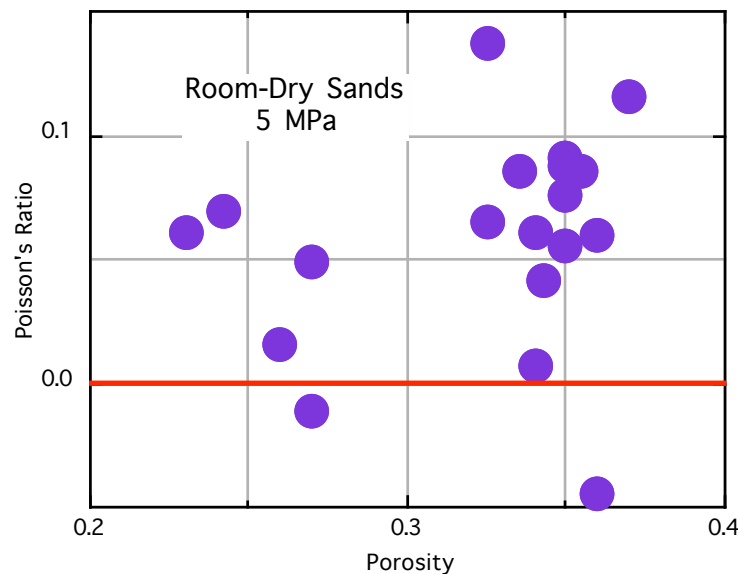> **Theoretically, PR may vary between -1 and 0.5**

$$-1 \leq \nu \leq 0.5$$

$$\nu = -1 \Rightarrow M = \frac{4}{3}G; \quad V_p = \frac{2}{\sqrt{3}}V_s; \quad K = 0; \quad \lambda = -\frac{2}{3}G$$

$$\nu = 0 \Rightarrow M = 2G; \quad V_p = \sqrt{2}V_s; \quad K = \frac{2}{3}G; \quad \lambda = 0$$

$$\nu = 0.5 \Rightarrow V_s = 0 | K = \infty$$

> **Sometimes, the dynamic PR of dry sand at low pressure may appear negative.**
>
> **Plot below shows PR versus porosity in dry unconsolidated sand at low differential pressure. This may mean (a) wrong data or (b) anisotropic rock.**

ROCK PHYSICS BASICS

Static and Dynamic Moduli

By definition, the *dynamic moduli* of rock are those calculated from the elastic-wave velocity and density. The *static moduli* are those directly measured in a deformational experiment.

The static and dynamic moduli of the same rock may significantly differ from each other. The main reason is likely to be the difference in the *deformation (strain) amplitude* between the *dynamic* and *static* experiments.

In the dynamic wave propagation experiment the strain is about 10^{-7} while static strain may reach 10^{-2}.

σ Radial Strain ε_r **Stress**

$$E = \sigma / \varepsilon_a$$

$$\nu = \varepsilon_r / \varepsilon_a$$

ε_a Axial Strain

Typical Plastic Behavior

Stress vs. Log Strain

Load — DYNAMIC — STATIC — Unload

10^{-7} 10^{-2}

Log Strain
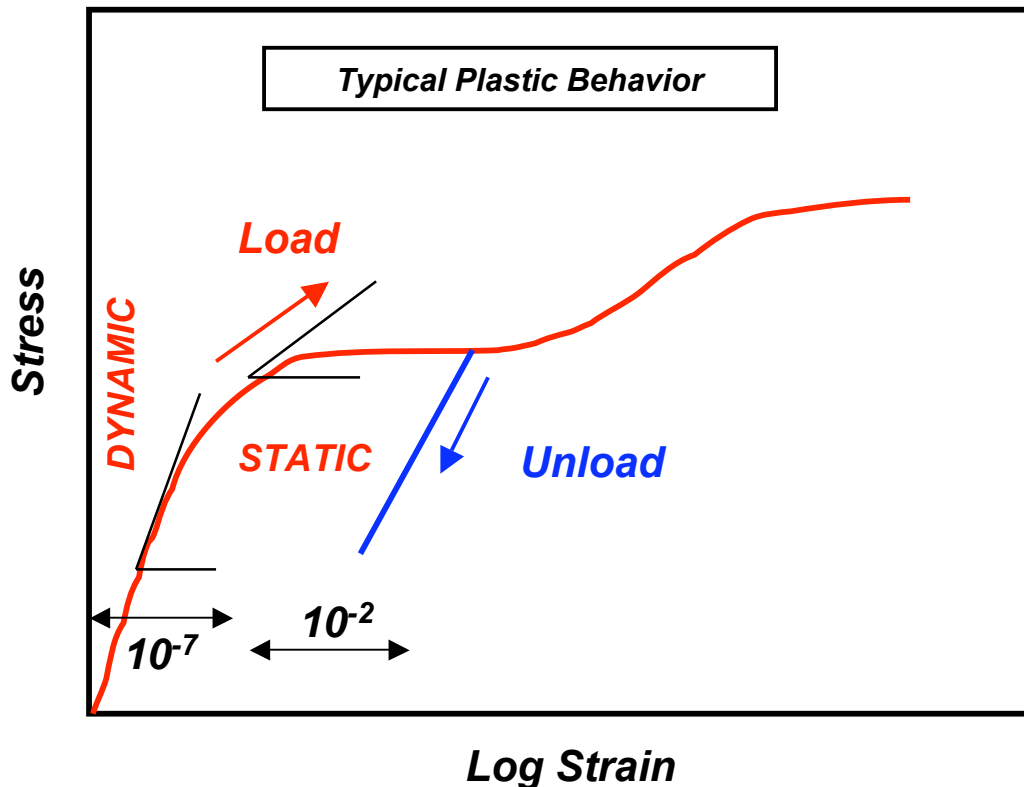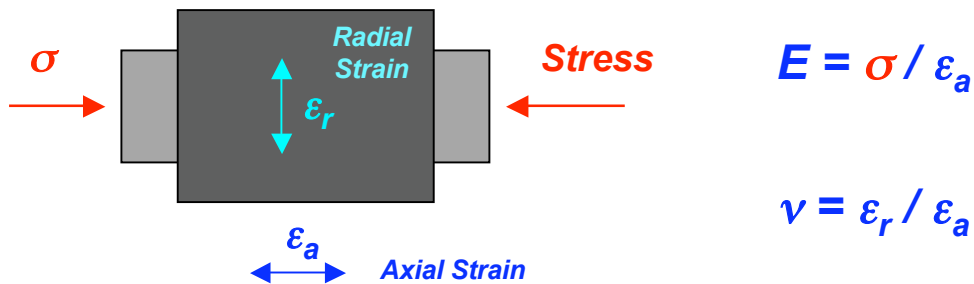
ROCK PHYSICS BASICS

Need for Static Moduli

Static moduli are often used in wellbore stability and in-situ stress applications to evaluate the possibility of breakouts, elevated pore pressure, and tectonic stress distribution. For example, a common method of calculating the horizontal stress in earth is by assuming that the earth is elastic and does not deform in the horizontal direction.

S_V

S_H
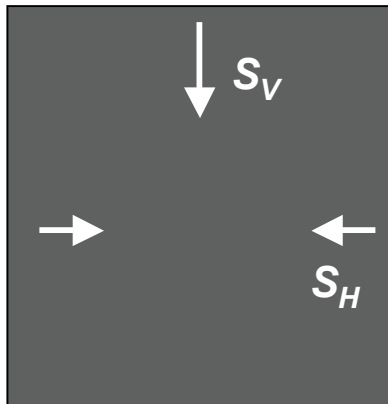
Horizontal Stress

Poisson's Ratio

$$S_H = S_V \frac{\nu}{1 - \nu}$$

Vertical (Overburden) Stress

Hydrofracture can be approximated by a 2D elliptical crack who's dimensions depend on the static Young's modulus and Poisson's ratio.

$$b = 2c \frac{1 - \nu^2}{E}(P - SH_{\min})$$

Importance of Static Young's Modulus and Poisson's Ratio for Hydrofracture Design

