BRIDGING COURSES IN MATHEMATICS

2 UNIT COURSE

School of Mathematics and Statistics

Mathematics Learning Centre

THE UNIVERSITY OF SYDNEY

Revision exercises

The course assumes reasonable familiarity with the following elementary algebra.

1. If $a = -1$, $b = -3$, $c = 0$ find the value of:

   (a) $2a^2b$
   (b) $5a + 2b$
   (c) abc
   (d) $\dfrac{7b}{a}$
   (e) $\dfrac{ac}{b}$
   (f) $\sqrt{2a - b^3}$

2. Simplify the following expressions by removing brackets and collecting terms:

   (a) $(4x + 2y) - (3x + 5y)$
   (b) $3(y - 5) - 2(x - 7)$
   (c) $(x + 2y + 7) - (7x - 2y + 1)$
   (d) $x(2x - 3) - 3x(5 - 2x)$
   (e) $x(y - z) - y(x + z)$
   (f) $4(x^2 + 7x + 5) + 2(x^2 - x - 1) - (x^2 + 2x + 3)$

3. Simplify:

   (a) $\dfrac{3a + 3b}{3}$
   (b) $\dfrac{x^2 - 5x}{x - 5}$
   (c) $\dfrac{2y^3 + 4y}{8y}$

4. Solve the following equations:

   (a) $2x + 7 = -5$
   (b) $5y + 3 = 2 + y$
   (c) $5(2x - 3) = 30$
   (d) $3(y + 1) - 4(y - 2) = 5$
   (e) $\dfrac{2t + 3}{7} = \dfrac{3t + 1}{3}$
   (f) $\dfrac{s - 4}{s + 2} = -1$

5. Use your calculator to evaluate each expression.

   (a) 123.4×987.6
   (b) $989 \div 63.25$
   (c) $\sqrt{(0.000006)}$
   (d) $\dfrac{1}{256.5} + \dfrac{7}{3.455}$
   (e) $(24.7)^2$

6. Use the formula $S = 0.007184 \times W^{0.425} \times H^{0.725}$, where S is the body surface area in m^2, W is the body mass in kg, H is the body height in cm, to find the surface area (skin area) of a person 150 cm tall, mass 76 kg.

7. Solve the inequalities:

   (a) $\dfrac{x}{2} > 3$

   (b) $2x - 3 \leq 9$

   (c) $5 - x > 1$

   (d) $1 < x + 2 < 5$

Answers to revision exercises

1. (a) -6 (b) -11 (c) 0 (d) 21 (e) 0 (f) 5

2. (a) $x - 3y$

   (b) $3y - 2x - 1$

   (c) $-6x + 4y + 6$

   (d) $8x^2 - 18x = 2x(4x - 9)$

   (e) $-z(x + y)$

   (f) $5x^2 + 24x + 15)$

3. (a) $a + b$ (b) x (c) $\dfrac{y^2 + 2}{4}$

4. (a) -6 (b) $-\dfrac{1}{4}$ (c) $\dfrac{9}{2}$ (d) 6 (e) $\dfrac{2}{15}$ (f) 1

5. (a) $1.218\ldots \times 10^5$

   (b) $15.\overline{63}$

   (c) 2.45×10^{-3} (correct to 3 significant figures)

   (d) 2.0 (2 S.F.)

   (e) 6.1×10^2 (2 S.F.)

6. $1.7\,m^2$

7. (a) $x > 6$

   (b) $x \leq 6$

   (c) $x < 4$

   (d) $-1 < x < 3$

Exercises Set 1

1. For the function $n = f(a) = 60a - 900$ described above, discuss the following:

(i) Does the formula make any sense for $a = 10$?

(ii) Would you expect the formula to apply for an adult?

(iii) At what rate is the size of the child's vocabulary changing?

2. An object is dropped from the top of a cliff and allowed to fall freely under gravity. After t seconds, its distance from the top of the cliff (in metres) is a function of time:

$$d = f(t) = 4.9t^2$$

(i) How far does the object fall in the first three seconds? In the next three seconds?

(ii) Discuss the speed (that is, the rate of change of distance as t increases) of the falling object.

(iii) Find the average speed over the first 3 seconds, and over the next 3 seconds.

(iv) Can you estimate the speed at the instant when $t = 3$?

3. (i) If $f(x) = 5(x - 3)$ find $f(0)$, $f(-6)$, $f(3.7)$, $f(a)$.

(ii) If $g(t) = t^3 - 5t^2$ find $g(-2)$, $g(2)$, $g(\frac{1}{2})$.

(iii) If $f(x) = 4x - 2$ find x for which $f(x) = 7$.

4. Draw a pair of coordinate axes and plot the following points:

(i)	$(2, 3)$	(ii)	$(-3, 2)$	(iii)	$(0, 1)$
(iv)	$(-2, 0)$	(v)	$(-1, -4)$	(vi)	$(1, -2)$

5. (i) Draw up a table as follows,

x :	-3	-2	-1	0	1	2	3
y :							

and fill in the y-values for each of the following functions.

(a)	$y = 3x + 1$	(b)	$y = 5 - \frac{1}{2}x$	(c)	$y = x^2 - 3$
(d)	$3y = x^3$	(e)	$2x - y = 7$	(f)	$x + y = 1$

(ii) Sketch each of the functions.

6. The formula $B = 0.006S^2$ can be used under some conditions to find the average braking distance B metres, for a car travelling at S km/h. B metres is the distance travelled by a car from the moment the brakes are applied.

(i) Graph the function for a useful set of values for S. Use graph paper if available.

(ii) A driver sees a potentially dangerous situation ahead. Would the distance in which the car actually comes to a stop (stopping distance) be the same as the braking distance? What needs to be accounted for?

Exercises Set 2

1. (*i*) For each of the equations below, express y as a (linear) function of x:

(a) $x = 2 - y$ (b) $x - 3y + 3 = 0$

(c) $3x - 2y + 6 = 0$ (d) $2x + y - 1 = 0$

(e) $x + 4y - 2 = 0$ (f) $y + 1 = 3(2x - 1)$

(*ii*) Write down the gradient of the graph of each equation.

(*iii*) Find where each graph cuts the x and y axes.

(*iv*) Sketch the graphs of the equations in part (*i*).

2. (*i*) Find the equation of each of the following lines:

(a) line through $(4, 1)$ with gradient 2

(b) line through $(1, -3)$ with gradient $\frac{1}{3}$

(c) line through $(2, 1)$ with gradient -3

(d) line through (p, q) with gradient -1

(*ii*) Find where each line cuts the x and y axes.

(*iii*) Sketch the graphs of the first three lines.

3. (*i*) Find the gradient of each of the following lines:

(a) line through $(2, 3)$ and $(4, 6)$

(b) line through $(-2, 1)$ and $(1, -2)$

(c) line through $(-1, 2)$ and $(3, 1)$

(d) line through $(4, -2)$ and $(-3, -2)$

(e) line through $(3, 0)$ and $(0, -2)$

(*ii*) Find the equation of each line.

(*iii*) Find where each cuts the x and y axes and sketch the graphs.

4. Find the equation of the line through $(4, 7)$ parallel to $y = 3x + 7$.

5. Where does the line through $(2, 5)$ and $(5, 9)$ intersect the vertical line $x = 7$?

Exercises Set 3

1. Factorise the following expressions:

(*i*)	$2a^2 + 10a$	(*ii*)	$5x^2y - 20xy$
(*iii*)	$ab - 4a^2b^2$	(*iv*)	$3p^2q + 6p^3$

2. Expand the following. Look for patterns which would help in factorising.

(*i*)	(a)	$(x+3)(x+3)$	(*ii*)	(a)	$(x-3)(x-3)$
	(b)	$(x+7)(x+7)$		(b)	$(x-7)(x-7)$
	(c)	$(x+5)(x+5)$		(c)	$(x-5)(x-5)$
	(d)	$(x+4)^2$		(d)	$(x-4)^2$
	(e)	$(x+6)^2$		(e)	$(x-6)^2$
	(f)	$(x+a)^2$		(f)	$(x-a)^2$
(*iii*)	(a)	$(x+3)(x-3)$	(*iv*)	(a)	$(x+3)(x+2)$
	(b)	$(x-4)(x+4)$		(b)	$(x+3)(x-2)$
	(c)	$(x-7)(x+7)$		(c)	$(x-3)(x+2)$
	(d)	$(x-a)(x+a)$		(d)	$(x-3)(x-2)$
(*v*)	(a)	$(x+7)(x+1)$	(*vi*)	(a)	$(2x+1)(x+3)$
	(b)	$(x+7)(x-1)$		(b)	$(2x+1)(x-3)$
	(c)	$(x-7)(x+1)$		(c)	$(2x-1)(x+3)$
	(d)	$(x-7)(x-1)$		(d)	$(2x-1)(x-3)$

3. Factorise the following.

(a)	$x^2 + 4x + 3$	(b)	$x^2 + 7x + 10$	(c)	$x^2 + 13x + 12$
(d)	$x^2 + 7x + 12$	(e)	$x^2 + 8x + 15$	(f)	$x^2 - 3x - 10$
(g)	$x^2 - 3x - 28$	(h)	$x^2 - 7x - 8$	(i)	$x^2 - 6x - 27$
(j)	$x^2 - x - 12$	(k)	$x^2 - 11x + 10$	(l)	$x^2 - 8x + 15$
(m)	$x^2 - 16x + 15$	(n)	$x^2 - 3x + 2$	(o)	$x^2 - 6x + 5$
(p)	$x^2 + 2x - 3$	(q)	$x^2 + 9x - 10$	(r)	$x^2 + 5x - 24$
(s)	$x^2 + 5x - 14$	(t)	$x^2 + x - 2$	(u)	$a^2 - 14a - 15$
(v)	$m^2 + 6m + 9$	(w)	$2x^2 + 7x + 3$	(x)	$3x^2 + 11x + 10$

4. Factorise and solve for x.

(a) $x^2 - 7x + 12 = 0$

(b) $x^2 + 3x = 0$

(c) $x^2 + 7x + 10 = 0$

(d) $4x^2 - 4x = 0$

(e) $x^2 - 4x - 5 = 0$

(f) $x^2 - 7x = 0$

(g) $x^2 + 6x + 5 = 0$

(h) $5x - x^2 = 0$

(i) $x^2 - 3x + 2 = 0$

(j) $4x^2 - 2x = 0$

(k) $x^2 - 81 = 0$

(l) $x^2 - 49 = 0$

(m) $x^2 - 1 = 0$

(n) $9x^2 - 16 = 0$

(o) $x^2 - 2x + 1 = 0$

(p) $x^2 + 8x + 16 = 0$

(q) $x^2 + 10x + 25 = 0$

(r) $x^2 - 14x + 49 = 0$

(s) $10 + 3x - x^2 = 0$

(t) $14 - 5x - x^2 = 0$

(u) $2x^2 - 7x + 5 = 0$

(v) $6x^2 - 25x - 9 = 0$

5. Solve the following equations for x.

(a) $x^2 - 3x - 5 = 0$

(b) $x^2 + x - 4 = 0$

(c) $x^2 + 6x + 2 = 0$

(d) $x^2 + 4x + 1 = 0$

(e) $3x^2 - x - 3 = 0$

(f) $x^2 + 5x + 7 = 0$

6. A farmer has 300 metres of fencing material, and wants to fence off a rectangular area of 10000 square metres, using an existing fence as one side of the rectangle. What should the dimensions of the rectangle be?

