2. Electrical resistivity methods

- The resistivity method is used in the study of horizontal and vertical discontinuities in the electrical properties of the ground.
- It utilizes direct currents or low frequency alternating currents to investigate the electrical properties (resistivity) of the subsurface.
- A resistivity contrast between the target and the background geology must exist.

Possible applications of resistivity surveying

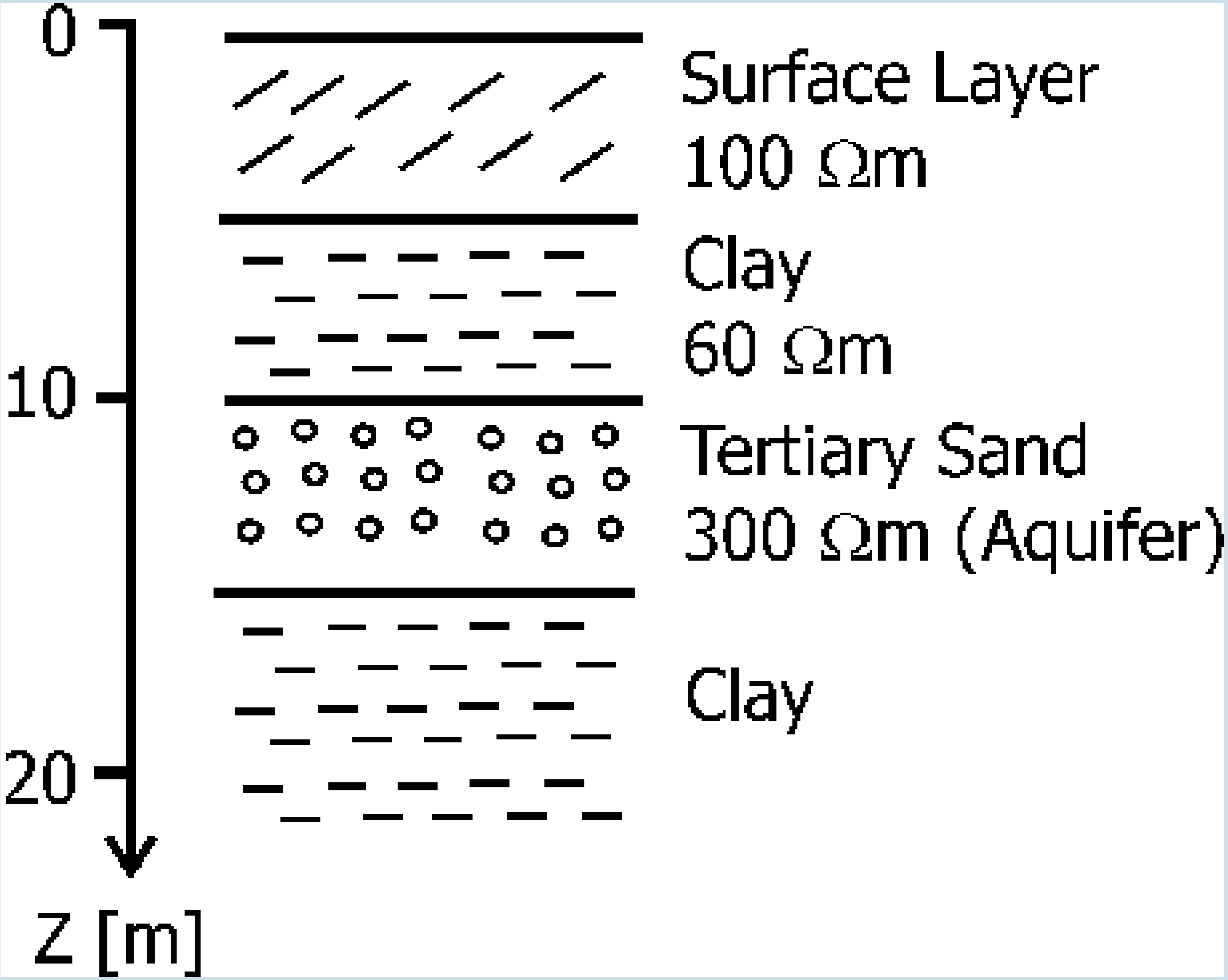


Fig. 1: Groundwater exploration

Possible applications of resistivity surveying

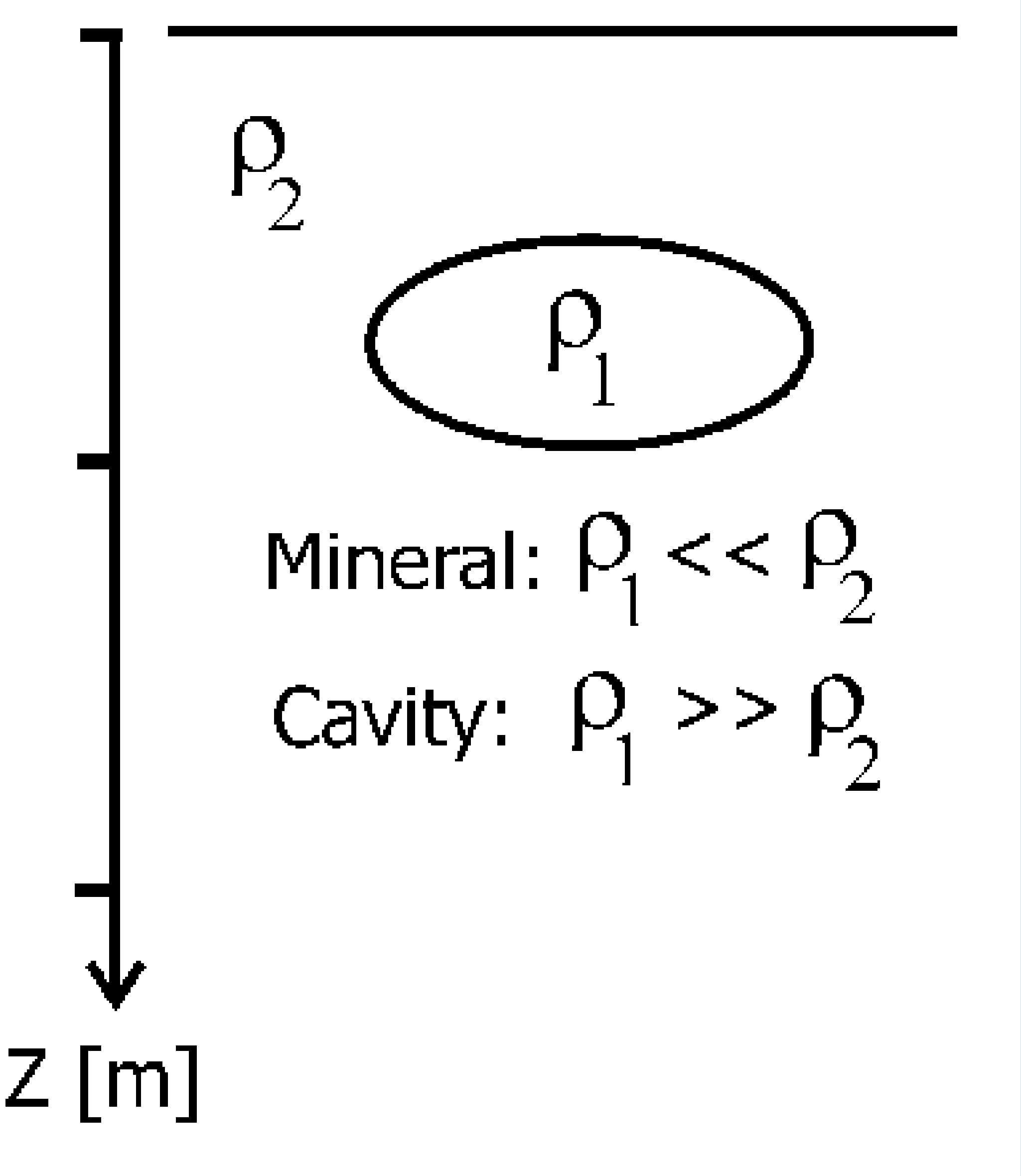


Fig. 2: Mineral exploration, detection of cavities

Possible applications of resistivity surveying

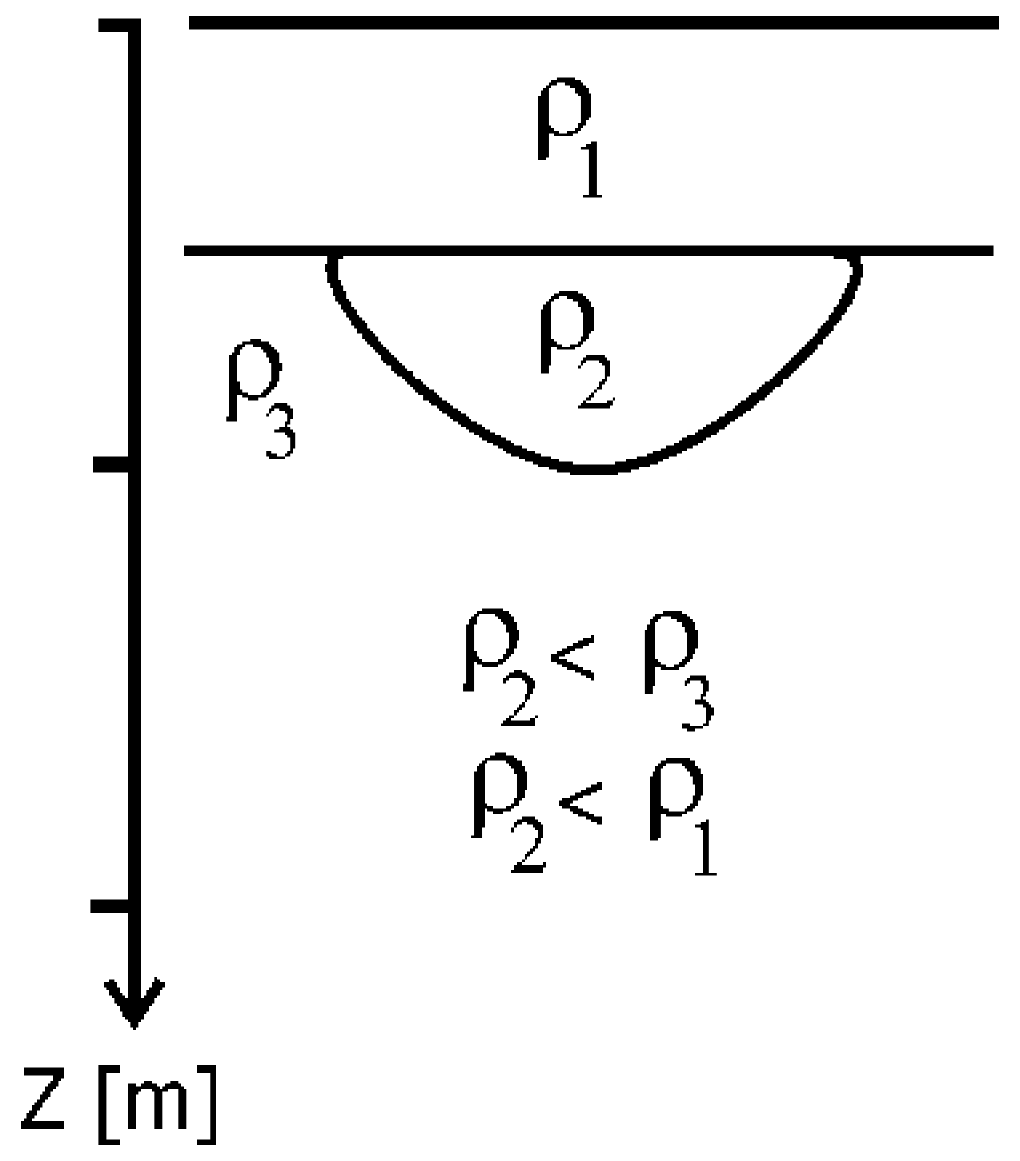


Fig. 3: Waste site exploration

Possible applications of resistivity surveying

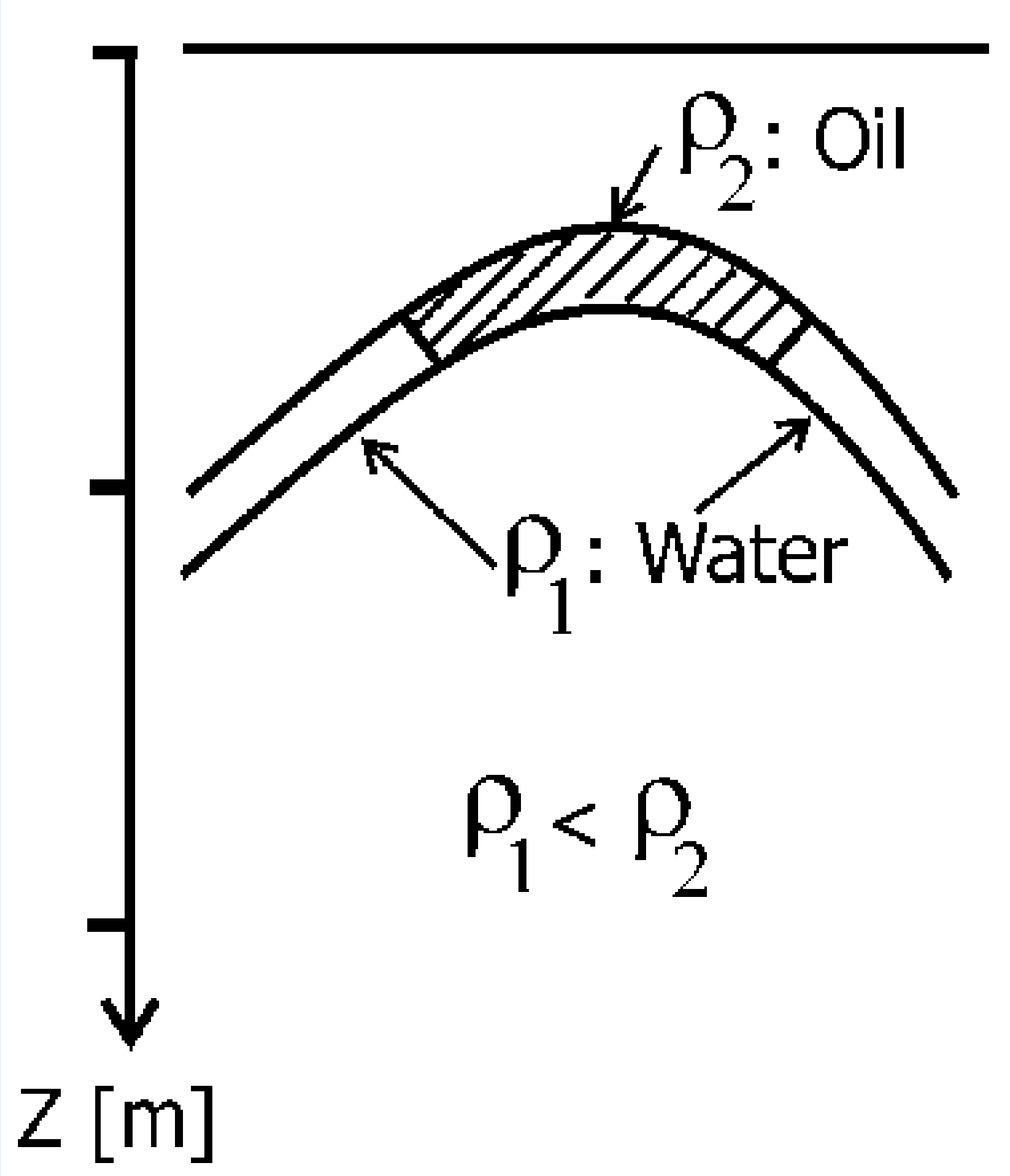


Fig. 4: Oil exploration

2.1 Resistivity

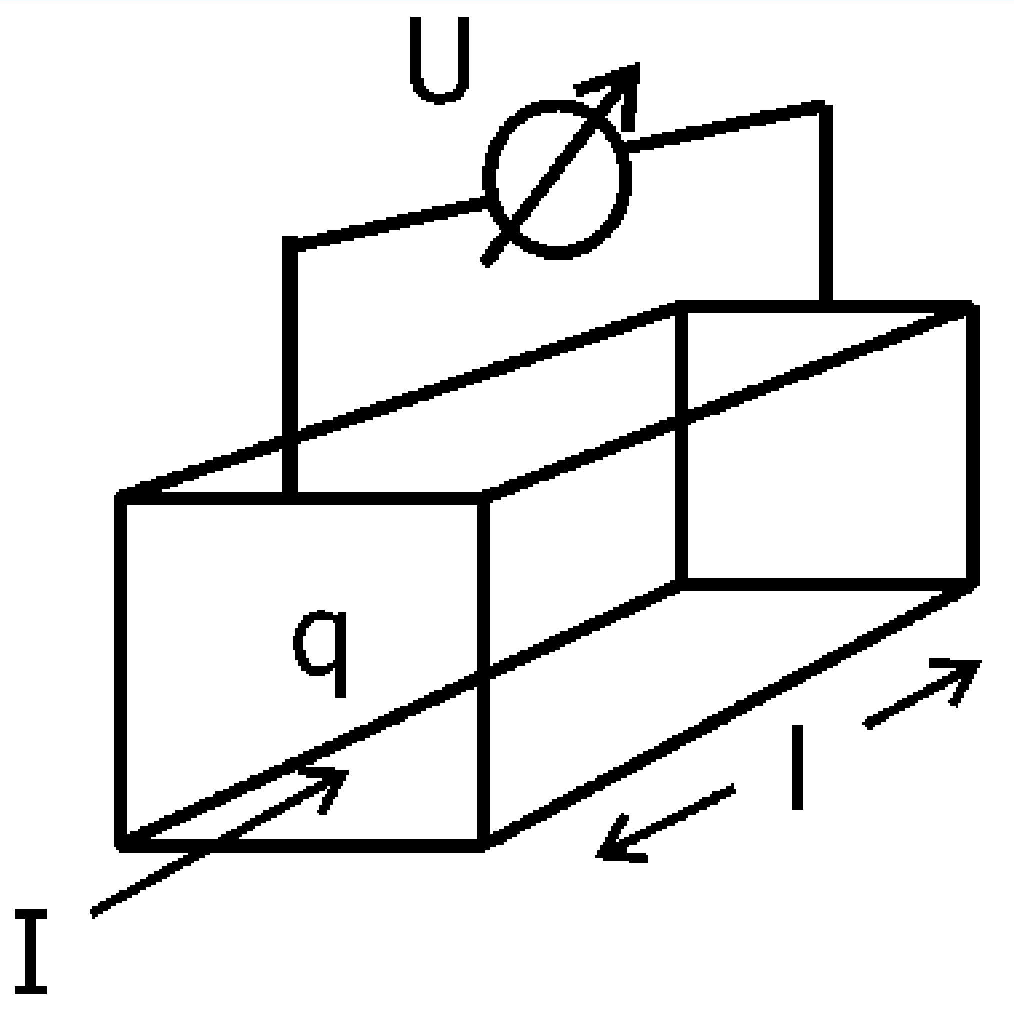


Fig. 5: Ohm's law

A direct current with strength I (A) flows through a conductor of a limited size.

$$I = \frac{qV}{\rho l} \qquad (2.1)$$

I: current strength (A)
V: Voltage (V)
q: cross section (m^2)
l: length
ρ: resistivity $(\Omega\,m)$
$\sigma = 1/\rho$: conductivity

Resistivity

This can be written alternatively in terms of field strength (E (V/m)) and current density (j (A/m^2)).

$$\rho = \mathrm{E}/\mathrm{j} \quad \left[\Omega\,\mathrm{m}\right]$$

Resistivity is one of the most variable physical properties.

$$1.6\times10^{-8}\,\Omega\,\mathrm{m} < \rho < 10^{16}\,\Omega\,\mathrm{m}$$

native silver

pure sulfur

Rock types and resistivity

Igneous rocks → highest resistivities

Sedimentary rocks → tend to be the most conductive due to their high fluid content

Metamorphic rocks → have intermediate but overlapping resistivities

Age of the rock is also important for the resistivity. For example:

Young volcanic rock (Quaternary) $\approx 10-200\,\Omega\,\mathrm{m}$

Old volcanic rock (Precambrian)$\approx 100-2000\,\Omega\,\mathrm{m}$

Rock types and resistivity

Most rock-forming minerals are insulators:

$10^{8} - 10^{16}\,\Omega\,\mathrm{m}$

However, measurement in-situ:

sedimentary rocks: $5 - 1000\,\Omega\,\mathrm{m}$

metamorphic/crystalline rocks: $100 - 10^{5}\,\Omega\,\mathrm{m}$

Reason: Rocks are usually porous and pores are filled with fluids, mainly water. As the result, rocks are electrolytic conductors. Electrical current is carried through a rock mainly by the passage of ions in pore waters.

→ Most rocks conduct electricity by electrolytic rather than ohmic processes.

Law of Archie

Observation: $\rho \approx 1/\phi^2$ where ϕ: porosity

Empirical law of Archie

$$\boxed{\rho = a\,\phi^{-m} S^{-n} \rho_w} \qquad (2.2)$$

ϕ : fractional pore volume (porosity)

S : fraction of the pores containing water

ρ_w: resistivity of water

$$n \approx 2$$

$$0.5 < a < 2.5$$

$$1.3 < m < 2.5$$

Example for the application of Archie's law

$$S=1 \quad a=1.5 \quad m=2$$

$$\rho/\rho_w=1.5\times10^4 \ \text{for} \ \phi=0.01$$

$$\rho/\rho_w=150 \ \text{for} \ \phi=0.1$$

$$\rho/\rho_w=17\times10^4 \ \text{for} \ \phi=0.3$$

$$\rho/\rho_w=6 \ \text{for} \ \phi=0.5$$

Schematic current flow in soil sample

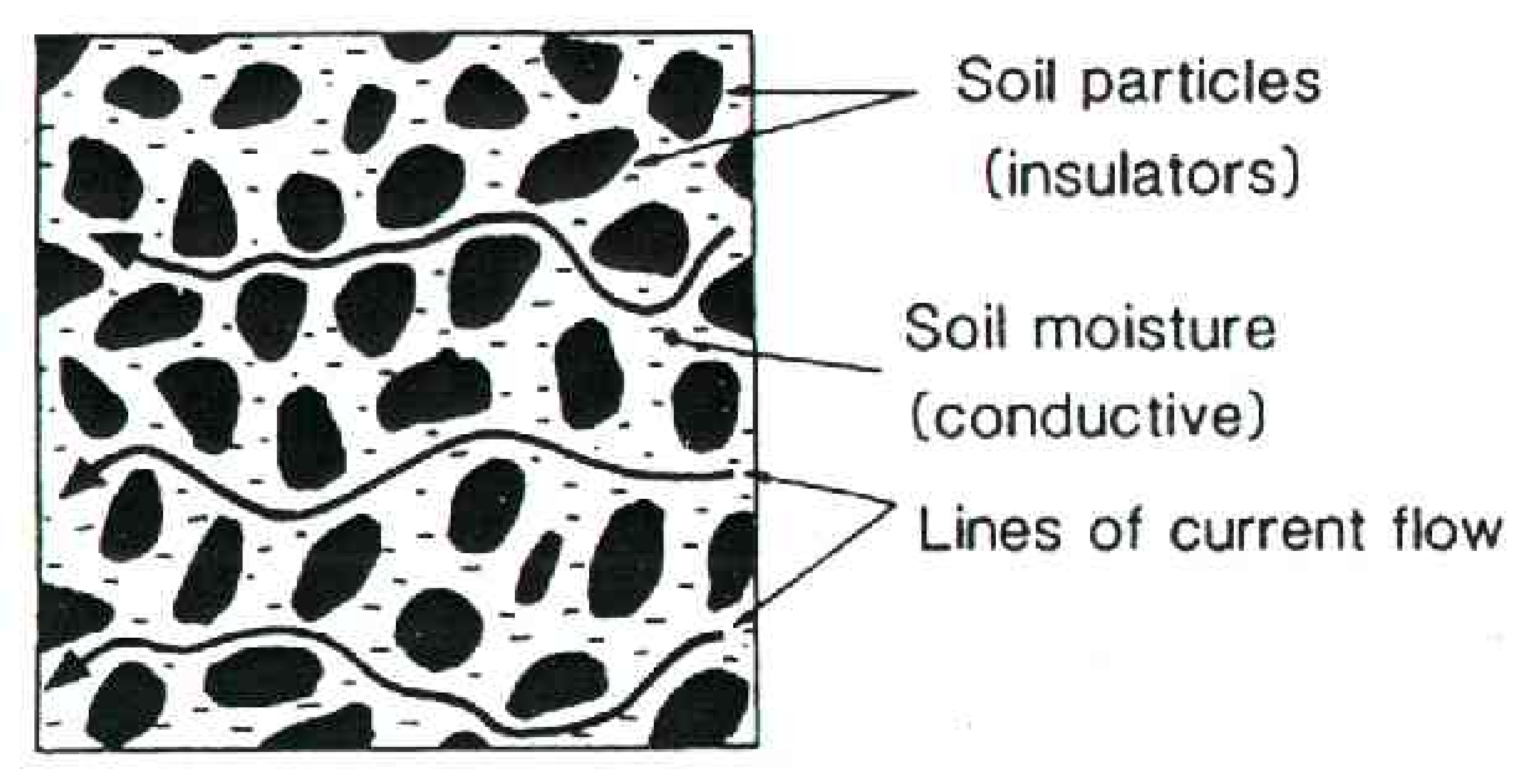


Fig. 6

➔ An increase in the number of ions in soil water (groundwater contamination) linearly decreases the soil resistivity.

