## Name:

## Teacher:



COLLEGE

## Unit 1 Maths Methods (CAS) Exam 2013

## Thursday June 6th 1.50-3.20 pm

Reading time: 10 Minutes
Writing time: $\mathbf{8 0}$ Minutes

## Instruction to candidates:

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a single bound exercise book containing notes and class-work, CAS calculator.

## Materials Supplied:

Question and answer booklet, detachable multiple choice answer sheet at end of booklet.

## Instructions:

- Write your name and that of your teacher in the spaces provided.
- Answer all short answer questions in this booklet where indicated.
- Always show your full working where spaces are provided.
- Answer the multiple choice questions on the separate answer sheet.

| Section A | Section B | Total exam |  |
| ---: | ---: | ---: | :---: |
|  | $/ 20$ |  |  |

## Section A - Multiple choice questions (20 marks)

## Question 1

Which of the following correctly shows the transformation of the equation $E=m c^{2}$ to make $c$ the subject?
a) $E=m c^{2}$
b) $c=m E^{2}$
c) $c= \pm \sqrt{m E}$
d) $c= \pm \sqrt{\frac{m}{E}}$
e) $c= \pm \sqrt{\frac{E}{m}}$

## Question 2

A straight line has the equation $4 x+6 y=12$. The intercepts of this line are at:
a) $x=3, y=2$
b) $x=-3, y=-2$
c) $x=3, y=-2$
d) $x=2, y=3$
e) $x=4, y=6$

## Question 3

A straight line segment joins the points $(2,2)$ and $(7,14)$. The distance between the points is:
a) 5
b) 12
c) 13
d) 14
e) 16

## Question 4

The equation of the line that passes through the two points $(1,-1)$ and $(4,5)$ is:
a) $y=2 x+1$
b) $y=2 x-3$
c) $y=\frac{4 x}{3}-\frac{1}{3}$
d) $y=\frac{x}{2}-1$
e) $y=4 x-5$

## Question 5

The equation of the linear function graphed here is:

a) $y=-5 x+2$
b) $y=-\frac{2 x}{5}+2$
c) $y=\frac{2 x}{5}+2$
d) $y=\frac{2 x}{5}-2$
e) $y=\frac{5 x}{2}+2$

## Question 6

The straight lines $y=\frac{x}{3}+1$ and $y=-\frac{2 x}{3}+4$ intersect at the point:
a) $(3,2)$
b) $(2,3)$
c) $(-3,2)$
d) $(-3,-2)$
e) $(-2,-3)$

## Question 7

The quadratic function $y=x^{2}-25$ has x intercepts at:
a) $x=25$
b) $x=-25$
c) $x=25$ and $x=-25$
d) $x=5$
e) $x=5$ and $x=-5$

## Question 8

The turning point form of the quadratic function $y=x^{2}-8 x+6$ is:
a) $y=(x-6)(x-8)$
b) $y=(x-8)^{2}+6$
c) $y=(x-4)^{2}-10$
d) $y=(x+4)^{2}-10$
e) $y=(x-4)^{2}+10$

## Question 9

The factorised form of the quadratic function $y=x^{2}-8 x+6$ is:
a) $y=(x-6)(x-8)$
b) $y=(x+6)(x-8)$
c) $y=(x-4-\sqrt{10})(x-4+\sqrt{10})$
d) $y=(x+4-\sqrt{10})(x+4+\sqrt{10})$
e) $y=(x-14)(x+6)$

## Question 10

A parabola is shown below. Which of the following quadratic equations correctly defines the parabola?

a) $y=-2(x+1)(x-3)$
b) $y=\frac{1}{2}(x+1)(x-3)$
c) $y=-\frac{1}{2}(x-1)(x+3)$
d) $y=-2(x-1)(x+3)$
e) $y=-\frac{1}{2}(x+1)(x-3)$