7. A frame of uniform width is to be made for a 40 cm by 100 cm rectangular painting. The area of the frame is to be half the area of the painting. How wide should the frame be?

8. A rock is thrown downwards from the top of a 150 metre high cliff with an initial velocity of 5 metres per second. After t seconds its distance, s, (in metres) from the top of the cliff is given by

$$s = 5t + 4.9t^2.$$

How long will it take for the rock to reach the ground?

Exercises Set 4

1. (i) Verify that the points $A = (1, 1)$, $B = (1.1, 1.21)$, $B' = (0.9, 0.81)$, $C = (1.01, 1.0201)$ and $C' = (0.99, 0.9801)$ all lie on the parabola $y = x^2$.

(ii) Find the slopes of the chords AB, AB', AC and AC'.

(iii) What do you think the slope of the parabola is at A?

(iv) Repeat parts (i) – (iii) for points close to $(2, 4)$, and to $(-3, 9)$.

2. (i) Find the derivative (gradient function) in each case:

(a)	$f(x) = x^2$	(b)	$f(x) = x^2 + x$
(c)	$f(x) = x^2 - 4x$	(d)	$f(x) = 3x^2 + 12x - 9$
(e)	$f(x) = x^2 + 5x + 7$	(f)	$f(x) = 6$
(g)	$f(x) = 4x - x^2$	(h)	$f(x) = 7x - 2$

(ii) Find the maximum or minimum value for each function.

3. Find $f'(0)$ for the following functions:

(a)	$f(x) = 2x + 5$	(b)	$f(x) = x^2$
(c)	$f(x) = 4x^2 - 7x + 6$	(d)	$f(x) = 9 - x^2$

4. Sketch the following parabolas.

(i) $y = x^2 - 4x + 3$ (ii) $y = x^2 - 5x + 6$

(iii) $y = 1 - x^2$ (iv) $y = x^2 + 2x + 3$

5. For each of the curves below find the gradient and the equation of the tangent to the curve at the given point.

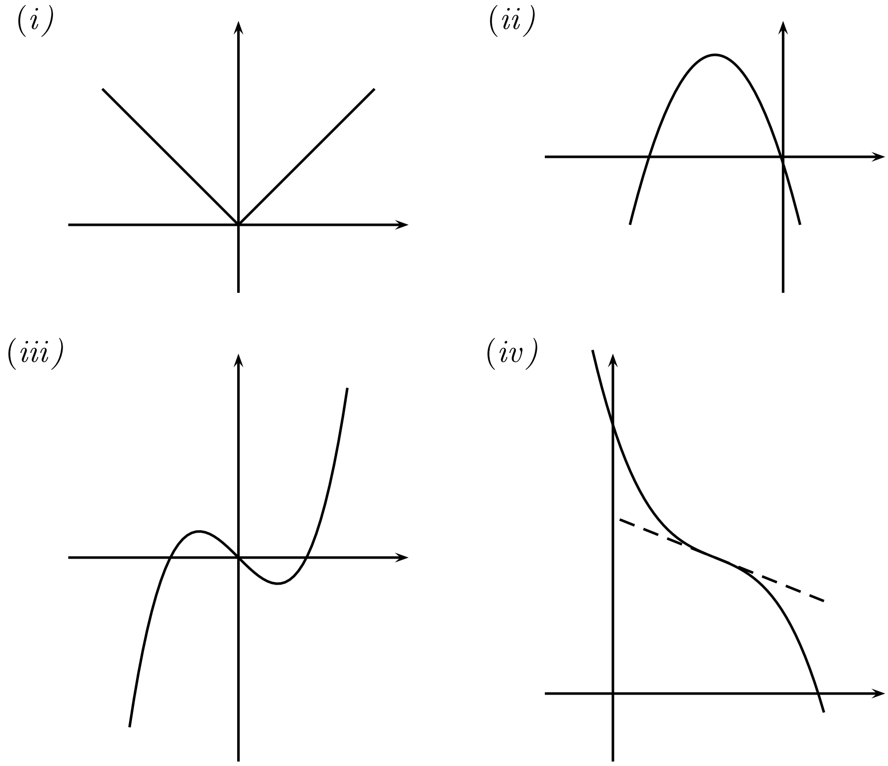
(i) $f(x) = x^2$ at the point $(1, 1)$

(ii) $f(x) = 2x^2$ at the point $(2, 8)$

(iii) $f(x) = x^2 - 3x + 1$ at the point $(0, 1)$

(iv) $f(x) = x^2 - 2x + 4$ at the point $(-1, 7)$

6. Sketch the gradient function for the following curves:



7. A rectangular field is to be fenced on three sides (the fourth boundary is formed by a river), with 300m of fencing. Find the dimensions of such a field if the area enclosed is to be as great as possible.

8. A magazine publisher has established that the profits obtained by selling the magazine are given by

$$P = -5000x^2 + 23200x - 24750$$

where x is the price of a single magazine. Show that the profit is maximized when the selling price is \$2.32.

Exercises Set 5

1. Take an arbitrary point (x, x^3) on the curve $y = x^3$. Take another point on the curve with x-coordinate $x + h$.

(i) Find the gradient of the chord joining these two points.

(ii) Determine what happens to this gradient as h tends to zero.

(iii) What is $\frac{d}{dx}(x^3)$?

2. Find the derivative (gradient function) in each case:

(i) $f(x) = x^3 + 6x$	(ii) $f(x) = 4x^5 - 3x^3 + 2x$
(iii) $f(x) = x - 7x^3$	(iv) $f(x) = 12x^6 + 4x^2 - x$
(v) $f(x) = x^3 - 4x^2 + 10x - 1$	(vi) $f(x) = 3 - 2x + 5x^4 + x^{10}$

3. Find $f'(0)$ for the following functions:

(a) $f(x) = x^3 + 1$ (b) $f(x) = 3x^4 + 2x^2 - x + 1$

4. Sketch the curve whose equation is $y = x^3 - 4x$, for x from -4 to 4. Find the coordinates of any stationary points and points where the graph crosses the axes.

5. Sketch the graph of $y = x^3(x - 2)$ for x between -1 and 3.

6. If $g(x) = x^3 - 4x + 3$, find $g'(x)$. Hence find $g'(0), g'(1)$ and $g'(-2)$.

7. (i) Find the y value of the point on the curve $y = x^2 + 5x - 8$ where the x value is 1.

(ii) Find the value of the derivative (the gradient of the curve) at the point.

(iii) Hence find the equation of the tangent to the curve $y = x^2 + 5x - 8$ at the point on it where $x = 1$.

8. Find two positive numbers whose sum is 20 and whose product is as large as possible.

9. Show that the rectangle that has maximum area for a given perimeter is a square.

Exercises Set 6

1. Find the first and second derivatives of the following functions:

(*i*) $f(x) = x^2 - 5x + 1$ (*ii*) $f(x) = x^3 - x^5 + 17$

(*iii*) $y = x^6 - 7x^4 + 2x^2 + 1$ (*iv*) $f(x) = x^2 + 2x + 1$

(*v*) $y = 5x^5 + 23$ (*vi*) $f(x) = -x^3$

2. Sketch each function. Use calculus to establish any stationary points and their nature. Identify any points of inflexion.

(*i*) $y = x^2 - 6$ (*ii*) $y = 2x^2 - 4x + 3$

(*iii*) $y = x^3 - 12x + 12$ (*iv*) $y = x^5$

(*v*) $y = x^6$ (*vi*) $y = (x - 1)^3$

(*vii*) $y = x^4 - 2x^2$

3. A rock blasted vertically upwards from the ground reaches a height of $s = 24t - 4.9t^2$ metres after t seconds.

(*i*) What is the initial velocity?

(*ii*) How high does the rock go?

(*iii*) After how many seconds does the rock fall back to the ground?

4. On Mars, an object allowed to fall freely under gravity falls a distance $s = 1.86t^2$ metres in t seconds. What is the acceleration due to gravity on Mars?

5. A body moves in a straight line according to the formula $s = t^3 - 4t^2 - 3t$ (where s = distance from the starting point in metres, and t = time in seconds). Find its acceleration when the velocity is zero.

Exercises Set 7

1. Simplify the following:

(i) $x^{12} \times x^{13} \div x^5$ (ii) $45a^{12} \div 9a^6$

(iii) $uv^{-2} \times \frac{u^{-1}}{v}$ (iv) $\left(\frac{1}{2}\right)^{-3} \times 2^{-1}$

2. Evaluate the following, where $a = 2$, $b = 3$ and $c = \frac{1}{2}$.

(i) $a^{-1} + b^{-2}$ (ii) $(ac)^{-1}$

(iii) $ab^{-1} + bc^{-1}$ (iv) $b^0 + (3b)^{\frac{1}{2}}$

3. Evaluate the following expressions:

(i) $64^{\frac{1}{2}}$ (ii) $64^{\frac{3}{2}}$ (iii) $16^{\frac{3}{4}}$ (iv) $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$

4. Simplify:

(i) $x^{\frac{2}{3}} \times x^{-\frac{5}{3}} \div x^{-1}$ (ii) $m^{\frac{1}{3}}(m^{\frac{1}{3}} + m^{-\frac{1}{3}})$

5. Simplify the following:

(i) $\dfrac{x^4\sqrt{x}}{x^3}$ (ii) $\dfrac{t^2t^{-3}}{\sqrt[3]{t}}$ (iii) $\dfrac{x^2x^5}{x^{-7}}$

(iv) $\dfrac{m^{\frac{3}{2}}\sqrt[3]{m}}{m^{-3}}$ (v) $\dfrac{x^{10}\sqrt{x^5}}{x}$ (vi) $\dfrac{x^{\frac{7}{5}}\sqrt[6]{x}}{x^{\frac{7}{6}}}$

(vii) $x^7x^{-5}x^2$ $(viii)$ $\dfrac{1}{x^{-2}\sqrt{x^{-1}}}$ (ix) $u^2u^5u^{\frac{6}{7}}$

6. For each function below, find $f'(x)$:

(i) $f(x) = x^8 + x^2$ (ii) $f(x) = 3x^9$

(iii) $f(x) = 2x^3 + x + 1$ (iv) $f(x) = 9x^{11} - 6x^7 + 5x - 4$

(v) $f(x) = x - \frac{1}{x}$ (Hint: $f(x) = x - x^{-1}$)

(vi) $f(x) = 1 + 2\sqrt{x}$ (Hint: $f(x) = 1 + 2x^{\frac{1}{2}}$)

7. For each of the equations below, find $\frac{dy}{dx}$.

(i) $y = -x^{13} + \dfrac{13}{x}$ (ii) $y = 6 - 5x + x^5$

(iii) $y = x^2 + \dfrac{2}{x^2}$ (iv) $y = 2\sqrt{x} - \dfrac{4}{\sqrt{x}}$

8. Find the derivatives of the following functions:

(*i*) $f(x) = 2\sqrt[3]{x} - 1$ (*ii*) $g(y) = 12y^2 - 7y + 4$

(*iii*) $f(x) = \sqrt{x^5}$ (*iv*) $h(u) = 13u^2 - 5u\sqrt{u}$

(*v*) $p(t) = 6t - \dfrac{12}{t^2}$ (*vi*) $f(x) = \dfrac{5}{x^3} + \dfrac{1}{x}$

9. Find the first and second derivatives of the following functions:

(*i*) $f(x) = x^{\frac{5}{2}}$ (*ii*) $y = x^2 + x^{\frac{7}{3}}$

(*iii*) $y = x^{\frac{1}{2}}$ (*iv*) $f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

(*v*) $y = x^{\frac{4}{3}}$ (*vi*) $y = -x^3$

(*vii*) $y = \dfrac{1}{x^2}$ (*viii*) $y = \dfrac{5x^3 + 2x^{\frac{1}{2}}}{x}$

10. An open tank is to be constructed with a square base of side x metres and with four rectangular sides. The tank is to have a capacity of 32 cubic metres.