The approximate resistivity values of common rock types

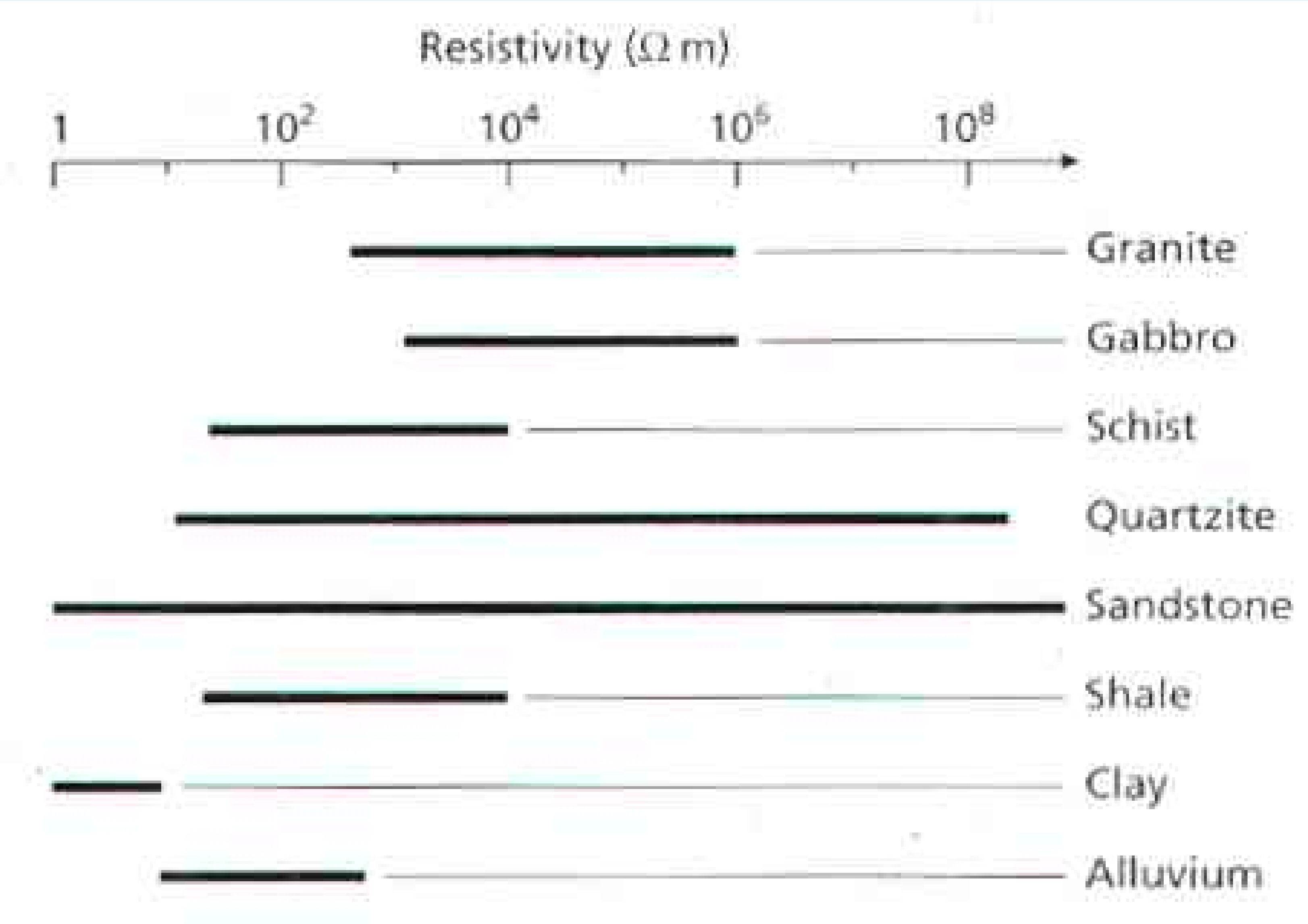


Fig. 7²

Discussion: Resistivity values

- There is considerable overlap between different rock types.
- Identification of a rock type is not possible solely on the basis of resistivity data.
- Resistivity of rocks depends on porosity, saturation, content of clay and resistivity of pore water (Archie's formula).

2.2 Current flow in a homogeneous earth

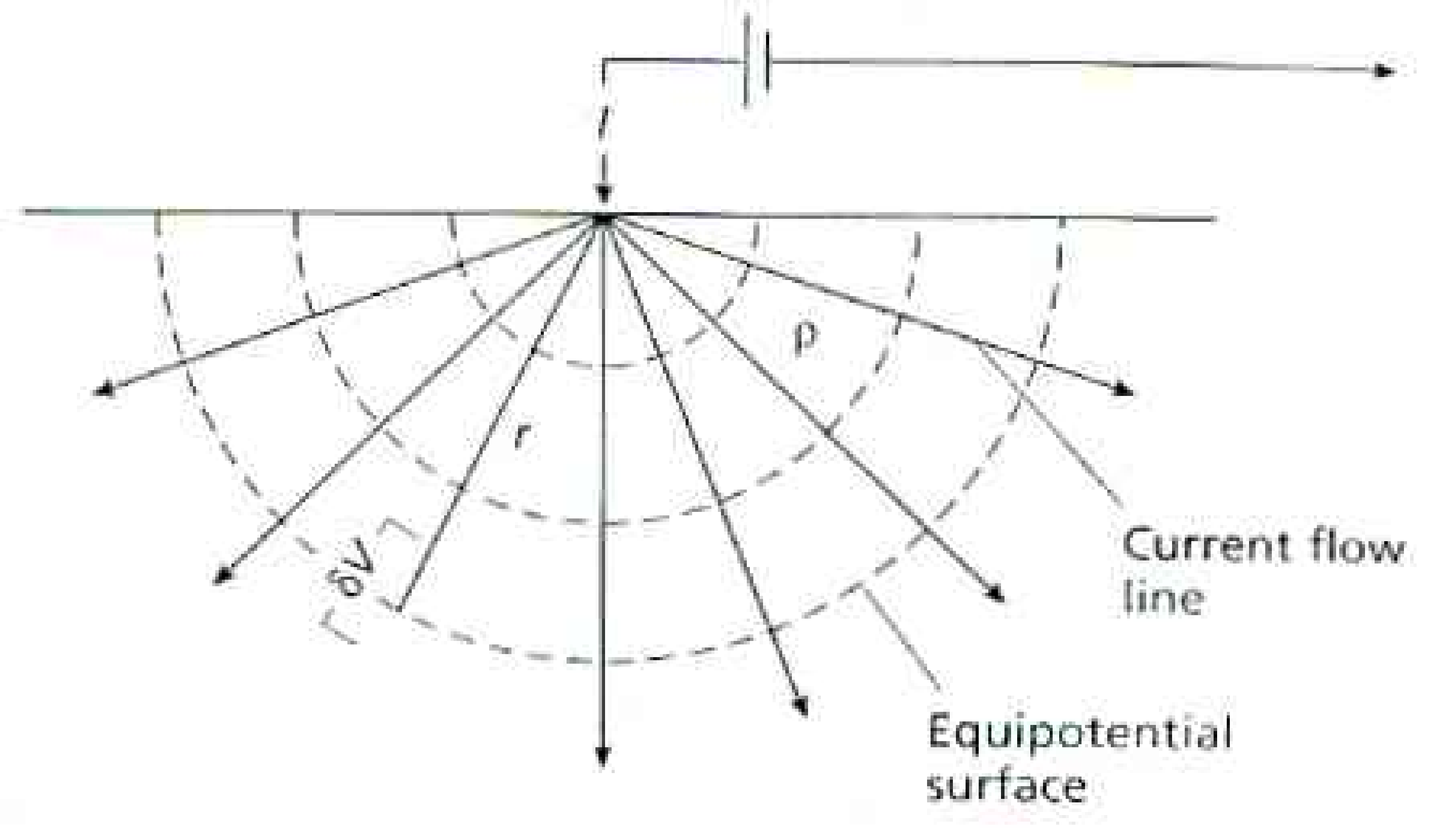


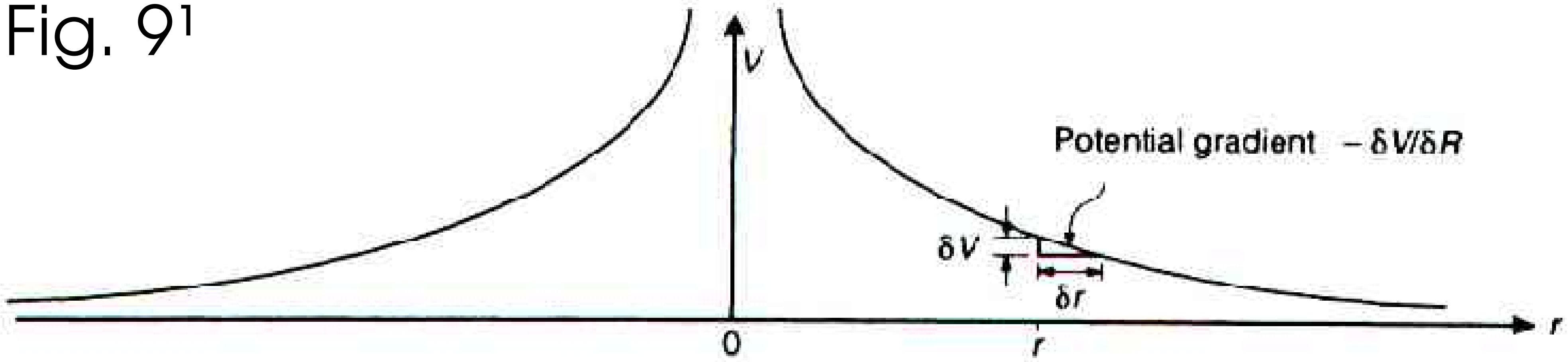
Fig. 8: Current flow for a single surface electrode²

Current flow in a homogeneous earth

- A single current electrode on the surface of a medium of uniform resistivity ρ is considered.
- The voltage drop between any two points on the surface can be described by the potential gradient.
- dV/dr is negative because the potential decreases in the direction of current flow.

Potential decay away from the point electrode

Fig. 9[1]



- Current flows radially away from the electrode so that the current distribution is uniform over hemispherical shells centered on the source.
- Lines of equal voltage (equipotentials) intersect the lines of equal current at right angles.

Potential of a point electrode

Ohm's law

$$\frac{\partial V}{\partial r} = -\frac{\rho I}{q} = -\frac{\rho I}{2\pi r^2}$$

Thus, the potential V_r at distance r is obtained by integration.

$$V_r = \int \partial V = -\int \frac{\rho I}{2\pi r^2} \partial r = \frac{\rho I}{2\pi r} \qquad (2.3)$$

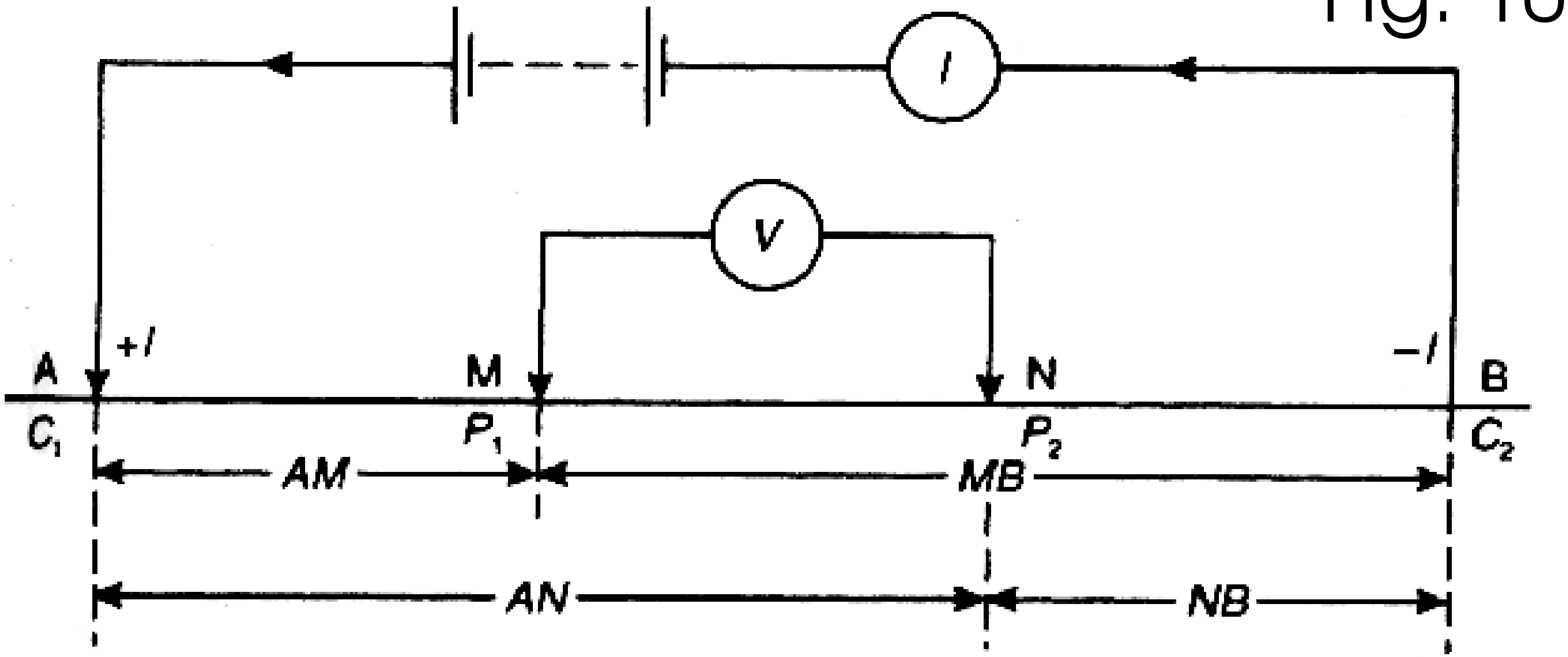
The circuit is completed by a current sink at a large distance from the electrode.

2.3 Electrode configurations and general case

2.3.1 General Case

The general case is considered, where the current sink is a finite distance from the source.

Fig. 10[1]



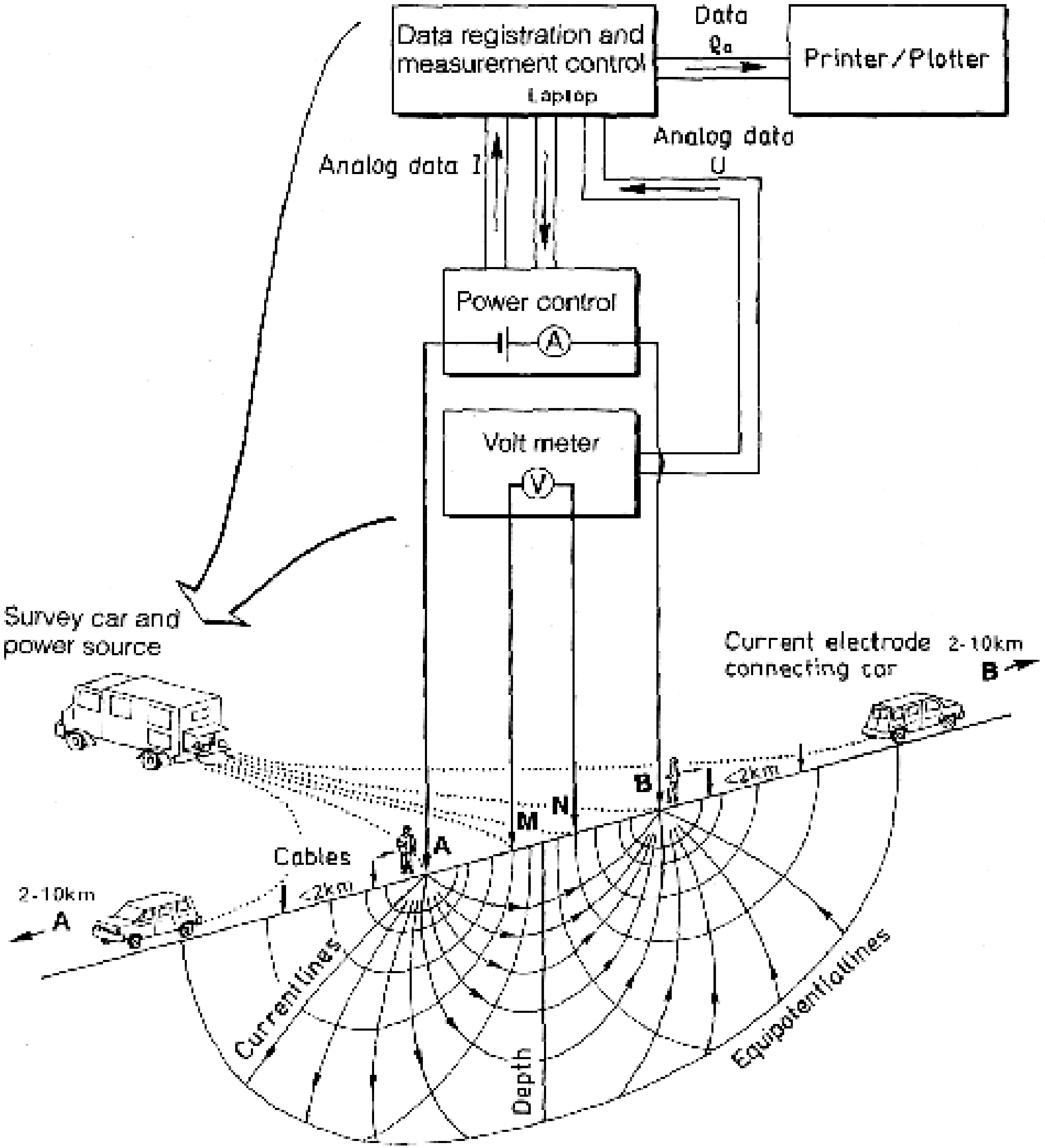


Fig. 11: Principle of measurement and potential field for for geoelectric DC surveys[3]

Potential for the general case

The potential V_M at the internal electrode M is the sum of the potential contributions V_A and V_B from the current source at A and the sink at B.

$$V_M = V_A + V_B$$

The potentials at electrode M and N are

$$V_M = \frac{\rho I}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB}\right] \qquad (2.4)$$

$$V_N = \frac{\rho I}{2\pi}\left[\frac{1}{AN} - \frac{1}{NB}\right]. \qquad (2.5)$$

Potential for the general case

Potential differences are measured

$$\Delta V_{MN} = V_M - V_N = \frac{\rho I}{2\pi}\left\{\left[\frac{1}{AM} - \frac{1}{MB}\right] - \left[\frac{1}{AN} - \frac{1}{NB}\right]\right\} \qquad (2.6)$$

$$\rightarrow \quad \rho = \frac{2\pi \Delta V_{MN}}{I}\left\{\left[\frac{1}{AM} - \frac{1}{MB}\right] - \left[\frac{1}{AN} - \frac{1}{NB}\right]\right\}^{-1} . \qquad (2.7)$$

Definition of the geometric factor

$$k = 2\pi\left\{\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right\}^{-1} \qquad (2.8)$$

$$\rho = \frac{\Delta V_{MN} k}{I} \quad . \qquad (2.9)$$

Discussion

- True resistivity of the subsurface if it is homogeneous.
- Where the ground is uniform, the resistivity should be constant and independent of both electrode spacing and surface location.
- When subsurface inhomogeneities exist, the resistivity will vary with the relative positions of electrodes.

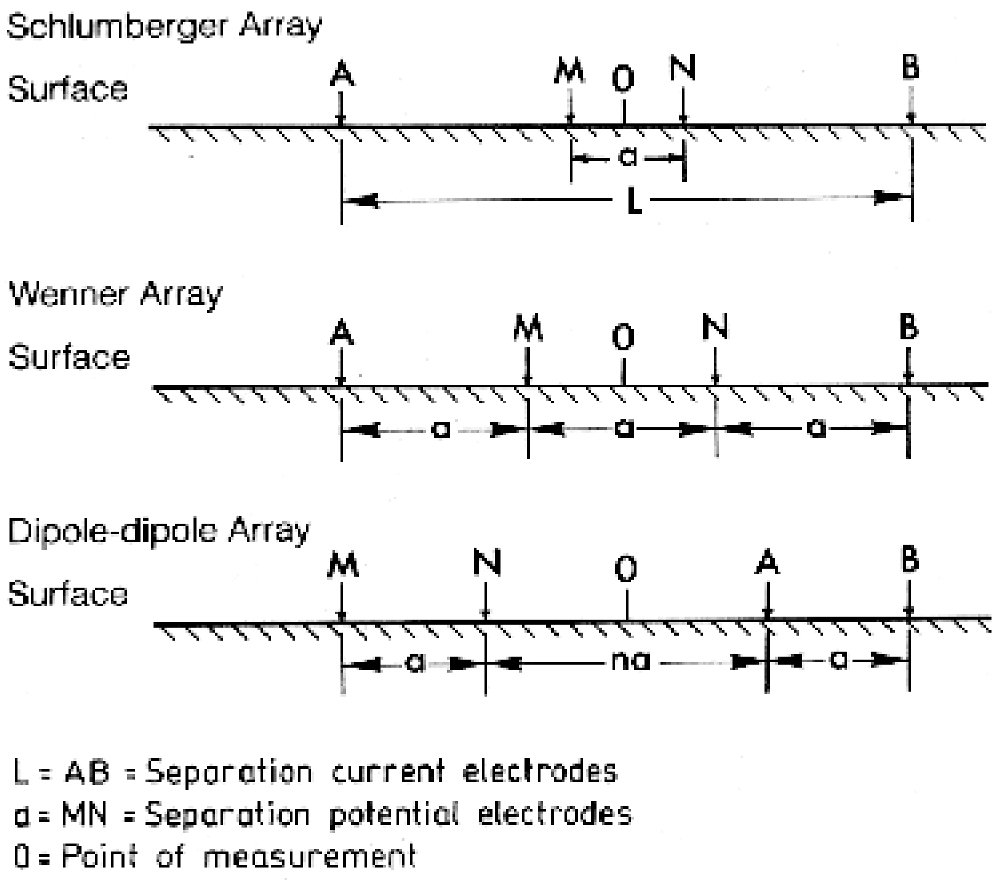
Discussion

- The calculated value is called apparent resistivity ρ_a .

$$\rho_a = \frac{\Delta V_{MN}}{I} k \qquad (2.10)$$

- In general, all field data are apparent resistivity. They are interpreted to obtain the true resistivities of the layers in the ground.

2.3.2 Electrode configurations



The apparent resistivity depends on the geometry of the array used (Eq. 2.8 and 2.9).

Fig. 12: Main types of electrode configurations [3]

Geometry factors for different configurations

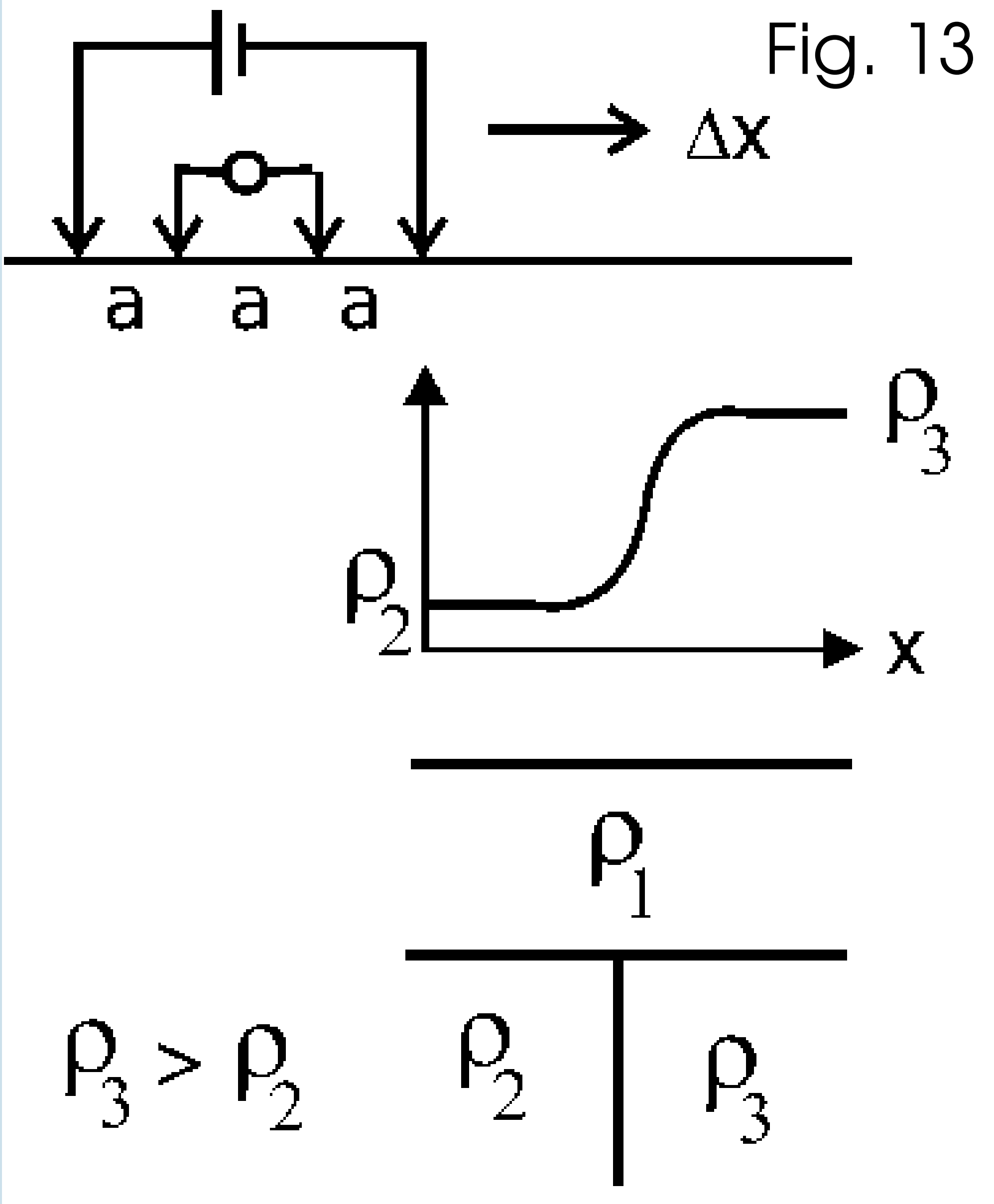
Derived geometric factors:

$$k=2\pi a$$ ... Wenner

$$k=\frac{\pi}{a}\left[\left(\frac{L}{2}\right)^2-\left(\frac{a}{2}\right)^2\right]$$ ... Schlumberger

$$k=\pi n(n+1)(n+2)a$$ ... Dipole-Dipole

2.3.4 Modes of deployment



- There are two main modes of deployment of electrode arrays.