P

$2b$

$2c$

S_{Hmin}
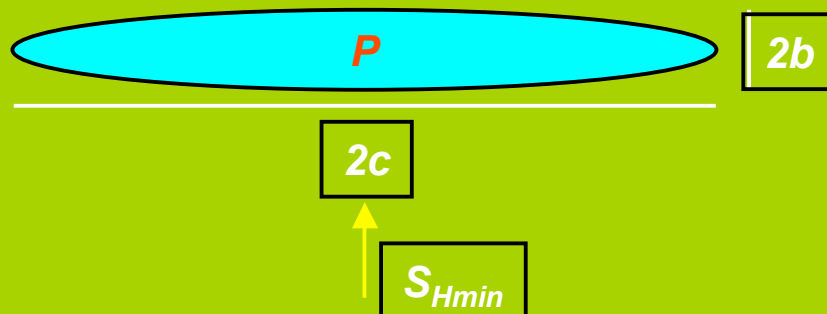
ROCK PHYSICS BASICS

Static and Dynamic Moduli in Sand
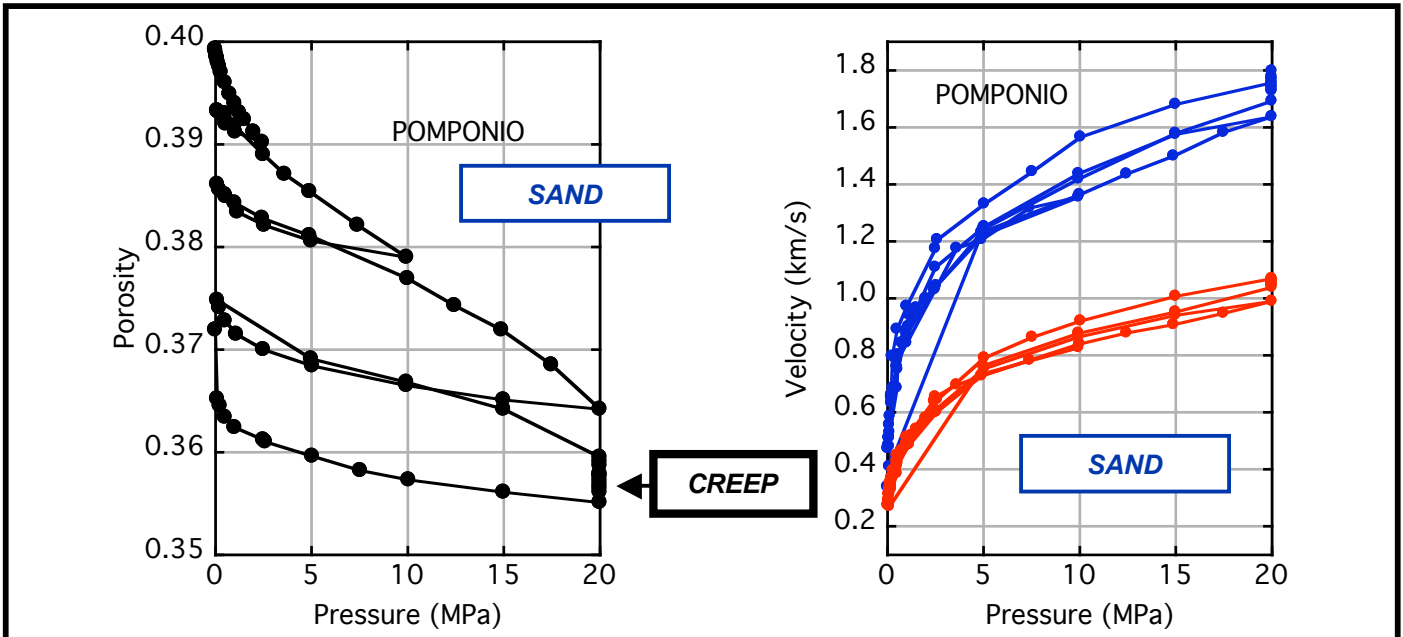
Porosity (left) and velocity (right) versus pressure in high-porosity room-dry sand sample from the Gulf of Mexico.
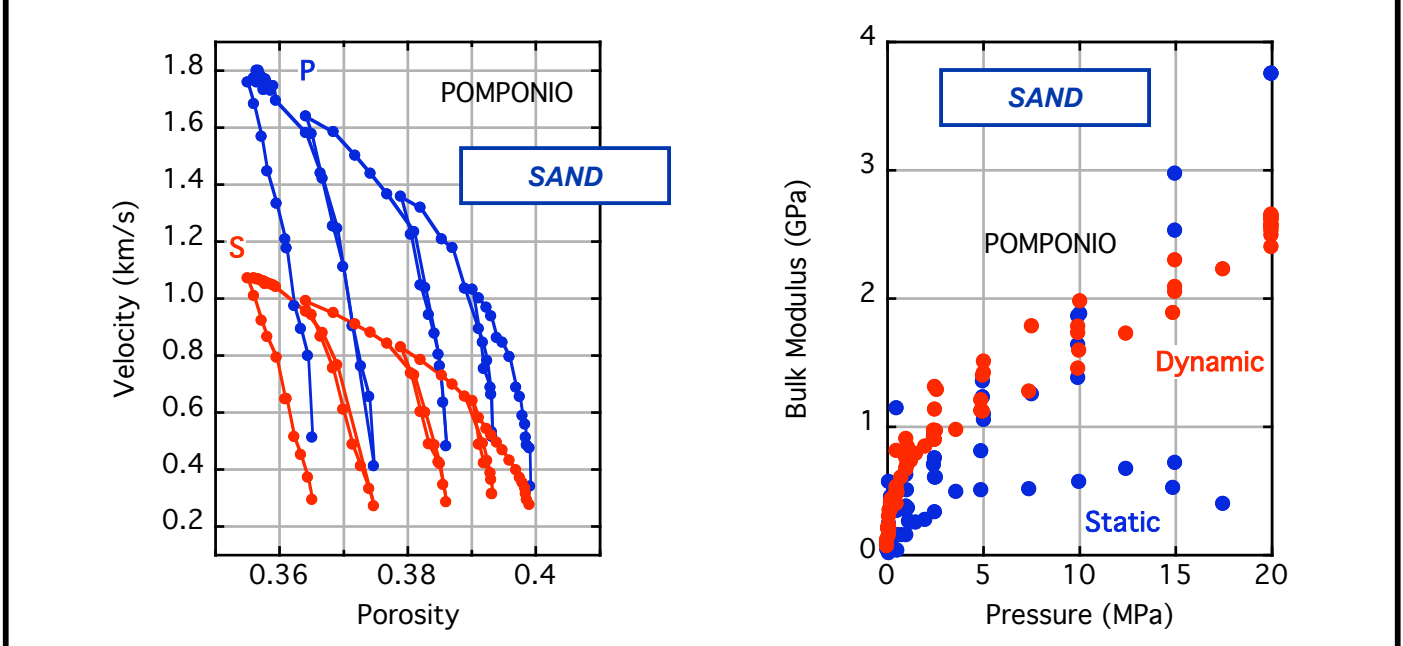
Velocity versus porosity (left) and dynamic and static bulk moduli calculated for the same sample.

Zimmer, M., 2003, Doctoral Thesis, Stanford University .

ROCK PHYSICS BASICS

Compaction Effects in Sand/Shale
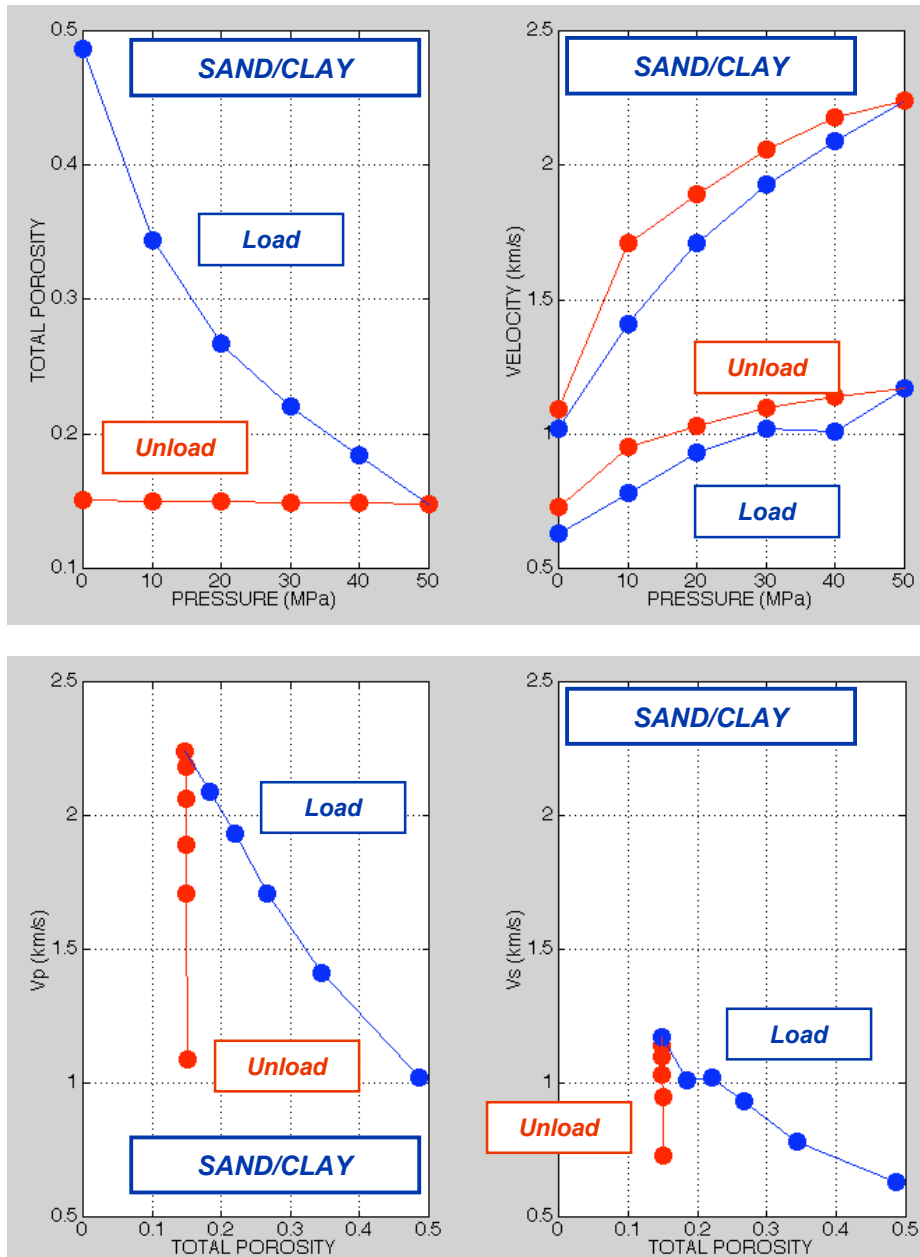
Velocity in shale depends on stress and porosity and deformation history.