(*i*) Show that the height of the tank is $\dfrac{32}{x^2}$.

(*ii*) Find the least area of sheet metal from which the tank can be constructed.

Exercises Set 8

1. On the same axes sketch the graphs of

(i) $y = 3^x$ (ii) $y = 3^{-x}$ (iii) $y = 4^x$ (iv) $y = e^x$

2. On the same axes sketch the graphs of

(i) $y = e^x$ (ii) $y = e^{0.5x}$ (iii) $y = e^{2x}$ (iv) $y = e^{-2x}$

3. Sketch graphs (very rough) to show the differences between

(a) $y = 100e^{0.1x}$ and $y = 100e^{0.7x}$.

(b) $y = 100e^{0.1x}$ and $y = 1000e^{0.1x}$.

4. Find $\dfrac{dy}{dx}$ for each of the following:

(i) $y = 3e^x$ (ii) $y = \dfrac{e^x + x}{2}$ (iii) $y = 2 - e^x$

5. The population, P, of a city increases according to the formula

$$P = Ae^{0.02t} \qquad \text{where } t \text{ represents time in years.}$$

In 1970 the population was 100,000. Find the population in 1980 and in 1990 (to the nearest thousand). (Hint: Take $t = 0$ in 1970.)

6. Radioactive carbon decays exponentially according to the formula

$$A = A_0 e^{-0.00012t}.$$

(A is the amount of radioactive carbon remaining after t years. A_0 is the initial amount.) Find the percentage of the initial amount remaining after

(i) 2000 years (ii) 6000 years (iii) 10000 years

7. Write each of the following functions as the composite of a function $u = f(x)$ and a function $g(u)$:

(i) e^{7x+2} (ii) $\sqrt{3 - x^3}$ (iii) $(x^5 + 1)^3$ (iv) $\dfrac{1}{3 - e^x}$ (v) e^{x^2+1}

Exercises Set 9

1. Use the chain rule to differentiate the following:

(i) $y = (x^3 + 2x - 1)^7$ (ii) $y = \sqrt{5 - x^2}$

(iii) $y = (3x + 4)^4$ (iv) $y = (2x + 8)^{\frac{1}{2}}$

(v) $y = \dfrac{1}{(2x + 3)^5}$ (vi) $y = \sqrt{4x - 1}$

(vii) $y = (2x + 3)^3$ (viii) $y = \dfrac{1}{(3x - 2)^4}$

2. Use the product rule (and the chain rule if necessary) to differentiate:

(i) $y = (x^4 + 3x)(x + x^2 + 1)$ (ii) $y = (x + 6)(x^2 + 3)$

(iii) $y = 2x^2(x^2 - 5)$ (iv) $y = (x^2 + 1)(x^2 - 6)$

(v) $y = x^3(1 + x)^3$ (vi) $y = (x + 1)^2(x + 2)^2$

3. Use the quotient rule to differentiate:

(i) $y = \dfrac{4x}{x^2 + 1}$ (ii) $y = \dfrac{2x + 1}{2x - 1}$ (iii) $y = \dfrac{x^2}{1 - x^2}$

(iv) $y = \dfrac{x}{x + 1}$ (v) $y = \dfrac{5x - 2}{3x + 5}$ (vi) $y = \dfrac{x^3 - 1}{x + 1}$

4. Find $\dfrac{dy}{dx}$:

(i) $y = \dfrac{1}{x + 1}$ (ii) $y = \left(\dfrac{1}{x^2}\right)^3$

(iii) $y = (x + 1)\sqrt{x}$ (iv) $y = \dfrac{x}{\sqrt{x^2 + 1}}$

5. Differentiate the following functions:

(i) $f(x) = e^{3x}$ (ii) $f(x) = e^{-2x}$

(iii) $f(x) = e^{4x+3}$ (iv) $f(x) = e^{(x^2)}$

(v) $f(x) = e^{1-x}$ (vi) $f(x) = e^{x^2-2x+7}$

(vii) $f(x) = xe^x$ (viii) $f(x) = x^2e^x$

6. The concentration of a certain drug in the blood at time t hours after taking the dose is x units, where $x = 0.3te^{-1.1t}$. Determine the maximum concentration and the time at which it is reached.

Plot the graph of $x = 0.3te^{-1.1t}$ for t=0, 0.5, 1, 2, 3 using graph paper.

The drug kills germs only if its concentration exceeds 0.06 units. Find, from the graph, the length of time during which the drug is able to kill germs.

7. Sketch the graph of $y = e^{-x^2}$.

Exercises Set 10

1. (i) Sketch the function $y = \cos x$ for $-2\pi \le x \le 2\pi$. (Use your calculator.)

(ii) Write down properties of the cosine function corresponding to those given above for $\sin x$.

2. Sketch the following functions.

(i) $y = 3\cos x$ (ii) $y = \cos 3x$

(iii) $y = 3 + \cos x$ (iv) $y = -\cos x$

(v) $y = \cos\left(x - \dfrac{\pi}{4}\right)$

3. Find the derivatives of the following functions:

(i) $y = x\sin(x)$ (ii) $y = \sin(x)\cos(x)$ (iii) $y = \sin^2(x)$

(iv) $y = \cos^2(x)$ (v) $y = x\cos(x)$

The second derivatives?

4. Sketch $\sin(2x)$, and try to give a rough sketch of its derivative. Can you think what this might be the graph of?

5. Differentiate the following using the chain rule

(i) $y = \cos 4x$ (ii) $y = \cos\left(x + x^2\right)$ (iii) $y = \sin\left(x^2\right)$

(iv) $y = (\sin x)^2$ (v) $y = \sin x^3 + \cos x^3$ (vi) $y = 2\sin(x + \pi)$

6. Differentiate using the product or quotient rule (and the chain rule if necessary)

(i) $y = \sin x \cos x$ (ii) $y = x^2 \sin x$

(iii) $y = \sin(3x)\cos\left(x^2\right)$ (iv) $y = \dfrac{x}{\sin x}$

(v) $y = \dfrac{3x}{1 + \cos x}$ (vi) $y = \dfrac{\sin x}{\cos x}$ $(= \tan x)$

7. Find $f'(x)$

(a) $f(x) = \sin x^3$ (b) $f(x) = \cos\left(x^2 - 2x + 1\right)$

(c) $f(x) = x\sin x$ (d) $f(x) = \sin\left(x^{-1}\right)$

(e) $f(x) = x^2\cos\left(x^2 + 4\right)$ (f) $f(x) = (\sin x)(\cos x)$

(g) $f(x) = 4\cos^2(x + 2)$ (h) $f(x) = \sin(\cos x)$

8. Simplify the following:

(i) $\sin\left(x + \frac{\pi}{2}\right)$ (ii) $\cos(x - \pi)$

(iii) $\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4}$ (iv) $\cos(2\pi - x)$

Exercises Set 11

1. Simplify:

(i) $e^{3\ln x}$ (ii) $e^{-\ln x}$ (iii) $\ln(e^{x+2})$

(iv) $\ln x^3 - \ln x$ (v) $\ln(5x) + \ln\left(\dfrac{2}{x}\right)$

2. Solve the following equations

(i) $\ln 2x = 1$ (ii) $\ln(3 + x) = 1$

(iii) $\ln(2x - 2) = 0$ (iv) $\ln(5x - 6) = 2$

3. Solve the equations

(i) $2e^x = 7$ (ii) $e^{3x-2} = 5$

(iii) $e^{2x} = 1$ (iv) $e^{x^2} = 10$

4. Write each of the following expressions as a single logarithm.

(i) $\ln(x + 1) + \ln(x - 2) + 2\ln(x - 3)$

(ii) $\ln(x - 4) - \ln(x + 5) + 4\ln x$

(iii) $\frac{1}{2}\ln(x + 1) - \frac{1}{2}\ln(x)$

5. (i) Write each of the following as sums, differences or multiples of simpler logarithmic quantities

(a) $\ln\left(\dfrac{x+1}{x+2}\right)$ (b) $\ln\sqrt{3x + 1}$

(c) $\ln\left(\dfrac{x^2\sqrt{x+1}}{\sqrt[3]{3x+4}}\right)$

(ii) Differentiate the functions in part (i).

6. Differentiate the following expressions:

(i) $\ln(3x + 2)$ (ii) $x\ln x$

(iii) $\ln(x^2)$ (iv) $\ln\sqrt{x - 1}$

(v) $(\cos x) \times (\ln x)$ (vi) $\sqrt{\ln x}$

7. If the population of a city increases at a rate which is proportional to the current population, and if the population was 2×10^6 in 1970, and 2.5×10^6 in 1980, find, in terms of t, the population t years after 1970.

8. In a certain bacterial culture, the rate of increase is proportional to the number of bacteria present.

(a) If the number doubles in 3 hours, by what factor is the number increased in one hour?

(b) How many bacteria are there after 9 hours, if the original population is 10^4?

(c) After how many hours are there 4×10^4 bacteria?

Exercises Set 12

1. Simplify the following;
$|3|$, $|-3|$, $|2-5|$, $|5-2|$, $-|-3|$, $\big||2|-|-5|\big|$, $\big||2|+|5|\big|$.

2. Find the values of x which satisfy the following:

(i) $|x-2|=3$ (ii) $|5x+1|=4$ (iii) $|2+x|=5$

3. Graph the following functions

(i) $y=|3x-2|$ (ii) $y=|x^2+x-2|$

4. (i) Find the equation of the straight line joining $(1,2)$ and $(-1,4)$.

(ii) Find the equation of the straight line through $(3,0)$ parallel to the line $2x+5y=10$.

5. (i) Solve for x:

(a) $2x^2-7x=0$ (b) $x^2-3x-40=0$

(ii) Find the point(s) of intersection of the hyperbola $y=\dfrac{1}{x-1}$ and the straight line $x-y=1$.

6. Find the gradient of the curve $y=x^3+2x^2$ at the point $(1,3)$. Hence find the equation of the tangent to the curve at this point.