## Question 11

Which of the following correctly lists all of the factors of the cubic function $y=x^{3}-x^{2}-6 x$ ?
a) $(x-6)^{3}$
b) $(x-1)(x+6)^{2}$
c) $x(x-3)(x+2)$
d) $x(x+3)(x-2)$
e) $(x+1)(x+3)(x-2)$

## Question 12

The complete set of solutions to the equation $27+\frac{x^{3}}{64}=0$ is:
a) $x=-12$
b) $x=+12$
c) $x=-\frac{3}{4}$
d) $x=\frac{3}{4}$ and $x=-\frac{3}{4}$
e) $x=12$ and $x=-12$

## Question 13

The graph of a cubic function is shown below. Which of the following equations describes the graph?
a) $y=(x-3)^{2}(x+1)$
b) $y=(x-3)(x+1)^{2}$
c) $y=(x-1)(x+3)^{2}$
d) $y=(x-1)^{2}(x+3)$
e) $y=(x-1)^{3}(x+3)^{3}$


## Question 14

The graph of a cubic function is shown below. Which of the following equations describes the graph?

a) $y=(x-2)^{3}+1$
b) $y=\frac{1}{8}(x-2)^{3}+1$
c) $y=\frac{1}{8}(x+2)^{3}-1$
d) $y=\frac{1}{8}(x-2)^{3}-1$
e) $y=-\frac{1}{8}(x+2)^{3}-1$

## Question 15

The expression $\log _{5} x^{3}+4 \log _{5} x$ can be simplified as:
a) $\log _{5} x$
b) 5
c) $5^{3}-5^{4}$
d) $7 \log _{5} x$
e) $4^{3}$

## Question 16

The rate of decay of a radioactive element is reducing by $5 \%$ every year. The time that it takes the rate to reduce to half is closest to:
a) 0.07 years.
b) 10.0 years.
c) 13.5 years.
d) 14.2 years.
e) 95 years.

## Question 17

The number of bacteria in a food sample is doubling every 45 minutes. By how much will the number have increased after 9 hours?
a) 1024 times
b) 2048 times
c) 4096 times
d) 8192 times
e) 16384 times

## Question 18

The expression $\frac{\left(a^{2} b^{3}\right)^{2}}{b} \times \frac{a b^{2}}{a^{4}}$ can be simplified as:
a) $\frac{\left(a^{2} b^{3}\right)^{2}}{b} \times \frac{a b^{2}}{a^{4}}$
b) $a b^{7}$
c) $a b^{5}$
d) 1
e) $\frac{a^{3} b^{5}}{a^{4} b}$

## Question 19

$x^{\frac{2}{3}}$ can be also be expressed as:
a) $\sqrt{x^{3}}$
b) $\sqrt[3]{x^{2}}$
c) $\frac{1}{\sqrt{x^{3}}}$
d) $\frac{1}{x^{\frac{3}{2}}}$
e) $-x^{\frac{2}{3}}$

## Question 20

The graph of the function $y=a^{x}$ is shown here.

Which of the following graphs below correctly shows the function $y=-a^{x}+2$ ?

a)

b)


d)

e)


## Section B - Short answer questions (30 marks)

## Question 1

The Shifty Corporation has released its new mobile phone plan. For a monthly connection cost of $\$ 20$, users pay 10 cents per minute for their calls.
a) Write a linear relation that describes the cost in dollars (c) as a function of the talk time (x) in minutes.
b) Calculate the total cost of connection \& calls for the following talk times:

| Time / month | Cost |
| :--- | :--- |
| 100 min |  |
| 200 min |  |
| 500 min |  |

c) Use this data to draw an appropriate graph of the cost vs talk time.


Low Budget has a plan that costs $\$ 55$ per month, but has unlimited talk time.
d) Draw this cost on the graph.
e) Calculate the talk time at which the Low Budget phone plan becomes cheaper than Shifty.
$\qquad$
$\qquad$
$\qquad$

## Question 2

Fully factorise the quadratic function $y=x^{2}+4 x-1$ by completing the square.