Yin, H., 1992, Doctoral Thesis, Stanford University .

ROCK PHYSICS BASICS

Elastic Composites and Elastic Bounds

Most physical bodies in nature are elastically heterogeneous, I.e., they are made of components that have different elastic moduli. The effective elastic moduli of a composite depend on (a) properties of individual components and (b) geometrical arrangement of these components in space.

It is often very hard, if not impossible, to theoretically calculate the effective elastic moduli of a composite.

However, elastic bounds help us contain the exact values of the effective elastic moduli.

The simplest bounds are the Voigt (stiffest) and Reuss (softest) bound. No matter how complex the composite is, its effective elastic moduli are contained within these bounds.

Composite   Voigt   Reuss
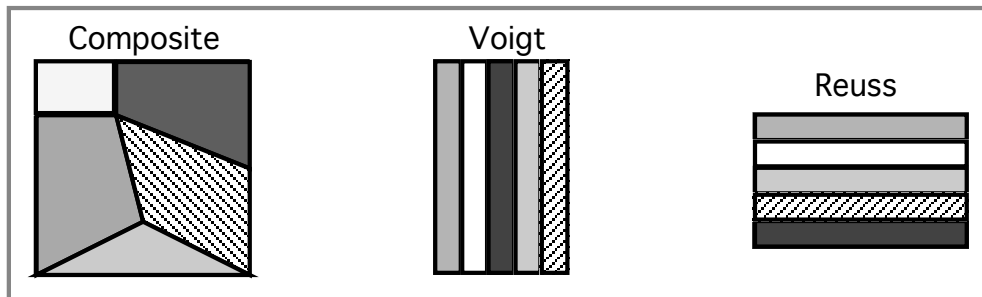
For an elastic modulus that may be either bulk or shear modulus of an N-component composite where the i-th component has modulus M_i and occupies volume fraction f_i, the Voigt (upper) bound is:

$$M_V = \sum_{i=1}^{N} f_i M_i$$

The Reuss (lower) bound is:

$$M_R = (\sum_{i=1}^{N} f_i M_i^{-1})^{-1}$$

Hill's average is simply the average between the Voigt and Reuss bounds:

$$M_H = \frac{M_V + M_R}{2} = \frac{1}{2}[\sum_{i=1}^{N} f_i M_i + (\sum_{i=1}^{N} f_i M_i^{-1})^{-1}]$$

The graph on the right shows the Voigt and Reuss bounds and Hill's average of a two-component composite.

The elastic modulus of the soft component is 10 and that of the stiff component is 20. The horizontal axis is the concentration of the stiff component.
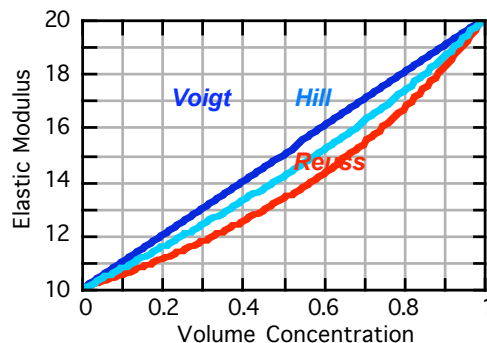
ROCK PHYSICS BASICS

Hashin-Shrikman Elastic Bounds

For an isotropic composite, the effective elastic bulk and shear moduli are contained within rigorous *Hashin-Shtrikman* bounds. These bounds have been derived for the bulk and shear moduli. The Hashin-Shtrikman bounds are tighter than the Voigt-Reuss bounds.

$$[\sum_{i=1}^{N} \frac{f_i}{K_i + \frac{4}{3}G_{min}}]^{-1} - \frac{4}{3}G_{min} \le K_{eff} \le [\sum_{i=1}^{N} \frac{f_i}{K_i + \frac{4}{3}G_{max}}]^{-1} - \frac{4}{3}G_{max},$$

$$[\sum_{i=1}^{N} \frac{f_i}{G_i + \frac{G_{min}}{6}\left(\frac{9K_{min} + 8G_{min}}{K_{min} + 2G_{min}}\right)}]^{-1} - \frac{G_{min}}{6}\left(\frac{9K_{min} + 8G_{min}}{K_{min} + 2G_{min}}\right) \le G_{eff} \le$$

$$[\sum_{i=1}^{N} \frac{f_i}{G_i + \frac{G_{max}}{6}\left(\frac{9K_{max} + 8G_{max}}{K_{max} + 2G_{max}}\right)}]^{-1} - \frac{G_{max}}{6}\left(\frac{9K_{max} + 8G_{max}}{K_{max} + 2G_{max}}\right),$$

In the Hashin-Shtrikman equations, the subscript "eff" is for the effective elastic bulk and shear moduli. The subscripts "min" and "max" are for the softest and stiffest components, respectively.

A physical realization of the Hashin-Shtrikman bounds for two components is the entire space filled by composite spheres of varying size. The outer shell of each sphere is the softest component for the lower bound and the stiffest component for the upper bound.

If one of the components is void (empty pore) the lower bound is zero.

Hashin-Shtrikman
Bounds: Realization
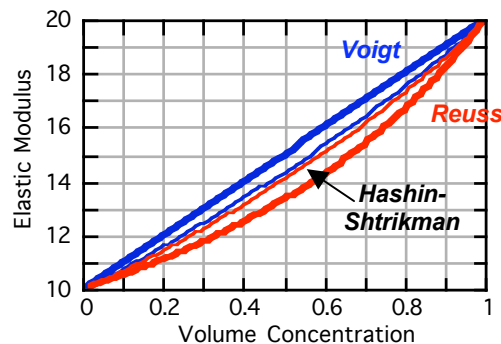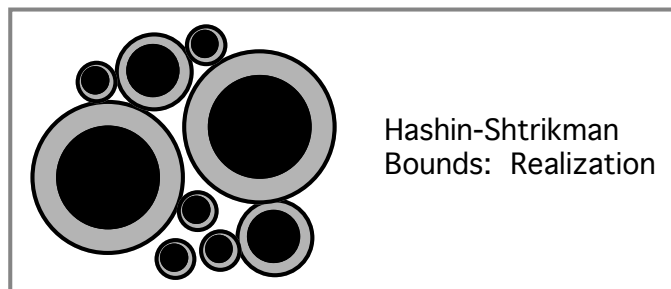
ROCK PHYSICS BASICS

Elastic Bounds for Water-Saturated Sand

> *Unfortunately, if the elastic moduli of the two components of the composite are very different, the bounds lie very far apart.*
>
> *They cannot be used for practical velocity prediction.*

> *In the example below, plotted is velocity versus porosity as calculated for a quartz/water mixture using the elastic bounds. The data points are for water-saturated sandstones. The data lie within the elastic bounds curves.*
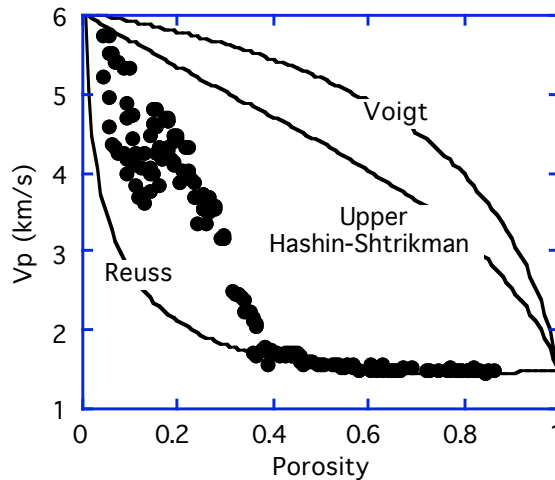
> *The bounds can still be used to quality control of the data.*

> *In the example below, the red data points lie below the lower bound curve for the quartz/water mix. The reason is that these data are for gas-saturated sands.*
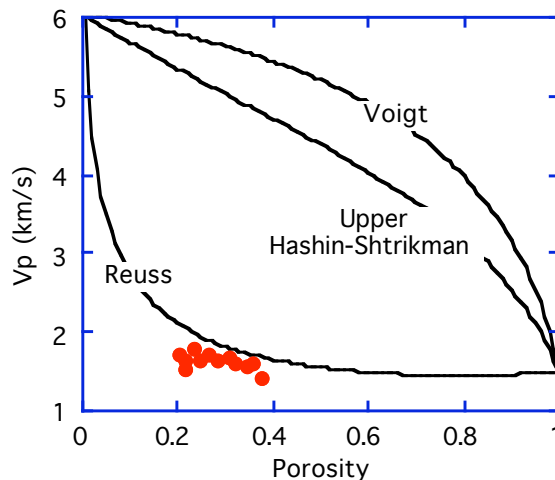
ROCK PHYSICS BASICS

Mixing Fluids

The main difference between fluid and solid is that fluid's shear modulus is zero. Therefore, the shear-wave velocity in fluid is zero.