7. Find the dimensions of a rectangle with area 96 cm^2 and perimeter 40 cms.

8. Simplify the following

(a) $v^3\sqrt{\dfrac{1}{v}}$ (b) $x^{\frac{3}{2}}x^{\frac{2}{3}}$ (c) $\dfrac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}}$

(d) $\sqrt[3]{\dfrac{x^7}{x^3}}$ (e) $\dfrac{t^5t^{-6}}{t^{-1}}$ (f) $\dfrac{u^3u^{-\frac{3}{2}}}{u^4u^{\frac{3}{2}}}$

(g) $\dfrac{\sqrt{u^7}u^2}{u}$ (h) $\dfrac{v^4}{v^{\frac{3}{2}}v^{-2}}$ (i) $\sqrt{\dfrac{u^7}{u^3}}$

(j) $\dfrac{w^5w^{-\frac{7}{2}}}{w^{-2}}$ (k) $\dfrac{\sqrt{w^2}w}{w^5}$ (l) $\dfrac{\sqrt[4]{x}}{x^2}$

(m) $\dfrac{\sqrt{x}}{x}$ (n) $y^2\sqrt[3]{y}y^{-1}$ (o) $\sqrt{y}y^{-2}\sqrt[3]{y}$

9. Find the derivative of the following functions using the product rule.

(i) $y = x^3(3x-4)$	(ii) $y = (2x+6)(x^2-2x)$
(iii) $y = 4x(2x^2-1)$	(iv) $y = (3x+6)(5x-3)$
(v) $f(x) = (5x^3+2)(2x-x^3)$	(vi) $y = (x+2)(x^2-4)$
(vii) $f(x) = (2x^2+3)(x^3-1)$	$(viii)$ $h(t) = (9t^2-6t)(2t+1)$
(ix) $u = (x-5)(3x^2+4x)$	(x) $y = (x^2-3x+1)(2x-6)$

As a check expand and then differentiate a few of the above.

10. Find the first derivatives of the following:

(i) x^4+2x^3-7x+5	(ii) $2x^{\frac{1}{2}}+3x^{-2}$
(iii) $\sqrt{x}-\dfrac{1}{\sqrt{x}}$	(iv) $x\sin x$
(v) $\dfrac{1}{e^x}$	(vi) e^{x^2+3}
(vii) $4\cos 3x$	$(viii)$ $e^{\sin x}$
(ix) $\tan\dfrac{1}{x}$	(x) $\dfrac{x+1}{x^2-2}$

11. A magazine advertisement is to contain 50 cm^2 of lettering with clear margins of 4 cm each at the top and bottom and 2 cm at each side. Find the overall dimensions if the total area of the advertisement is to be a minimum.

12. The cost per hour of running a pleasure cruiser is $\$\left(\dfrac{x^2}{40}+10\right)$, where x is the speed in knots.

(i) For a trip of 100 nautical miles show that the cost is

$$\$\left[\frac{100}{x}\left(\frac{x^2}{40}+10\right)\right].$$

(ii) What is the most economical speed for running the cruiser on this trip? (that is, the speed which minimises the cost.)

13. Sketch the curve $y = 2\cos 3x$.

14. Determine all critical points for the following functions, and hence sketch:

(i) $y = x^3 - 3x^2$ $\qquad$ (ii) $y = x^2 e^x$

15. Find the equation of the tangent to $y = \dfrac{x}{x+3}$ when $x = -1$.

16. Simplify the expressions:

(i) $e^{2\ln x}$ $\qquad$ (ii) $\log\left(\dfrac{10^x}{100^x}\right)$ $\qquad$ (iii) $\ln 2x + \ln\left(\dfrac{1}{x}\right)$

17. Solve the equations:

(i) $2^x = 5$ $\qquad$ (ii) $\ln(x^2+1) = 3$

(iii) $e^{3x} = 1$ $\qquad$ (iv) $e^2 \times e^{x+1} = 10$

18. The number of bacteria in a certain culture is initially 120,000, and the number, N, present t hours later is given by the formula $N = Ae^{0.25t}$. Find the value of A, and the number of bacteria present after 6 hours. At this time (i.e., after 6 hours), how fast is the number of bacteria increasing?

19. A population is growing exponentially (i.e. $P = Ae^{kt}$, t in years), and doubles in the first 5 years. Find

(i) the value of k.

(ii) the population after 20 years.

(iii) after how many years the population is four times the initial value.

20. Solve the simultaneous equations:

(i) $2x - 3y = 7, \qquad 4x - 5y = 11$

(ii) $x = y + 2, \qquad y = 3x - 10$

21. Find all values of x between 0 and 2π for which $\sin\left(3x - \dfrac{\pi}{2}\right)$

(i) $= 0$. $\qquad$ (ii) $= 1$.

22. For a period of its life, the increase in the diameter of a tree follows aproximately the law:

$$D(t) = Ae^{kt}$$

where $D(t)$ is the diameter of the tree t years after the beginning of this period.

(a) If initially the diameter is 50 cm, find the value of A.

(b) If $D'(t) = 0.1D(t)$, find the value of k.

(c) After how many years is the diameter 61 cm?

23. Sunlight transmitted through water loses intensity as it penetrates to greater depths according to the law

$$I(d) = I(0)e^{-kd}$$

where $I(d)$ is the intensity at depth d below the surface. If $I(300) = 0.3I(0)$ find

(i) the value of k

(ii) The depth at which the intensity would be decreased by half.

Answers to Exercises

Exercises Set 1

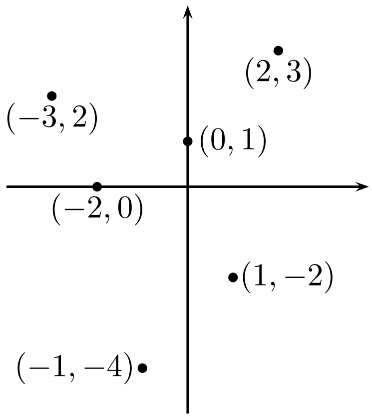
1. (*iii*) 60 words per month

2. (*i*) 44.1 m, 132.3 m (*iii*) 14.7 m/sec, 44.1 m/sec.

3. (*i*) $-15,\ -45,\ 3.5,\ 5(a-3)$

(*ii*) $-28,\ -12,\ -\frac{9}{8}$ (*iii*) $x = \dfrac{9}{4}$

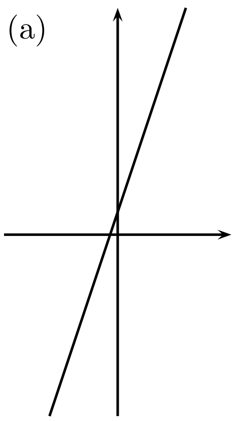
4.



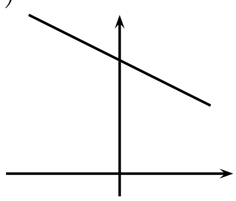
5. (*i*)

	$x:$	-3	-2	-1	0	1	2	3
(a)	$y:$	-8	-5	-2	1	4	7	10
(b)	$y:$	$6\frac{1}{2}$	6	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	$3\frac{1}{2}$
(c)	$y:$	6	1	-2	-3	-2	1	6
(d)	$y:$	-9	$-\frac{8}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{8}{3}$	9
(e)	$y:$	-13	-11	-9	-7	-5	-3	-1
(f)	$y:$	4	3	2	1	0	-1	-2

(*ii*) (a)



(b)



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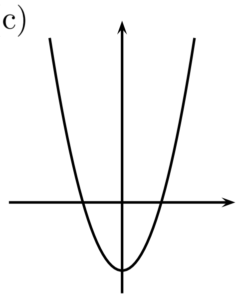

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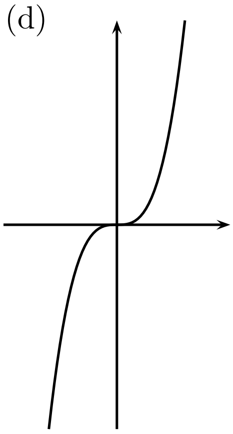

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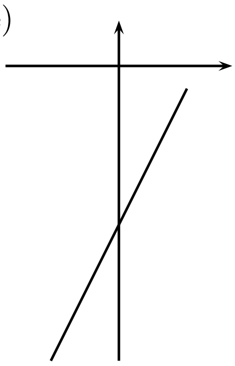
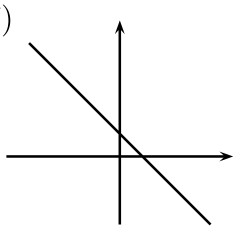
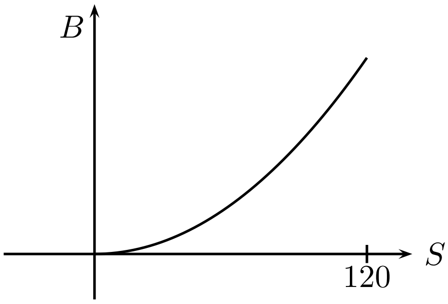


(f)



6. (i)



Exercises Set 2

1. (i) (a) $y = -x + 2$ (b) $y = \dfrac{x}{3} + 1$

(c) $y = \dfrac{3}{2}x + 3$ (d) $y = -2x + 1$

(e) $y = -\dfrac{1}{4}x + \dfrac{1}{2}$ (f) $y = 6x - 4$

(ii) (a) -1 (b) $\dfrac{1}{3}$ (c) $\dfrac{3}{2}$ (d) -2 (e) $-\dfrac{1}{4}$ (f) 6

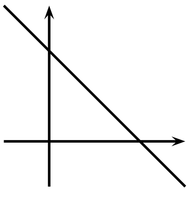
(iii) $x-$ intercept:

(a) 2 (b) -3 (c) -2 (d) $\dfrac{1}{2}$ (e) 2 (f) $\dfrac{2}{3}$

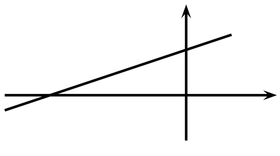
$y-$intercept:

(a) 2 (b) 1 (c) 3 (d) 1 (e) $\dfrac{1}{2}$ (f) -4

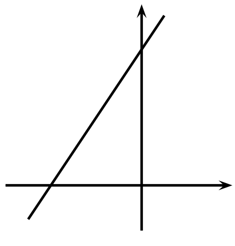
(iv) (a)



(b)



(c)



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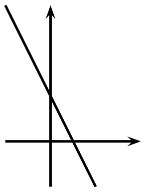

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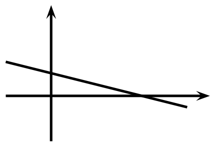

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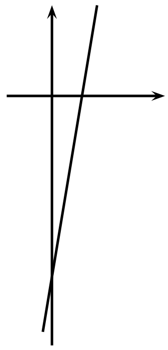


2. (i) (a) $y = 2x - 7$ (b) $3y = x - 10$

(c) $y = -3x + 7$ (d) $y = -x + p + q$

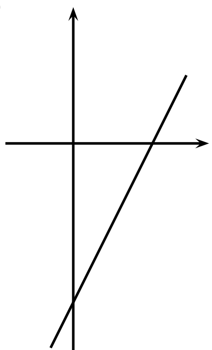
(ii) $x-$intercept:

(a) $\frac{7}{2}$ (b) 10 (c) $\frac{7}{3}$ (d) $p + q$

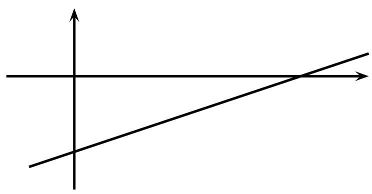
$y-$intercept:

(a) -7 (b) $-\frac{10}{3}$ (c) 7 (d) $p + q$

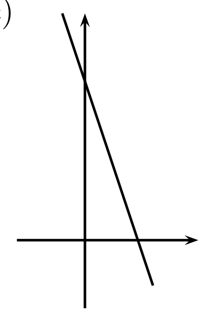
(a)



(b)



(c)



3. (i) (a) $\frac{3}{2}$ (b) -1 (c) $-\frac{1}{4}$ (d) 0 (e) $\frac{2}{3}$

(ii) (a) $3x - 2y = 0$ (b) $x + y + 1 = 0$ (c) $x + 4y - 7 = 0$

(d) $y + 2 = 0$ (e) $3y = 2x - 6$

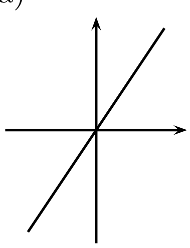
(iii) $x-$intercept:

(a) 0 (b) -1 (c) 7 (d) line $\parallel$ to $x-$axis (e) 3

$y-$intercept:

(a) 0 (b) -1 (c) $\dfrac{7}{4}$ (d) -2 (e) -2

(a)



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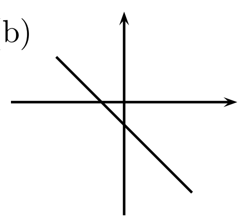

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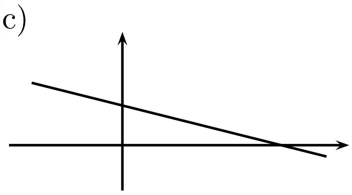

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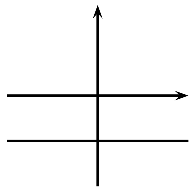
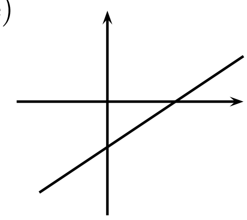


(e)



4. $3x - y - 5 = 0$

5. $x = 7, y = \dfrac{35}{3}$

Exercises Set 3

1. (i) $2a(a+5)$ (ii) $5xy(x-4)$

(iii) $ab(1-4ab)$ (iv) $3p^2(q+2p)$

2. (i) (a) x^2+6x+9 (ii) (a) x^2-6x+9

(b) $x^2+14x+49$ (b) $x^2-14x+49$

(c) $x^2+10x+25$ (c) $x^2-10x+25$

(d) $x^2+8x+16$ (d) $x^2-8x+16$

(e) $x^2+12x+36$ (e) $x^2-12x+36$

(f) $x^2+2ax+a^2$ (f) $x^2-2ax+a^2$

(iii) (a) x^2-9 (iv) (a) x^2+5x+6

(b) x^2-16 (b) x^2+x-6

(c) x^2-49 (c) x^2-x-6

(d) x^2-a^2 (d) x^2-5x+6

(v) (a) x^2+8x+7 (vi) (a) $2x^2+7x+3$

(b) x^2+6x-7 (b) $2x^2-5x-3$

(c) x^2-6x-7 (c) $2x^2+5x-3$

(d) x^2-8x+7 (d) $2x^2-7x+3$

3. (a) $(x+1)(x+3)$ (b) $(x+5)(x+2)$

(c) $(x+12)(x+1)$ (d) $(x+3)(x+4)$

(e) $(x+3)(x+5)$ (f) $(x-5)(x+2)$

(g)	$(x-7)(x+4)$	(h)	$(x-8)(x+1)$
(i)	$(x-9)(x+3)$	(j)	$(x-4)(x+3)$
(k)	$(x-10)(x-1)$	(l)	$(x-5)(x-3)$
(m)	$(x-15)(x-1)$	(n)	$(x-2)(x-1)$
(o)	$(x-5)(x-1)$	(p)	$(x+3)(x-1)$
(q)	$(x+10)(x-1)$	(r)	$(x+8)(x-3)$
(s)	$(x+7)(x-2)$	(t)	$(x+2)(x-1)$
(u)	$(a-15)(a+1)$	(v)	$(m+3)^2$
(w)	$(2x+1)(x+3)$	(x)	$(3x+5)(x+2)$

4. (a) $x = 3,\ 4$ (b) $x = 0,\ -3$ (c) $x = -5,\ -2$ (d) $x = 0,\ 1$

(e) $x = 5,\ -1$ (f) $x = 0,\ 7$ (g) $x = -5,\ -1$ (h) $x = 0,\ 5$

(i) $x = 2,\ 1$ (j) $x = 0,\ \dfrac{1}{2}$ (k) $x = \pm 9$ (l) $x = \pm 7$

(m) $x = \pm 1$ (n) $x = \pm\dfrac{4}{3}$ (o) $x = 1$ (p) $x = -4$

(q) $x = -5$ (r) $x = 7$ (s) $x = 5,\ -2$ (t) $x = 2,\ -7$

(u) $x = 1,\ \dfrac{5}{2}$ (v) $x = \dfrac{9}{2},\ -\dfrac{1}{3}$

5. (a) $x = \dfrac{3 \pm \sqrt{29}}{2}$ (b) $x = \dfrac{-1 \pm \sqrt{17}}{2}$ (c) $x = -3 \pm \sqrt{7}$

(d) $x = -2 \pm \sqrt{3}$ (e) $x = \dfrac{1 \pm \sqrt{37}}{6}$ (f) no real roots

6. Either 200 m × 50 m, or 100 m × 100 m.

7. 6.5 cm

8. 5.0 seconds

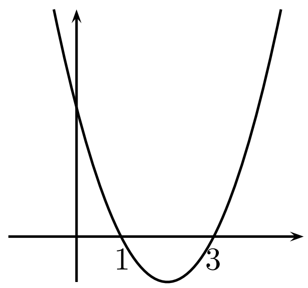
Exercises Set 4

2. (*i*) (a) $2x$ (b) $2x+1$ (c) $2x-4$ (d) $6x+12$
(e) $2x+5$ (f) 0 (g) $4-2x$ (h) 7

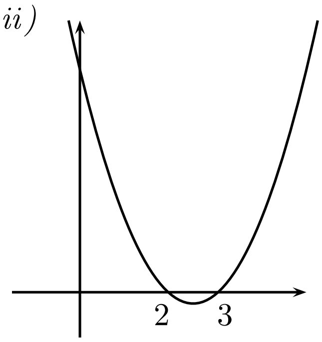
(ii)	(a)	min. of 0 at $x = 0$.	(b)	min. of $-\frac{1}{4}$ at $x = -\frac{1}{2}$.
	(c)	min. of -4 at $x = 2$.	(d)	min. of -21 at $x = -2$.
	(e)	min. of $\frac{3}{4}$ at $x = -\frac{5}{2}$.	(f)	no max. or min.
	(g)	max. of 4 at $x = 2$.	(h)	no max. or min.

3. (a) 2 (b) 0 (c) -7 (d) 0

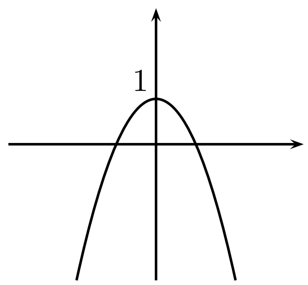
4. *(i)*



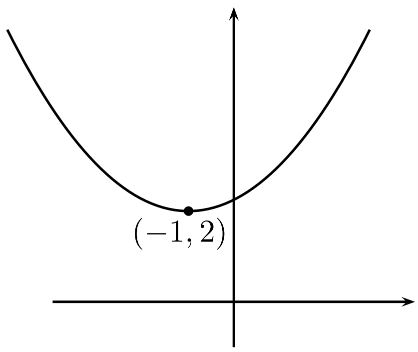
(ii)



(iii)



(iv)

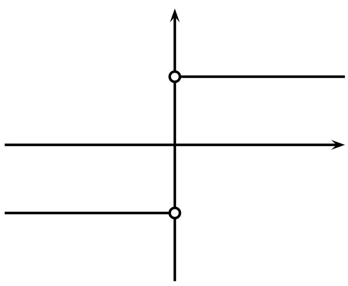


5. (i) 2; $y = 2x - 1$ (ii) 8; $y = 8x - 8$

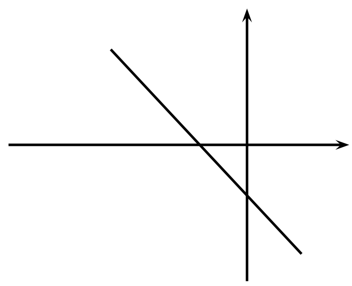
(iii) -3; $y + 3x = 1$ (iv) -4; $y + 4x = 3$

6.

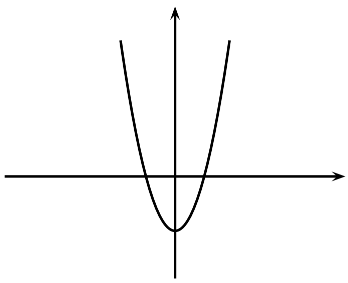
(i)

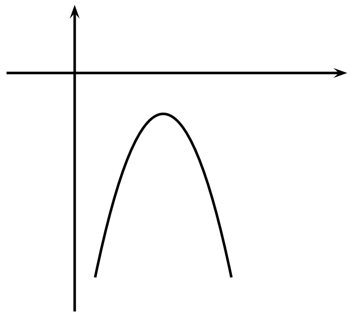


(ii)



(iii) *(iv)*





7. 150×75

Exercises Set 5

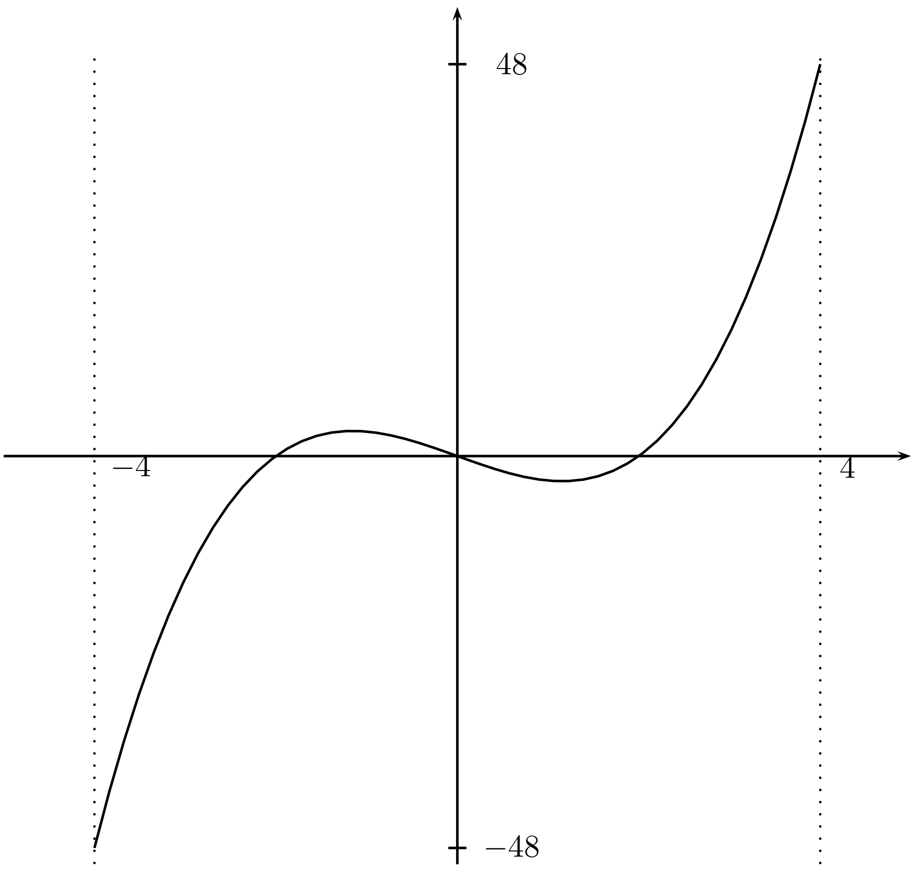
1. (iii) $3x^2$

2. (i) $3x^2 + 6$ (ii) $20x^4 - 9x^2 + 2$ (iii) $1 - 21x^2$
(iv) $72x^5 + 8x - 1$ (v) $3x^2 - 8x + 10$ (vi) $-2 + 20x^3 + 10x^9$