A) <u>Geoelectric mapping</u>: Determination of lateral variation of resistivity in defined horizons. The current and potential electrodes are maintained at a fixed separation and progressively moved along a profile.

Applications of geoelectric mapping

- This method is employed in mineral prospecting to locate faults or shear zones or to determine localized bodies of anomalous conductivity.
- It is used in geotechnical surveys to determine variations in bedrock depth and the presence of steep discontinuities.

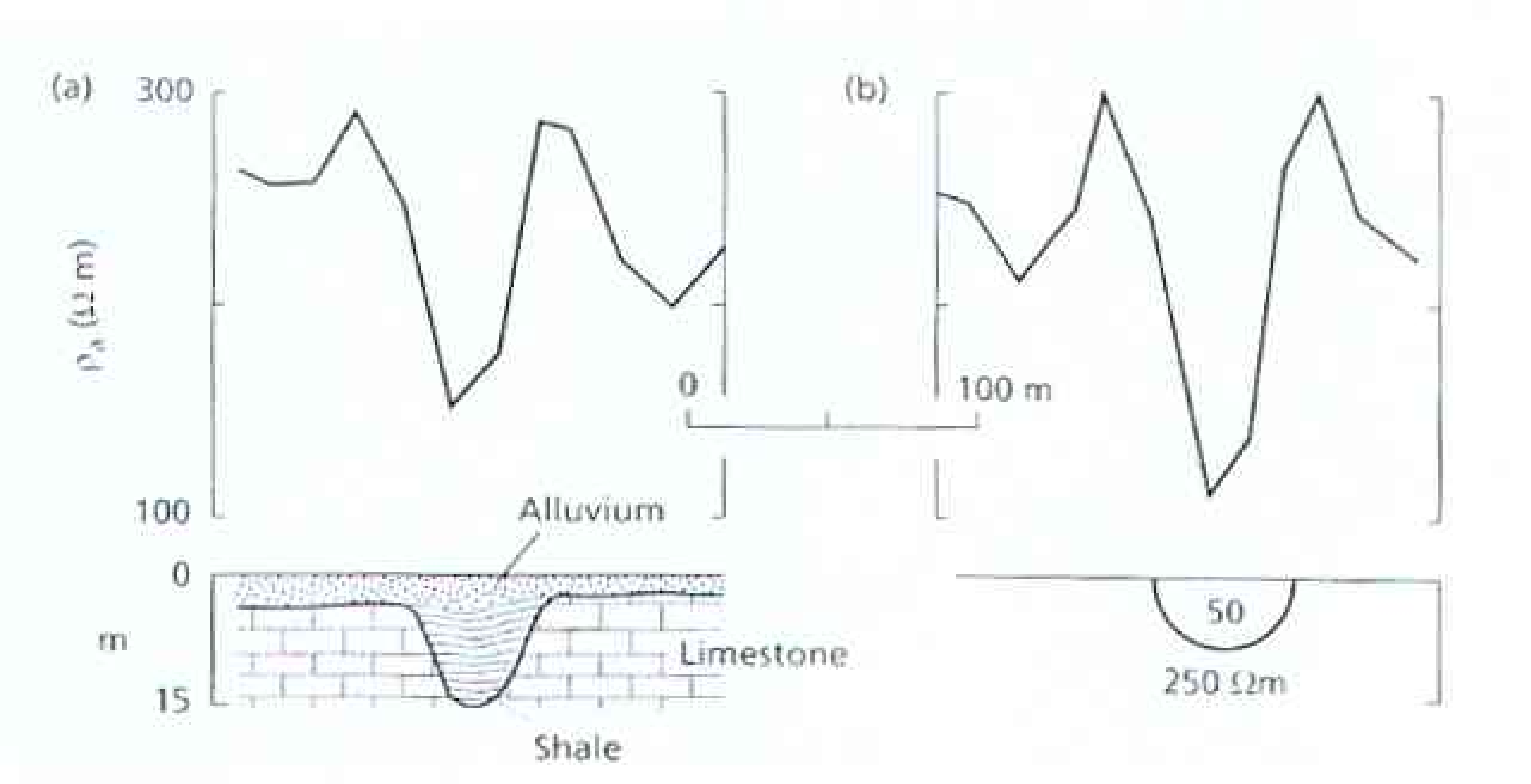


Fig. 14: (a) The observed Wenner resistivity profile over a shale-filled sink of known geometry in Kansas, USA. (b) The theoretical profile for a buried hemisphere.[2]

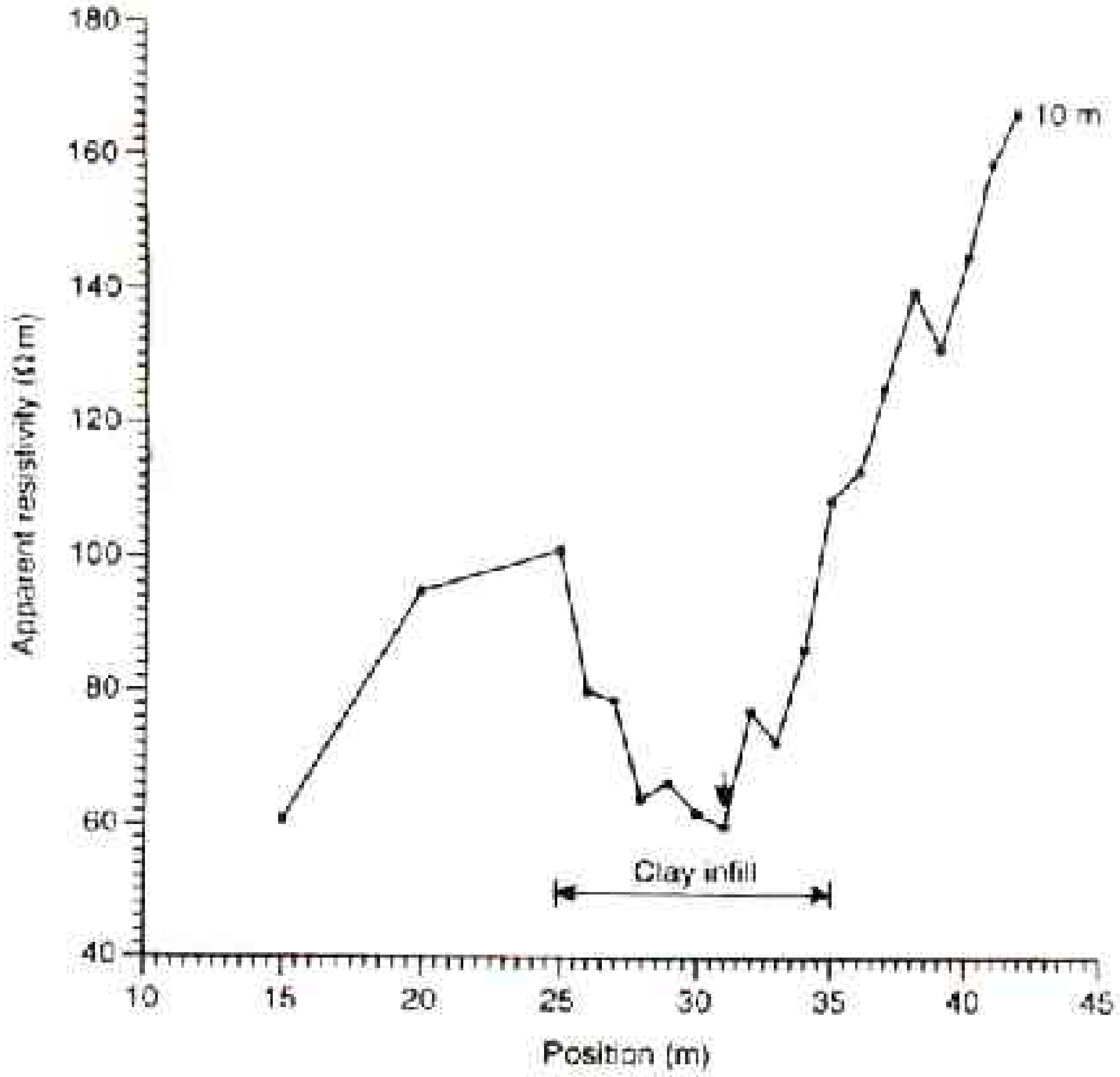


Fig. 15: A constant-separation traverse using a Wenner array with 10m electrode spacing over a clay-filled solution feature (position arrowed) in limestone.[1]

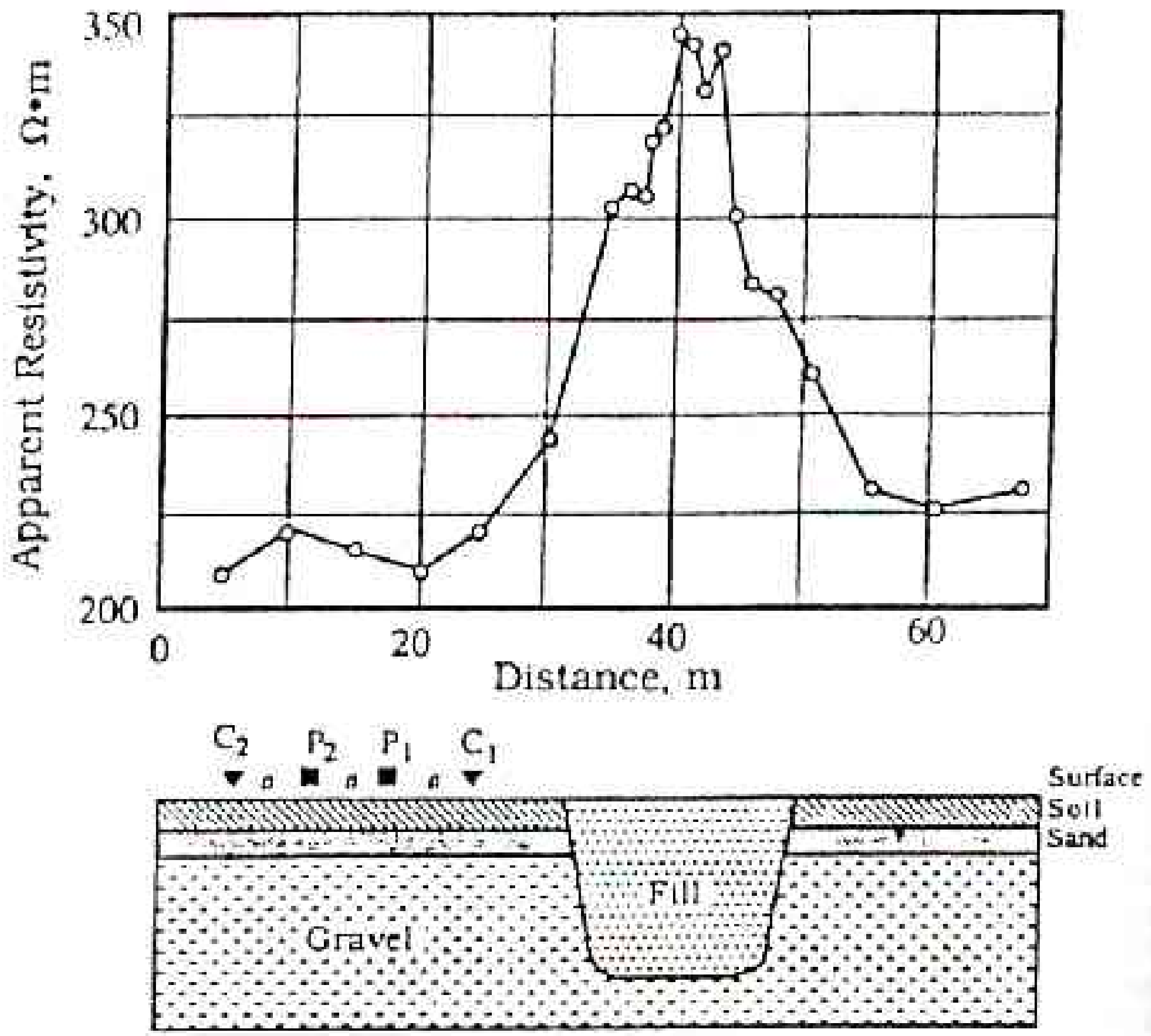


Fig. 16: Observed apparent resistivity profile across a resistive landfill using the Wenner array.[1]

Geoelectric Sounding

B) Geoelectric sounding: determination of the vertical variation of the resistivity.
The current and potential electrodes are maintained at the same relative spacing and the whole spread is progressively expanded about a fixed central point.

As the distance between the current electrodes is increased, so the depth to which the current penetrates is increased.

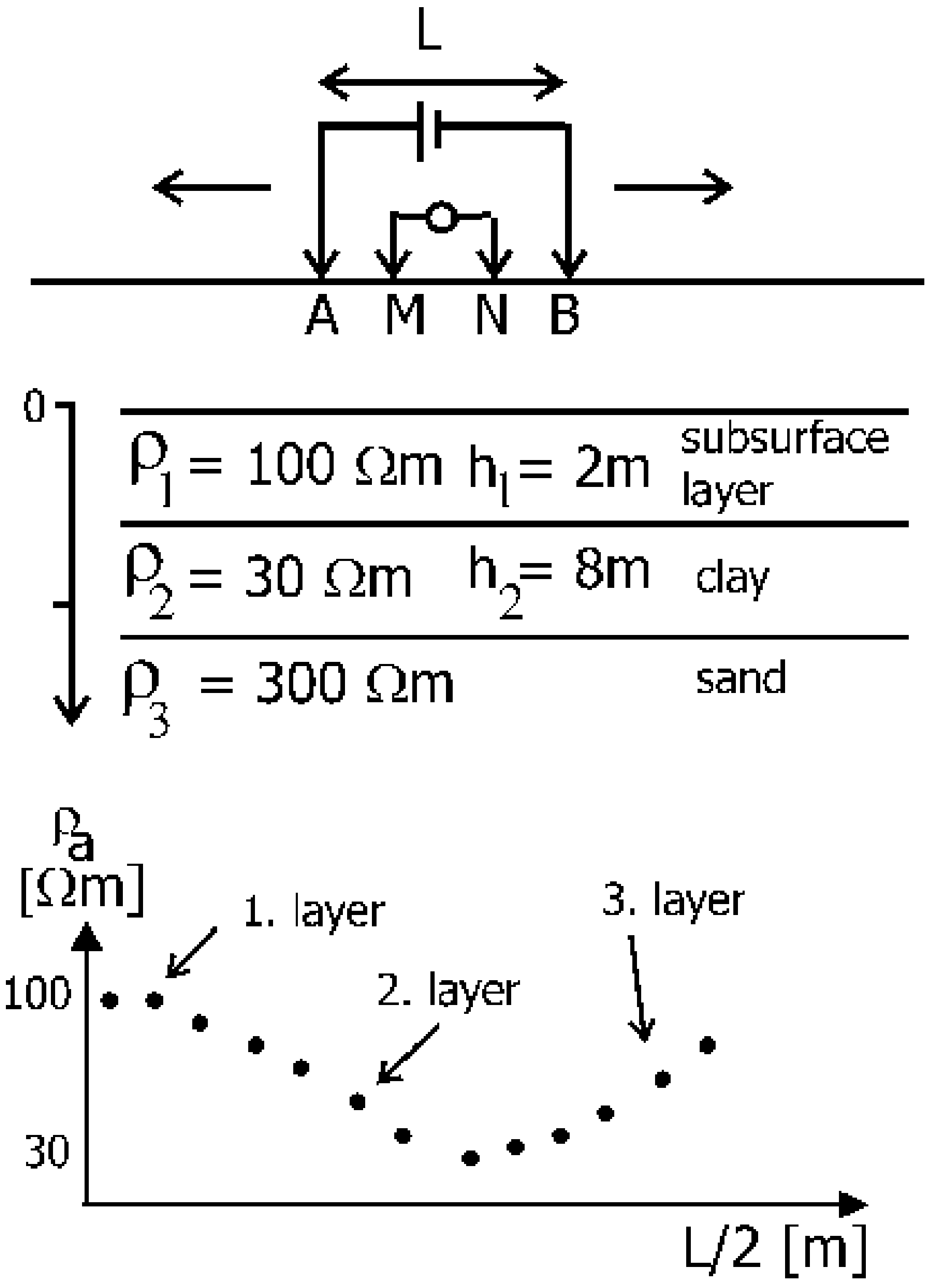


Fig. 17: Geoelectric Sounding

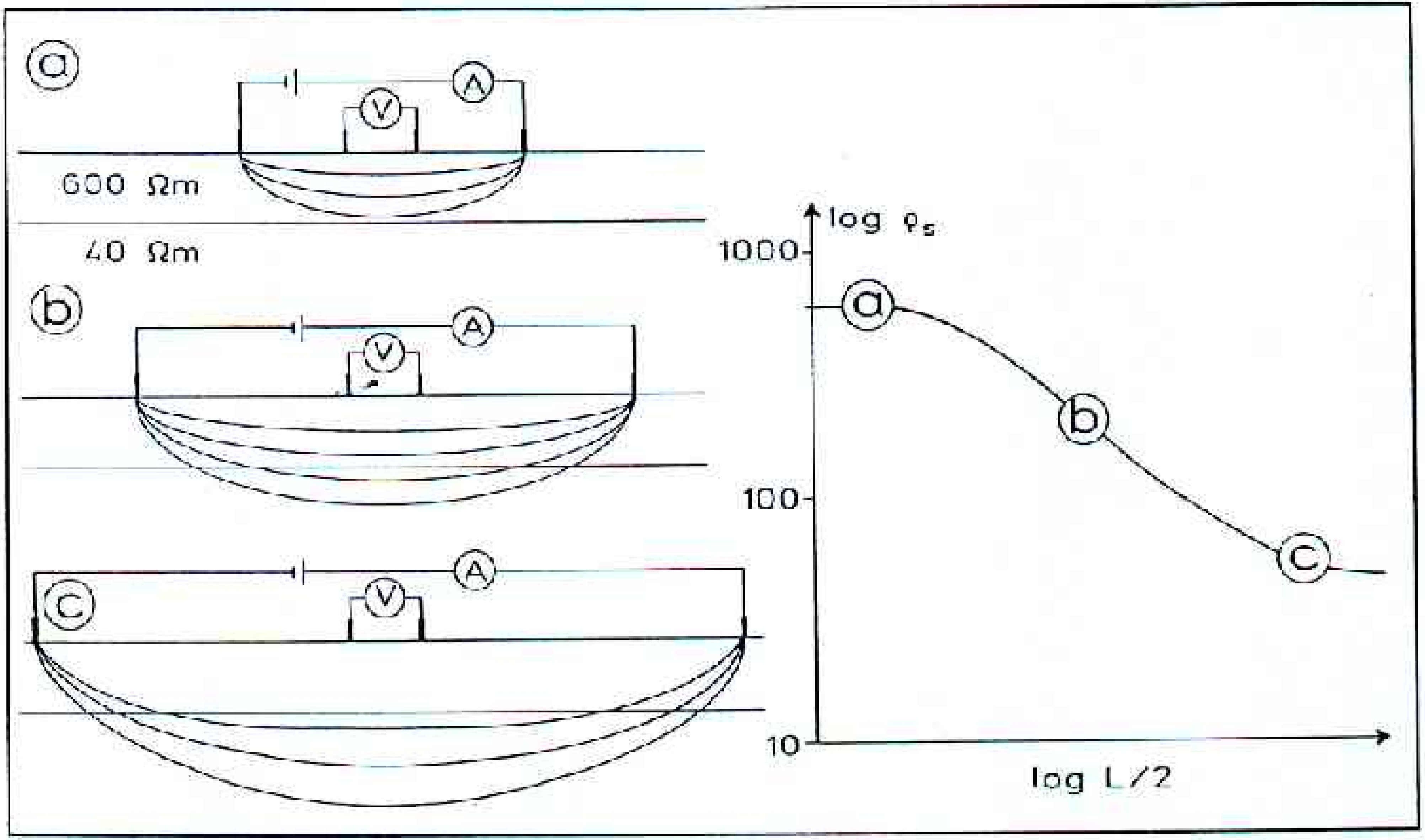


Fig. 18: Realization of a geoelectric sounding, development of a sounding curve.[4]

Multielectrode systems

- Soundings and mappings are very time consuming.
- Therefore multielectrode systems are developed. Typically 50 electrodes are laid out in two strings of 25 electrodes, with electrodes connected by a multi core cable to a switching box and resistance meter. The whole data acquisition procedure is software controlled from a laptop computer.

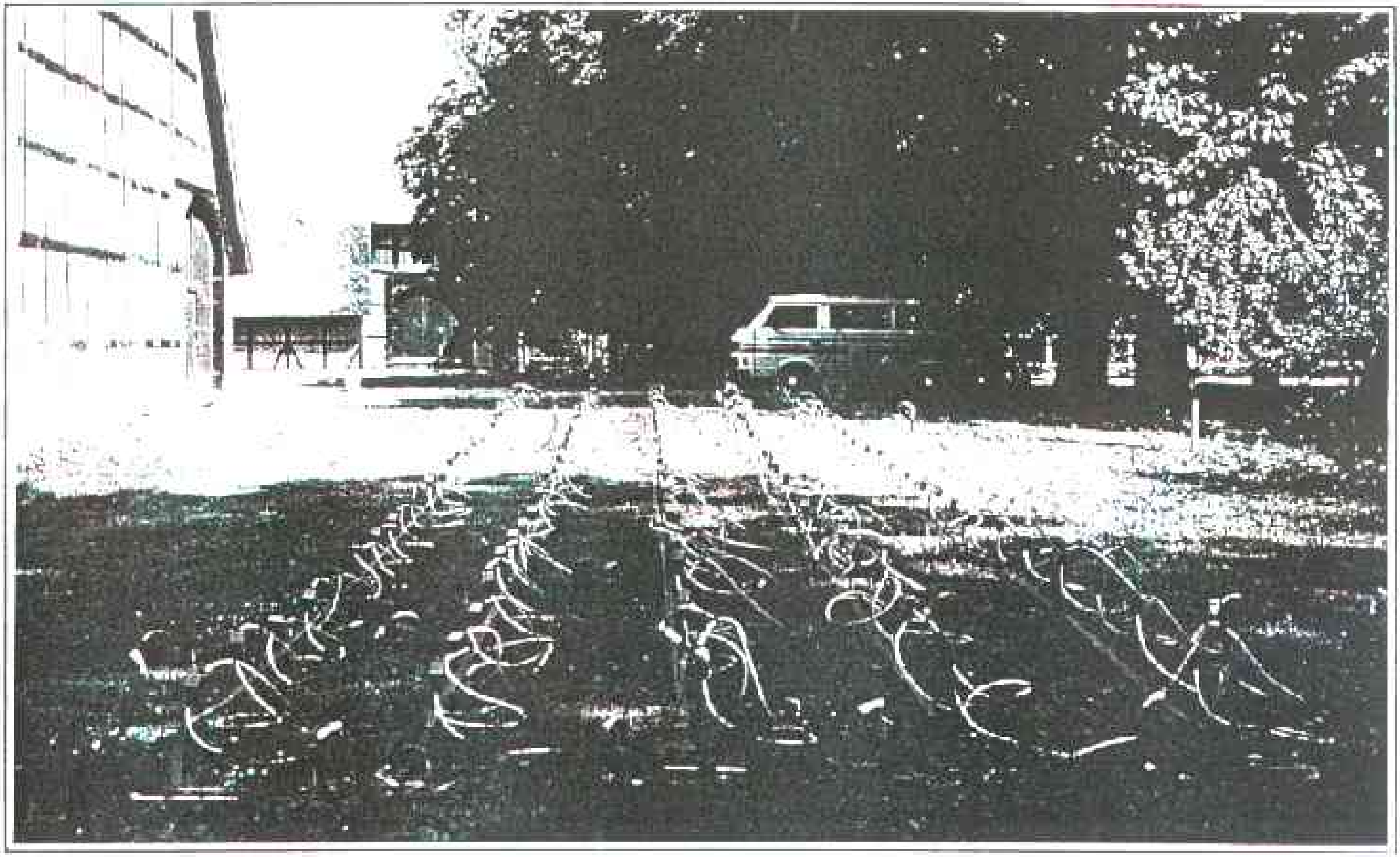


Fig. 19: Geoelectric mapping using a multi-electrode device.[4]

Continuous geoelectric mapping

- A new and quick mapping system is the pulled array continuous electrical profiling technique.

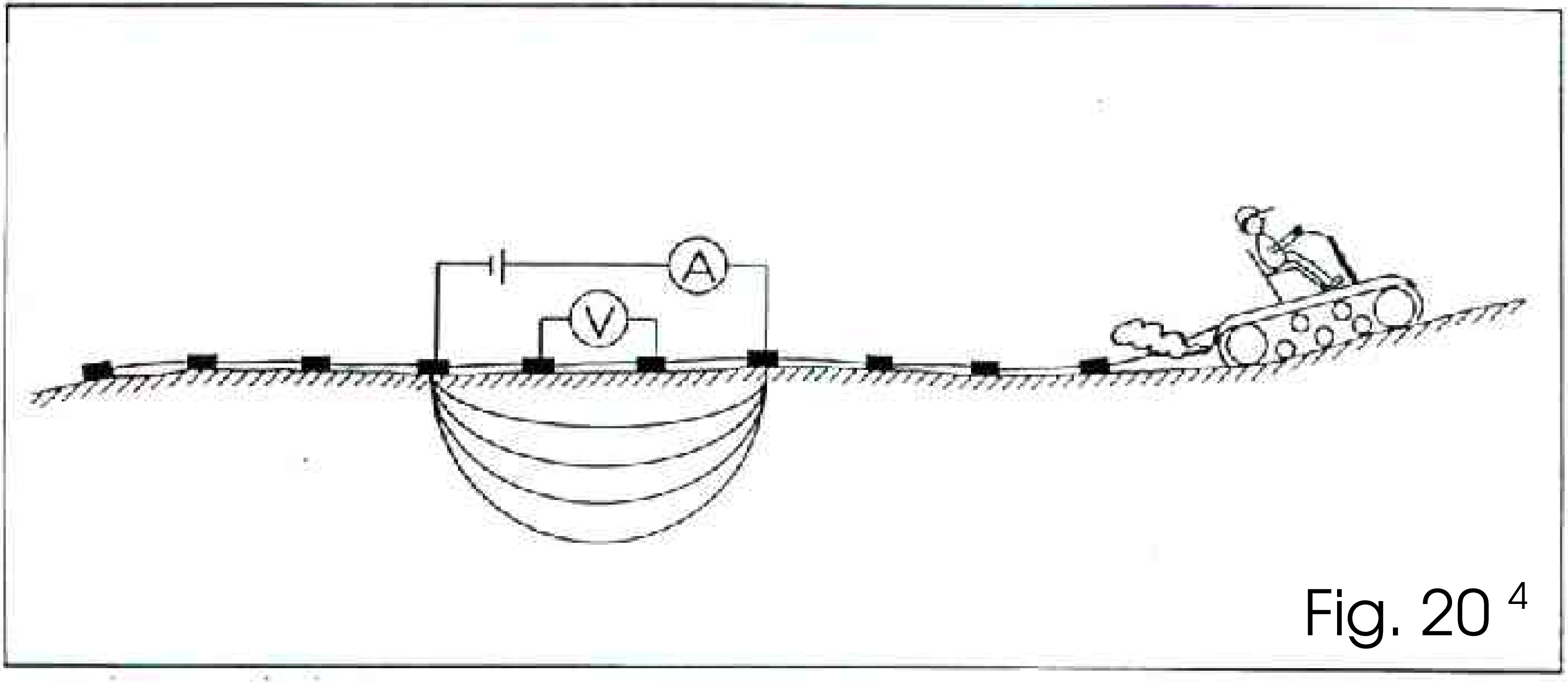


Fig. 20 [4]

Continuous geoelectric mapping

- An array of heavy steel electrodes each weighing 10–20 kg is towed behind a vehicle containing the measuring equipment. Measurement are made continuously. 10–15 line kilometers of profiling can be achieved in a day.

