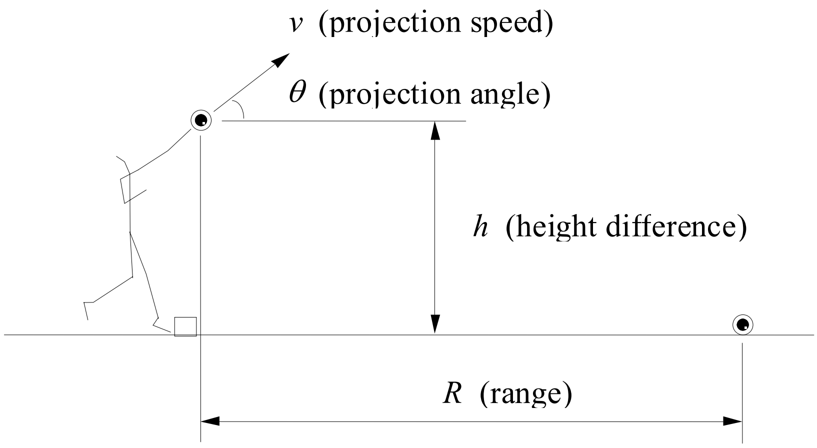
2. Optimum Projection Angle in the Shot Put

OBJECTIVES

- Appreciate the interaction between the mechanical principles that determine the optimum projection angle in the shot put.
- Use a simple model to determine the optimum projection angle for a typical shot putter.
- Examine how the optimum projection angle depends on the physical characteristics of the athlete.

INTRODUCTION

Most introductory biomechanics courses examine the motion of projectiles in free flight. One of the key results is that the optimum projection angle for achieving maximum horizontal range is 45°. However, actual performers in projectile-related sports seldom use an angle of 45°. For example, typical projection angles of world-class shot-putters are around 37°. Some researchers have noted that in shot-putting the landing is about 2 m lower than the launch. Even so, this produces only a small reduction in the calculated optimum projection angle (to about 42°).



(Linthorne, 2001)

The reason for the discrepancy between theory and practice is that the projection velocity and launch height attained by the athlete are *not* independent of the projection angle, as is assumed in the conventional calculation of the optimum projection angle. Experiments have shown that the projection velocity an athlete can generate *decreases* with increasing projection angle and that this substantially *reduces* the optimum projection angle (Linthorne, 2001).

In this learning session you will use spreadsheets and graphs to examine the optimum projection angle in the shot put. You will calculate the optimum projection angle using the conventional method and then re-calculate the angle using the correct method.

PROCEDURE

Go to one of the University computers and log on to the computer in the usual way. Start up **Microsoft Office Excel 2010**.

Conventional Calculation

In the first stage of the investigation you will calculate the optimum projection angle using the incorrect assumption that the projection velocity and launch height are independent of projection angle.
The horizontal range of a shot in free flight is given by

$$R = \frac{v^2 \sin\theta \cos\theta + v\cos\theta\sqrt{(v\sin\theta)^2 + 2gh}}{g} \qquad (1)$$

where v is the projection velocity, θ is the projection angle, h is the height difference between the launch and the landing, and g is the acceleration due to gravity.

Use equation 1 to produce curves of range as a function of projection angle for projection velocities of 8 m/s, 9 m/s, 10 m/s, ... up to 15 m/s. Assume a height difference of $h = 2.10$ m (typical for an elite male shot-putter).

Hints: First make a column of projection angles from 0° to 90° by using the **Fill Series** (**Linear**, **Step** 1) function from the **Editing** group on the **Home** tab. **Excel** does trigonometric calculations (sin, cos, tan, etc.) in radians, rather than in degrees, so you will have to convert the projection angles into radians. Remember, 1 revolution = 360° = 2π radians, so

$$\frac{\theta_{\text{radians}}}{2\pi \text{ radians}} = \frac{\theta_{\text{degrees}}}{360°}$$

$$\therefore \quad \theta_{\text{radians}} = \theta_{\text{degrees}}\left(\frac{2\pi}{360°}\right)$$