The P-wave velocity in fluid depends on the bulk modulus and density as

$$V_P = \sqrt{\frac{K}{\rho}}$$

If several types of fluid coexist in the same space (but are not mixed at the molecular level) the effective bulk modulus does not depend on the geometrical arrangement of the components. It is exactly the Reuss (isostress) average of the individual bulk moduli:

$$\frac{1}{K_{Fluid}} = \sum_{i=1}^{N} \frac{f_i}{K_i}$$

The same equation but written in terms of compressibility ($\beta = 1/K$) is:

$$\beta_{Fluid} = \sum_{i=1}^{N} f_i \beta_i$$

This is why even small fractions of free gas make the pore fluid very compressible (figure below, on the right).
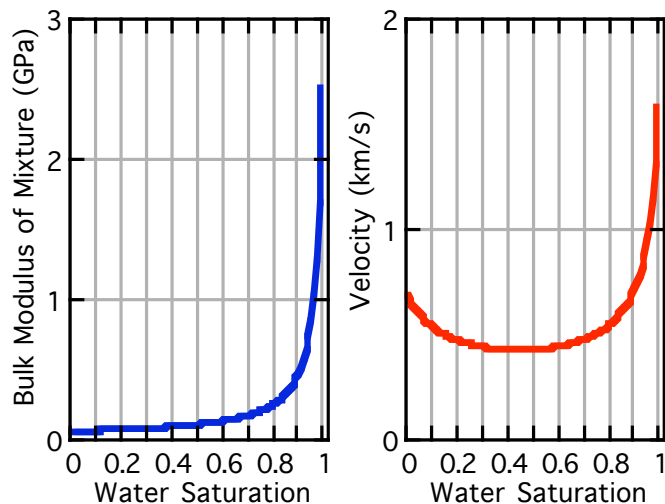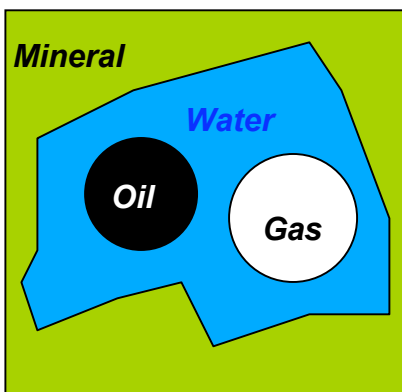
ROCK PHYSICS BASICS

Pressure and Rock Properties

When rock is extracted from depth and re-loaded in the lab, it exhibits a strong velocity-pressure dependence, especially at low confining pressure. The primary reason is microcracks that open during in-situ stress relief.

Below: Vp and Vs versus confining pressure in room-dry sandstone samples. The in-situ effective stress is about 15 MPa. Different colors correspond to different samples.
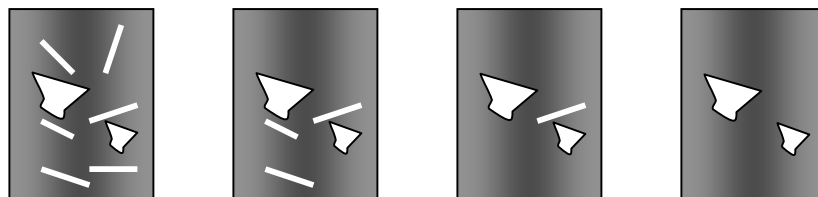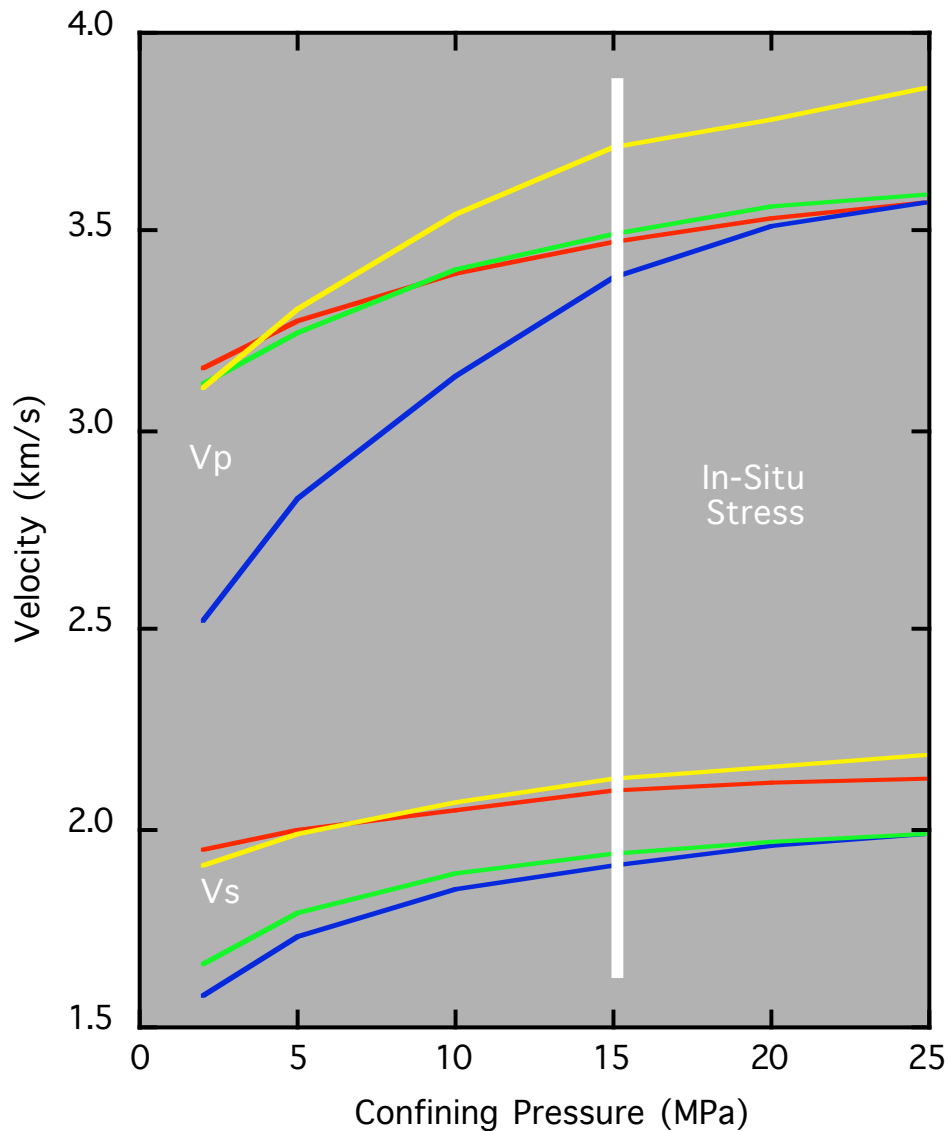
ROCK PHYSICS BASICS

Pressure and Rock Properties

Phenomenology of pressure effect on velocity in room-dry sandstone dataset.

The velocity-porosity trends become sharper as pressure increases.
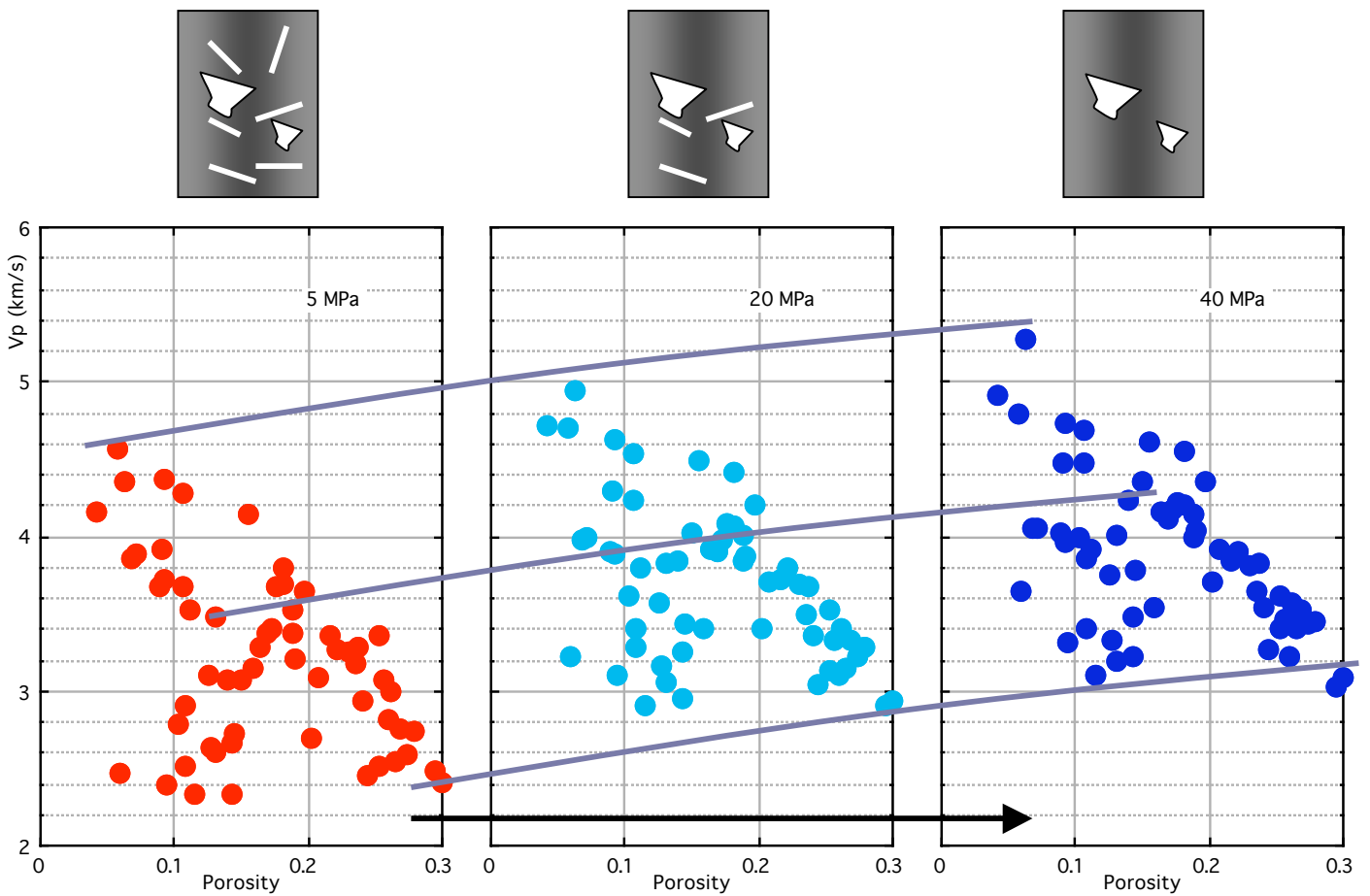
ROCK PHYSICS BASICS