3. (a) 0 (b) -1

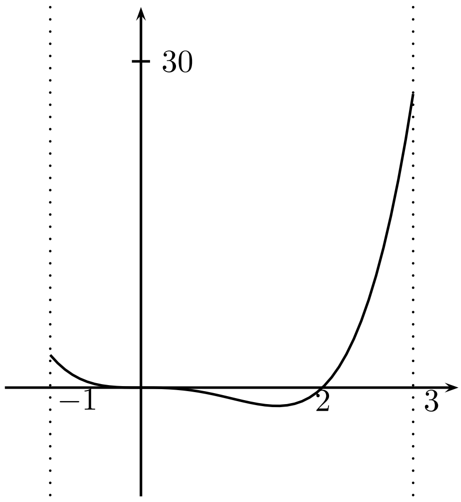
4. Stationary points at $(1.15, -3.08)$ $(-1.15, 3.08)$;

$y-$intercept $= 0$; $x-$intercepts at -2, 0, 2.



5. Minimum of $-\dfrac{27}{16}$ for $x = \dfrac{3}{2}$.

Note that the stationary point at $x = 0$ is not a turning point.



6. $3x^2 - 4,\ -4,\ -1,\ 8.$

7. (i) -2 (ii) 7 (iii) $y = 7x - 9$

8. 10 and 10

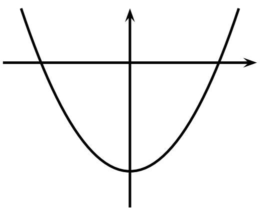
Exercises Set 6

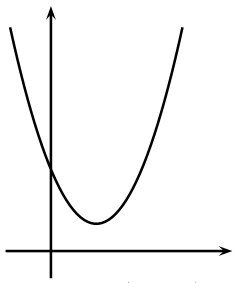
1. (i) $2x - 5$, 2 (ii) $3x^2 - 5x^4$, $6x - 20x^3$

(iii) $6x^5 - 28x^3 + 4x$, $30x^4 - 84x^2 + 4$ (iv) $2x + 2$, 2

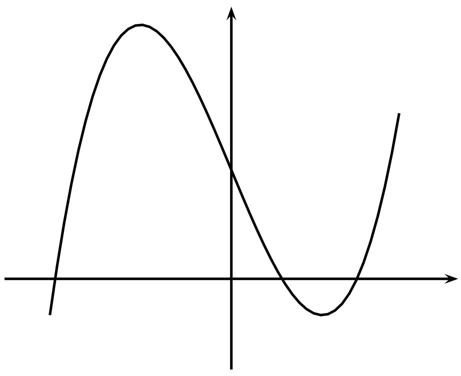
(v) $25x^4$, $100x^3$ (vi) $-3x^2$, $-6x$

2. (i) Min. at $(0, -6)$ (ii) Min. at $(1, 1)$

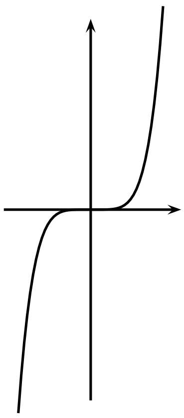


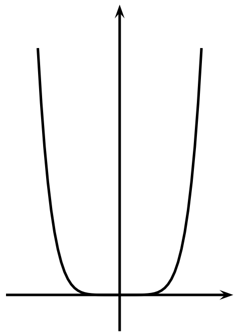


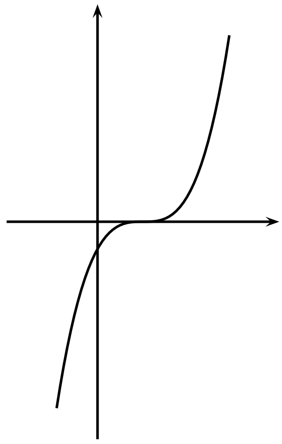
(iii) Min. at $(2, -4)$, max. at $(-2, 28)$, inflexion at $(0, 12)$.



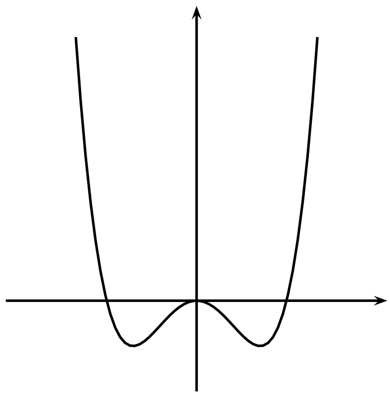
(iv) Inflexion at $(0,0)$ (v) Min. at $(0,0)$ (vi) Inflexion at $(1,0)$







(vii) Min. at $(-1,-1)$ and at $(1,-1)$, max. at $(0,0)$,
inflexions at $\left(\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$ and at $\left(-\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$.



3. (i) 24 m/sec (ii) approx. 29.4 metres (iii) about 5 secs

4. 3.72 m/sec^2

5. 10 m/sec^2

Exercises Set 7

1. (i) x^{20} (ii) $5a^6$ (iii) v^{-3} (iv) 4

2. (i) $\frac{11}{18}$ (ii) 1 (iii) $6\frac{2}{3}$ (iv) 4

3. (i) 8 (ii) 512 (iii) 8 (iv) $\frac{27}{8}$

4. (i) 1 (ii) $m^{\frac{2}{3}} + 1$

5. (i) $x^{\frac{3}{2}}$ (ii) $t^{-\frac{4}{3}}$ (iii) x^{14} (iv) $m^{\frac{29}{6}}$
(v) $x^{\frac{23}{2}}$ (vi) $x^{\frac{2}{5}}$ (vii) x^4 $(viii)$ $x^{\frac{5}{2}}$
(ix) $u^{\frac{55}{7}}$

6. (i) $8x^7 + 2x$ (ii) $27x^8$

(iii) $6x^2 + 1$ (iv) $99x^{10} - 42x^6 + 5$

(v) $1 + \dfrac{1}{x^2}$ (vi) $\dfrac{1}{\sqrt{x}}$

7. (i) $-13x^{12} - \dfrac{13}{x^2}$ (ii) $-5 + 5x^4$

(iii) $2x - \dfrac{4}{x^3}$ (iv) $\dfrac{1}{\sqrt{x}} + \dfrac{2}{\sqrt{x^3}}$

8. (i) $\dfrac{2}{3\sqrt[3]{x^2}}$ (ii) $24y - 7$

(iii) $\dfrac{5}{2}\sqrt{x^3}$ (iv) $26u - \dfrac{15}{2}\sqrt{u}$

(v) $6 + \dfrac{24}{t^3}$ (vi) $-\dfrac{15}{x^4} - \dfrac{1}{x^2}$

9. (i) $\frac{5}{2}x^{\frac{3}{2}}$, $\frac{15}{4}x^{\frac{1}{2}}$ (ii) $2x + \frac{7}{3}x^{\frac{4}{3}}$, $2 + \frac{28}{9}x^{\frac{1}{3}}$

(iii) $\frac{1}{2}x^{-\frac{1}{2}}$, $-\frac{1}{4}x^{-\frac{3}{2}}$ (iv) $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{-3}{2}}$, $-\frac{1}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

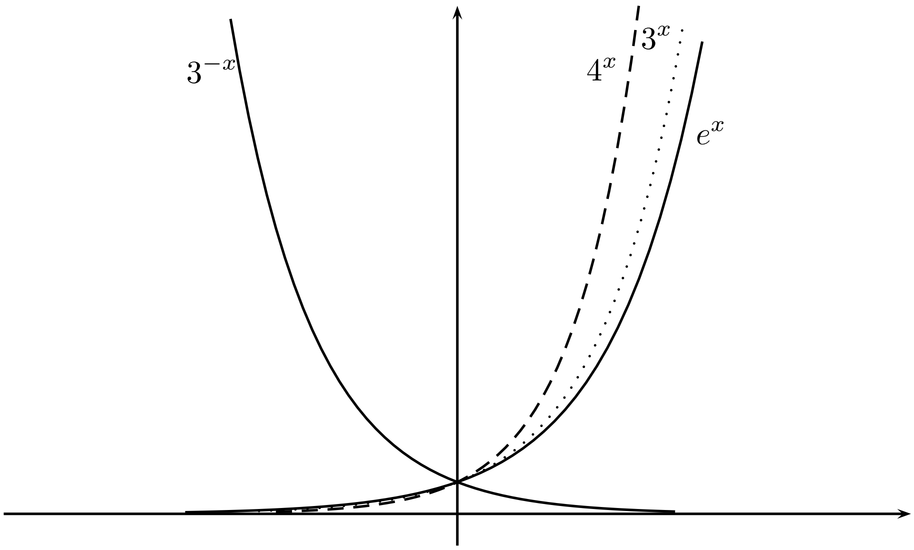
(v) $\frac{4}{3}x^{\frac{1}{3}}$, $\frac{4}{9}x^{-\frac{2}{3}}$ (vi) $-3x^2$, $-6x$

(vii) $-\dfrac{2}{x^3}$, $\dfrac{6}{x^4}$ $(viii)$ $10x - x^{-\frac{3}{2}}$, $10 + \frac{3}{2}x^{-\frac{5}{2}}$

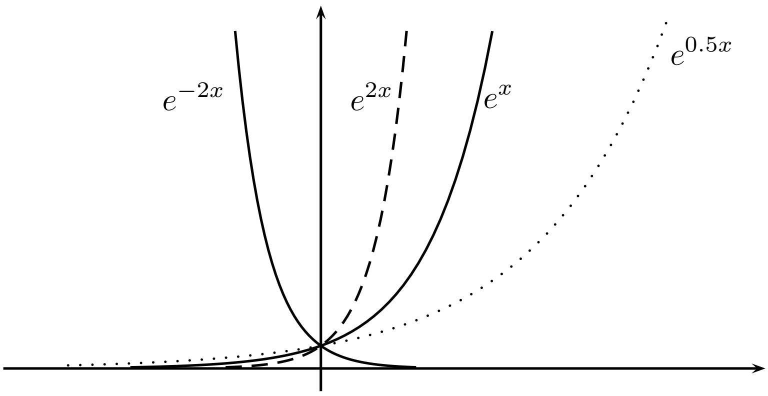
10. (ii) $48\ \mathrm{m}^2$

Exercises Set 8

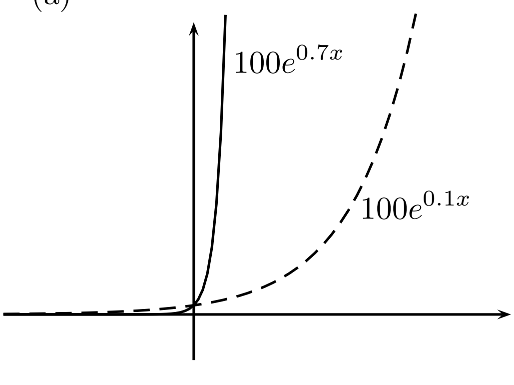
1.