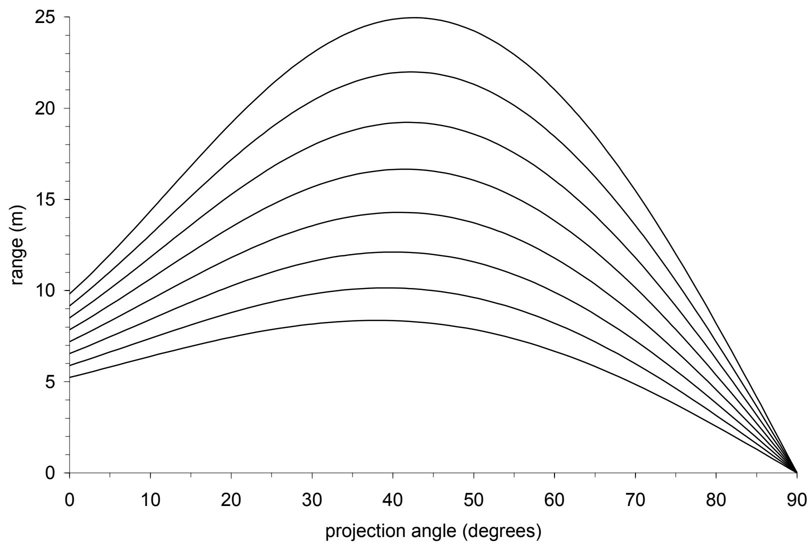
For π, **Excel** uses the constant **PI**. (A much quicker method to convert degrees into radians is to use the **RADIANS** formula).

Start with a projection velocity of 8 m/s. To calculate the range of the shot, type in equation 1 for a projection angle of 0 radians, then use the **Fill Down** function to calculate the range for the remaining projection angles. See the screen shot below if you are having difficulty with equation 1. (I have made use of constants; Vel1 = 8, Grav = 9.80665, and HtDiff = 2.10, but you could type the numbers directly into the equation if you wish.)

F2 =(Vel1^2*SIN(B2)*COS(B2)+Vel1*COS(B2)*SQRT((Vel1*SIN(B2))^2+2*Grav*HtDiff))/Grav

	A	B	C	D	E	F	G	H	I	J	K	L
1	Angle (degrees)	Angle (radians)	Constants	Projection velocity (m/s)		v = 8 m/s	v = 9 m/s	v = 10 m/s	v = 11 m/s	v = 12 m/s	v = 13 m/s	v = 14 m/s
2	0	0.000	9.80665	8		5.24	5.89	6.54	7.20	7.85	8.51	9.16
3	1	0.017	2.10	9		5.35	6.03	6.72	7.42	8.11	8.81	9.52
4	2	0.035		10		5.46	6.18	6.91	7.64	8.38	9.12	9.88
5	3	0.052		11		5.58	6.33	7.09	7.86	8.65	9.44	10.25
6	4	0.070		12		5.70	6.48	7.28	8.09	8.92	9.77	10.64
7	5	0.087		13		5.81	6.63	7.46	8.32	9.20	10.10	11.03
8	6	0.105		14		5.93	6.78	7.65	8.56	9.48	10.44	11.42
9	7	0.122		15		6.05	6.93	7.85	8.79	9.77	10.78	11.83
10	8	0.140		16		6.16	7.08	8.04	9.03	10.06	11.13	12.24
11	9	0.157		17		6.28	7.23	8.23	9.27	10.35	11.48	12.65
12	10	0.175				6.39	7.38	8.42	9.51	10.64	11.83	13.07
13	11	0.192				6.51	7.53	8.61	9.75	10.94	12.18	13.49
14	12	0.209				6.62	7.68	8.80	9.98	11.23	12.53	13.91
15	13	0.227				6.73	7.83	8.99	10.22	11.52	12.89	14.32
16	14	0.244				6.84	7.97	9.18	10.46	11.81	13.24	14.74
17	15	0.262				6.95	8.12	9.37	10.69	12.10	13.59	15.16
18	16	0.279				7.05	8.26	9.55	10.92	12.38	13.93	15.57
19	17	0.297				7.15	8.40	9.73	11.15	12.66	14.27	15.98
20	18	0.314				7.25	8.53	9.90	11.37	12.94	14.61	16.38
21	19	0.332				7.35	8.66	10.08	11.59	13.21	14.94	16.78

Plot curves of range versus projection angle for the selected projection velocities. (The graph you should obtain is shown below.) This graph shows how the traditional idea arises that the optimum projection angle is just under 45°. Note also that as the shot-putter throws with a higher projection velocity, the range increases, but the optimum projection angle remains at just under 45°.



Projection Velocity

The previous set of calculations contains a serious error. The calculations did not include the fact that an athlete *cannot* throw with the same velocity at all projection angles. A shot-putter who has a maximum projection velocity of (say) 15 m/s at a projection angle of 0° will not be able to produce 15 m/s at angles of 30°, 60° and 90°. The projection velocity an athlete can generate steadily *decreases* as the athlete tries to throw with a higher and higher projection angle.

