

2-way ANOVA Example Solution:

Researchers have sought to examine the effects of various types of music on agitation levels in patients who are in the early and middle stages of Alzheimer's disease. Patients were selected to participate in the study based on their stage of Alzheimer's disease. Three forms of music were tested: Easy listening, Mozart, and piano interludes. While listening to music, agitation levels were recorded for the patients based on heart rate in beats per minute. Scores are recorded below.

Group	Piano Interlude	Mozart	Easy Listening
Early Stage Alzheimer's	103.5	69.9	95.6
	104.7	78.5	99.3
	100.2	88.2	101.4
	105.9	73.4	120.1
	Mean: 103.6	Mean: 77.5	Mean: 104.1
Middle Stage Alzheimer's	99.7	70.8	119.5
	100.5	80.9	116.4
	110.1	75.6	98.6
	115.1	90.2	96.5
	Mean: 106.4	Mean: 79.4	Mean: 107.8

1. Propose null hypotheses to test whether types of music or stage of Alzheimer's have an effect on agitation level.

Null hypotheses assume a default position. In this case, no treatment or stage affects agitation levels. Furthermore, no interactions between factors comprise a third null hypothesis. Alternative hypotheses suggest that the null hypotheses are not true. In summary,

$H_{0,T}$: All types of music don't affect agitation levels

$H_{a,T}$: $H_{0,T}$ isn't true

$H_{0,B}$: Stage of Alzheimer's doesn't affect agitation levels

$H_{a,B}$: $H_{0,B}$ isn't true

$H_{0,I}$: No interactions between types of music and stage of Alzheimer's

$H_{a,I}$: $H_{0,I}$ isn't true

2. Using the various equations for calculating a two-way ANOVA, explain how to calculate the values in each box of the following ANOVA table. Then, fill it out using the attached MATLAB script.

Source	Sum of Sq.	Degrees of Freedom	Mean of Squares	F-statistic	P-value
Type of Music					
Degree of Alzheimer's					
Alzheimer's x Music type					
Within				X	X
Total			X	X	X

Sum of Squares:

A. Type of music: $SS_{\text{music}} = nb \sum_{i=1}^3 (\bar{y}_i - \bar{y})^2$

- Where y_i is the mean of a type of treatment, n represents the number of data values in a treatment, b represents the number of stages of Alzheimer's, and y is the weighted average of all means.

B. Degree of Alzheimer's: $SS_{\text{Alzheimers}} = ra \sum_{j=1}^2 (\bar{y}_j - \bar{y})^2$

- Where y_j is the mean of a stage of Alzheimer's, n represents the number of data values in a stage, a represents the number of types of music, and y is the weighted average of all means.

C. Interaction: $SS_{\text{Interaction}} = n \sum_{i=1}^3 \sum_{j=1}^2 (\bar{y}_{ij} - \bar{y}_i - \bar{y}_j - \bar{y})^2$

- Where y_{ij} is the mean of the data values of a type of music and a stage of Alzheimer's, y_i is the mean of the data values of that type of music, y_j is the mean of the data values of that stage of Alzheimer's, and y is the weighted average of the all means.

D. Within: $SS_{\text{within}} = \sum_{i=1}^3 \sum_{j=1}^2 \sum_{k=1}^4 (\bar{y}_{ijk} - \bar{y}_{ij})^2$

- Where y_{ijk} is a specific data value of a type of music and a stage of Alzheimer's, k represents the index of a data value for a type of music and a stage of Alzheimer's, and y_{ij} is the mean of the data values of a type of music and a stage of Alzheimer's.

E. Total: $SS_{\text{Total}} = \sum_{i=1}^3 \sum_{j=1}^2 \sum_{k=1}^4 (\bar{y}_{ijk} - \bar{y})^2$

- Where y_{ijk} is a specific data value of a type of music and a stage of Alzheimer's, k represents the index of a data value for a type of music and a stage of Alzheimer's, and y is the weighted average of all means.