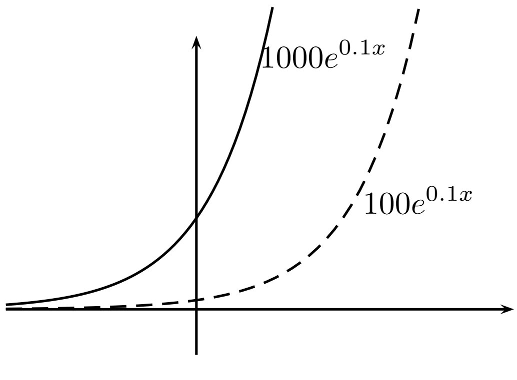
2.



3. (a)



(b)



4. $(i)\quad 3e^x \qquad (ii)\quad \dfrac{e^x+1}{2} \qquad (iii)\quad -e^x$

5. 122,000; 149,000

6. $(i)\quad 80\% \qquad (ii)\quad 50\% \qquad (iii)\quad 30\%$

7. (i) $u = f(x) = 7x + 2,\ g(u) = e^u$ (ii) $u = f(x) = 3 - x^3,\ g(u) = \sqrt{u}$

(iii) $u = f(x) = x^5 + 1,\ g(u) = u^3$ (iv) $u = f(x) = 3 - e^x,\ g(u) = \dfrac{1}{u}$

(v) $u = f(x) = x^2 + 1,\ g(u) = e^u$

Exercises Set 9

1. (i) $7(x^3 + 2x - 1)^6(3x^2 + 2)$ (ii) $\dfrac{-x}{\sqrt{5 - x^2)}}$

(iii) $12(3x + 4)^3$ (iv) $(2x + 8)^{-\frac{1}{2}}$

(v) $\dfrac{-10}{(2x + 3)^6}$ (vi) $\dfrac{2}{\sqrt{4x - 1}}$

(vii) $6(2x + 3)^2$ $(viii)$ $\dfrac{-12}{(3x - 2)^5}$

2. (i) $(x^4 + 3x)(1 + 2x) + (x + x^2 + 1)(4x^3 + 3)$
$= 6x^5 + 5x^4 + 4x^3 + 9x^2 + 6x + 3$

(ii) $(x + 6) \times 2x + (x^2 + 3) \times 1 = 3(x^2 + 4x + 1)$

(iii) $8x^3 - 20x$ (iv) $4x^3 - 10x$

(v) $x^3 \times 3(1 + x)^2 + (1 + x)^3 \times 3x^2 = 3x^2(1 + x)^2(1 + 2x)$

(vi) $(x + 1)^2 \times 2(x + 2) + (x + 2)^2 \times 2(x + 1) = 2(x + 1)(x + 2)(2x + 3)$

3. (i) $\dfrac{4 - 4x^2}{(x^2 + 1)^2}$

(ii) $\dfrac{(2x - 1) \times 2 - (2x + 1) \times 2}{(2x - 1)^2} = \dfrac{-4}{(2x - 1)^2}$

(iii) $\dfrac{(1 - x^2) \times 2x - x^2 \times -2x}{(1 - x^2)^2} = \dfrac{2x}{(1 - x^2)^2}$

(iv) $\dfrac{(x + 1) \times 1 - x \times 1}{(x + 1)^2} = \dfrac{1}{(x + 1)^2}$

(v) $\dfrac{(3x + 5) \times 5 - (5x - 2) \times 3}{(3x + 5)^2} = \dfrac{31}{(3x + 5)^2}$

(vi) $\dfrac{(x + 1) \times 3x^2 - (x^3 - 1) \times 1}{(x + 1)^2} = \dfrac{2x^3 + 3x^2 + 1}{(x + 1)^2}$

4. (i) $\dfrac{-1}{(x + 1)^2}$ (ii) $-\dfrac{6}{x^7}$

(iii) $(x+1) \times \dfrac{1}{2\sqrt{x}} + \sqrt{x} \times 1 = \dfrac{3x+1}{2\sqrt{x}}$

(iv) $\dfrac{\sqrt{x^2+1} - x \times \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x}{x^2+1} = \dfrac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \dfrac{1}{(x^2+1)^{\frac{3}{2}}}$

5. (i) $3e^{3x}$ (ii) $-2e^{-2x}$

(iii) $4e^{4x+3}$ (iv) $2xe^{x^2}$

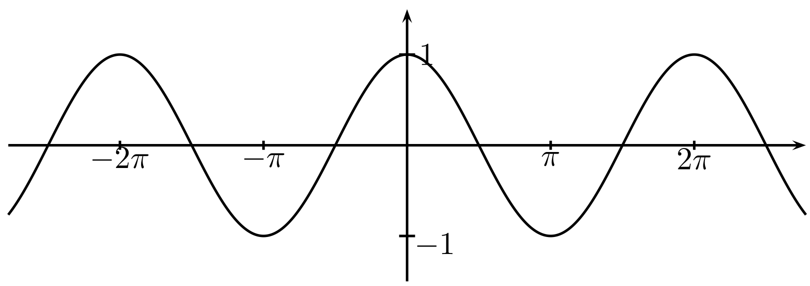
(v) $-e^{1-x}$ (vi) $2(x-1)e^{x^2-2x+7}$

(vii) $x \times e^x + e^x \times 1 = (1+x)e^x$ $(viii)$ $x^2 \times e^x + 2x \times e^x = x(x+2)e^x$

6. When $t = \dfrac{1}{1.1} = 0.\overline{90}$ hours $=$ 54 minutes 32 seconds, the maximum value of $\dfrac{0.3e^{-1}}{1.1} = 0.1003\ldots$ units is reached.

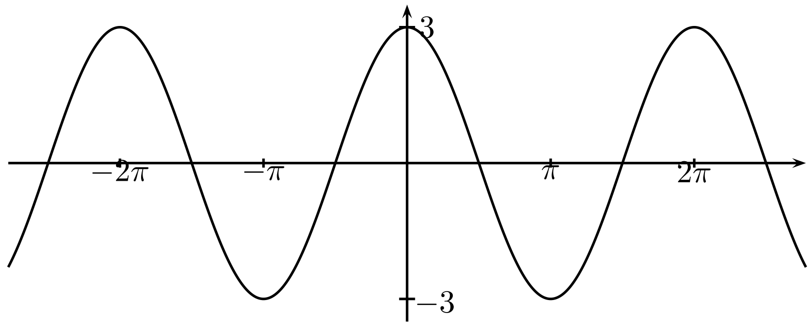
Exercises Set 10

1. (i)

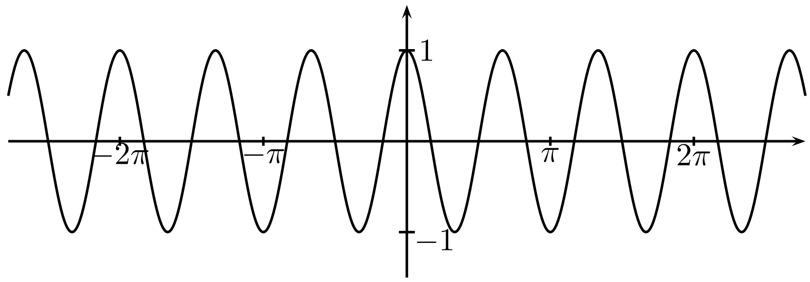


(ii) $-1 \le \cos x \le 1, \quad \cos(x+2\pi) = \cos x, \quad \cos(-x) = \cos x.$

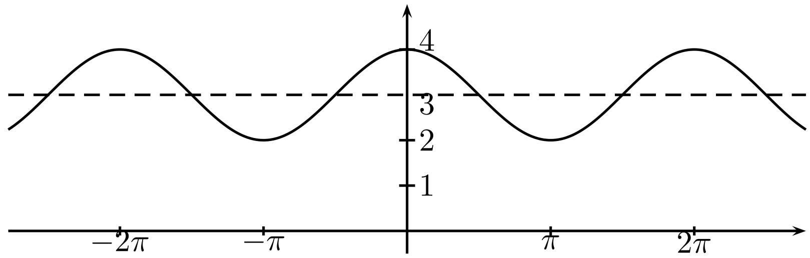
2. (i)



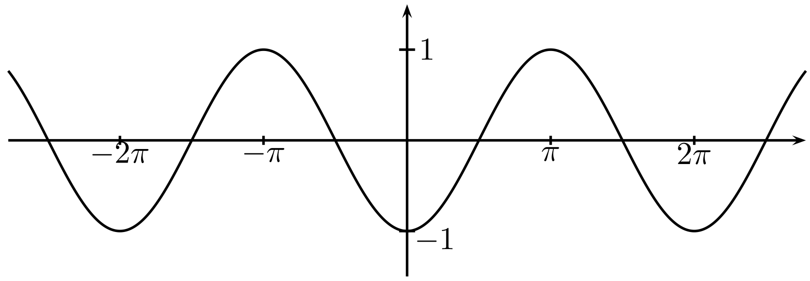
(ii)



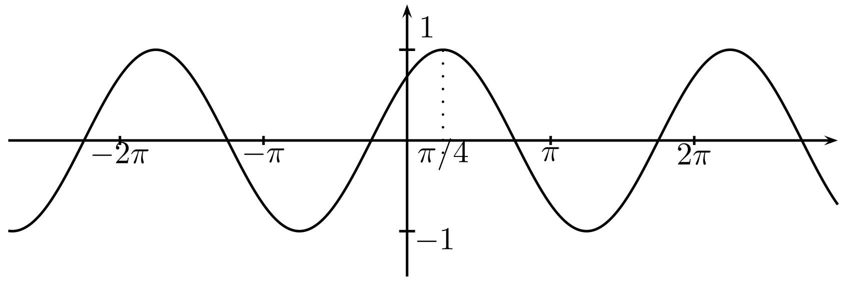
(iii)



(iv)



(v)



3. (i) $\sin(x) + x\cos(x)$ $\qquad$ (ii) $\cos^2(x) - \sin^2(x)$

(iii) $2\sin(x)\cos(x)$ $\qquad$ (iv) $-2\sin(x)\cos(x)$

(v) $\cos(x) - x\sin(x)$

The second derivatives: (i) $2\cos(x) - x\sin(x)$ $\qquad$ (ii) $-4\sin(x)\cos(x)$