The decrease in projection velocity with increasing projection angle is a result of two factors.

1. When throwing with a high projection angle, the shot-putter must expend a greater effort during the delivery phase to overcome the weight of the shot, and so less effort is available to accelerate the shot (i.e. produce projection velocity).
2. The structure of the human body favours the production of putting force in the horizontal direction more than in the vertical direction. Considering just upper body strength, most athletes can lift more weight in a bench press exercise than in a shoulder press exercise.

The observed relation between projection velocity v (in m/s) and projection angle θ (in degrees) is described mathematically by

$$v = \sqrt{\frac{2\,(F - a\,\theta)\,l}{m}} \qquad (2)$$

where F is the force (in newtons) exerted on the shot for a horizontal release angle, a is a constant that characterizes the rate of force decrease with increasing release angle, l is the acceleration path length (in metres) of the shot during the delivery, and m is the mass of the shot (7.26 kg) (Linthorne, 2001).

Plot a curve of projection velocity versus projection angle using equation 2 and the data for Athlete 1 in Table 1.

To calculate the effect of the athlete's velocity-angle relation on the optimum projection angle, substitute the athlete's expression for v into equation 1, and re-calculate the range of the shot. That is, wherever you see v in equation 1, replace it with equation 2, and plot a curve of range versus projection angle. (Assume a launch height of $h = 2.10$ m.) You should find that the optimum projection angle for Athlete 1 is about 34°, rather than about 42°.

Table 1. Data for three shot-putters.

Variable	Athlete 1	Athlete 2	Athlete 3
Force, F (N)	500	430	565
Rate, a (N/degree)	3.5	1.5	5.5
Acceleration length, l (m)	1.65	1.65	1.65
Shoulder height, $h_{shoulder}$ (m)	1.70	1.70	1.70
Arm length, l_{arm} (m)	1.00	1.00	1.00

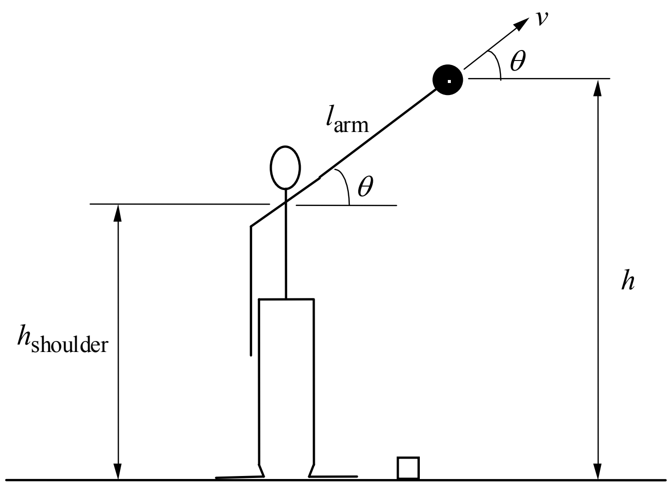
Launch Height

This is not the whole story. In shot-putting the launch height increases slightly with increasing projection angle. This relation arises from the geometry of the throwing action. When using a high projection angle, the angle of the athlete's arm to the horizontal is greater, and therefore at the instant of launch the shot is at a greater height above the ground.

The observed relation between launch height h and projection angle θ is described mathematically by

$$h = h_{shoulder} + l_{arm} \sin \theta \qquad (3)$$

where $h_{shoulder}$ is the height of the athlete's shoulders when standing upright, and l_{arm} is the length of the athlete's outstretched throwing arm and shoulder.



(Linthorne, 2001)

Use equation 3 and the data in Table 1 to plot a curve of launch height versus projection angle for Athlete 1.

To calculate the effect of the athlete's height-angle relation on the optimum projection angle, substitute the athlete's expressions for h (and v) into equation 1, and re-calculate the range of the shot. That is, wherever you see h (and v) in equation 1, replace it with equation 3 (and equation 2) and plot curves of range versus projection angle. You should find that introducing the relation between launch height and projection angle has very little effect on the optimum projection angle (about 34°).

Physical Characteristics of the Athletes

Athletes can have markedly different optimum projection angles, depending on their physical size (height, length of limbs, etc.), the types of strength exercises used in training, and the athlete's throwing technique. Here we examine three athletes that are the same size, but different in muscle strengths and coordination patterns.