Pressure and Rock Properties

Phenomenology of pressure effect on velocity in room-dry sandstone dataset.

Both Vp and Vs may significantly change with changing pressure.
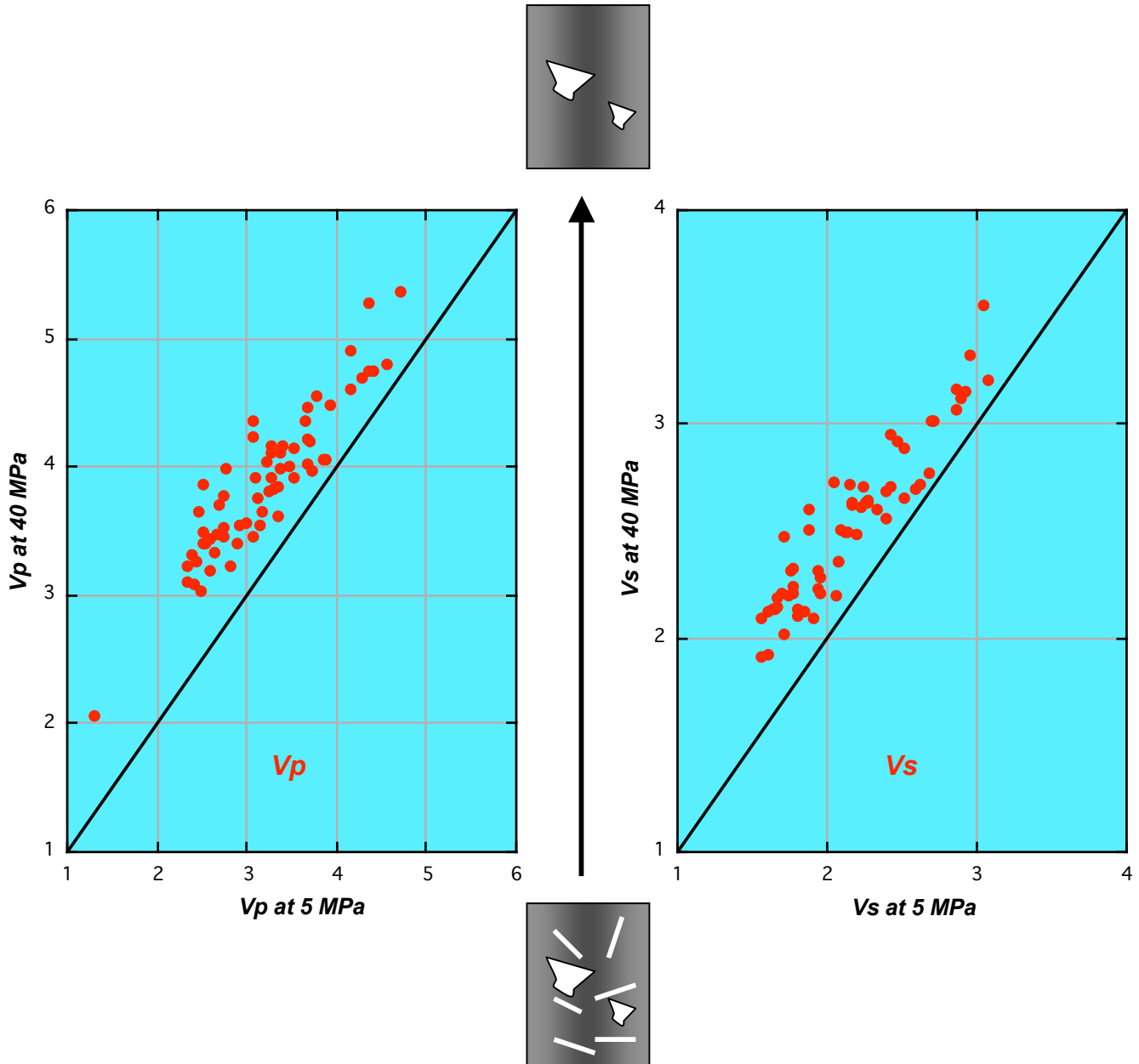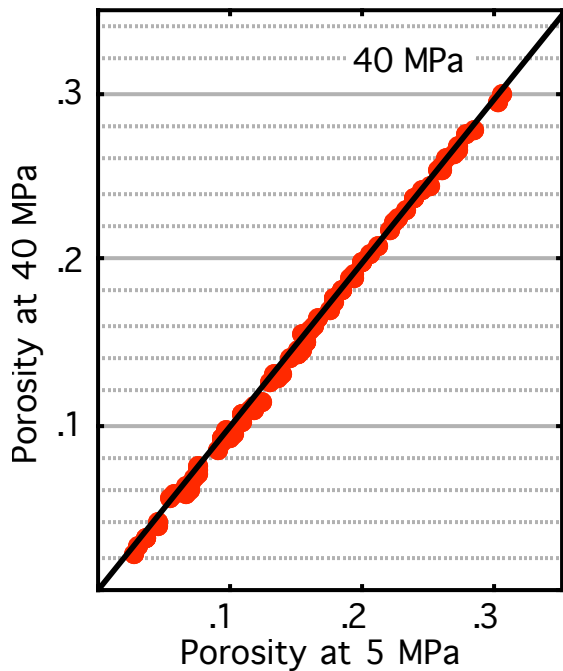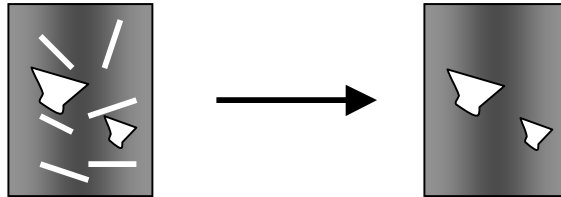
ROCK PHYSICS BASICS

Pressure and Rock Properties

> *The Vp and Vs versus pressure changes are not necessarily scalable -- the Vp/Vs ratio and Poisson's ratio change as well.*
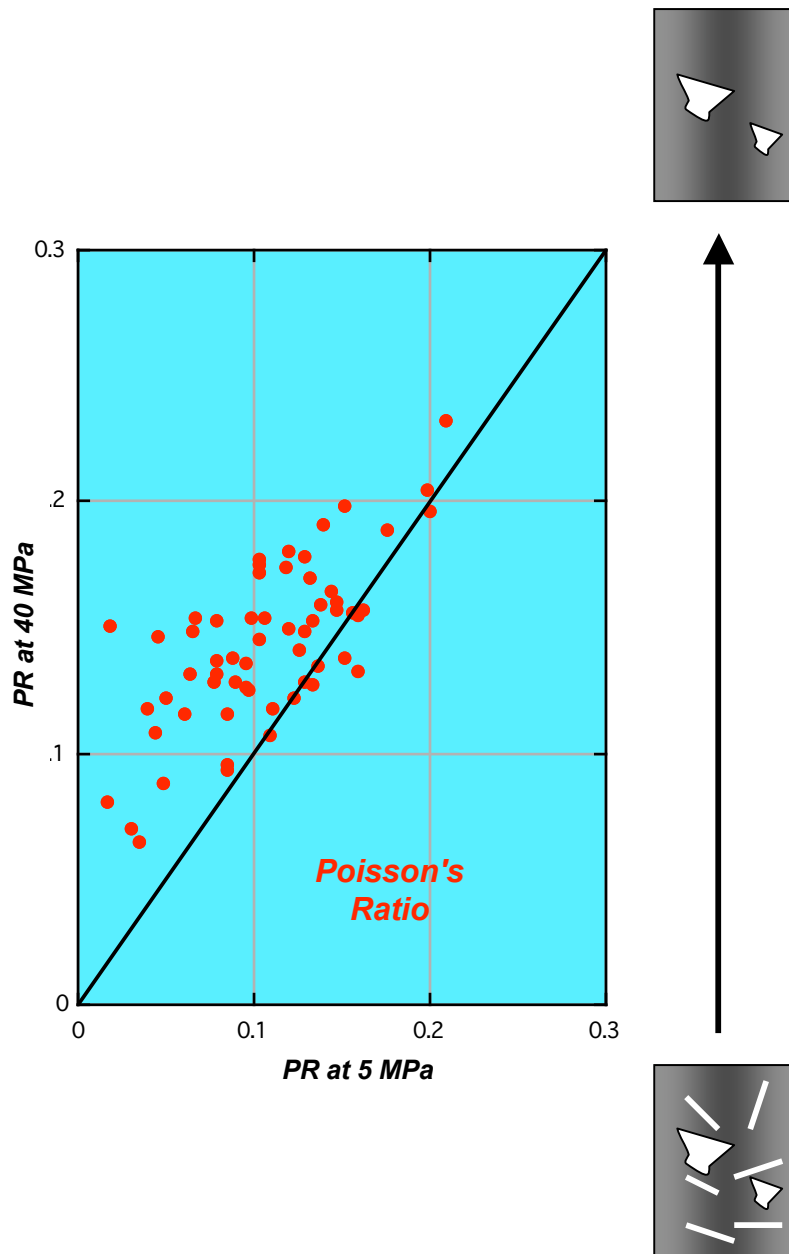
ROCK PHYSICS BASICS