2.4 Interpretation of geoelectric data

Aim of the interpretation: Determination of the resistivity and thickness of each layer from the observed resistivities.

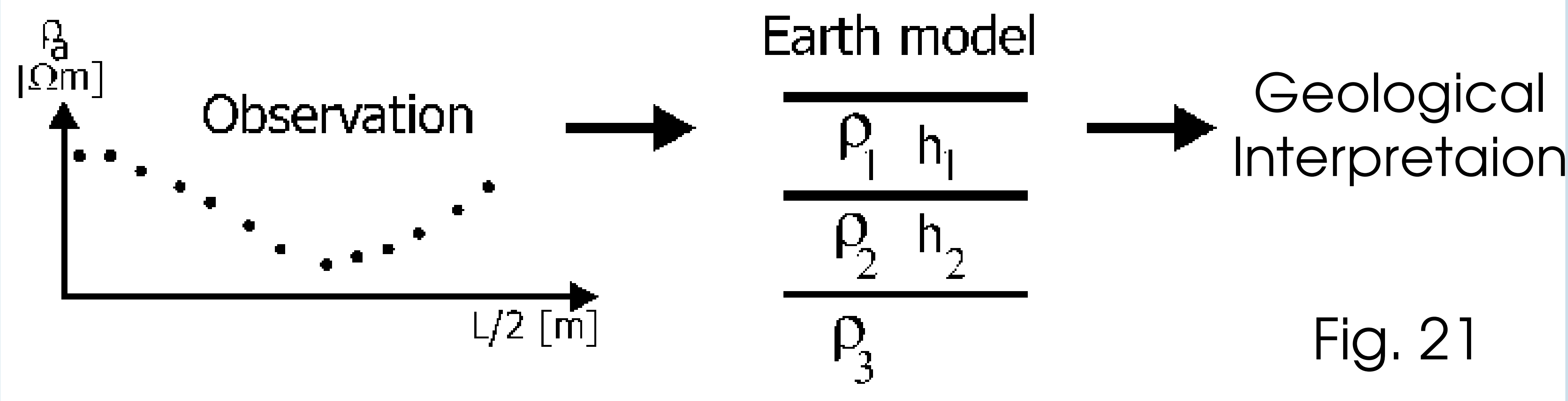


Fig. 21

Vertical electrical soundings can be interpreted by using:

a) graphical model curves (master curves) → little used

b) computer modeling (inversion calculation)

Master curves

Master curves: The master curves are prepared in a dimensionless form for a number of reflection coefficients $k=(\rho_2-\rho_1)/(\rho_2+\rho_1)$ or for ρ_2/ρ_1 by dividing ρ_a/ρ_1 and by dividing a/z_1 . z_1 is the thickness of the upper layer (for two layer case!).

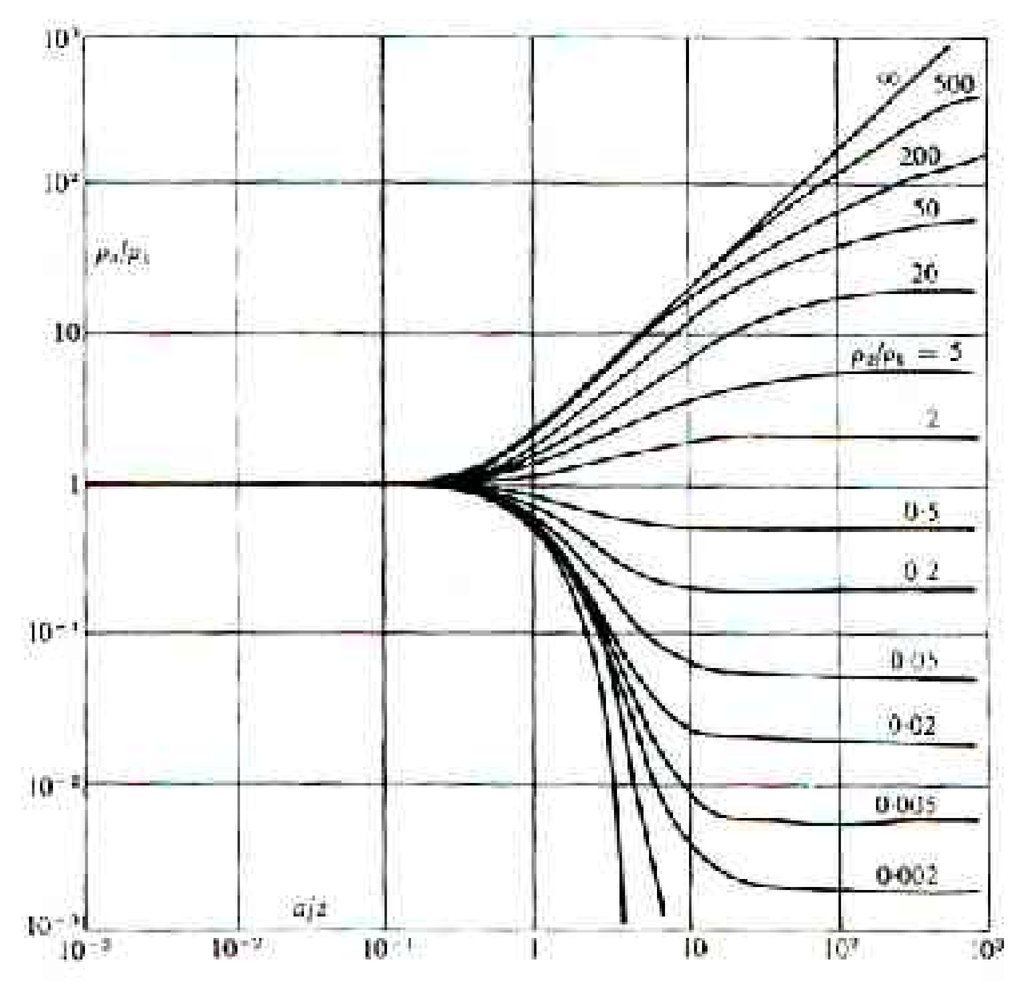


Fig. 22[5]

Usage of the master curves

The field curve to be interpreted is plotted on transparent logarithmic paper with the same modulus as the master curve. It is then shifted over the master curve keeping the coordinate axes parallel, until a reasonable match is obtained with one of the master curves or with an interpolated curve.

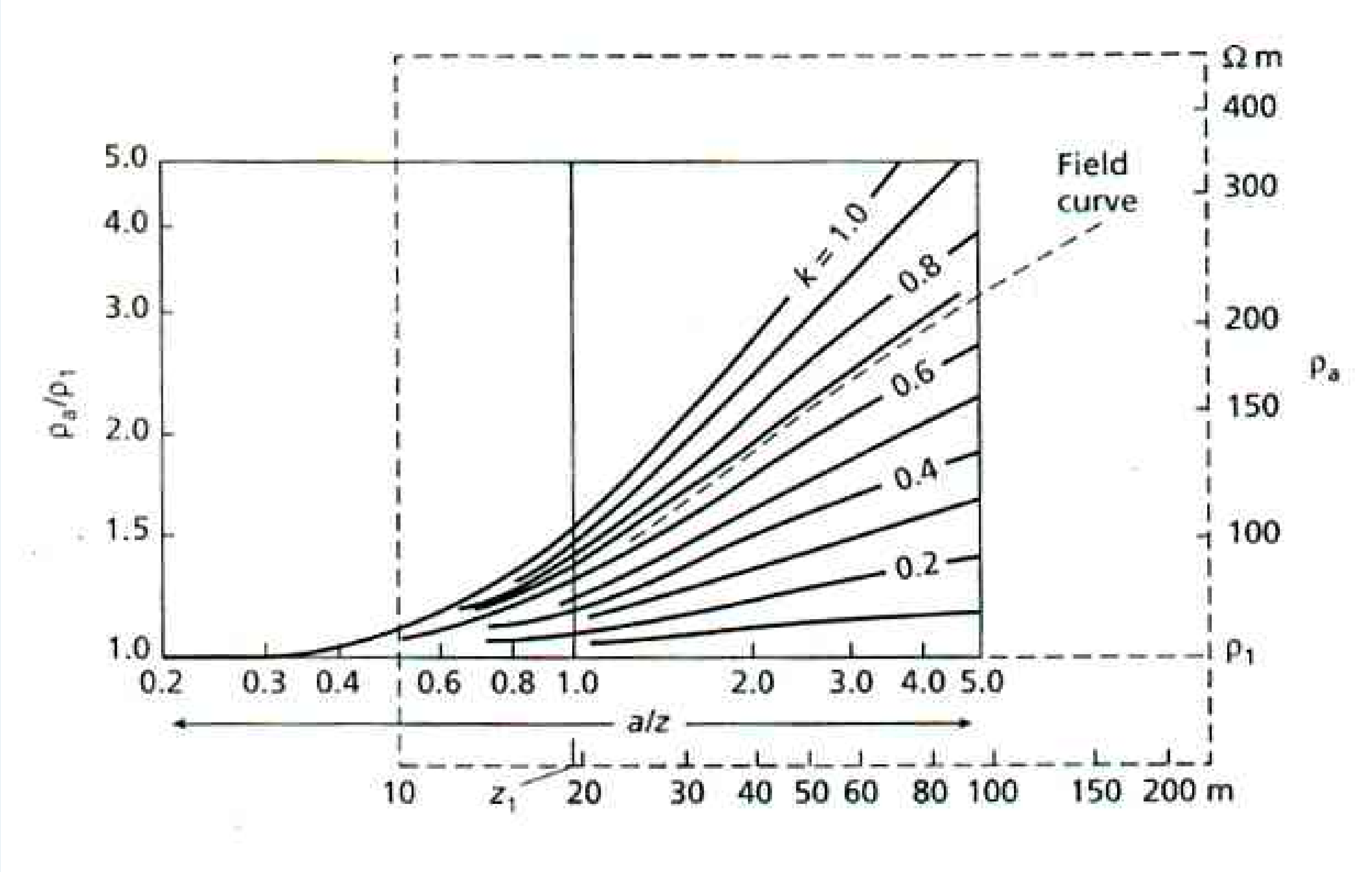


Fig. 23: The interpretation of a two-layer apparent resistivity graph by comparison with a set of master curve. The upper layer resistivity ρ_1 is 68 $\Omega\,\mathrm{m}$ and its thickness z_1 is 19.5 m. [2]

3 layer case

3 layer case: Much larger sets of curves are required to represent the increased number of possible combinations of resistivities and layer thicknesses.

$$\rho_a / \rho_1 = f(\rho_{2,} \rho_{3,} k_{1,} z_{1,} a)$$

Direct curve fitting is time consuming, better use auxiliary point techniques.

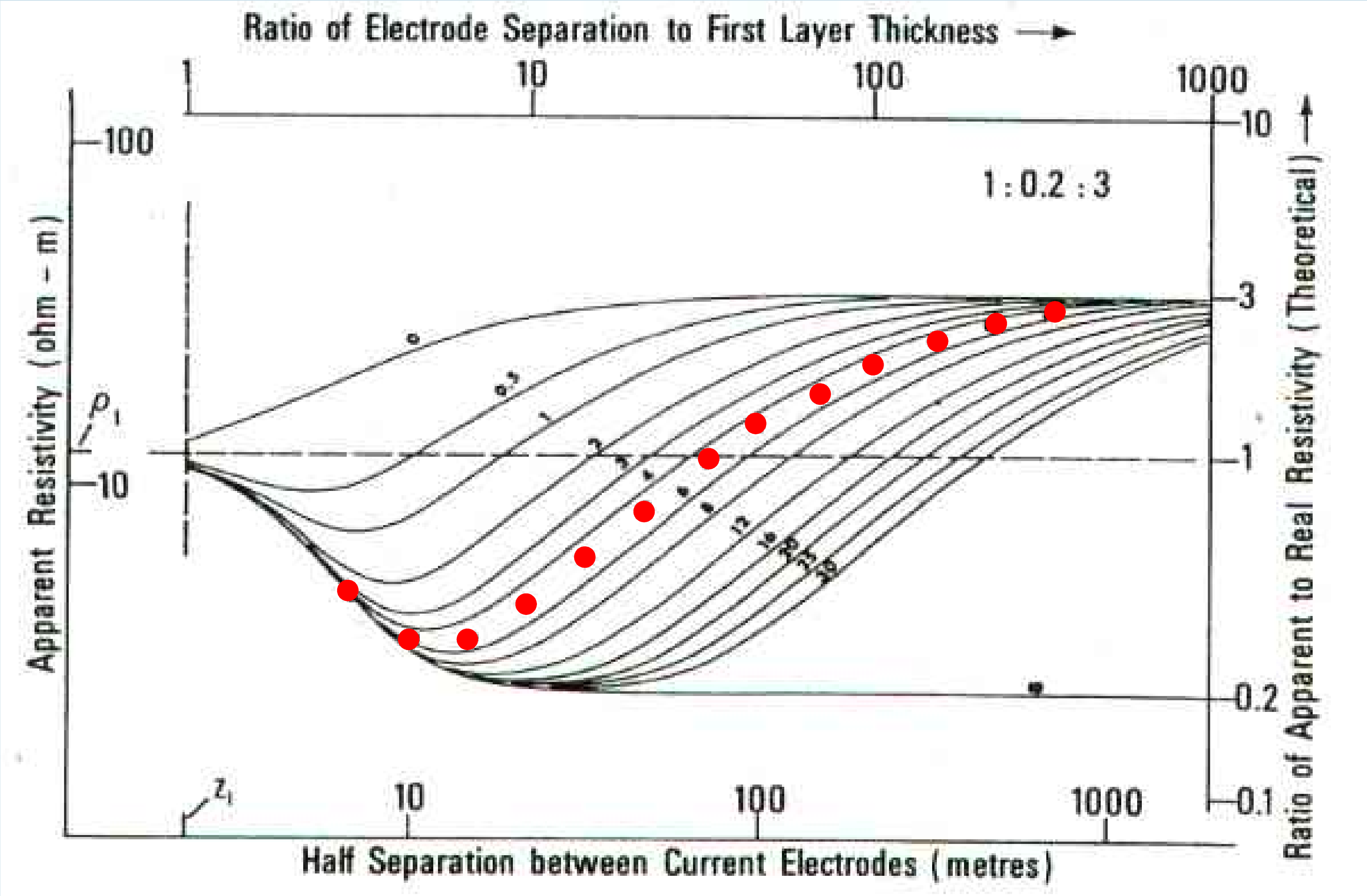


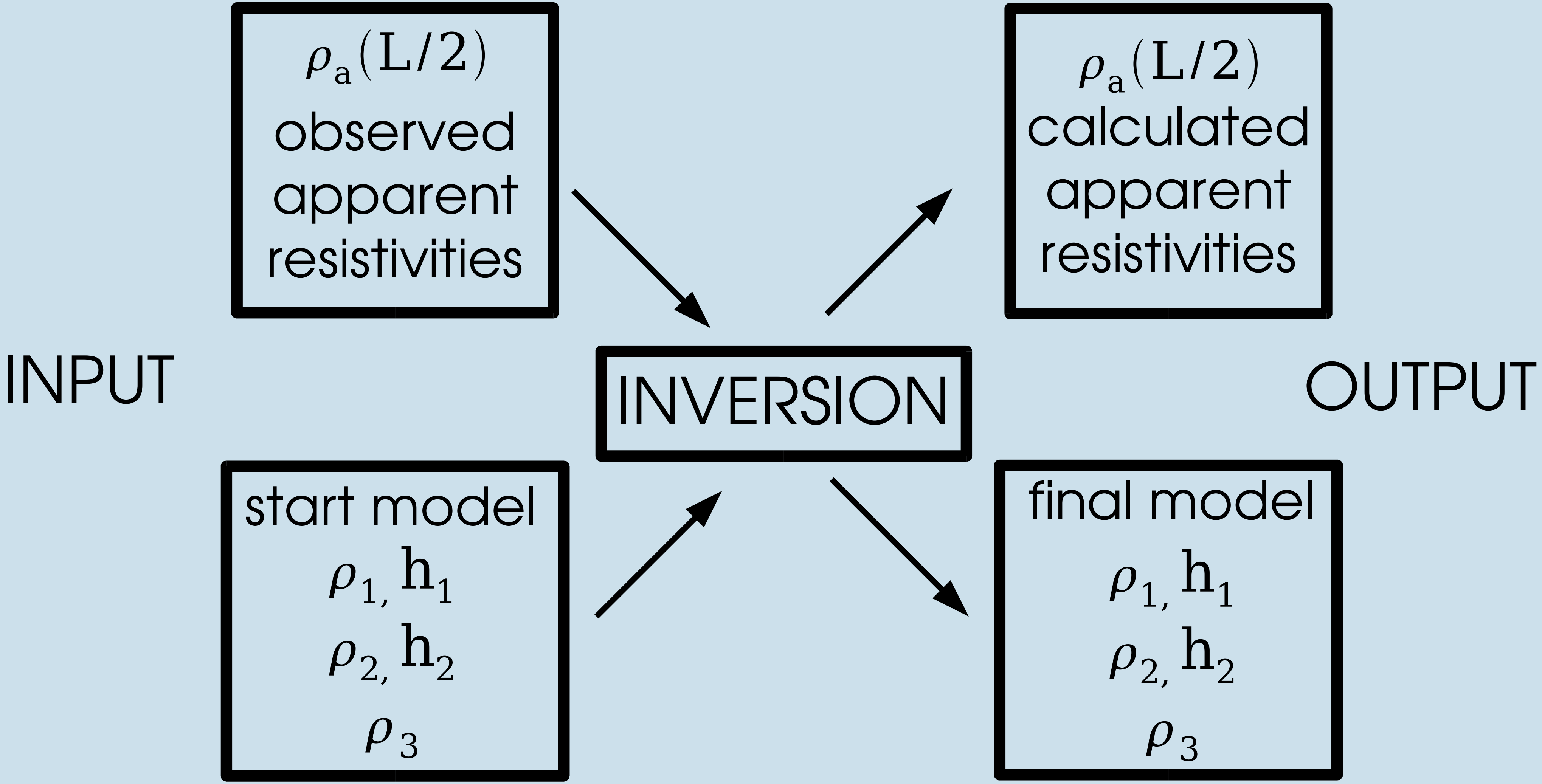
Fig. 24: Example of curve fitting for three layers; experimental points are based upon actual measurements in the Arctic using a Schlumberger array.[6]

Fig. 24 (continued): $\rho_2/\rho_1=0.2, \rho_3/\rho_1=3$. Number on curves are values of z_2/z_1. To obtain the field parameters the axes (dashed lines) of the theoretical curves are extended to intersect the axes (full lines) of the field curve. The points of intersection give the field values of ρ_1 and z_1 . ρ_2 and ρ_3 follow from the ratios given for this family of curves. z_2 is found from the ratio number given on the best fitting curve. In this case an intepolation has been made between curves for 4 and 6. Final results therefore give:

$$\rho_1=13\,\Omega\,\mathrm{m}, \rho_2=1.6\,\Omega\,\mathrm{m}, \rho_3=39\,\Omega\,\mathrm{m}$$

$$z_1=2.2\,\mathrm{m}, z_2=11\,\mathrm{m}$$

Computer modeling



INPUT

$\rho_a(L/2)$

observed apparent resistivities

start model

$\rho_{1,}\, h_1$

$\rho_{2,}\, h_2$

ρ_3

INVERSION

OUTPUT

$\rho_a(L/2)$

calculated apparent resistivities

final model

$\rho_{1,}\, h_1$

$\rho_{2,}\, h_2$

ρ_3

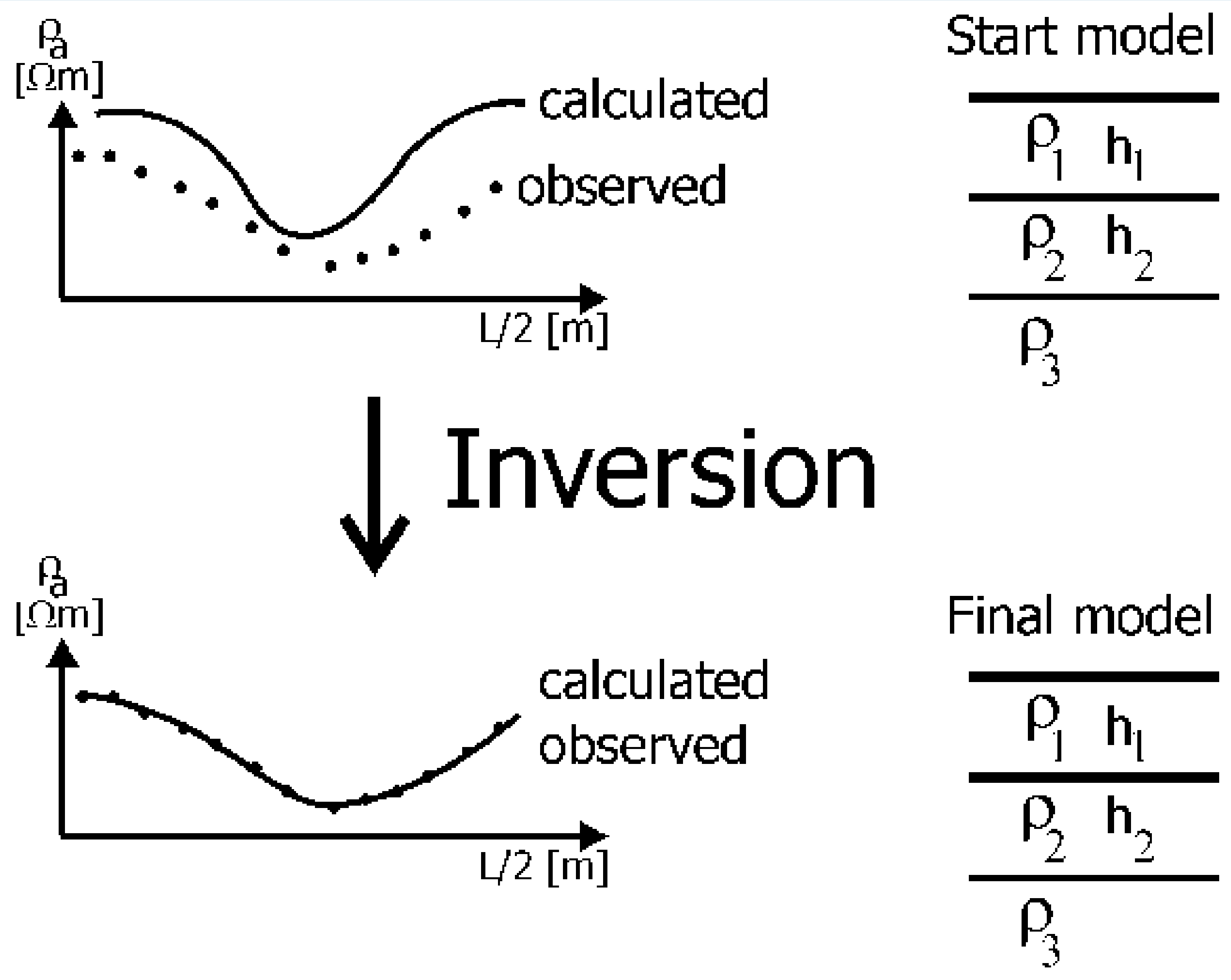


Fig. 25: Inversion scheme in geoelectric sounding

2.4.1 Possible interpretation errors

a) Equivalent models

- Resistivities and thicknesses of each layer can be derived from the apparent resistivity curve clearly.
- In the field measurement errors occur $(\rho_a \pm \Delta \rho_a)$.
- The apparent resistivity curve can be interpreted by different resistivity models.

The **principle of equivalence**: The thickness and resistivity can not be derived independently.