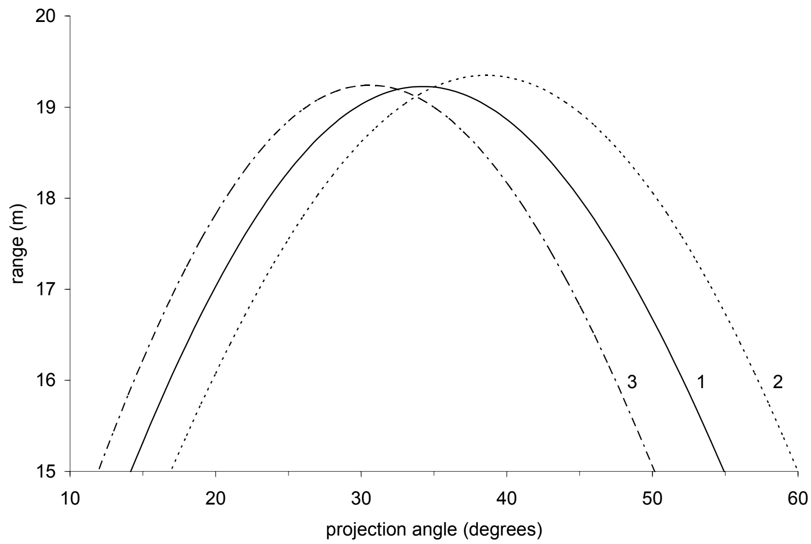
Plot curves of projection velocity versus projection angle for Athletes 1, 2, and 3 using the data in Table 1. (Note that the variables that depend on the size of the athlete are identical.)

Suggest why an athlete might have projection velocity versus projection angle curves like those shown for Athletes 1, 2, and 3. Consider the strength training exercises that would tend to produce a curve like that for Athlete 2 or like that for Athlete 3.

To see how the shape of the velocity-angle relation influences the optimum projection angle, calculate the range for Athletes 2 and 3, (you should already have Athlete 1) and plot curves of range versus projection angle for the three athletes. If you are having difficulty with the calculations, see the screen shot and graph below.

S2 =(P2^2*SIN(B2)*COS(B2)+P2*COS(B2)*SQRT((P2*SIN(B2))^2+2*Grav*Q2))/Grav

	A	O	P	Q	R	S	T	U	V	W	X	Y	
1	Projection angle (degrees)	Athlete 1	Projection velocity (m/s)	Launch height (m)	Range (m) (vel)	Range (m) (vel&h)	Athlete 2	Projection velocity (m/s)	Launch height (m)	Range (m) (vel&h)	Athlete 3	Projection velocity (m/s)	La heig
2	0	500	15.08	1.70	9.87	8.88	430	13.98	1.70	8.23	565	16.03	
3	1	3.5	15.02	1.72	10.24	9.30	1.5	13.96	1.72	8.61	5.5	15.95	
4	2	1.70	14.97	1.73	10.62	9.73	1.70	13.93	1.73	9.00	1.70	15.87	
5	3	1.00	14.92	1.75	11.01	10.17	1.00	13.91	1.75	9.40	1.00	15.79	
6	4	7.26	14.86	1.77	11.40	10.61		13.88	1.77	9.80		15.71	
7	5	1.65	14.81	1.79	11.79	11.06		13.86	1.79	10.21		15.63	
8	6		14.76	1.80	12.19	11.50		13.83	1.80	10.62		15.55	
9	7		14.70	1.82	12.58	11.95		13.81	1.82	11.03		15.47	
10	8		14.65	1.84	12.97	12.40		13.78	1.84	11.45		15.39	
11	9		14.59	1.86	13.37	12.84		13.76	1.86	11.86		15.31	
12	10		14.54	1.87	13.75	13.27		13.73	1.87	12.27		15.23	
13	11		14.48	1.89	14.14	13.70		13.71	1.89	12.69		15.14	
14	12		14.43	1.91	14.51	14.12		13.68	1.91	13.09		15.06	
15	13		14.37	1.92	14.88	14.53		13.66	1.92	13.49		14.98	
16	14		14.32	1.94	15.24	14.93		13.63	1.94	13.89		14.89	
17	15		14.26	1.96	15.59	15.32		13.61	1.96	14.28		14.81	
18	16		14.21	1.98	15.92	15.69		13.58	1.98	14.66		14.72	
19	17		14.15	1.99	16.24	16.05		13.56	1.99	15.03		14.64	
20	18		14.09	2.01	16.55	16.40		13.53	2.01	15.39		14.55	
21	19		14.04	2.03	16.85	16.72		13.51	2.03	15.74		14.47	



References

Linthorne, N. P. (2001). Optimum release angle in the shot put. *Journal of Sports Sciences*, *19*, 359–372.