Pressure and Rock Properties

> The effect of pressure on porosity is not as large as on velocity. The main reason is that rock's elasticity is affected by thin compliant cracks that do not occupy much of the pore space volume.
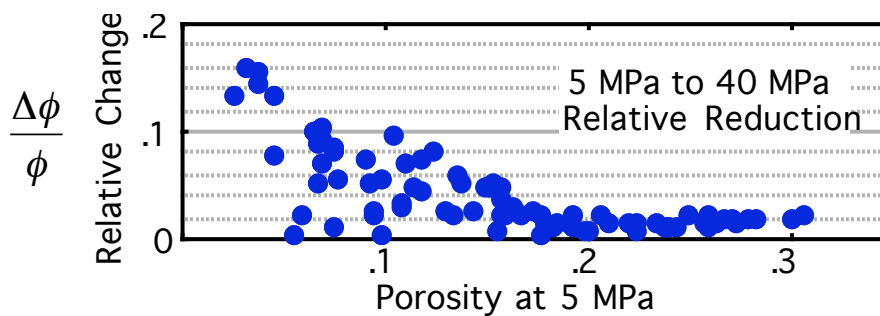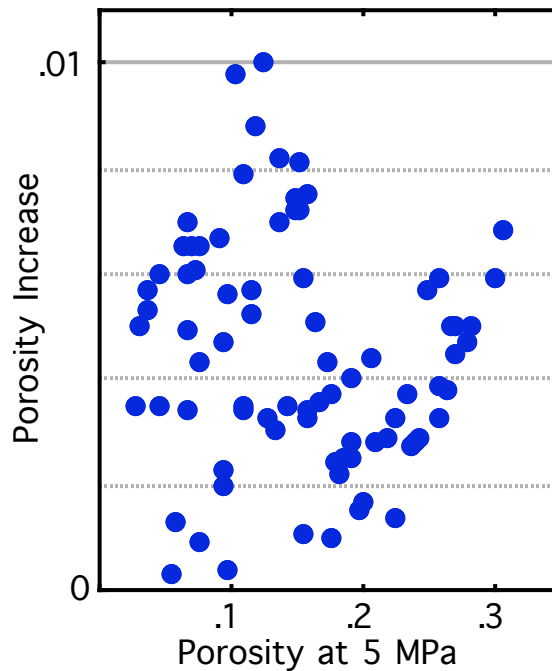
ROCK PHYSICS BASICS

Stress-Induced Anisotropy

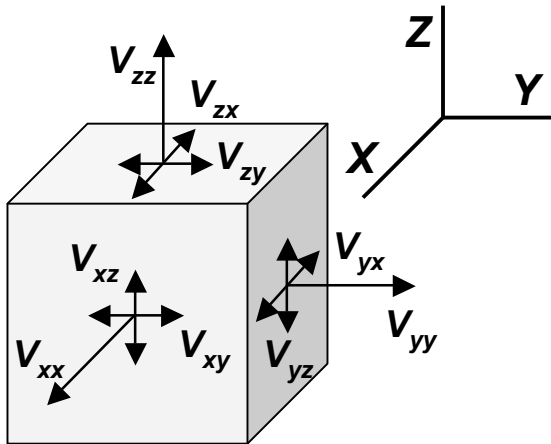**Thomsen's anisotropic formulation (weak transverse isotropy)**

θ
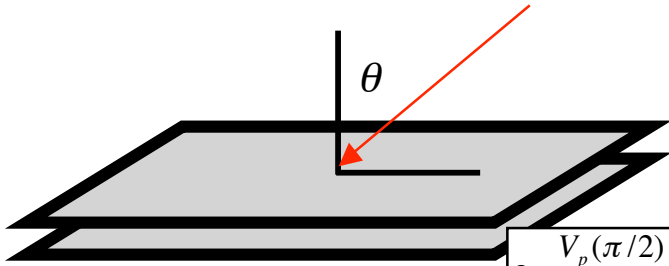
$$V_p(\theta) \approx \alpha(1 + \delta \sin^2\theta \cos^2\theta + \varepsilon \sin^4\theta)$$
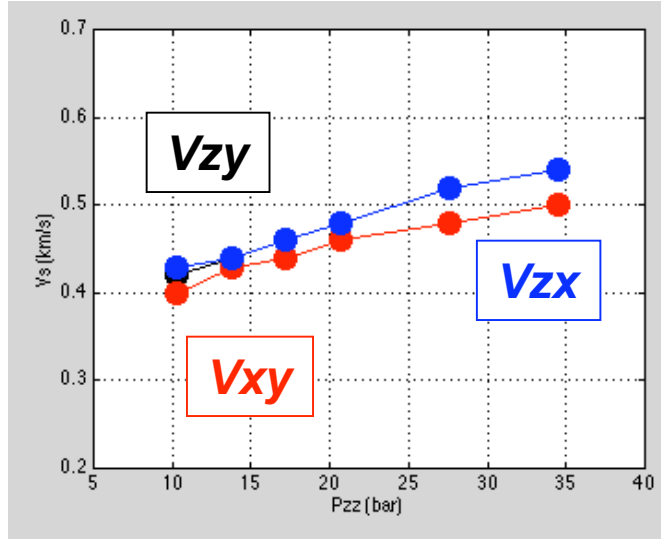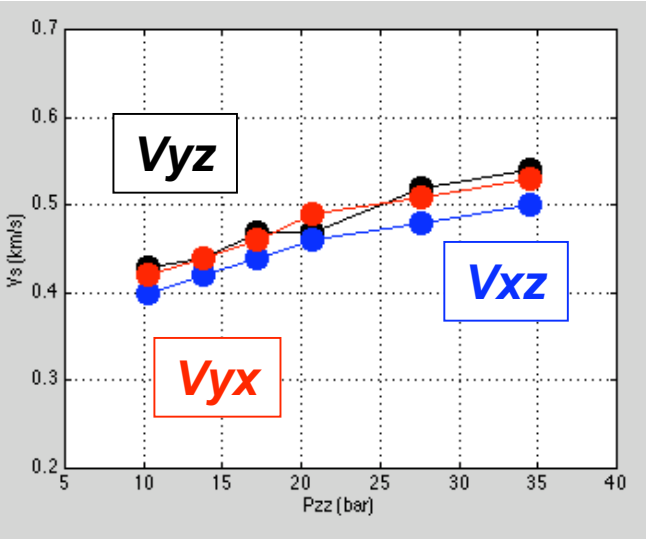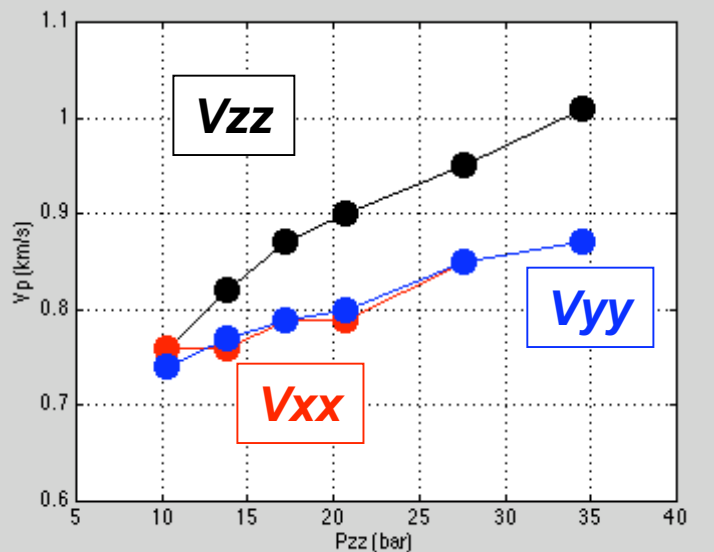
$$V_{sV}(\theta) \approx \beta[1 + \frac{\alpha^2}{\beta^2}(\varepsilon - \delta)\sin^2\theta \cos^2\theta]$$

$$V_{sH}(\theta) \approx \beta[1 + \gamma \sin^2\theta]$$

$$\varepsilon = \frac{V_p(\pi/2) - V_p(0)}{V_p(0)} \qquad \gamma = \frac{V_{sH}(\pi/2) - V_{sV}(\pi/2)}{V_{sV}(\pi/2)} = \frac{V_{sH}(\pi/2) - V_{sH}(0)}{V_{sH}(0)}$$

V_{zz} V_{zx} V_{zy} V_{xz} V_{yx} V_{xx} V_{xy} V_{yz} V_{yy} Z Y X

Ottawa sand. Uniaxial compression. Pxx = Pyy = 1.72 bar. Yin (1992).