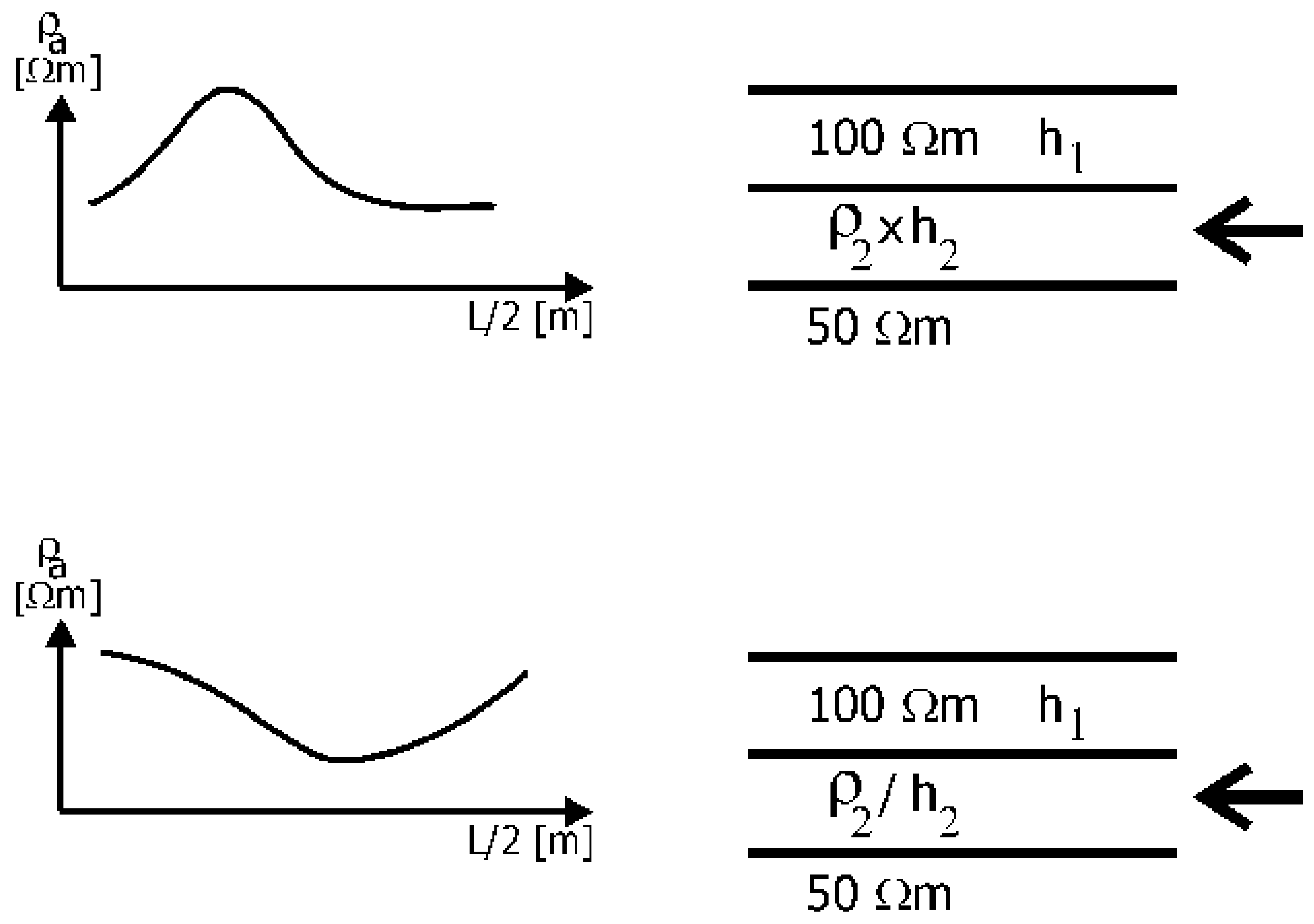
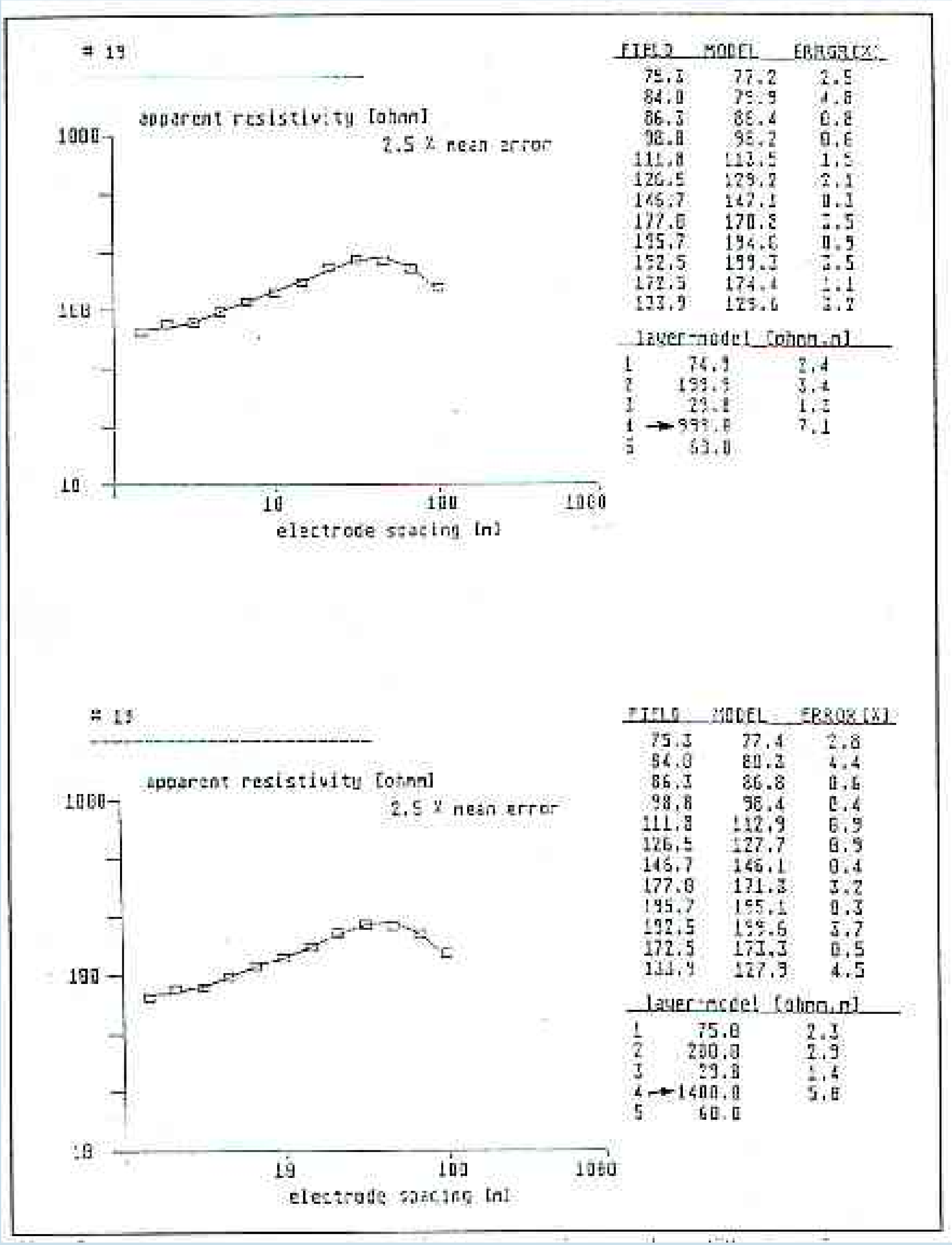

ρ_a [Ωm]
L/2 [m]
100 Ωm
h_1
$\rho_2 \times h_2$
50 Ωm
ρ_a [Ωm]
L/2 [m]
100 Ωm
h_1
ρ_2 / h_2
50 Ωm

Fig. 26



FIELD	MODEL	ERROR(%)
75.3	77.2	2.5
84.0	79.9	4.8
86.3	86.4	0.8
98.8	95.2	0.6
111.8	113.5	1.5
126.5	129.2	2.1
146.7	147.1	0.3
177.0	170.3	3.5
195.7	194.6	0.5
192.5	199.3	3.5
172.5	174.4	1.1
133.9	129.6	3.2

layer-model [ohm.m]

1	74.9	2.4
2	199.9	3.4
3	29.8	1.3
4	→ 999.0	7.1
5	63.0	

FIELD	MODEL	ERROR(%)
75.3	77.4	2.8
84.0	80.2	4.4
86.3	86.8	0.6
98.8	98.4	0.4
111.8	112.9	0.9
126.5	127.7	0.9
146.7	146.1	0.4
177.0	171.3	3.2
195.7	195.1	0.3
192.5	199.6	3.7
172.5	173.3	0.5
133.9	127.9	4.5

layer-model [ohm.m]

1	75.0	2.3
2	200.0	2.9
3	29.8	1.4
4	→ 1400.0	5.8
5	60.0	

Fig. 27: Example of a geoelectric sounding on a gravel deposit. Depending on the assumed apparent resistivity of the target the thickness of the deposit varies. [4]

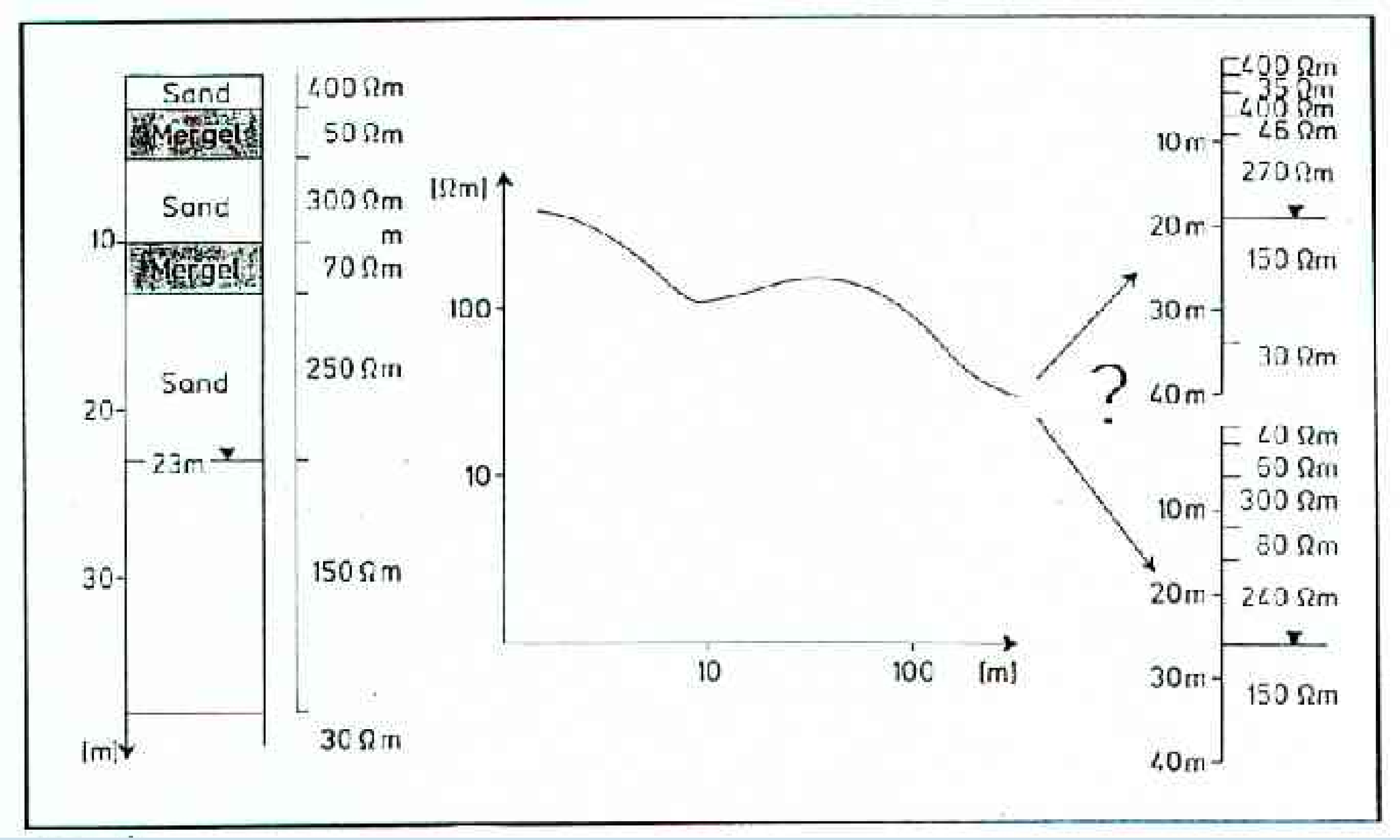


Fig. 28: Due to the principle of equivalence different depth of the groundwater table can be derived from the data. [4]

Model selection

- The geophysicist has to select the model, that agrees best with the known geological and hydrogeological structures of the ground. Another selective criterion is the comparison with neighboring soundings.

b) Supression

This is particularly a problem when three or more layers are present and their resistivities are ascending or descending with depth. The middle intermediate layer may not be evident on the field curve.

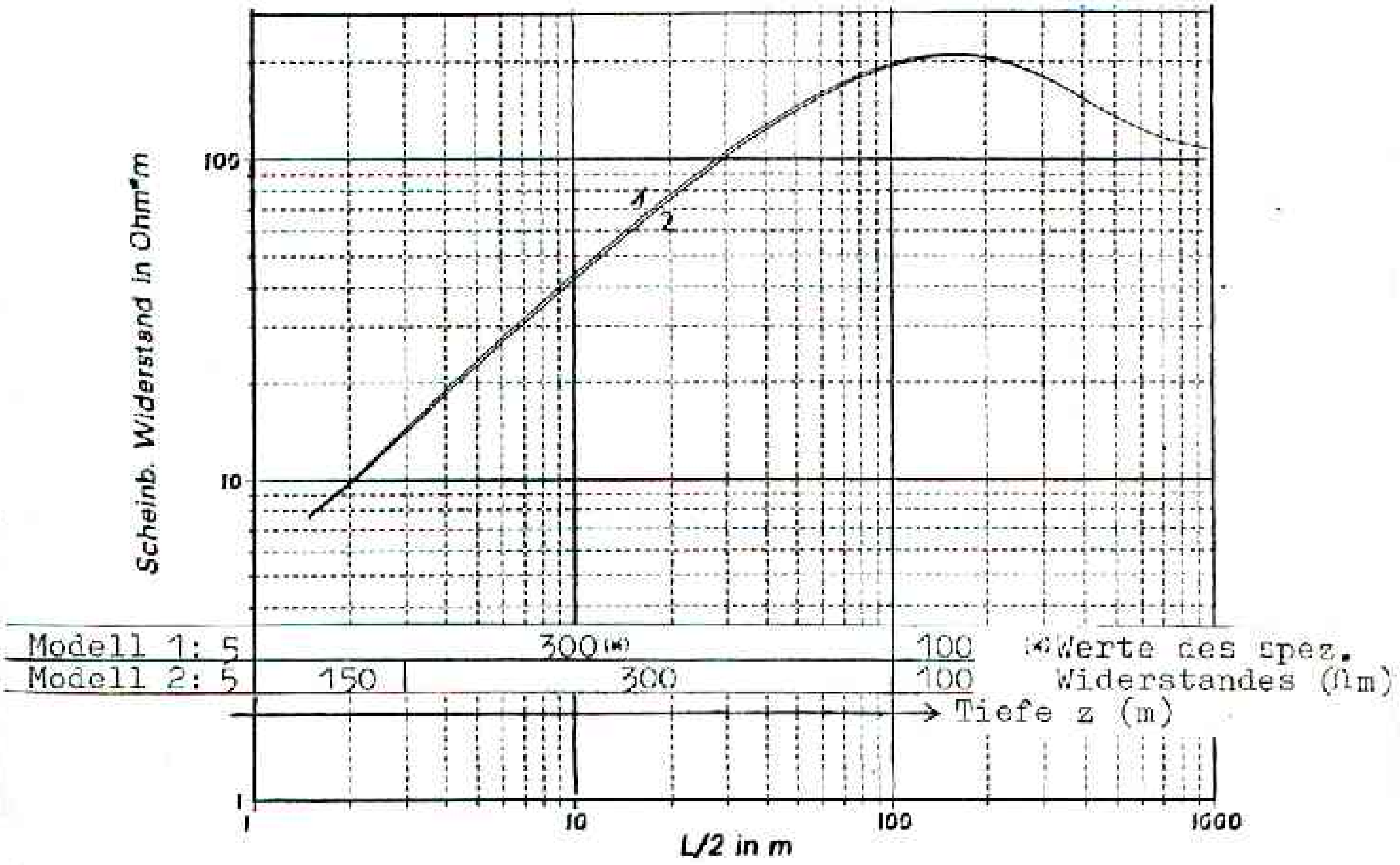


Fig. 29: Example for supression. [7]

c) The effect of anisotropy

In sediments such as clay or shale the resistivity perpendicular to the layering is usually greater than parallel to the direction of layering.

Anisotropy coefficient $\lambda=\sqrt{\rho_t/\rho_l}$

Averaged resistivity $\rho=\sqrt{\rho_t\rho_l}$

Anisotropy results in too large thicknesses being assigned to layers.

$$h'=\lambda h$$

Sand, gravel $\lambda=1.3$

Coal $\lambda=2$

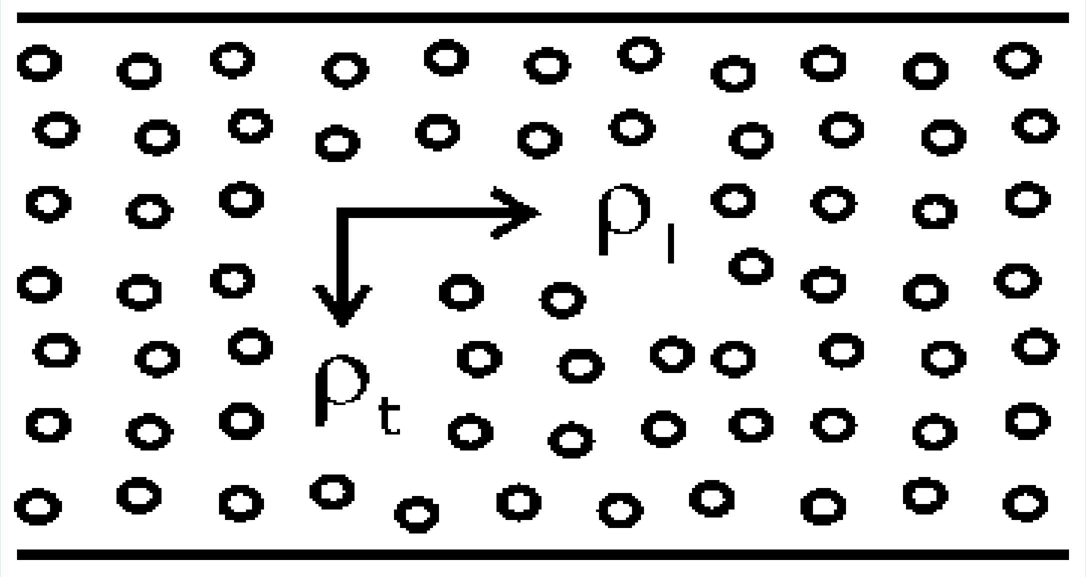


Fig. 30

d) Non-horizontal layering

1D interpretation is valid, if the dip of the layers is not greater than 15°.

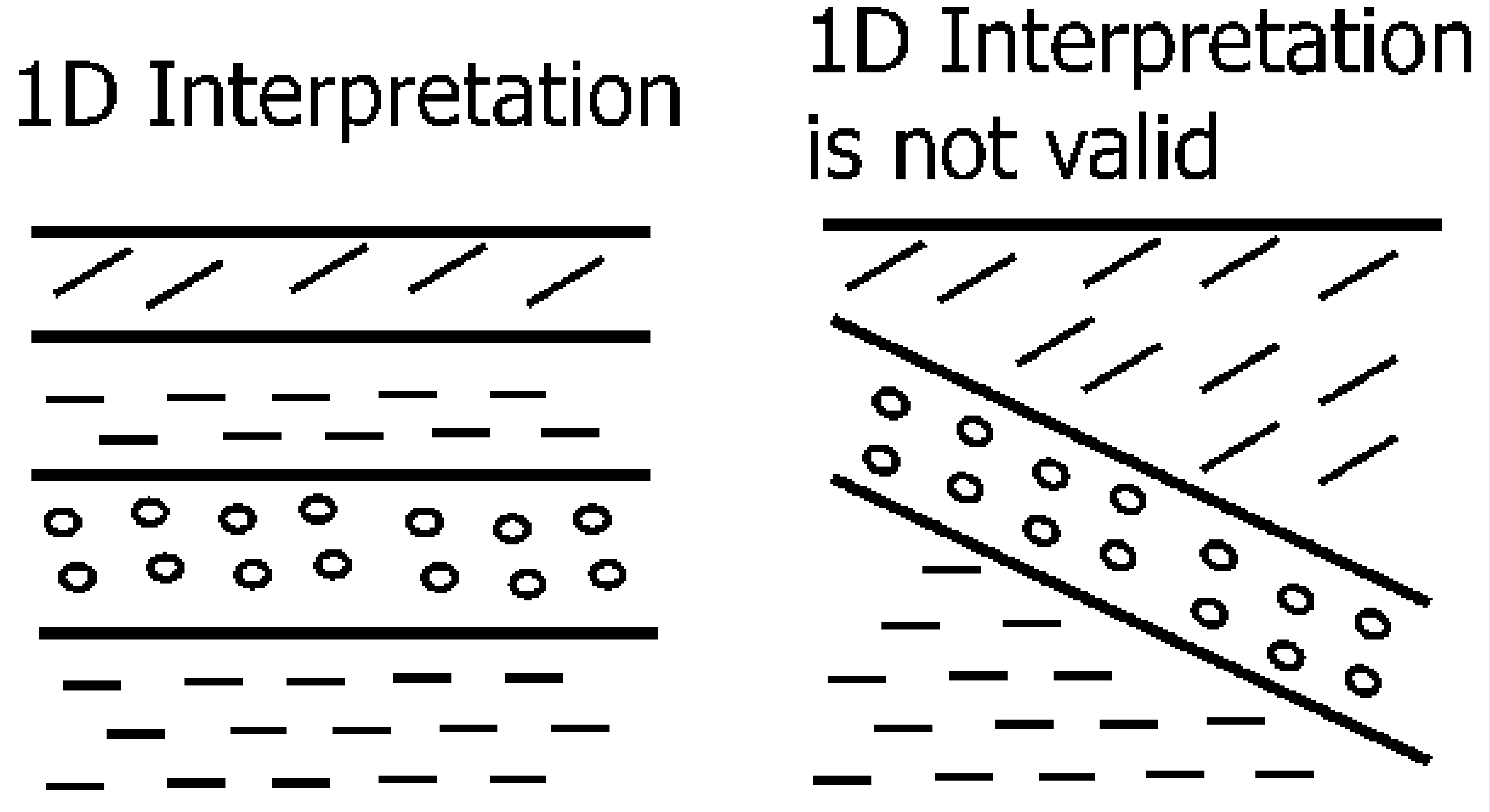


Fig. 31

e) Interpretation errors caused by faults

- Measurements perpendicular to the strike direction of the fault.
- The location of the fault can be determined.

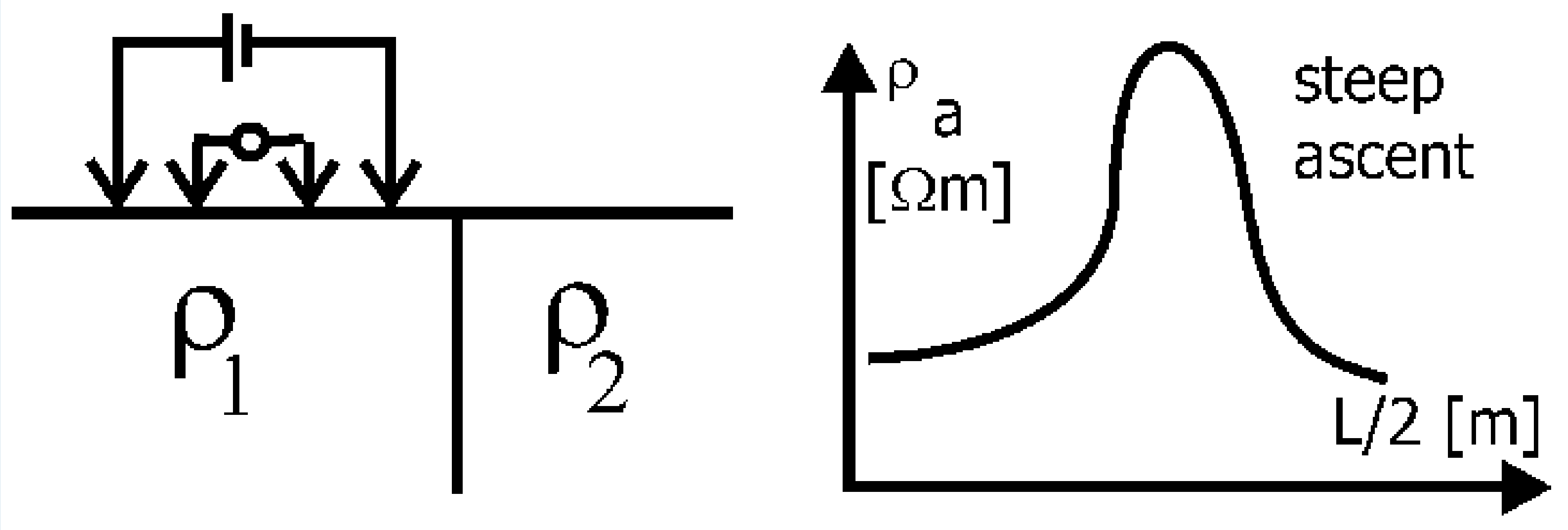


Fig. 32

- Measurements parallel to the strike direction of the fault.
- No effect of the fault can be seen on the apparent resistivity curve.