(iii) $2\cos^2(x) - 2\sin^2(x)$ (iv) $-2\cos^2(x) + 2\sin^2(x)$

(v) $-2\sin(x) - x\cos(x)$

5. (i) $-4\sin(4x)$ (ii) $-(1+2x) \times \sin(x+x^2)$

(iii) $(\cos(x^2)) \times 2x$ (iv) $2\sin(x) \times \cos(x)$

(v) $3x^2\cos(x^3) - 3x^2\sin(x^3)$ (vi) $2\cos(x+\pi)$

6. (i) $\cos^2(x) - \sin^2(x)$ (ii) $x^2\cos(x) + 2x\sin(x)$

(iii) $-2x\sin(x^2) \times \sin(3x) + 3\cos(3x) \times \cos(x^2)$

(iv) $\dfrac{\sin(x) - x\cos(x)}{\sin^2(x)}$

(v) $\dfrac{3 + 3\cos(x) + 3x\sin(x)}{(1+\cos(x))^2}$ (vi) $\dfrac{1}{\cos^2 x}$ $(= \sec^2 x)$

7. (a) $3x^2\cos x^3$ (b) $-(2x-2)\sin(x^2-2x+1)$

(c) $\sin x + x\cos x$ (d) $-x^{-2}\cos x^{-1}$

(e) $2x\cos(x^2+4) - 2x^3\sin(x^2+4)$ (f) $\cos^2 x - \sin^2 x$

(g) $-8\cos(x+2).\sin(x+2)$ (h) $-(\cos(\cos x)).\sin x$

8. (i) $\cos x$ (ii) $-\cos x$ (iii) 1 (iv) $\cos x$

Exercises Set 11

1. (i) x^3 (ii) $\dfrac{1}{x}$ (iii) $x+2$ (iv) $2\ln x$ (v) $\ln 10$

2. (i) $\dfrac{e}{2}$ (ii) $e-3$ (iii) $\frac{3}{2}$ (iv) $\dfrac{e^2+6}{5}$

3. (i) $\ln\frac{7}{2}$ (ii) $\dfrac{2+\ln 5}{3}$ (iii) 0 (iv) $\pm\sqrt{\ln 10}$

4. (i) $\ln(x+1)(x-2)(x-3)^2$

(ii) $\ln\dfrac{x^4(x-4)}{x+5}$

(iii) $\ln\sqrt{\dfrac{x+1}{x}}$

5. (*i*) (a) $\ln(x+1) - \ln(x+2)$ (b) $\frac{1}{2}\ln(3x+1)$

(c) $2\ln(x) + \frac{1}{2}\ln(x+1) - \frac{1}{3}\ln(3x+4)$

(*ii*) (a) $\dfrac{1}{x+1} - \dfrac{1}{x+2}$ (b) $\dfrac{3}{2(3x+1)}$

(c) $\dfrac{2}{x} + \dfrac{1}{2(x+1)} - \dfrac{1}{3x+4}$

6. (*i*) $\dfrac{3}{3x+2}$ (*ii*) $1 + \ln x$ (*iii*) $\dfrac{2}{x}$

(*iv*) $\dfrac{1}{2(x-1)}$ (*v*) $\dfrac{\cos x}{x} - (\sin x) \times (\ln x)$ (*vi*) $\dfrac{1}{2x\sqrt{\ln x}}$

7. $P = 2 \times 10^6 e^{0.022t}$

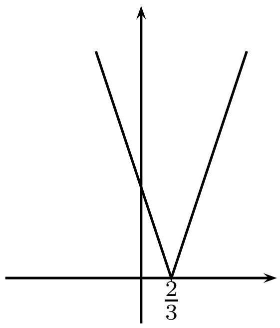
8. (a) $2^{\frac{1}{3}} = 1.259\ldots$ (b) 8×10^4 (c) 6 hours

Exercises Set 12

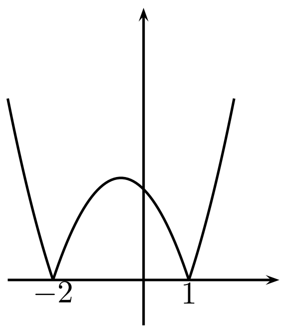
1. 3, 3, 3, 3, −3, 3, 7

2. (*i*) −1 or 5 (*ii*) −1 or $\frac{3}{5}$ (*iii*) −7 or 3.

3. (*i*)



(*ii*)



4. (*i*) $x + y - 3 = 0$ (*ii*) $2x + 5y = 6$

5. (*i*) (a) $x = 0,\ \frac{7}{2}$ (b) $x = 8,\ -5$

(*ii*) $(0, -1)$, $(2, 1)$

6. $y = 7x - 4$

7. 8 cm by 12 cm

8. (a) $v^{\frac{5}{2}}$ (b) $x^{\frac{13}{6}}$ (c) $x^{\frac{5}{6}}$
(d) $x^{\frac{4}{3}}$ (e) 1 (f) u^{-4} (g) $u^{\frac{9}{2}}$
(h) $v^{\frac{9}{2}}$ (i) u^2 (j) $w^{\frac{7}{2}}$ (k) w^{-3}
(l) $x^{-\frac{7}{4}}$ (m) $x^{-\frac{1}{2}}$ (n) $y^{\frac{4}{3}}$ (o) $y^{-\frac{7}{6}}$

9. (*i*) $3x^2(3x-4)+x^3(3)=12x^2(x-1)$

(*ii*) $2(x^2-2x)+(2x+6)(2x-2)=2(3x^2+2x-6)$

(*iii*) $4(2x^2-1)+4x(4x)=4(6x^2-1)$

(*iv*) $3(5x-3)+(3x+6)(5)=30x+21$

(*v*) $(15x^2)(2x-x^3)+(5x^3+2)(2-3x^2)=-30x^5+40x^3-6x^2+4$

(*vi*) $(1)(x^2-4)+(x+2)(2x)=3x^2+4x-4=(3x-2)(x+2)$

(*vii*) $(4x)(x^3-1)+(2x^2+3)(3x^2)=x(10x^3+9x-4)$

(*viii*) $(18t-6)(2t+1)+(9t^2-6t)(2)=6(9t^2-t-1)$

(*ix*) $(1)(3x^2+4x)+(x-5)(6x+4)=9x^2-22x-20$

(*x*) $(2x-3)(2x-6)+(x^2-3x+1)(2)=2(3x^2-12x+10)$

10. (*i*) $4x^3+6x-7$ (*ii*) $x^{-\frac{1}{2}}-6x^{-3}$

(*iii*) $\frac{1}{2}\left(\frac{1}{\sqrt{x}}+\frac{1}{x\sqrt{x}}\right)$ (*iv*) $\sin x + x\cos x$

(*v*) $-\frac{1}{e^x}$ (*vi*) $2xe^{x^2+3}$

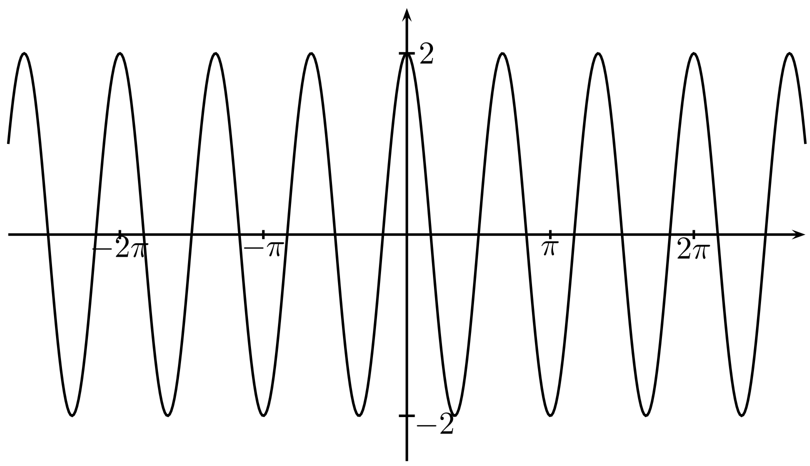
(*vii*) $-12\sin 3x$ (*viii*) $\cos x . e^{\sin x}$

(*ix*) $-\frac{1}{x^2}\sec^2\frac{1}{x}$ (*x*) $-\frac{x^2+2x+2}{(x^2-2)^2}$

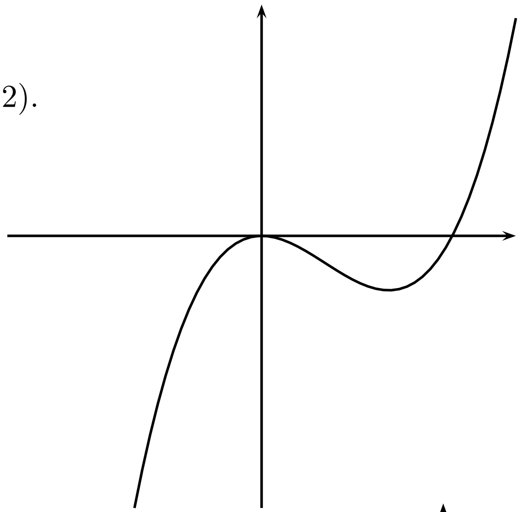
11. 9 cm wide, 18 cm high

12. (*ii*) 20 knots

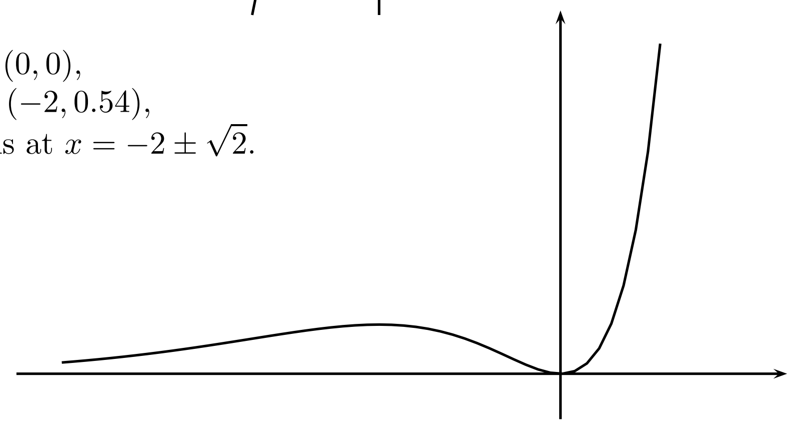
13.



14. (i) Max. at $(0, 0)$,
min. at $(2, -4)$,
inflexion at $(1, -2)$.



(ii) Min. at $(0, 0)$,
max. at $(-2, 0.54)$,
inflexions at $x = -2 \pm \sqrt{2}$.



15. $3x - 4y + 1 = 0$

16. (i) x^2 (ii) $-x$ (iii) $\ln 2$

17. (i) $x = 2.32$ (ii) $x = \pm 4.37$ (iii) $x = 0$ (iv) $x = -0.70$

18. $A = 120,000$. After 6 hours, $N = 537,803$, and $\dfrac{dN}{dt} = 134451$.

19. (i) $k = \dfrac{\ln 2}{5}$ (ii) $16A$ (iii) after 10 years

20. (i) $x = -1$, $y = -3$ (ii) $x = 4$, $y = 2$

21. (i) $x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$ (ii) $x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

22. (a) $A = 50$ cm. (b) $k = \frac{1}{10}$ (c) 2 years.

23. (i) $k = -\dfrac{\ln 0.3}{300} = 0.004$ (ii) 173 years.