Vzz Vyy Vxx
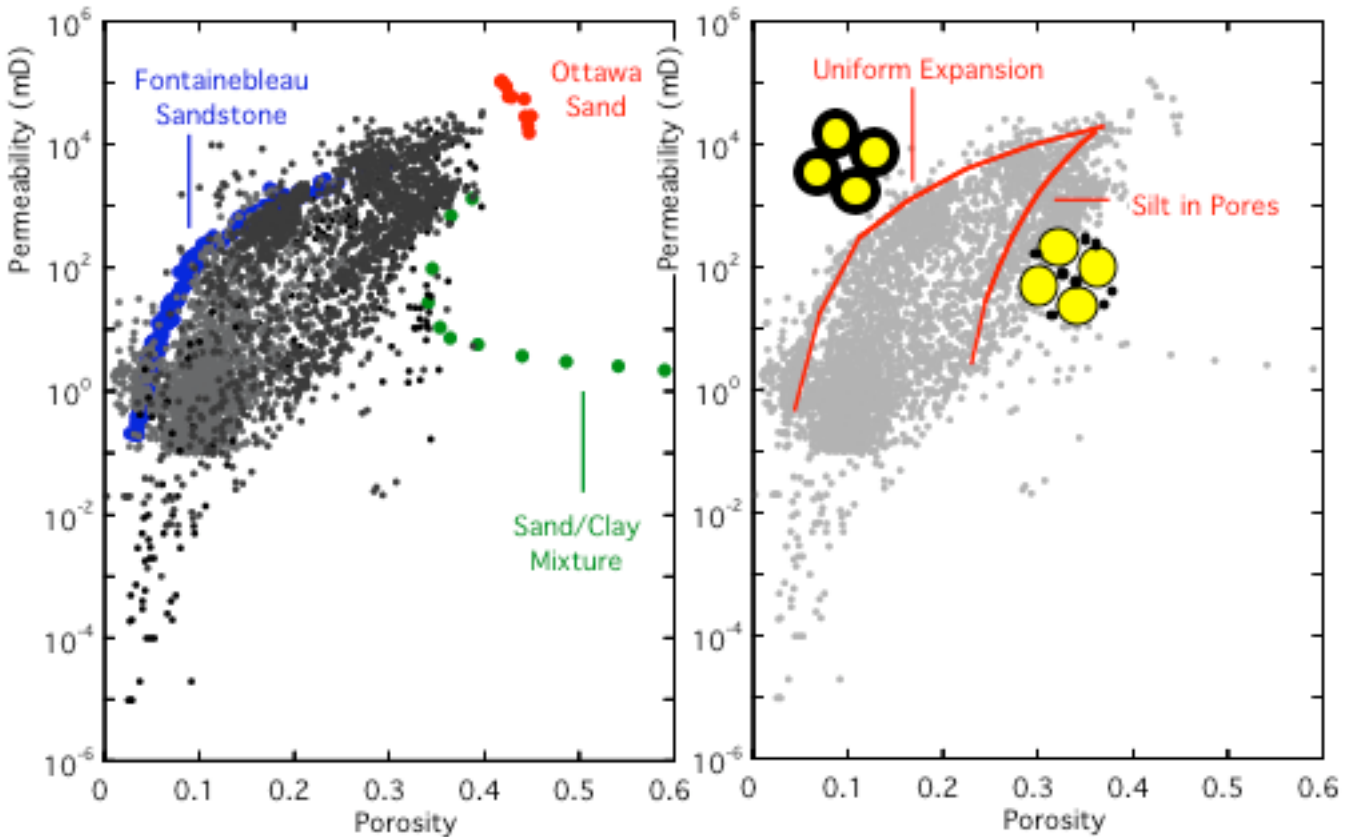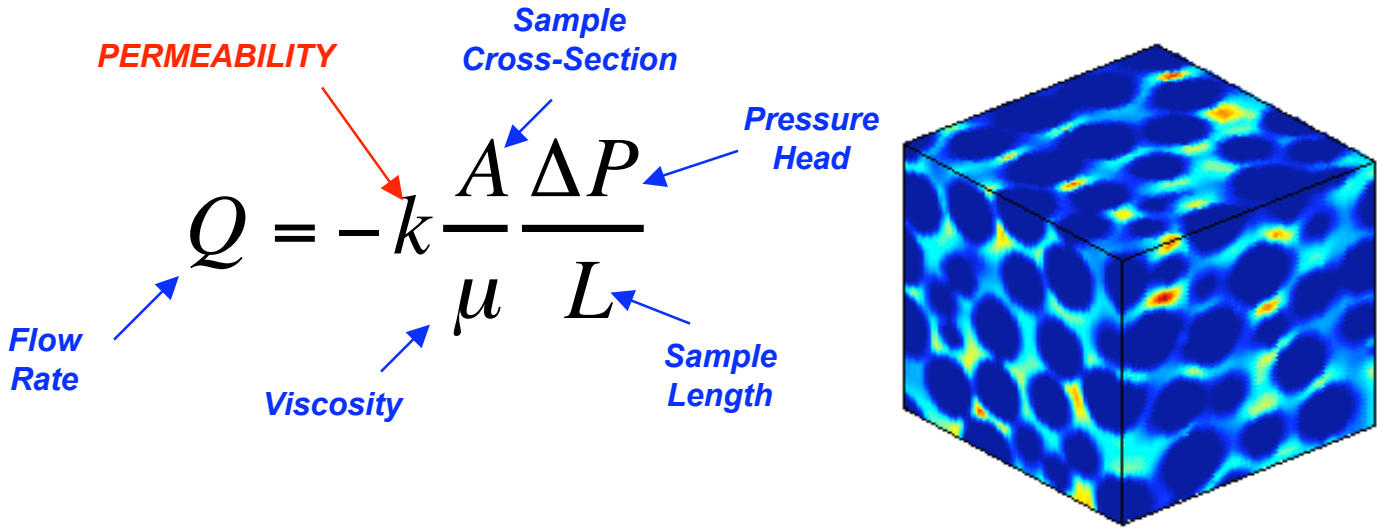
Vyz Vxz Vyx

Vzy Vzx Vxy

Anisotropic stress field induces anisotropy in otherwise isotropic rock.

ROCK PHYSICS BASICS

Permeability

Permeability is a fluid flow property of a porous medium. Its definition comes from Darcy's law which states that the flow rate is linearly proportional to the pressure gradient.

PERMEABILITY

Sample Cross-Section

Pressure Head

$$Q = -k \frac{A}{\mu} \frac{\Delta P}{L}$$

Flow Rate

Viscosity

Sample Length

Fontainebleau Sandstone

Ottawa Sand

Sand/Clay Mixture

Uniform Expansion

Silt in Pores

ROCK PHYSICS BASICS

Permeability -- Kozeny-Carman Equation

Permeability depends on porosity and grain size.

Kozeny-Carman Equation (SI UNITS)