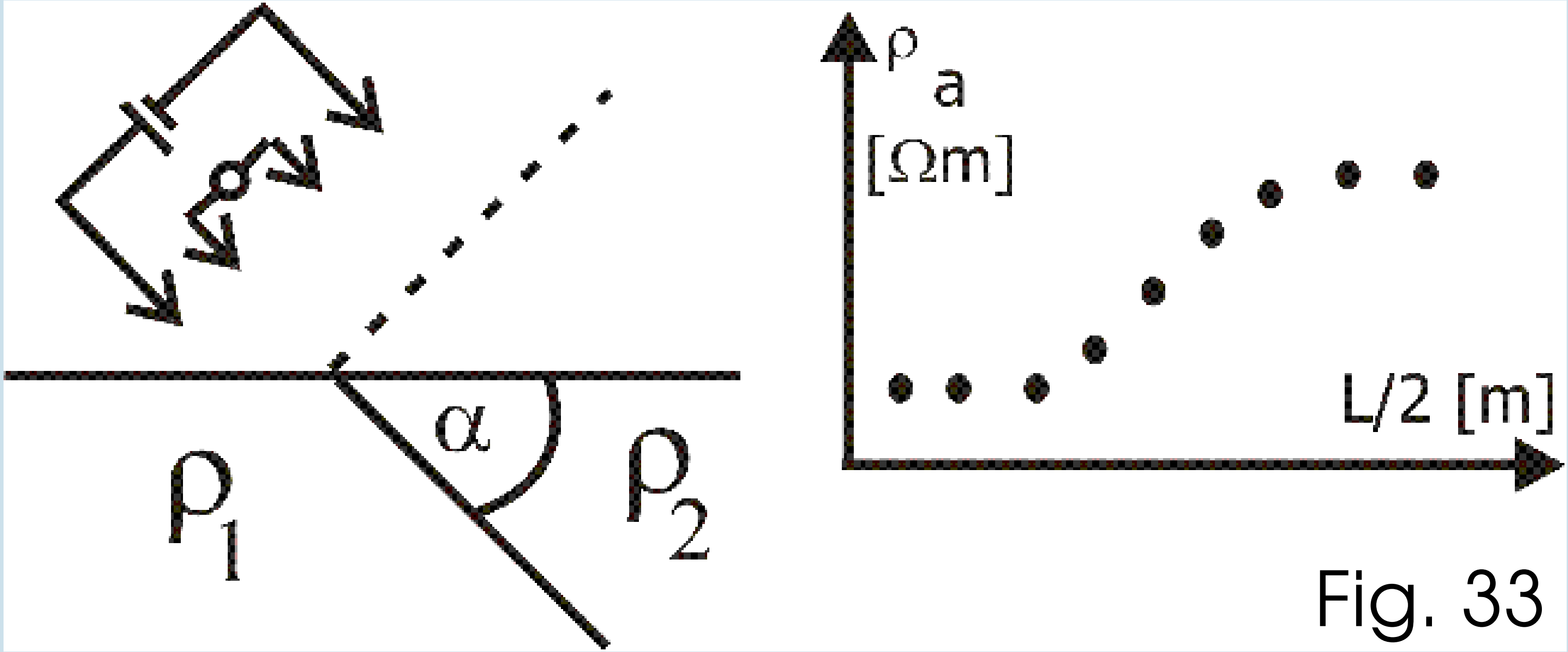


Fig. 33

- Interpretation error. The curve can be interpreted as a 2 layer case. However there is only one layer!

2.4.2 Methods for the determination of lateral variations of resistivity in the earth

- Half Schlumberger measurements

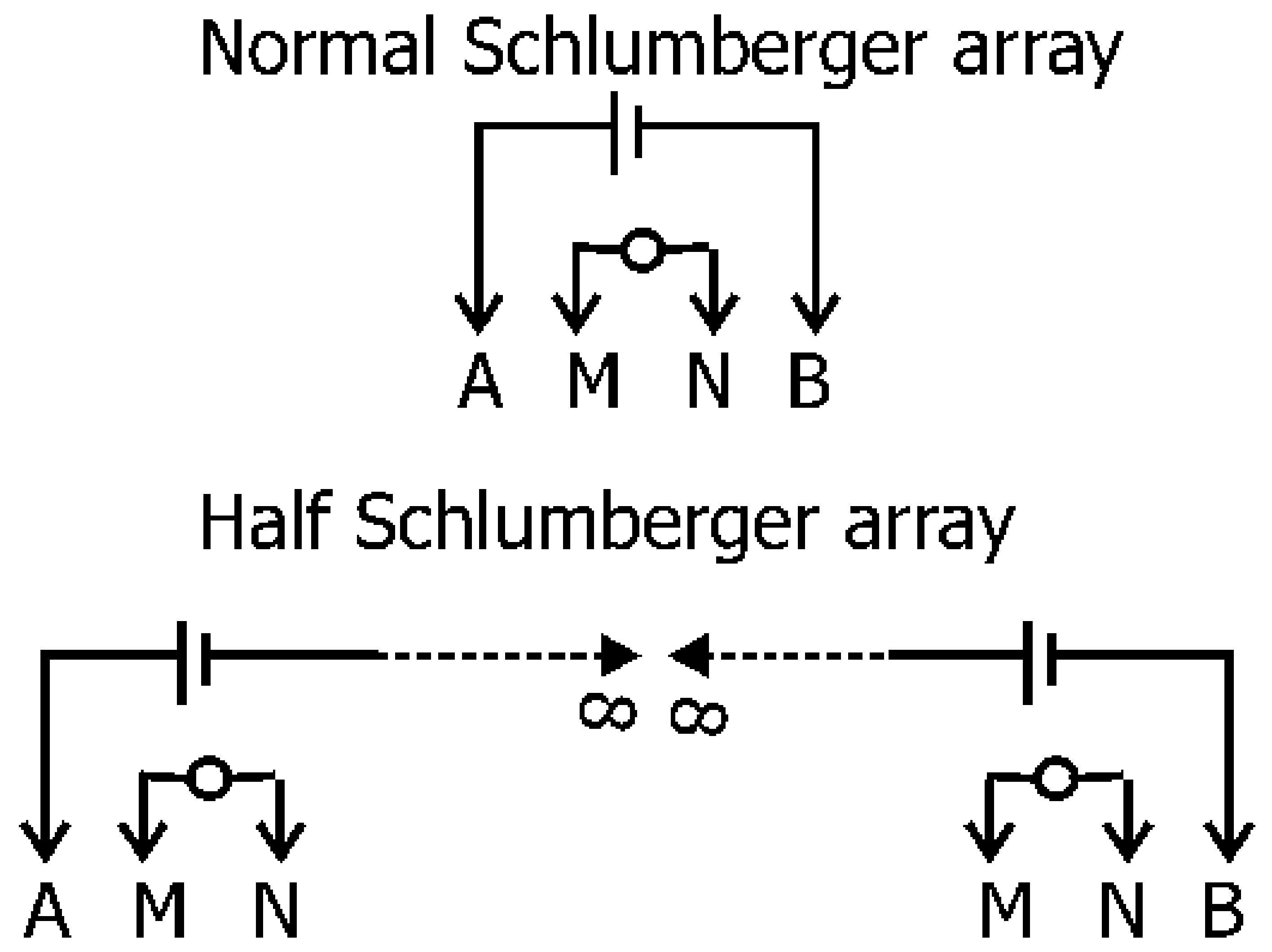
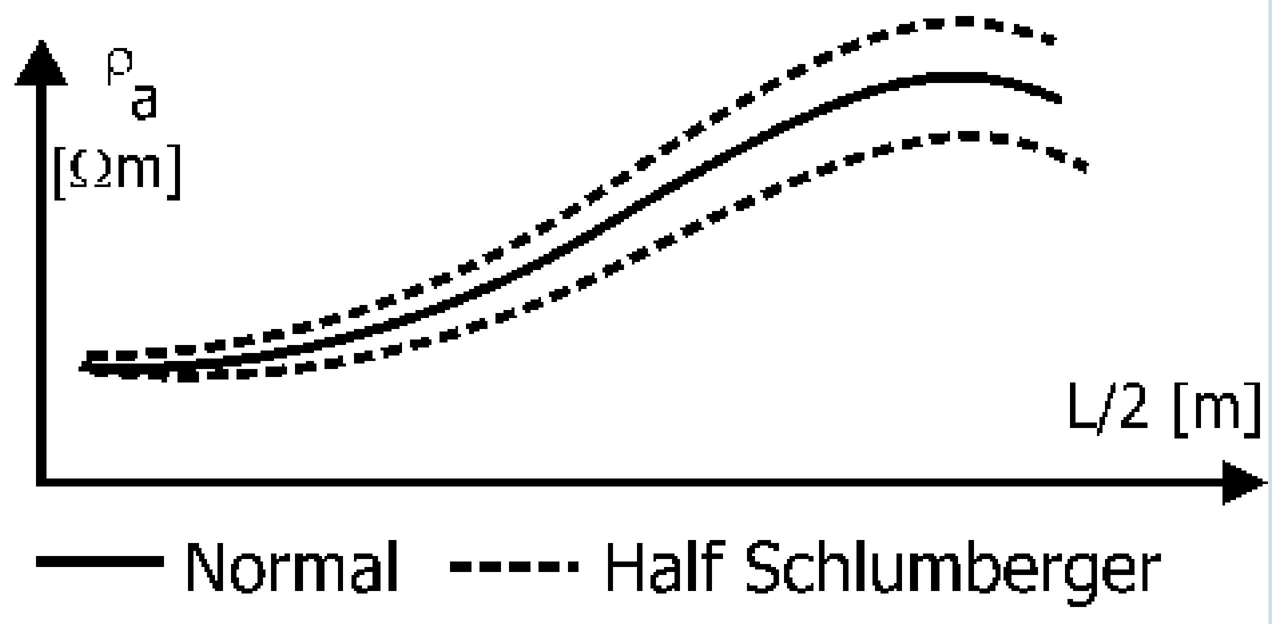


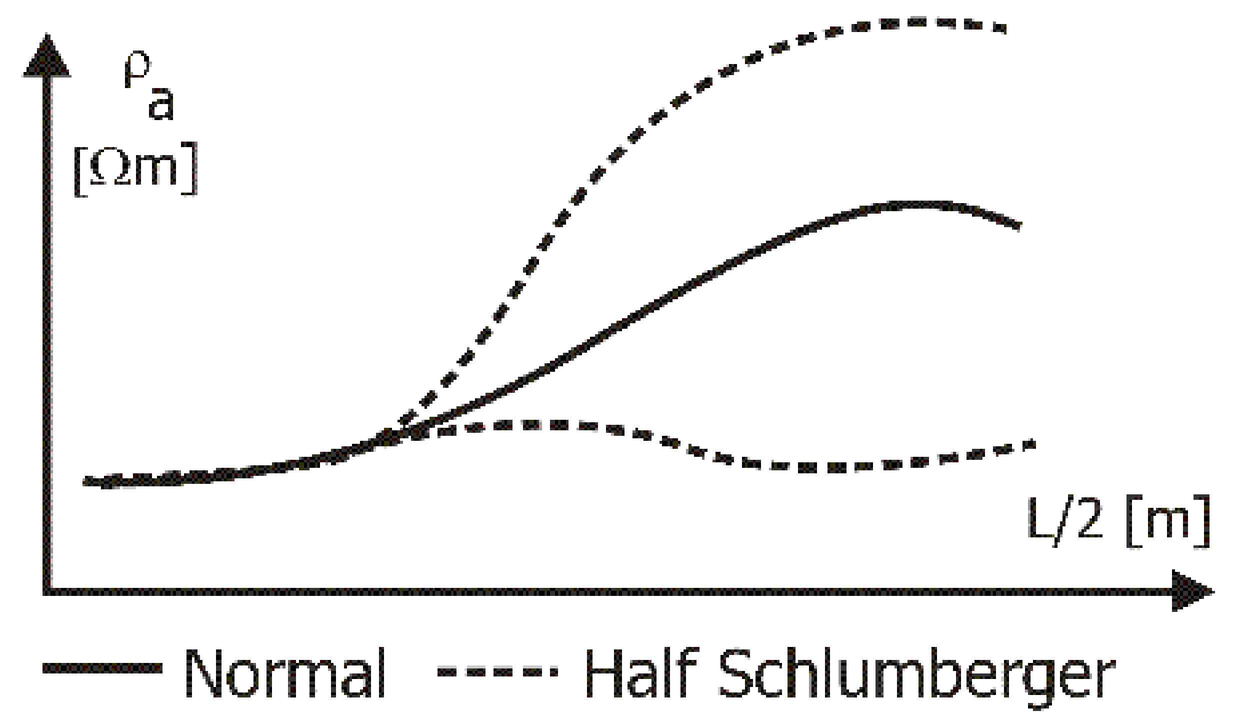
Fig. 34

Measurement on the same location with 3 arrays



- No fault, no lateral variation of resistivity!

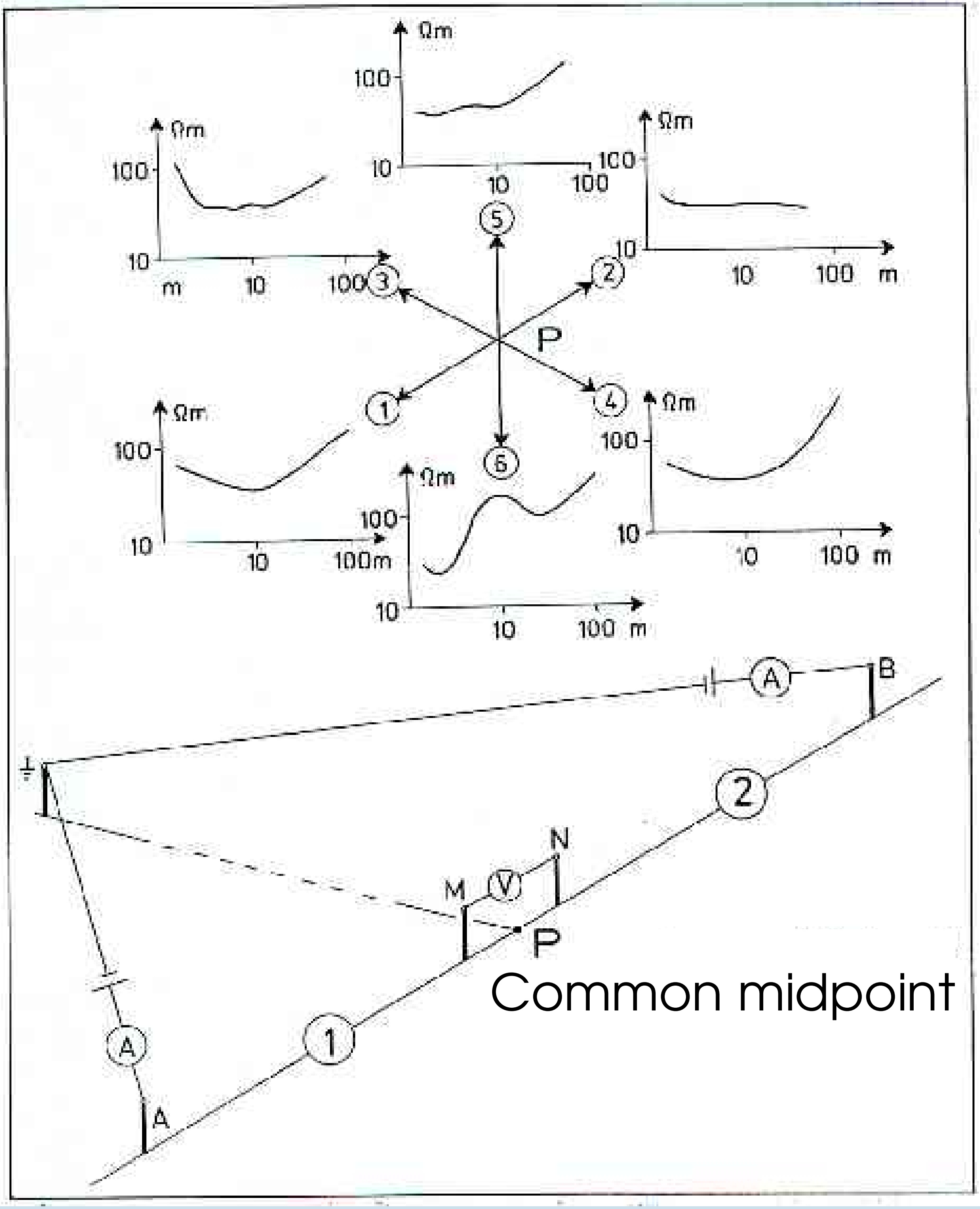
Fig. 35



- Fault or a lateral change in the earth exists.

Fig. 36

➔ Circular geoelectric measurements



Common midpoint

Fig. 37: Circular geoelectric sounding curves in a disturbed environment. [4]

2.4.3 Reciprocity

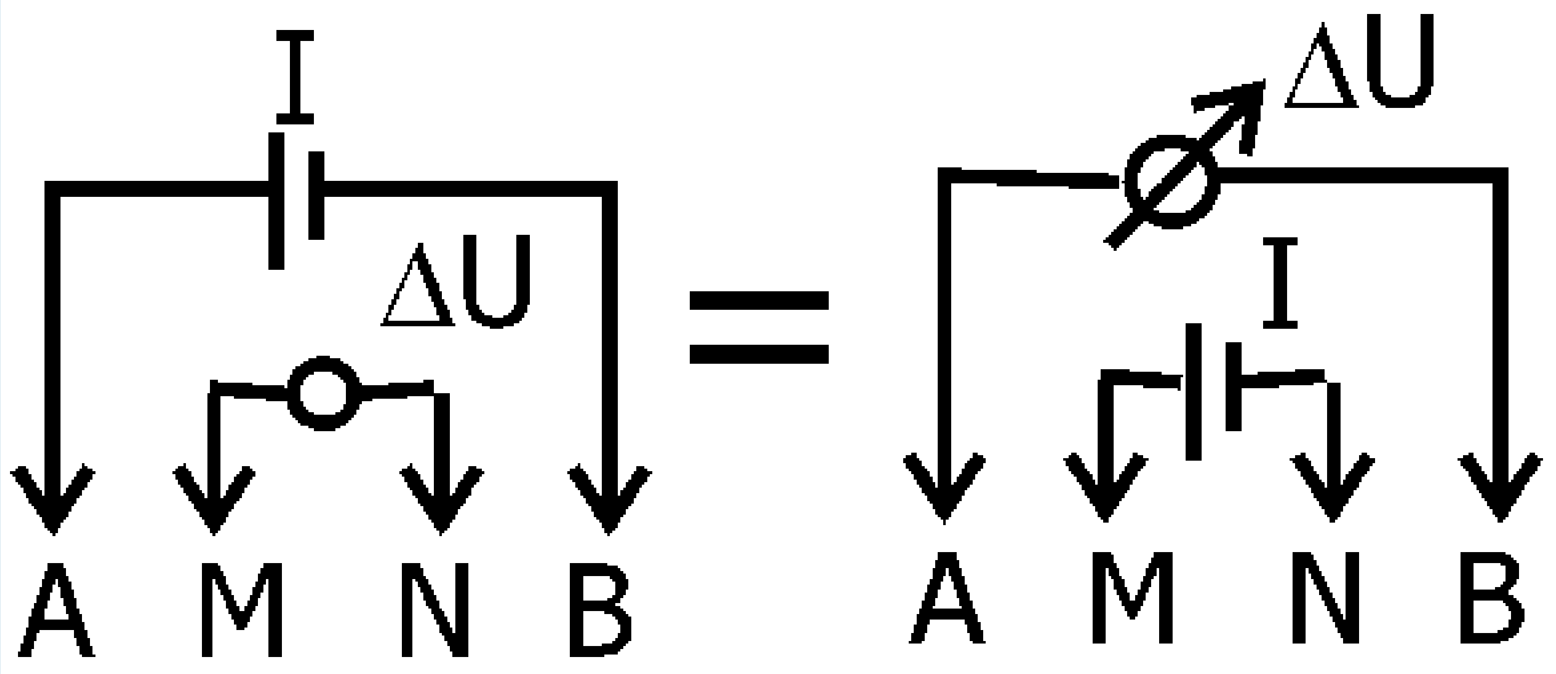


Fig. 38

$$\frac{\Delta U_{MN}}{I_{AB}} = \frac{\Delta U_{AB}}{I_{MN}}$$

2.5 Case Histories: Waste Sites

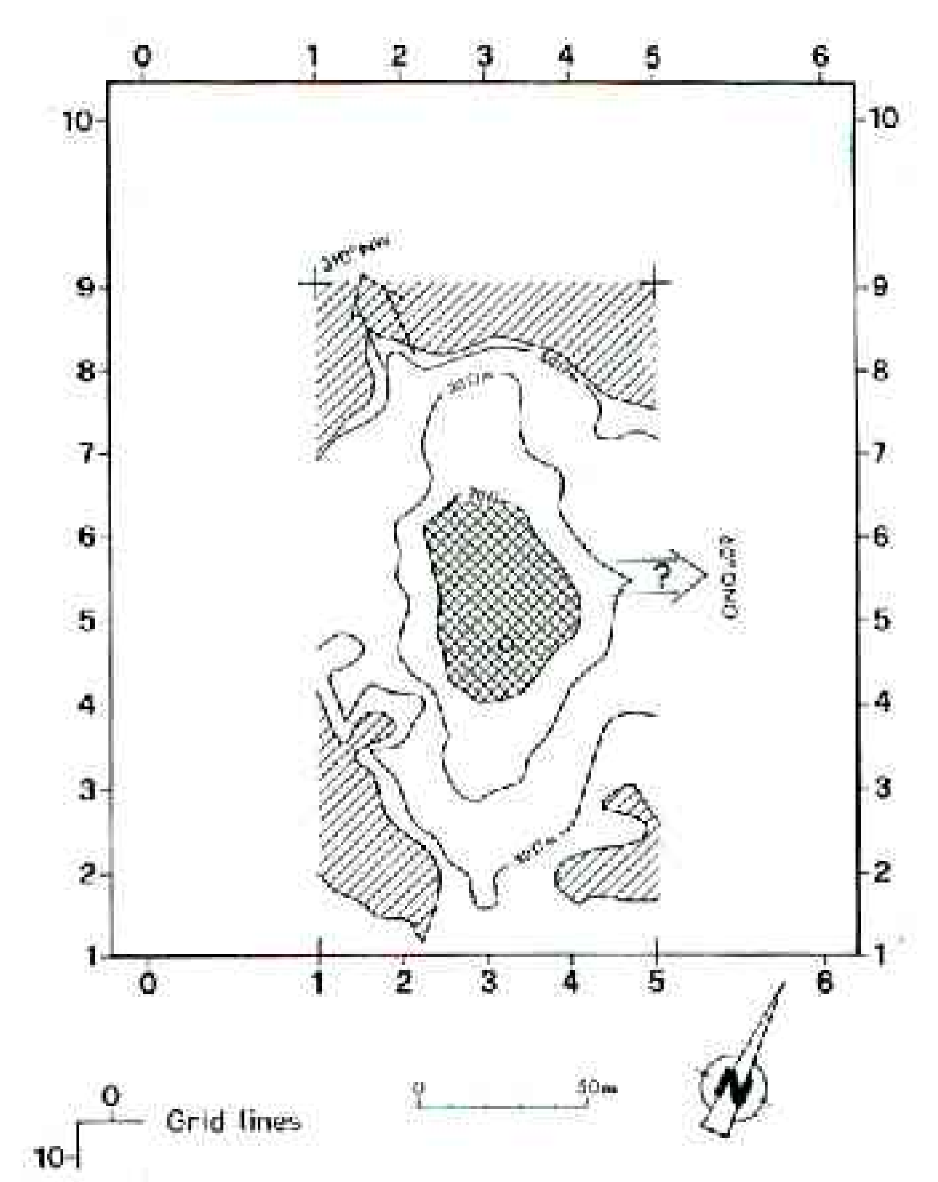
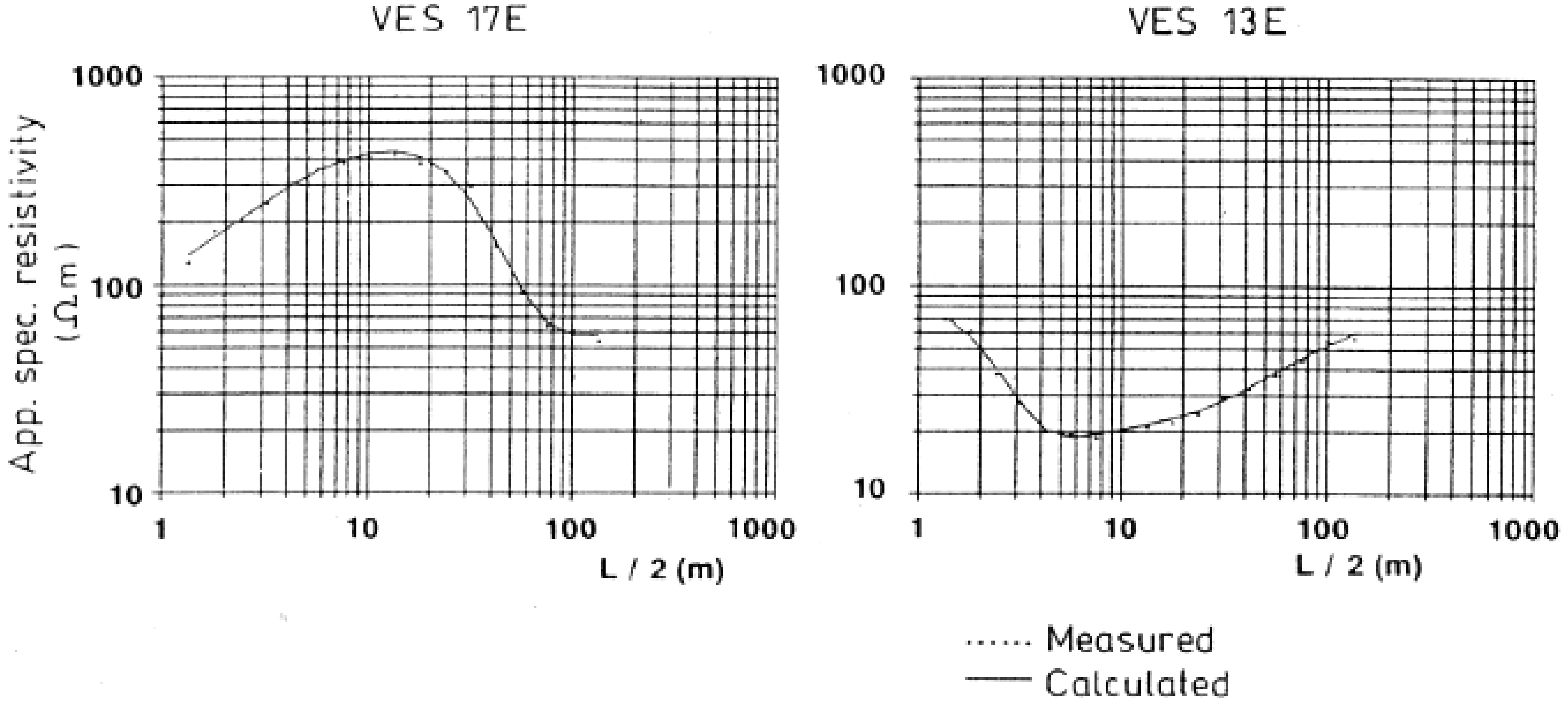