## Question 3

A ball is thrown off a balcony. The path of the ball is described by the equation $y=-\frac{1}{10}(x+10)(x-20)$, where y is the vertical height and x is the horizontal distance in front of the balcony (both measured in metres).

a) Calculate the height of the ball at the point where it was thrown from.

(2 marks)
b) State the distance that the ball travels horizontally before hitting the ground.

(1 mark)
c) Calculate the maximum height reached by the ball.

d) Use a CAS calculator to find the horizontal position (to the nearest cm ) when the ball is 21 metres above the ground.


## Question 4

Fully factorise the cubic function $p(x)=x^{3}+4 x^{2}-4 x-16$ by using the factor theorem and long division.

## Question 5

The value of an investment was found to grow according to the equation $V(t)=\$ 15000 \times 1.07^{t}$, where t is the time (measured in years).
a) Find the initial value of the investment.
$\qquad$
$\qquad$
(1 mark)
b) What percentage of value does the investment gain every year?
$\qquad$
$\qquad$
c) Find the value (to the nearest dollar) one year after investing the money.
$\qquad$
$\qquad$
d) Find the time (to the nearest month) for the value of the investment to triple.
$\qquad$
$\qquad$
$\qquad$
(2 marks)

Another investor deposits $\$ 20,000$ at the same time, which grows at an annual rate of $5 \%$.
e) Write an equation to describe the value of this investment.
f) Use a CAS calculator or other means to find the time (to the nearest month) when both investments have the same value.
(1 mark)

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Answer sheet for section A

| 1. | a | b | C | d | e |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2. | a | b | c | d | e |
| 3. | a | b | C | d | e |
| 4. | a | b | c | d | e |
| 5. | a | b | C | d | e |
| 6. | a | b | C | d | e |
| 7. | a | b | C | d | e |
| 8. | a | b | c | d | e |
| 9. | a | b | c | d | e |
| 10. | a | b | C | d | e |
| 11. | a | b | C | d | e |
| 12. | a | b | c | d | e |
| 13. | a | b | C | d | e |
| 14. | a | b | C | d | e |
| 15. | a | b | c | d | e |
| 16. | a | b | C | d | e |
| 17. | a | b | C | d | e |
| 18. | a | b | c | d | e |
| 19. | a | b | c | d | e |
| 20. | a | b | c | d | e |

## Answer sheet for section $A$

| 1. | a | b | C | d | e |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2. | a | b | c | d | e |
| 3. | a | b | c | d | e |
| 4. | a | b | C | d | e |
| 5. | a | b | C | d | e |
| 6. | a | b | C | d | e |
| 7. | a | b | C | d | e |
| 8. | a | b | c | d | e |
| 9. | a | b | c | d | e |
| 10. | a | b | C | d | e |
| 11. | a | b | C | d | e |
| 12. | a | b | C | d | e |
| 13. | a | b | C | d | e |
| 14. | a | b | C | d | e |
| 15. | a | b | C | d | e |
| 16. | a | b | C | d | e |
| 17. | a | b | C | d | e |
| 18. | a | b | C | d | e |
| 19. | a | b | C | d | e |
| 20. | a | b | C | d | e |

## Section B - Short answer questions (30 marks)

## Question 1

The Shifty Corporation has released its new mobile phone plan. For a monthly connection cost of \$20, users pay 10 cents per minute for their calls.
a) Write a linear relation that describes the cost in dollars (c) as a function of the talk time (x).

$$
C(x)=0.1 x+20
$$

b) Calculate the total cost of connection \& calls for the following talk times:

| Time / month | Cost |
| :--- | :--- |
| 100 min | $C(x)=0.1 \times 100+20=\$ 30$ |
| 200 min | $C(x)=0.1 \times 200+20=\$ 40$ |
| 500 min | $C(x)=0.1 \times 500+20=\$ 70$ |

c) Use this data to draw an appropriate graph of the cost vs talk time.