Permeability   Porosity

$$\frac{k}{d^2} = \frac{1}{72} \frac{\phi^3}{(1-\phi)^2 \tau^2}$$

Grain Size   Tortuosity

Drastic drop in permeability as porosity increases but clay content increases too (grain size decreases). Yin (1992).
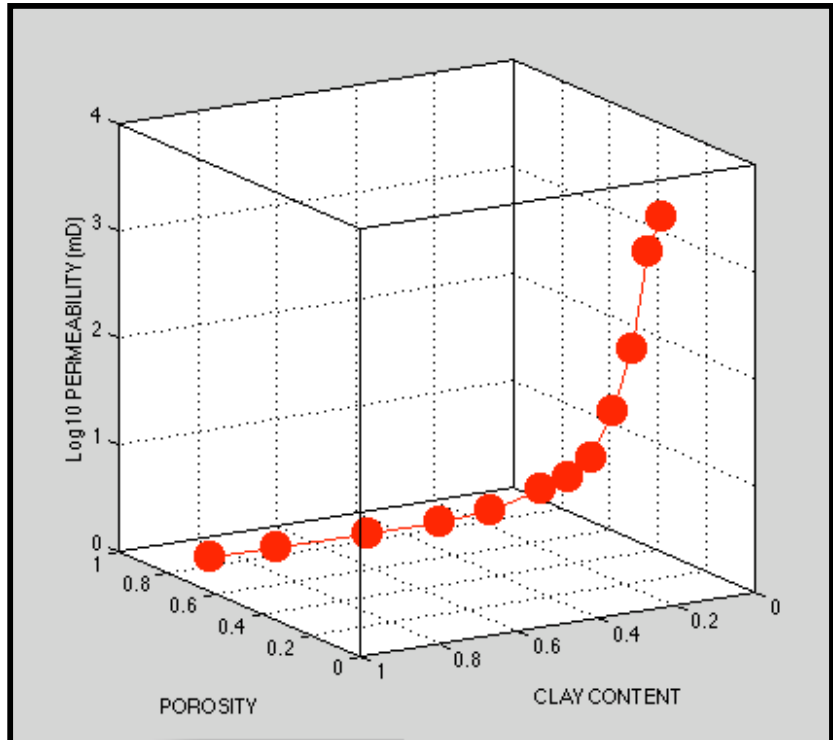
Permeability may be almost zero in carbonate where large vuggy pores are not connected. This pore geometry is a topological inverse of the pore geometry of clastic sediment. Permeability equations that work in clastics may not work in carbonates.
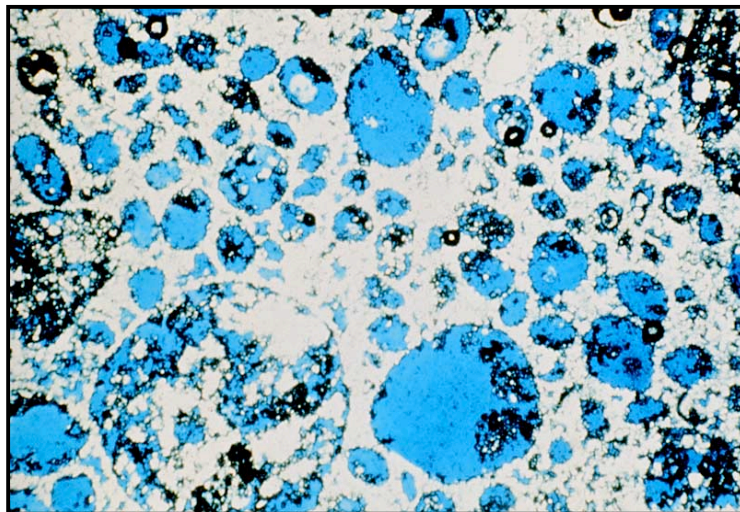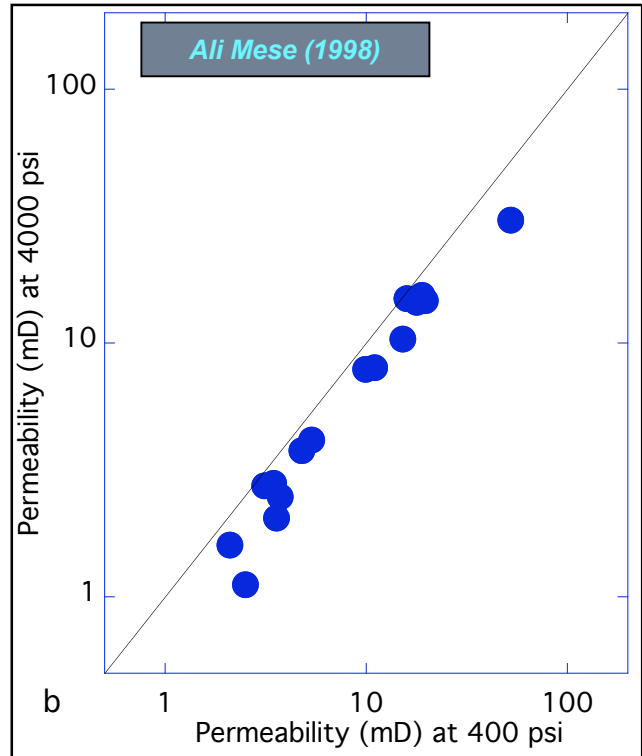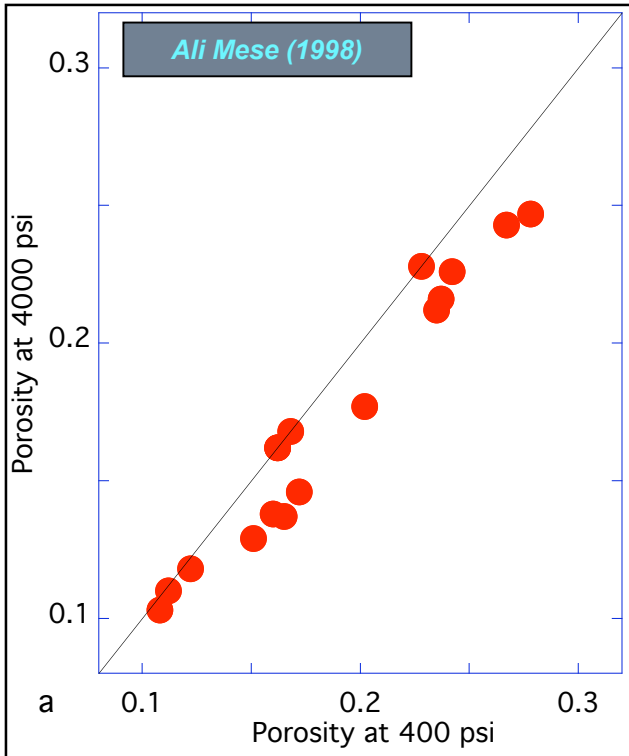

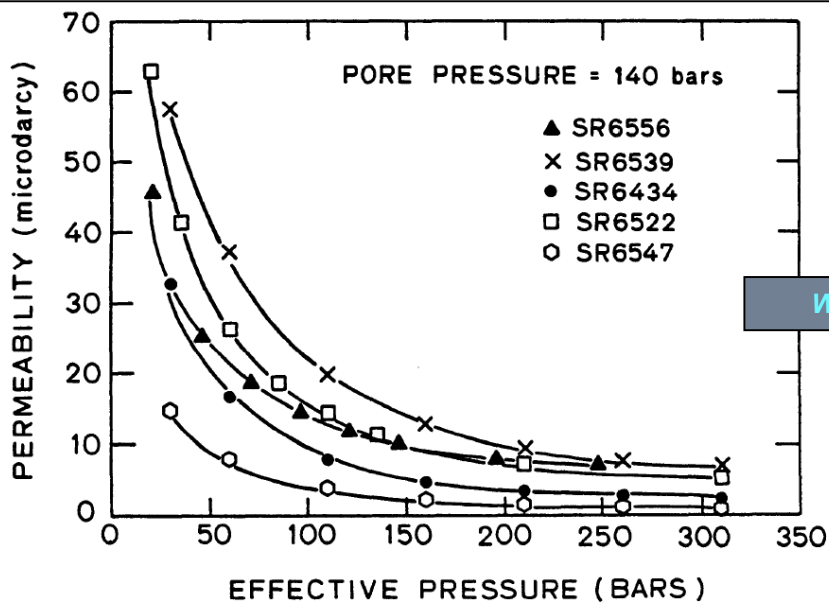*Permeability depends on porosity and grain size and also (critically) on the pore-space geometry.*

ROCK PHYSICS BASICS

Permeability and Stress

Permeability in medium-to-high porosity clastic sediment is weakly dependent on stress.
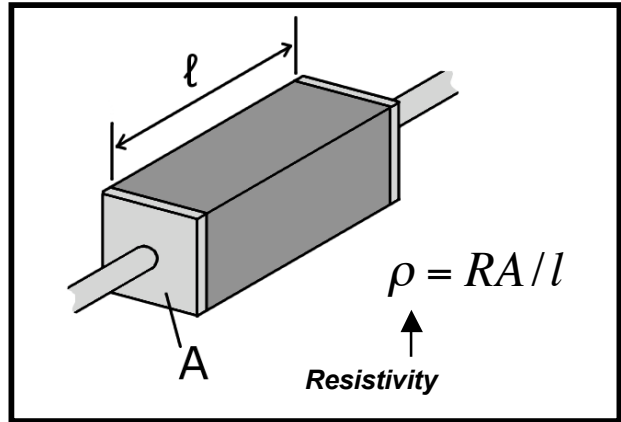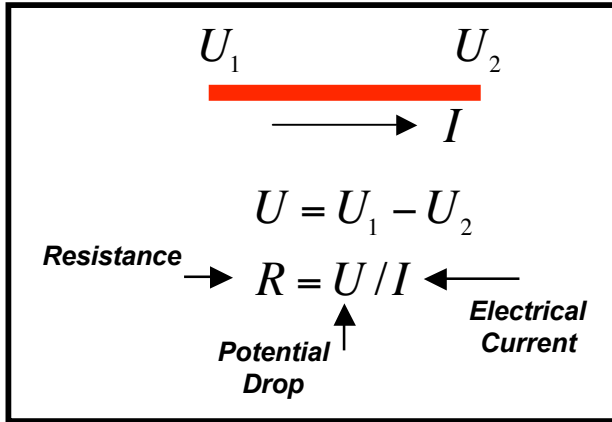

Permeability in tight sandstone samples. Porosity 0.03 to 0.07.

Relative changes of high permeability with pressure are smaller than those for low permeability. Both may have implications for fluid transport.

DEFINITIONS OF RESISTIVITY

$$U_1 \qquad U_2$$

$$I$$

$$U = U_1 - U_2$$

Resistance $\rightarrow \quad R = U/I \quad \leftarrow$ **Electrical Current**

Potential Drop

$$\ell$$

$$\rho = RA/l$$

A **Resistivity**

$$[R] = \Omega \equiv \mathrm{Ohm}$$
$$[\rho] = \Omega \cdot \mathrm{m} \equiv \mathrm{Ohm} \cdot \mathrm{m}$$

$$\sigma = 1/\rho$$

Conductivity