Fig. 39: Contours of apparent specific resistivity, industrial/domestic waste dump. Area hatched > 60 Ohm-m, cross hatched < 20 Ohm-m. arrows = possible seepage paths [3]



Model:

Layer	Resistivity (Ω m)	Thickness
1	91.12	0.77
2	651.39	10.10
3	47.87	47.78
4	63.92	

Layer	Resistivity (Ω m)	Thickness
1	94.35	1.02
2	8.00	0.93
3	22.69	19.25
4	86.58	

Fig 40 a: Geoelectric Soundings (Schlumberger array) of a hazardous waste site on Rhine island. Sounding 17E lies outside, 13E inside the dump, displaying profound differences in resistivity between waste and sediment. [3]

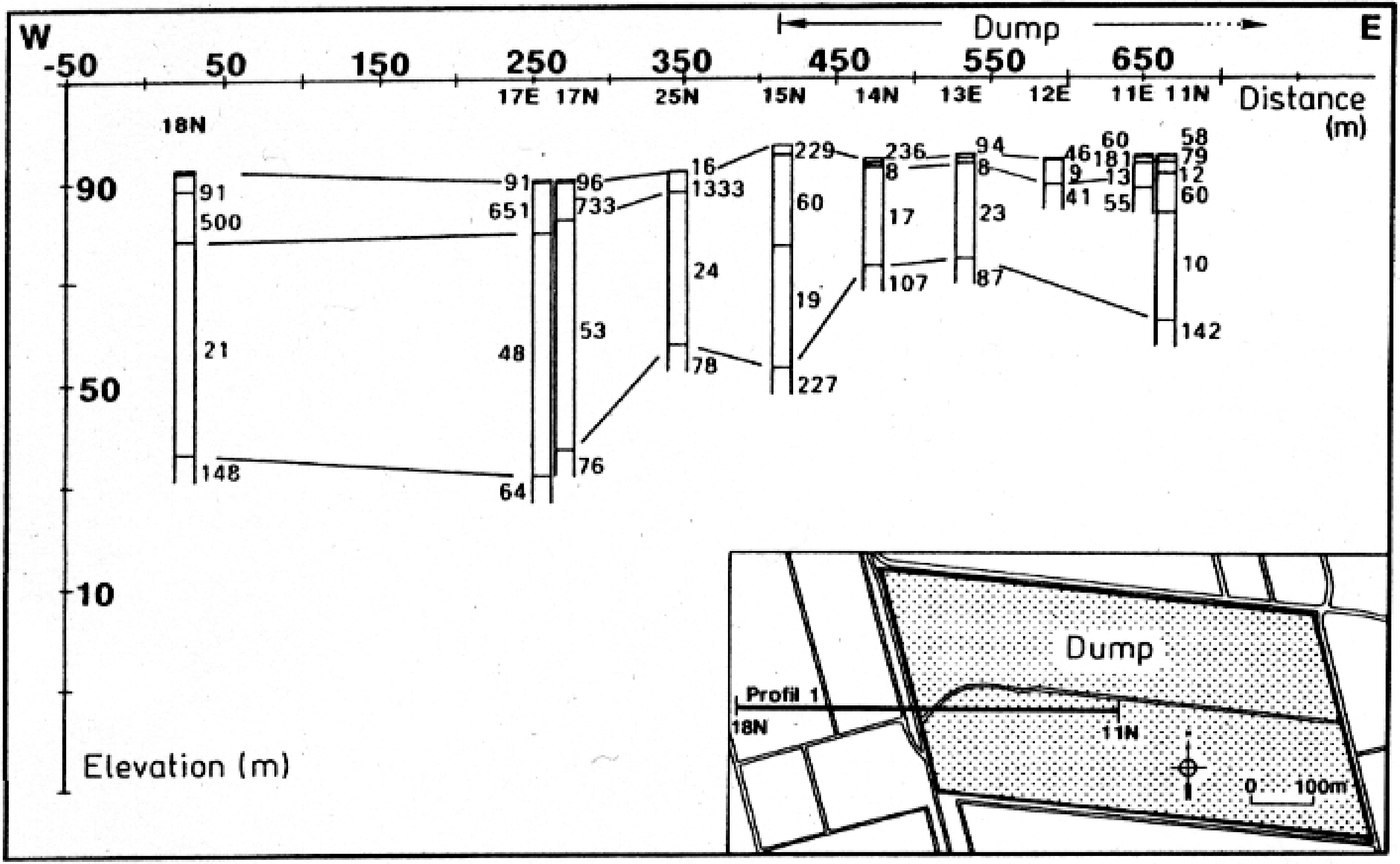
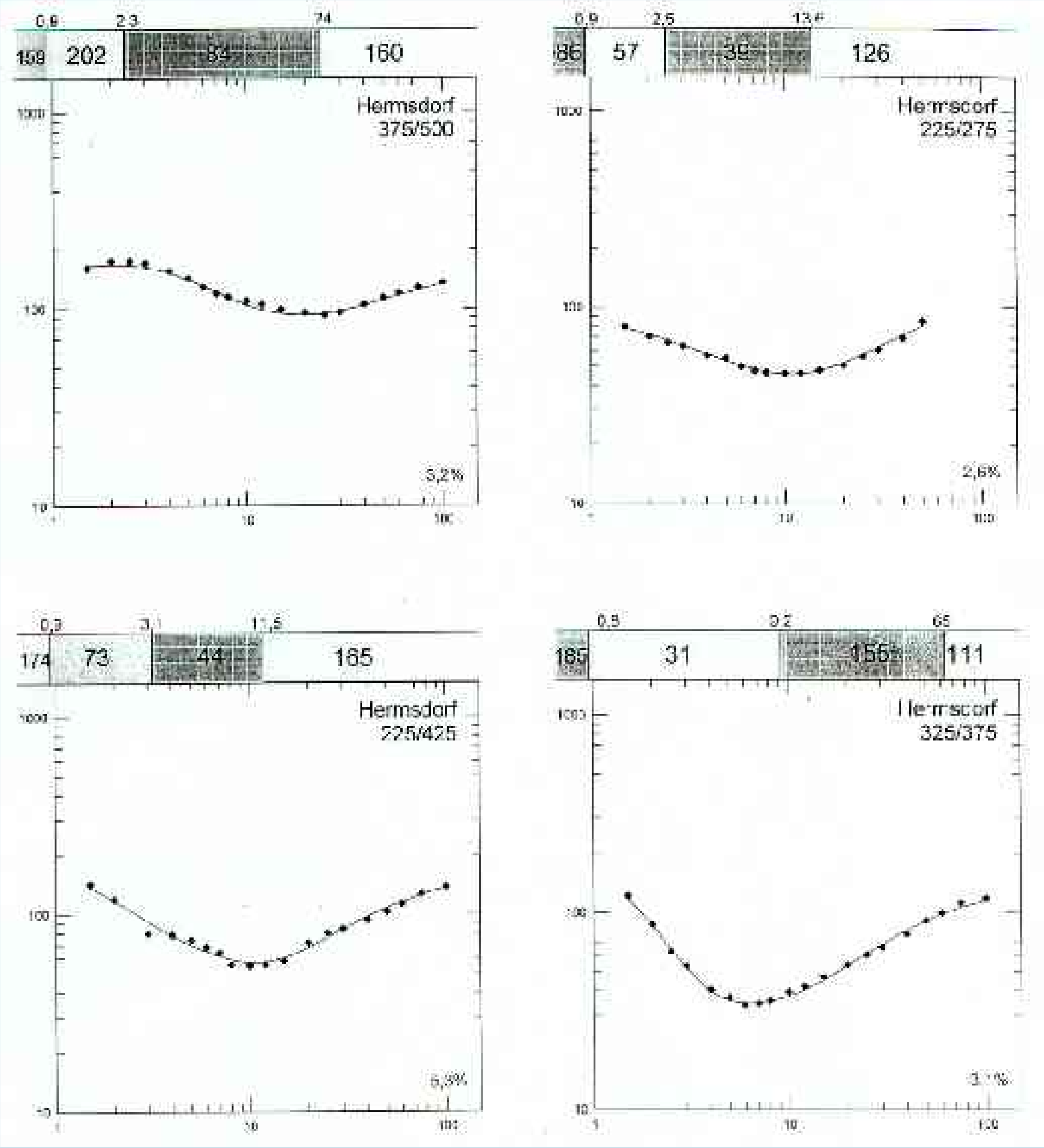


Fig 40 b: Geoelectric Soundings (Schlumberger array) of a hazardous waste site on Rhine island. [3]

Fig 41: Hermsdorf DC



2.5 Case Histories: Groundwater

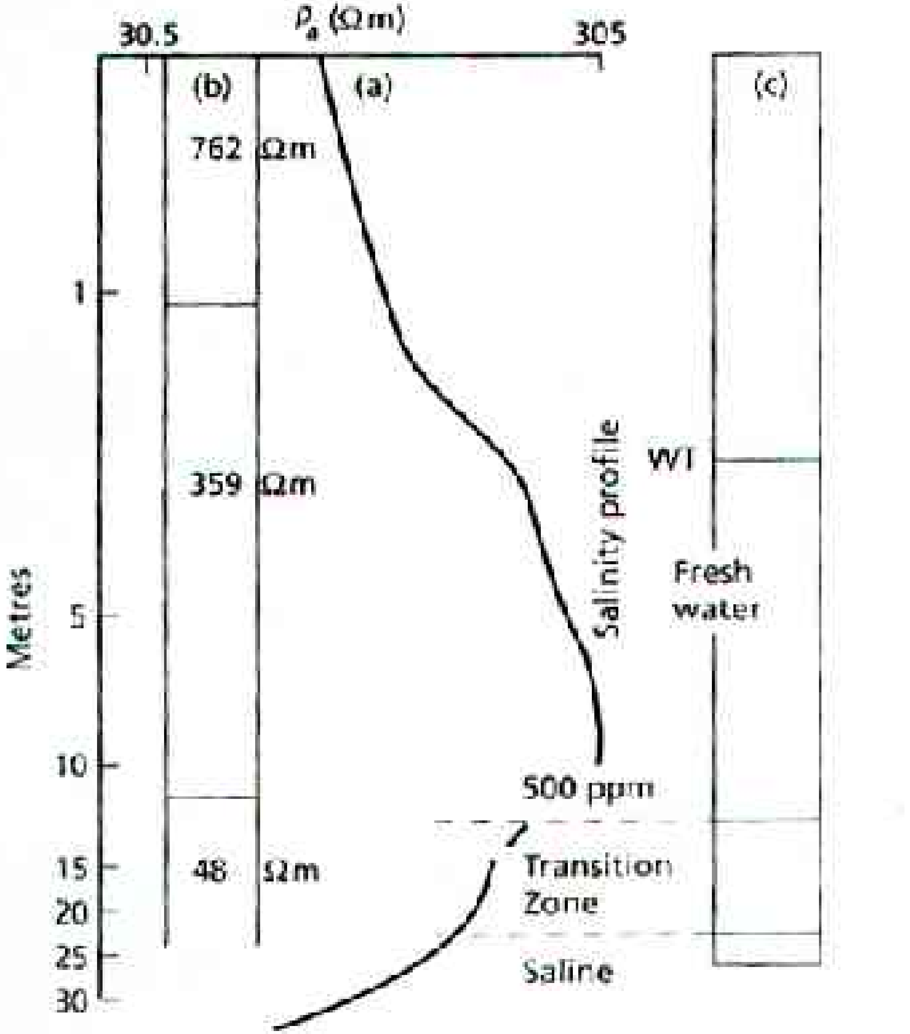
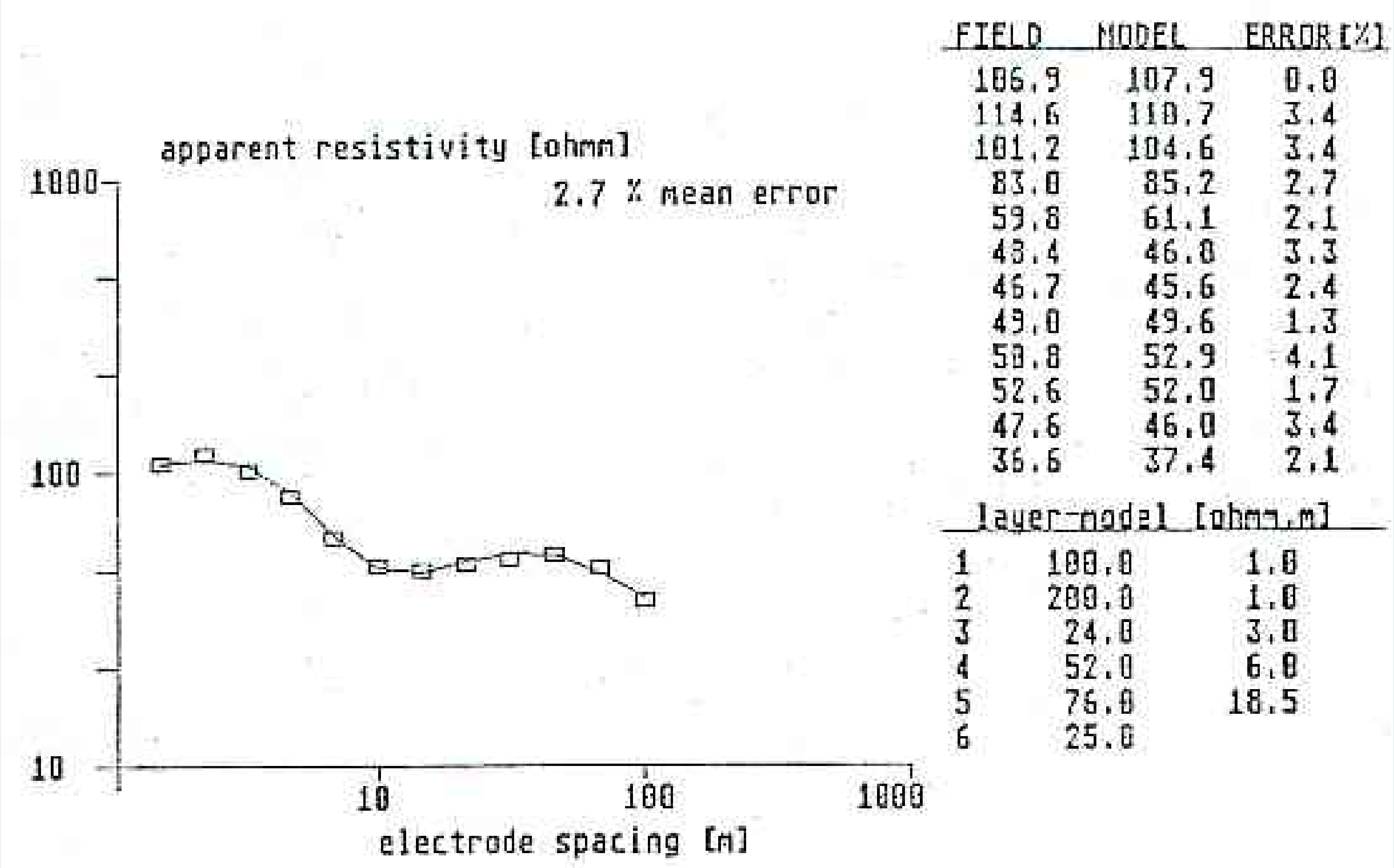


Fig 42: (a) Vertical electrical sounding adjacent to test borehole in the Central Lens, Grand Cayman. (b) Layered model interpretation of the VES. (c) Interpreted salinity profile.[2]

2.5 Case Histories: Geology



FIELD	MODEL	ERROR[%]
106.9	107.9	0.0
114.6	110.7	3.4
101.2	104.6	3.4
83.0	85.2	2.7
59.8	61.1	2.1
48.4	46.0	3.3
46.7	45.6	2.4
49.0	49.6	1.3
50.8	52.9	4.1
52.6	52.0	1.7
47.6	46.0	3.4
36.6	37.4	2.1

layer-model [ohmm.m]

1	100.0	1.0
2	200.0	1.0
3	24.0	3.0
4	52.0	6.0
5	76.0	18.5
6	25.0	

Fig 43: Example of a sounding curve to locate the marl layer. 4

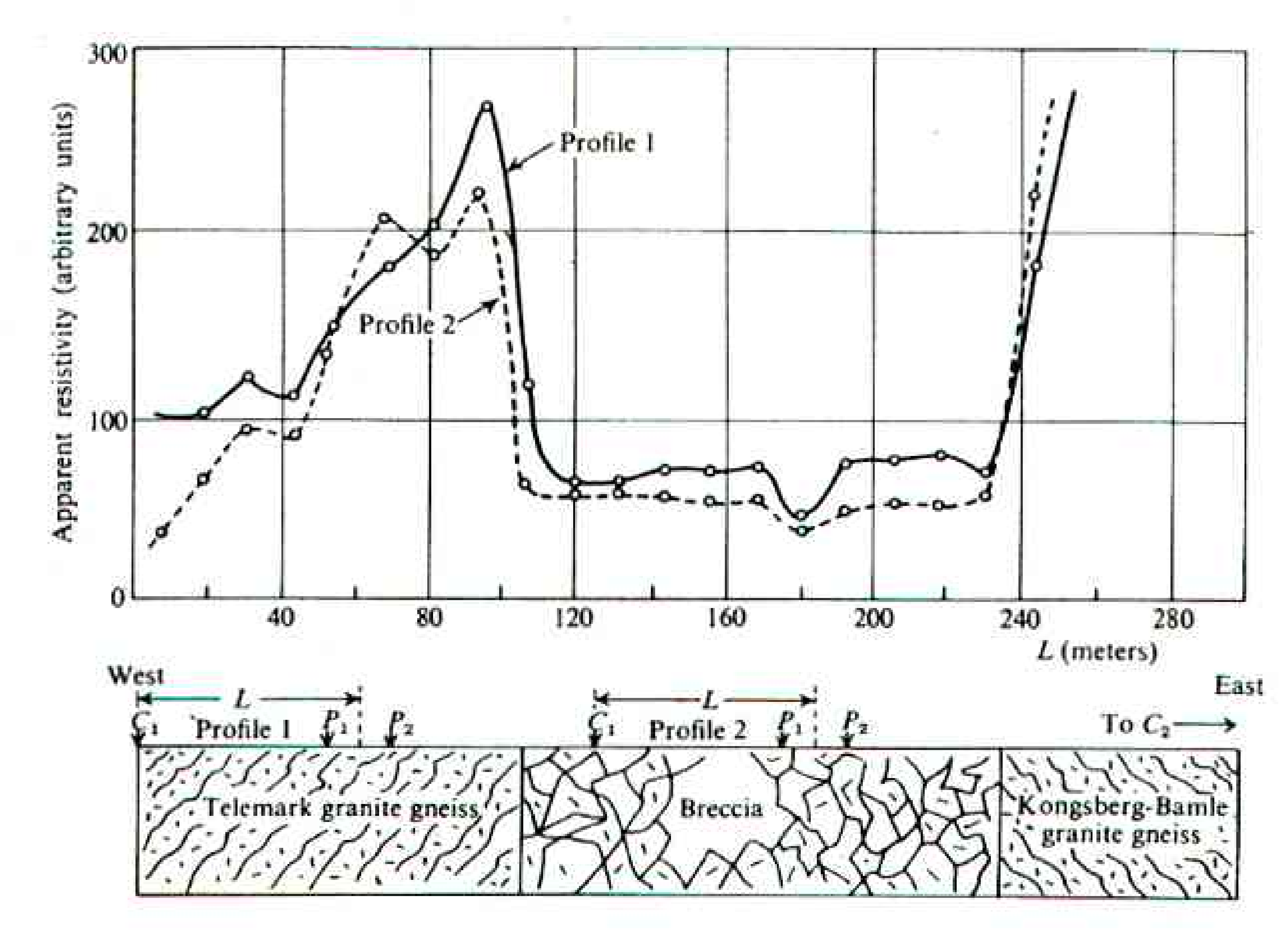


Fig 44: Mapping vertical contacts with the half-Schlumberger (gradient) array, Kongsberg, Norway. [5]

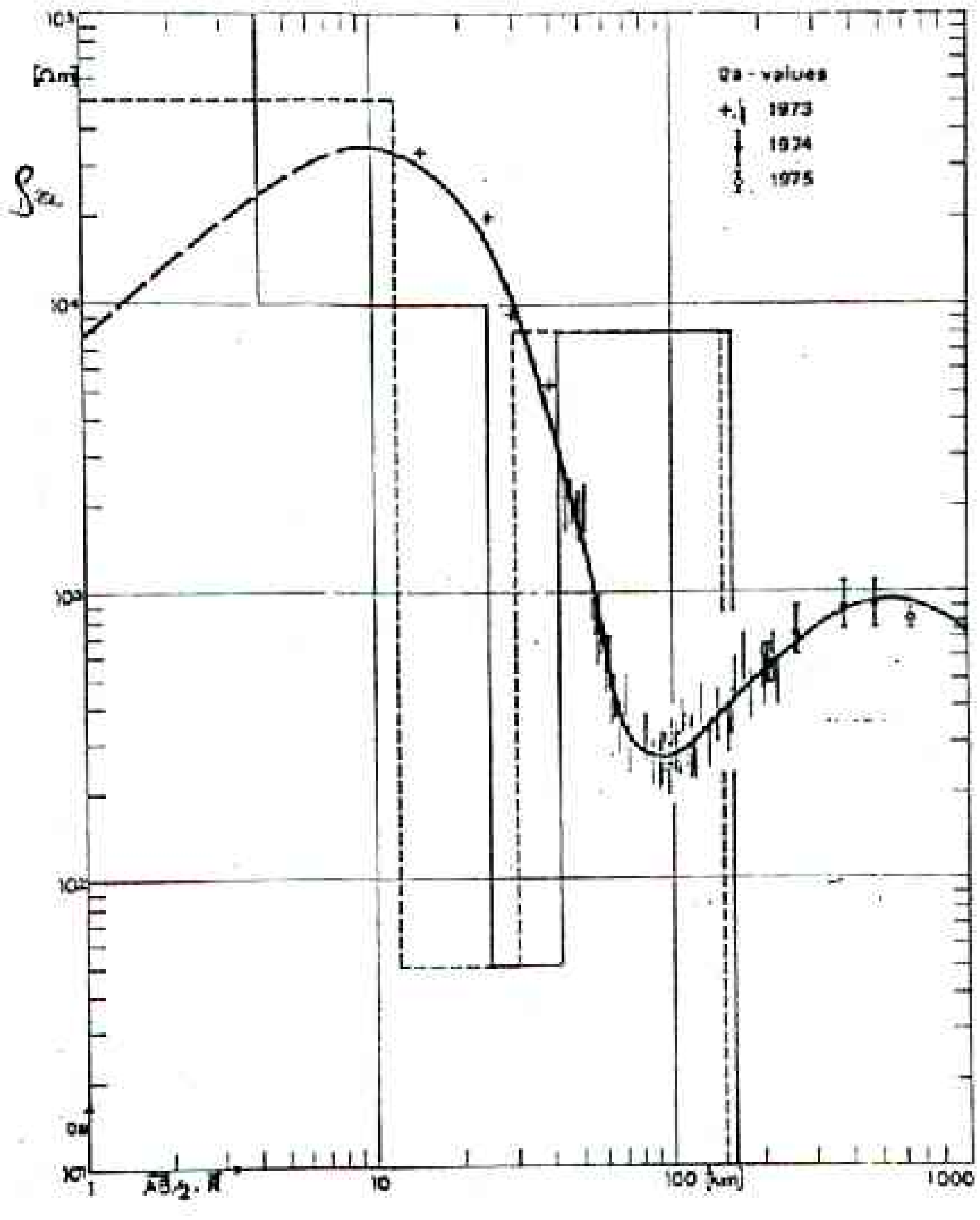


Fig. 45: Schlumberger sounding curve in South Africa.