Degrees of Freedom:

- A. Type of music: $DF_{\text{Music}} = a - 1$
 - Where a is the number of types of music
- B. Degree of Alzheimer's: $DF_{\text{Alzheimer's}} = b - 1$
 - Where b is the number of stages of Alzheimer's
- C. Interaction: $DF_{\text{Interaction}} = (a - 1)(b - 1)$
- D. Within: $DF_{\text{Within}} = ab(n - 1)$
 - Where n is the number of data values for a type of music and a stage of Alzheimer's
- E. Total: $DF_{\text{Total}} = N - 1$
 - Where N is the total number of data points

Mean of Squares:

- A. Type of Music: $MS_{\text{Music}} = \frac{SS_{\text{Music}}}{a - 1}$
- B. Degree of Alzheimer's: $MS_{\text{Alzheimer's}} = \frac{SS_{\text{Alzheimer's}}}{b - 1}$
- C. Interaction: $MS_{\text{Interaction}} = \frac{SS_{\text{Interactions}}}{(a - 1)(b - 1)}$
- D. Within: $MS_{\text{Within}} = \frac{SS_{\text{Within}}}{ab(n - 1)}$
- E. Total: None

F-statistics:

- A. Type of Music: $F_{\text{Music}} = \frac{MS_{\text{Music}}}{MS_{\text{Within}}}$
- B. Degree of Alzheimer's: $F_{\text{Alzheimer's}} = \frac{MS_{\text{Alzheimer's}}}{MS_{\text{Within}}}$
- C. Interaction: $F_{\text{Interaction}} = \frac{MS_{\text{Interaction}}}{MS_{\text{Within}}}$
- D. Within: None
- E. Total: None

P-values:

Now that you have all the necessary components, using your F-statistic values that you calculated above, an appropriate α value, and the correct degrees of freedom, look up the p-values for each category.

Now, running the attached MATLAB script (AnovaIntex.m), the following table is filled out:

```
% ANOVAIntex.m

clear all; format short e

%% First, create a matrix with the data
%Music Type: 1    2    3
Agitationlvl= [103.5 69.9 95.6;... %Early Stage Alzheimer's
              104.7 78.5 99.3;...
              100.2 88.2 101.4;...
              105.9 73.4 120.1;...
              99.7 70.8 119.5;... %Middle Stage Alzheimer's
              100.5 80.9 116.4;...
              110.1 75.6 98.6;...
              115.1 90.2 96.5];

%% Now use the ANOVA command to run it

anovaagitate=anova2(Agitationlvl,4) %The 4 represents the number of
data points in each group

% After you run it, a figure should pop up that'll look something like
this:

%Source          SS          df          MS          F          Prob>F
%-----
%Columns          3893.51      2          1946.75     25.65      0
%Rows             45.93       1          45.93      0.61      0.4468
%Interaction       3.15       2           1.58      0.02      0.9795
%Error            1366.24     18          75.9
%Total            5308.82     23
```

Source	Sum of Sq.	Degrees of Freedom	Mean of Squares	F-statistic	P-value
Type of Music	3893.51	2	1946.75	25.65	0
Degree of Alzheimer's	45.93	1	45.93	0.61	0.4468
Alzheimer's x Music type	3.15	2	1.58	0.02	0.9795
Within	1366.24	18	75.9	X	X
Total	5308.82	23	X	X	X

3. Interpret your F-statistic and P-values. Are there any significant main effects or an interaction effect?

Test results show that there is a significant main effect for type of music, due to the p-value, indicating very strong evidence against the null hypothesis. However, it appears that there is no significant difference in agitation levels between early and middle stages of Alzheimer's disease. There is also little or no real evidence of interactions between type of music and degree of Alzheimer's in agitation levels.

4. Now that you have all the information you need, come up with a conclusion statement and discuss the relationship between the factors.

Without post hoc tests, it is not possible to know exactly where the significant differences lie specifically. However, it appears that patients in both the early and middle stages of Alzheimer's disease experience significantly less agitation while listening to Mozart. Patients in the early stages of Alzheimer's disease exhibited the greatest agitation while listening to easy listening music and patients in the middle stages of Alzheimer's disease exhibited the greater agitation while listening to easy listening music as well.

Source: <http://www2.webster.edu/~woolfm/8banswer.html>