Low Budget has a plan that costs $\$ 50$ per month, but has unlimited talk time.
d) Draw this cost on the graph.
e) Calculate the talk time at which the Low Budget phone plan becomes cheaper than Shifty.

$$
\begin{aligned}
& 55=20+0.1 x \\
& \hline 35=0.1 x \\
& x=350
\end{aligned}
$$

## Question 2

Fully factorise the quadratic function $y=x^{2}+4 x-1$ by completing the square.
$y=x^{2}+4 x-1$
$y=\left(x^{2}+4 x+4\right)-4-1$
$y=(x+2)^{2}-5$
$y=(x+2+\sqrt{5})(x+2-\sqrt{5})$

$$
y=(x+2+\sqrt{5})(x+2-\sqrt{5})
$$

(2 marks)

## Question 3

A ball is thrown off a balcony. The path of the ball is described by
the equation $y=-\frac{1}{10}(x+10)(x-20)$, where y is the vertical height and x is the horizontal distance in front of the balcony (both measured in metres).

a) Calculate the height of the ball at the point where it was thrown from.
$h(0)=-\frac{1}{10}(0+10)(0-20)=20$
20 m
(2 marks)
b) State the distance that the ball travels horizontally before hitting the ground.

20 m
(1 mark)
c) Calculate the maximum height reached by the ball.

The maximum height occurs half way between the two $x$ intercepts: $x=5$.
$h(5)=-\frac{1}{10}(5+10)(5-20)=\frac{225}{10}=22.5 \mathrm{~m}$
d) Use a CAS calculator to find the horizontal position (to the nearest cm ) when the ball is 21 metres above the ground.

$$
x=1.13 \mathrm{~m} \text { and } x=8.87 \mathrm{~m}
$$

## Question 4

Fully factorise the cubic function $p(x)=x^{3}+4 x^{2}-4 x-16$ by using the factor theorem and long division.
$p(2)=2^{3}+4 \times 2^{2}-4 \times 2-16=0$
so $(x-2)$ is a factor

$$
\begin{array}{r}
\quad \begin{array}{r}
x^{2}+6 x+8 \\
\frac{x^{3}+4 x^{2}-4 x-16}{} \\
\frac{x^{3}-2 x^{2}}{6 x^{2}-4 x-16} \\
\frac{6 x^{2}-12 x}{8 x-16} \\
\frac{8 x-16}{0}
\end{array}
\end{array}
$$

The other two factors from $x^{2}+6 x+8$ are $(x+2)$ and $(x+4)$.
$y=(x-2)(x+2)(x+4)$
(3 marks)

## Question 5

The value of an investment was found to grow according to the equation $V(t)=\$ 15000 \times 1.07^{t}$, where t is the time (measured in years).
a) Find the initial value of the investment.
$V(0)=\$ 15000 \times 1.07^{0}$
$V(0)=\$ 15000$
b) What percentage of value does the investment gain every year?

## 7\%

c) Find the value (to the nearest dollar) one year after investing the money.
$V(1)=\$ 15000 \times 1.07^{1}$
$V(1)=\$ 16050$
d) Find the time (to the nearest month) for the value of the investment to triple.
$3=1.07^{x}$
$x=\frac{\log 3}{\log 1.07}$
$x=16.23$ years $=16$ years and 3 months

Another investor deposits $\$ 20,000$ at the same time, which grows at an annual rate of $5 \%$.
e) Write an equation to describe the value of this investment.
$V(t)=\$ 20000 \times 1.05^{t}$
f) Use a CAS calculator or other means to find the time (to the nearest month) when both investments have the same value.
$\$ 20000 \times 1.05^{t}=\$ 15000 \times 1.07^{t}$

$$
\begin{aligned}
& t=15.24 \\
& t=15 \text { years and } 3 \text { months }
\end{aligned}
$$