2)

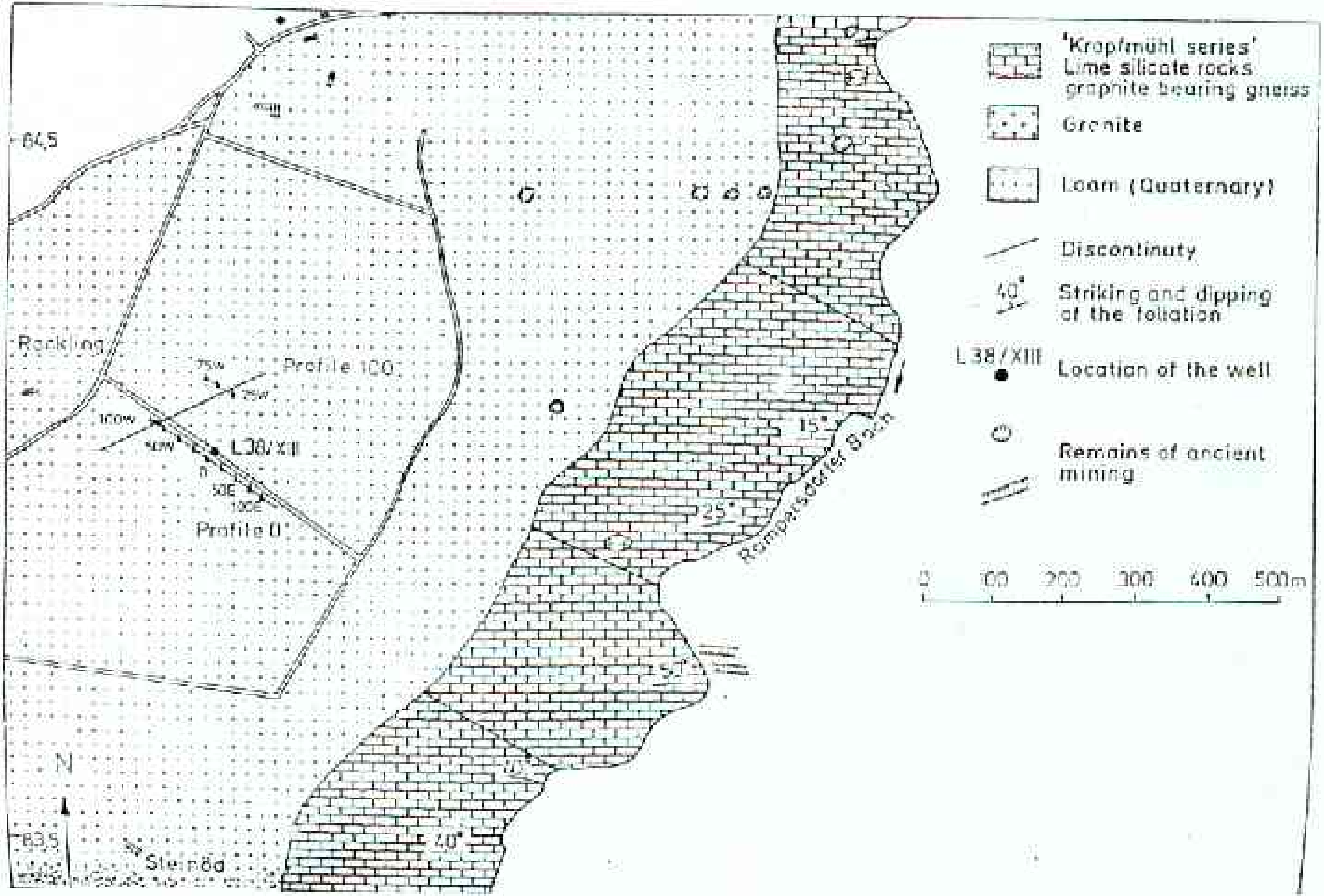
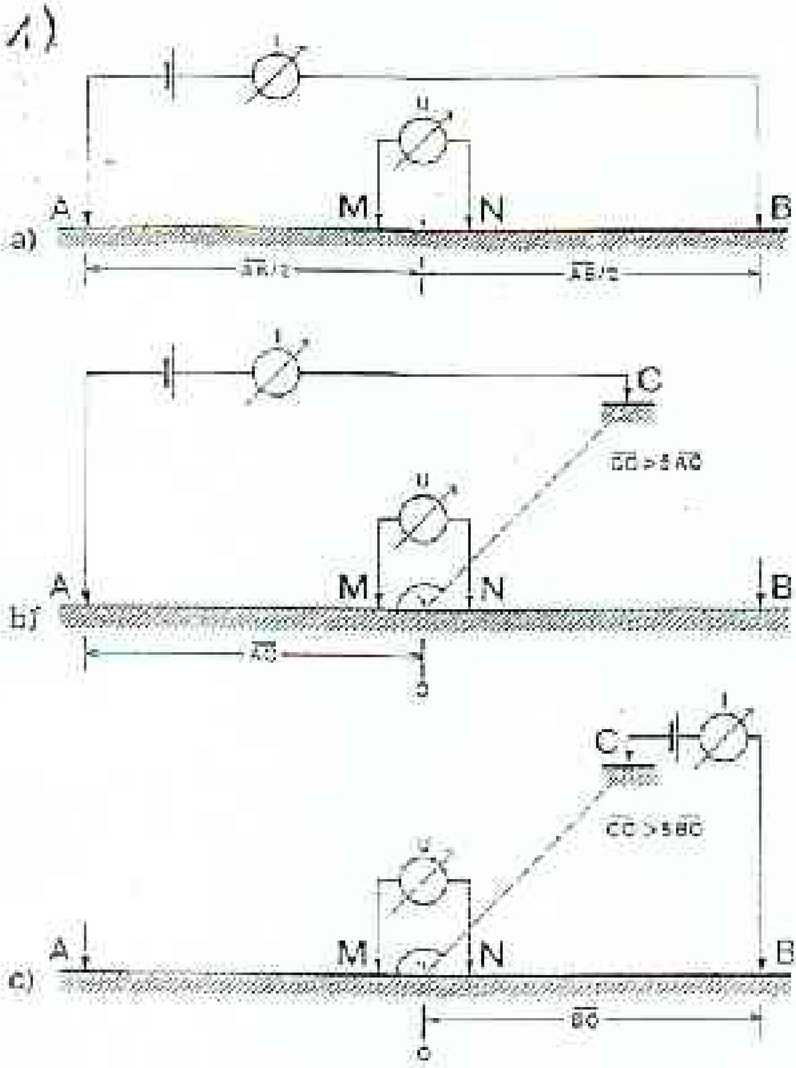
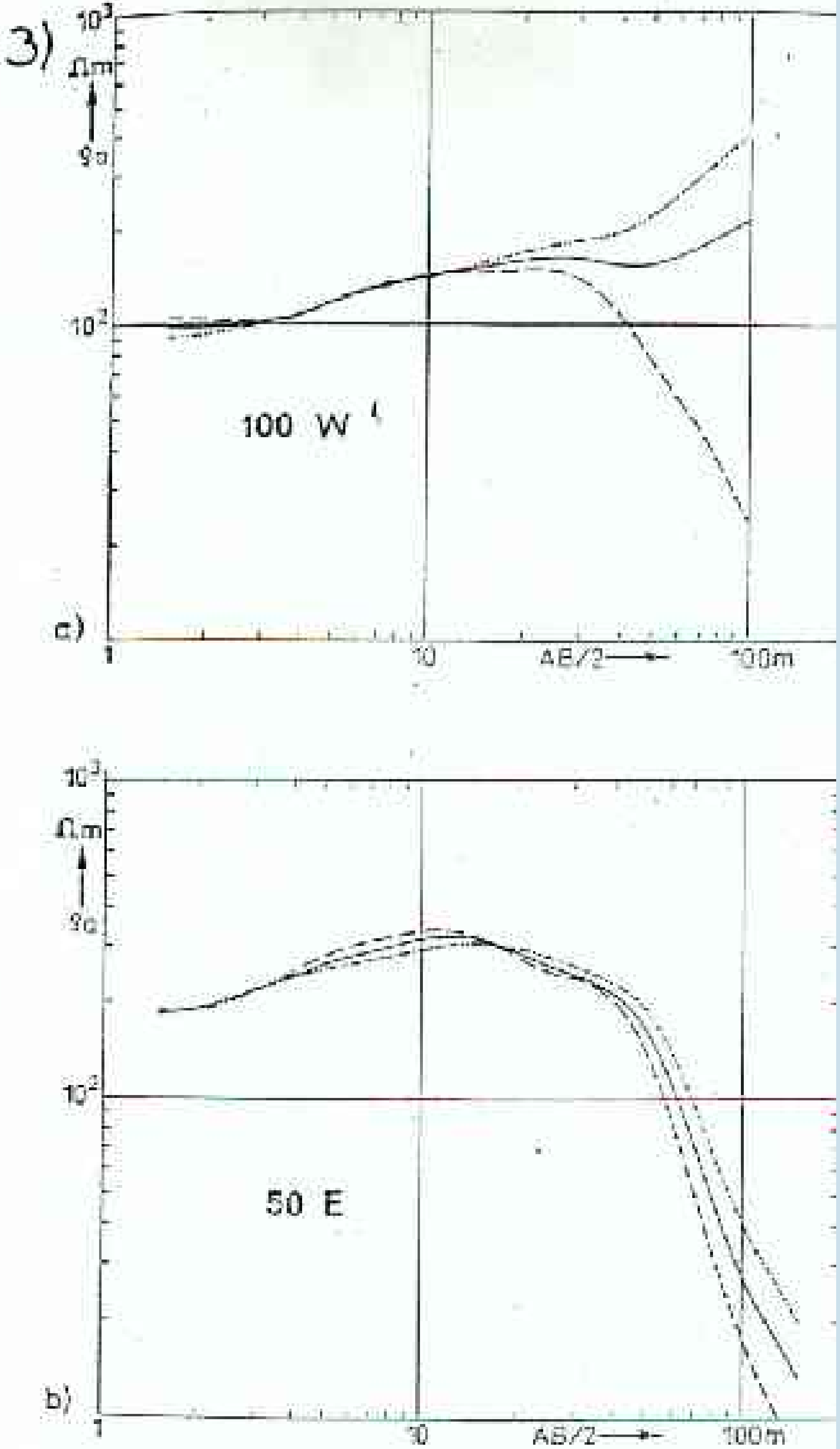

'Kropfmühl series'
Lime silicate rocks
graphite bearing gneiss
Granite
Loam (Quaternary)
Discontinuity
40°
Striking and dipping of the foliation
L 38/XIII
Location of the well
Remains of ancient mining
0
100
200
300
400
500m
64,5
Rockling
Profile 100
75W
25W
100W
50W
L38/XII
50E
100E
Profile 0
15°
25°
57°
40°
Rampersdorfer Bach
N
63,5
Steinöd

Fig. 46

Fig. 47


1)
I
U
A
M
N
B
a)
AB/2
AB/2
C
CO > 5 AO
b)
AO
O
C
CO > 5 BO
c)
BO
O


3)
10³
Ωm
ρa
10²
100 W
a)
1
10
AB/2
100m
10³
Ωm
ρa
10²
50 E
b)
1
10
AB/2
100m

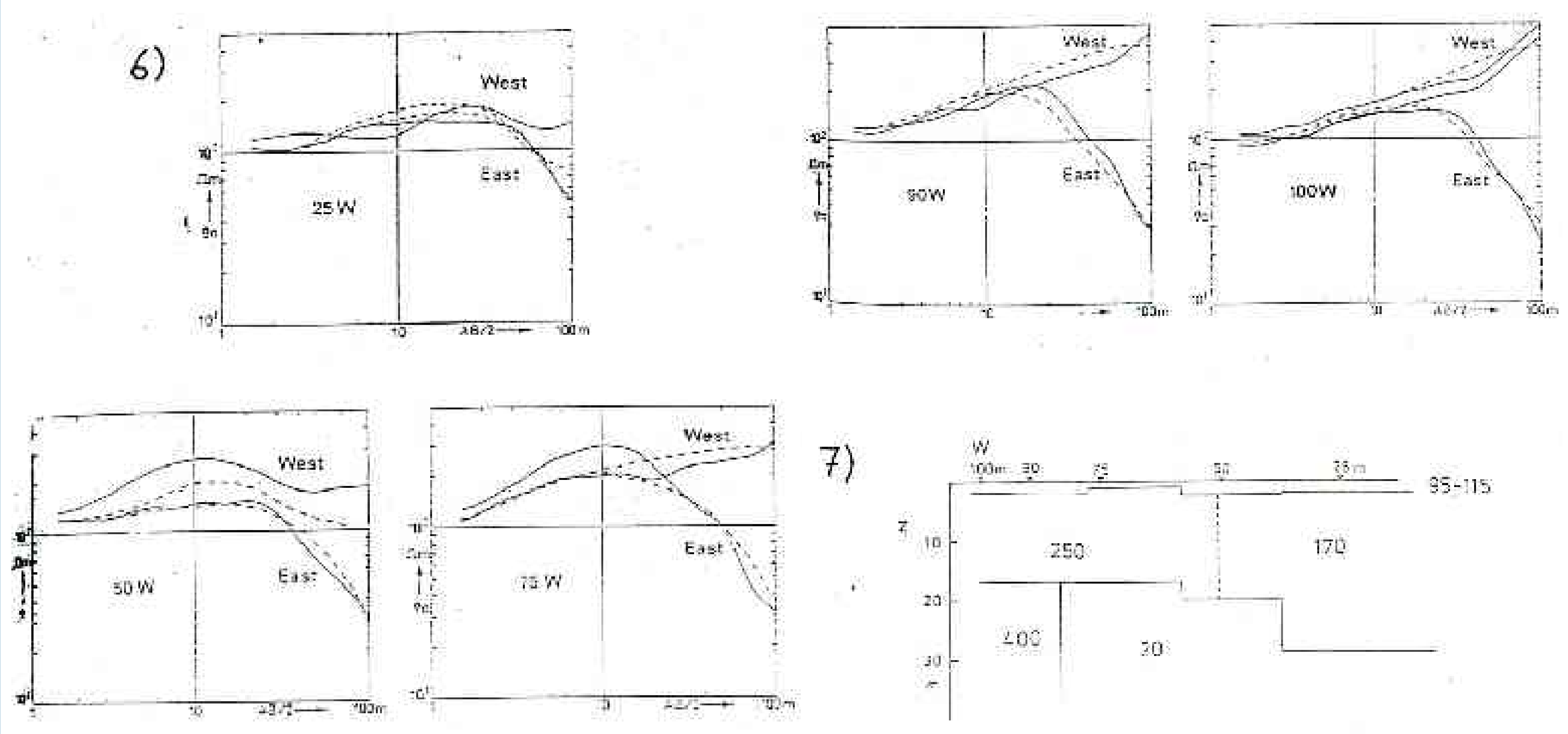

6)
West
East
25 W
50 W
75 W
90W
100W
AB/2 →
10
100 m
10^1
10^2
Ωm
7)
W
100m
50
25
50
25 m
95-115
250
170
400
30
z
10
20
30

Fig. 48

2.5 Case Histories: Archaeology

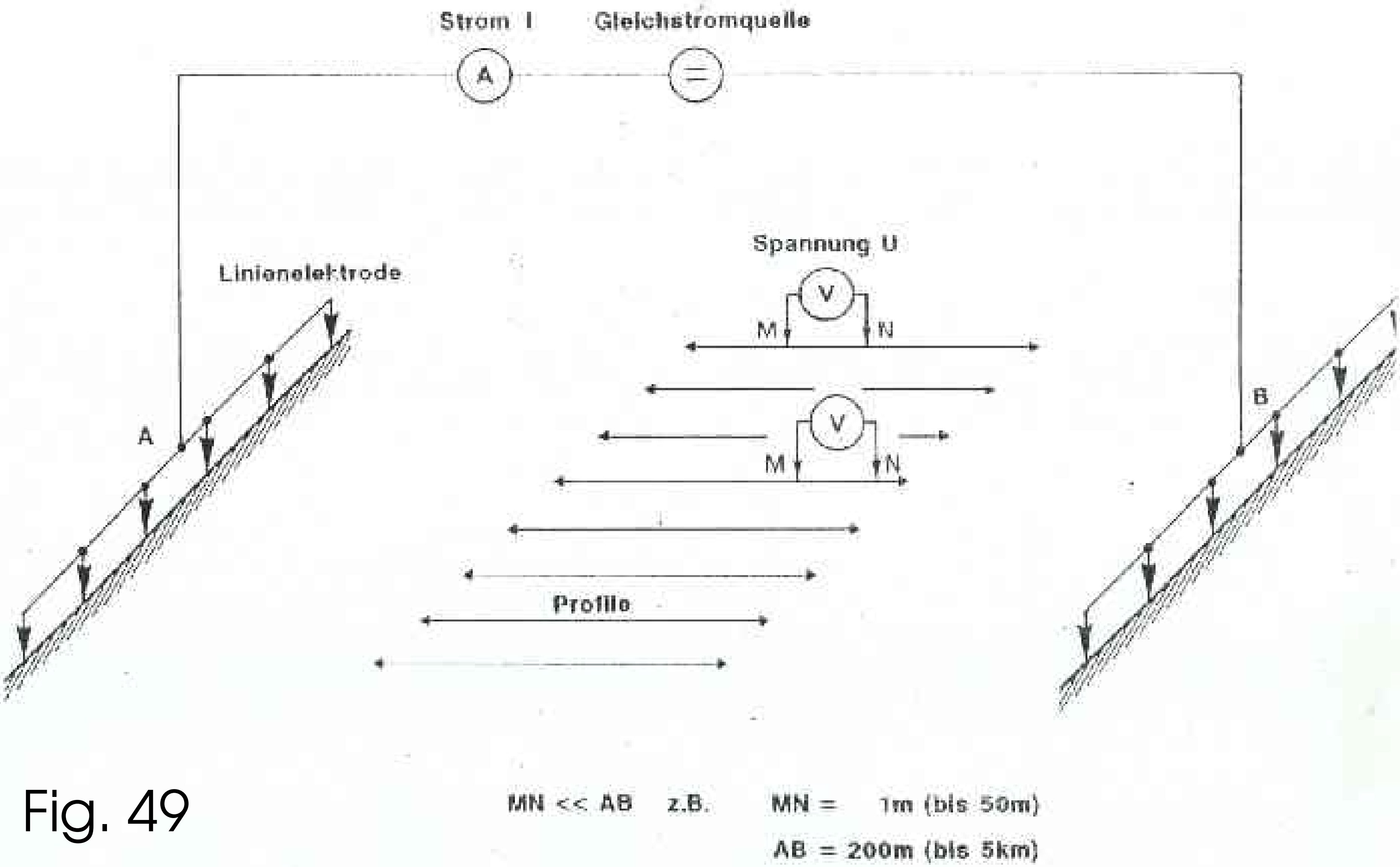
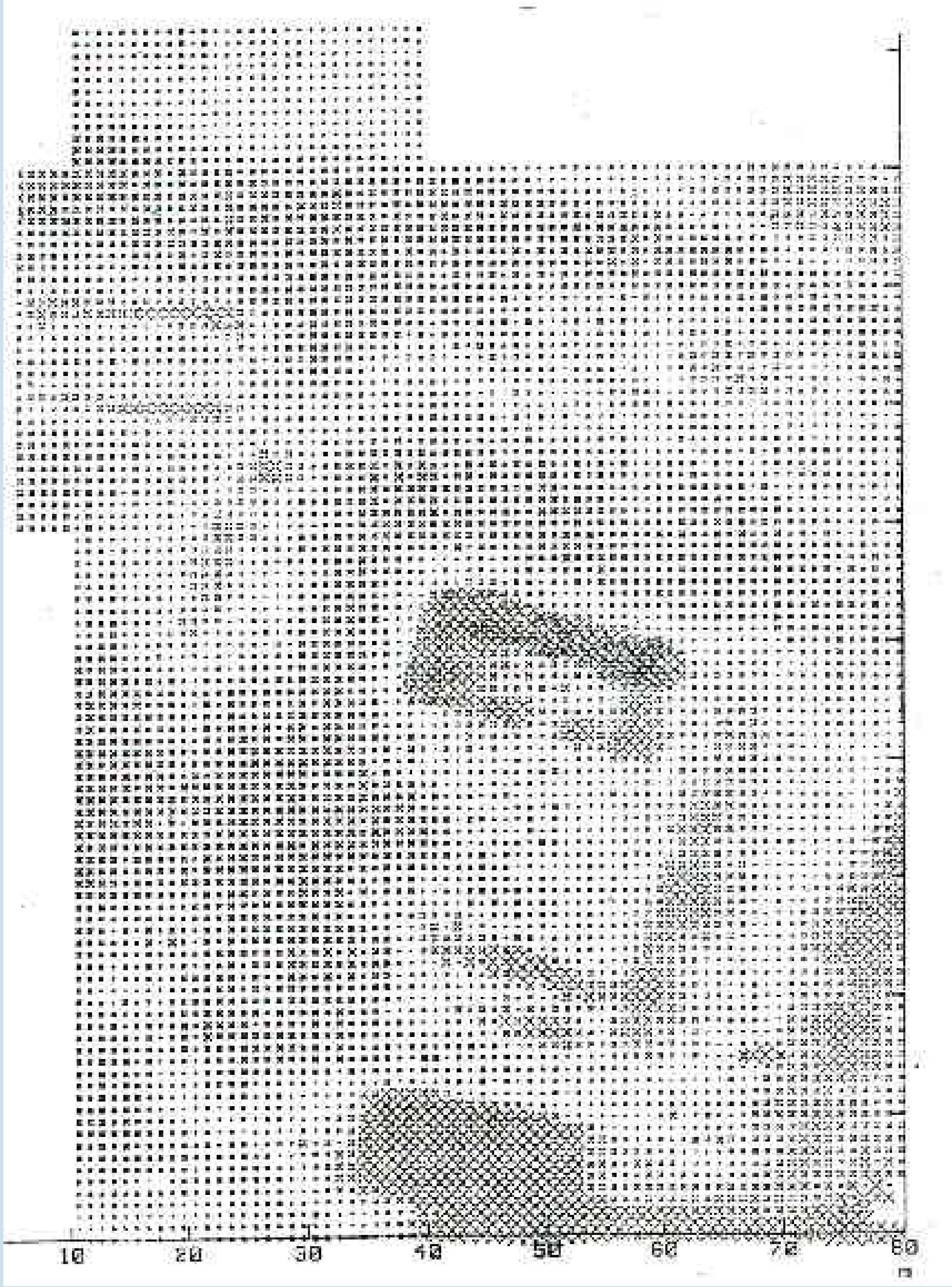


Fig. 49

Fig. 50



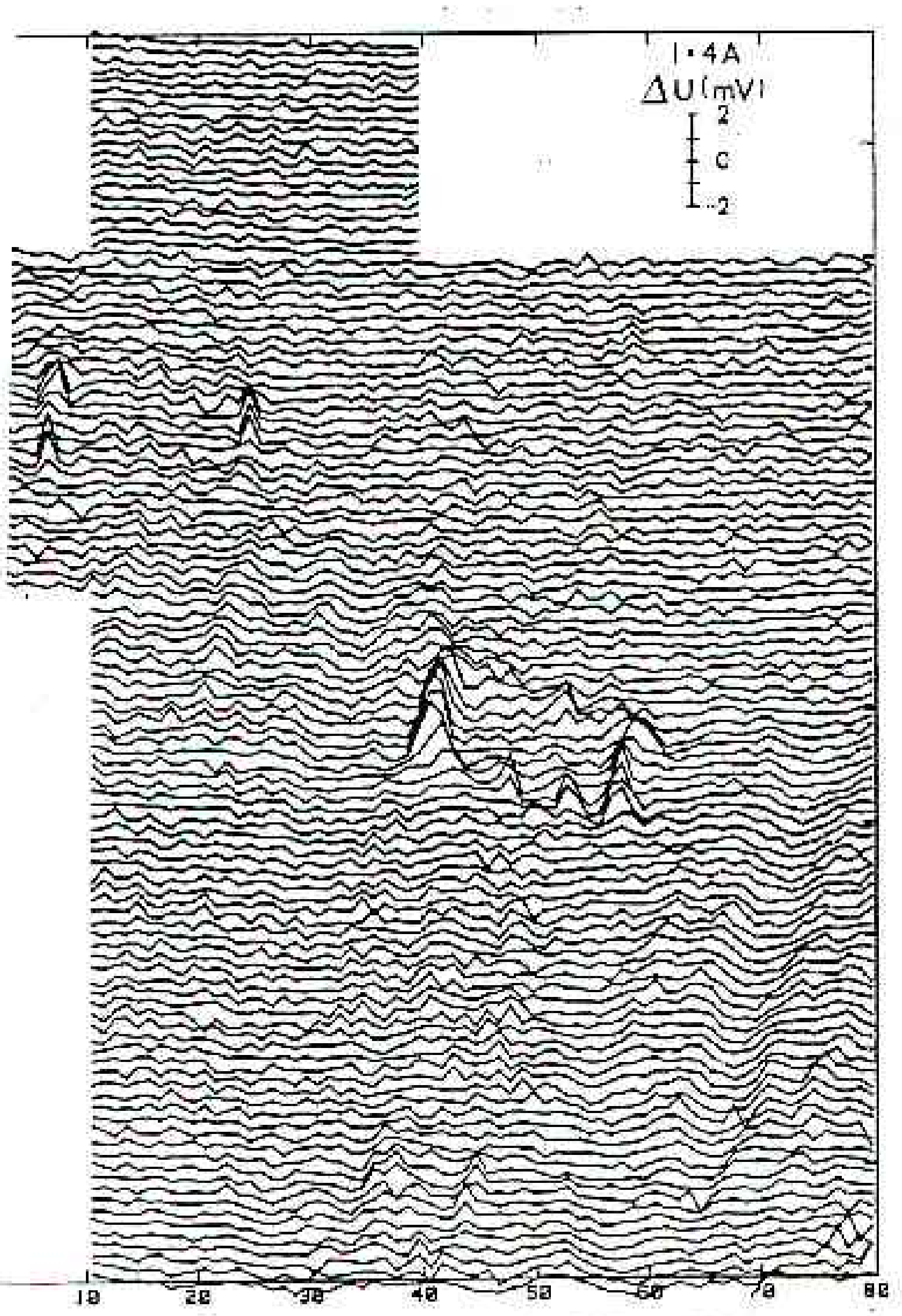


Fig. 51

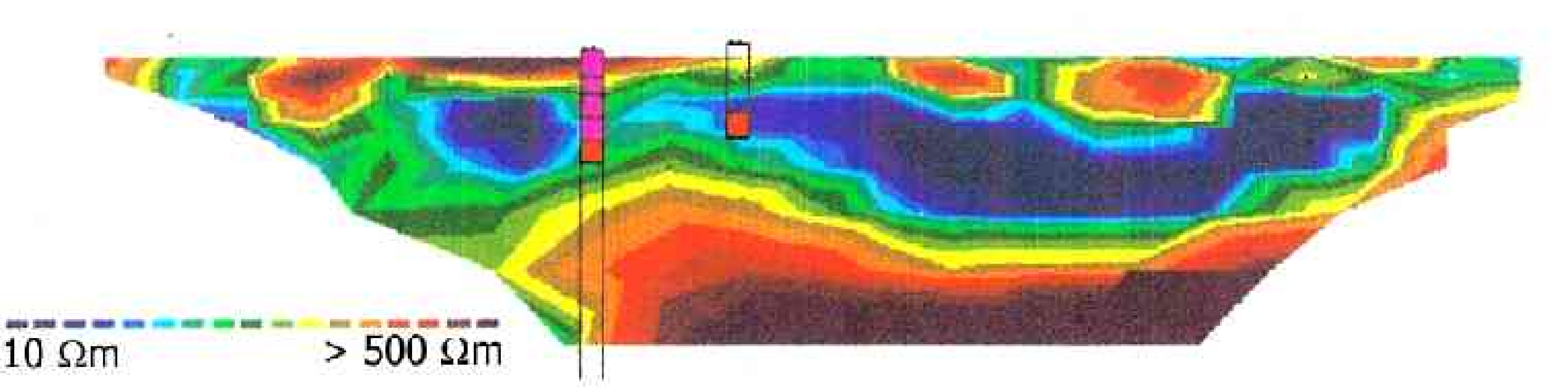


Fig. 52: Cologne 2D inversion

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